

# Special Issue “Mathematical Methods for Operations Research Problems”

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This Special Issue of *Mathematics* is dedicated to the application of Operations Research methods to a wide range of problems. Operations Research uses mathematical modeling and algorithms for supporting decision processes and finding optimal solutions in many fields. For this Issue, high-quality papers were solicited to address both theoretical and practical issues in the wide area of Operations Research. In particular, submissions presenting new theoretical results, models and algorithms were welcome. Some topics mentioned in the Call for Papers for this Issue were linear and nonlinear programming, optimization problems on graphs, project management, scheduling, logistics and transportation, queuing theory and simulation, to name a few.

After a careful refereeing process, 15 papers were selected for this Issue. As a rule, all submissions were reviewed by three experts in the corresponding area. The authors of the accepted papers come from 16 countries: Hungary, Turkey, Spain, France, Japan, Mexico, Czech Republic, Germany, Thailand, Chile, India, Korea, Croatia, Chile, USA and Lithuania. Subsequently, the published papers were surveyed in increasing order of their publication dates for this Special Issue.

The first accepted paper [1] deals with body-centered cubic lattices which are important grids appearing in nature. The authors formulate the shortest path problem on higher dimensional body-centered grids as an integer programming problem. Finally, a Gomory cut is applied to guarantee an integer solution, and some comments on Hilbert bases of rational polyhedral cones are given.

The second paper [2] studies an alternative mechanism for using mathematical programming to incorporate negative learning into a widely used ant colony optimization. The authors compare their approach with existing negative learning approaches from the literature on two combinatorial optimization problems: the minimum dominating set problem and the multi-dimensional knapsack problem. It is shown that the new approach outperforms the existing ant colony algorithms and negative learning mechanisms.

In the third paper [3], the authors cluster the Pareto Front for a multi-objective optimization problem in a given number of clusters and identify isolated points. In particular,  $K$ -center problems and some variants are investigated and a unified formulation is given, where both discrete and continuous variants, partial  $K$ -center problems and their min-sum  $K$ -radii on a line are considered. In the case of dimension two, a polynomial dynamic programming algorithm is given, while for a higher dimension, the associated problem is  $NP$ -hard. For some variants, including the  $K$ -center problem and min-sum  $K$ -radii variants, further improvements are discussed. In addition, parallel implementations lead to a speed-up in practice.

Paper [4] deals with a graph-theoretic subject. In particular, the authors develop lower and upper bounds on the global total  $k$ -domination number of a graph. It is the minimum cardinality of a so-called global total  $k$ -dominating set of this graph. The results were obtained by using algebraic connectivity in graphs. Moreover, the authors present an approach to obtain a global total  $(k + 1)$ -dominating set from a global total  $k$ -dominating set.

In the fifth paper [5], three methods for deriving a priority vector in the theoretical framework of pairwise comparisons are investigated with respect to sensitivity and order



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violation, namely, the Geometric Mean Method, the Eigenvalue Method and the Best–Worst Method. The authors apply a One-Factor-at-a-Time sensitivity analysis via Monte Carlo simulations. The investigations show that the Best–Worse Method is statistically significantly more sensitive and, thus, less robust than the other two methods.

Paper [6] investigates a parallel machine scheduling problem on uniform machines with identical processing times as well as given release and delivery times with a minimizing makespan, which is the time when the last job is delivered to the customer. The authors present a polynomial algorithm which is based on the ‘branch less cut more’ framework developed earlier. This algorithm generates a tree similar to a branch and bound algorithm, but the branching and cutting criteria depend on structural properties and are not based on lower bounds. The algorithm finds an optimal solution if for any pair of jobs a specified inequality with respect to the release and delivery times is satisfied. If these conditions are not satisfied or for the case of non-identical processing times, the algorithm can be used as an approximate one.

Paper [7] considers short-term scheduling of the pressing process which occurs in the fabrication of a multi-layer printed circuit board. For this problem, a mixed-integer linear programming formulation is given for the case of minimizing the makespan. In addition, a three-phase heuristic is also given. It turns out that the MILP model can solve small and medium-sized instances. On the one hand, the MILP model could not solve most of the large-sized instances within a time limit of two hours, but the heuristic found an optimal solution for all instances, for which the MILP model could find an optimal solution in much smaller times.

Paper [8] deals with particle swarm optimization. For bio-inspired algorithms, where a proper setting of the initial parameters by an expert is required. In this paper, the authors suggest a hybrid approach allowing the adjustment of the parameters based on a state deducted by the swarm algorithm. The state deduction is reached by a classification of the observations using a hidden Markov model. Extensive tests for the set covering problem show that the presented hybrid algorithm finds better regions in the heuristic space than the original particle swarm optimization, and it shows an overall good performance.

Paper [9] investigates a perishable inventory system with an  $(s, Q)$  ordering policy together with a finite waiting hall. The single server only begins serving when  $N$  customers have arrived. This is known as  $N$ -policy. The authors investigate the impatient demands which are caused by the  $N$ -policy server to an inventory system. In particular, the steady-state vector is investigated. In addition, some measures of the performance of the system are analyzed and the expected cost rate in the steady state is given.

Paper [10] deals with cryptocurrency portfolio selection and applies a multi-criteria approach based on PROMETHEE II. The authors found that their model gave the best cryptocurrency portfolio when considering the daily return, the standard deviation, the value-at-risk, the conditional value-at-risk, the volume, the market capitalization as well as nine cryptocurrencies for the period from January 2017 to February 2020. It turned out that the proposed model won against all other models considered.

In the eleventh paper [11], a game decision-making model for a low-carbon e-commerce supply chain is derived. The paper analyzes the influence of carbon trading on the regional sustainable development. It turns out that the total carbon emission is positively related to the commission rate. The empirical analysis conducted by the authors confirms that the implementation of carbon trading is conducive to the regional sustainable development and that controlling the environmental governance intensity promotes carbon productivity. In the future, the inclusion of more factors is intended to make the model more realistic.

In the twelfth paper [12], a new cuckoo search algorithm is presented which is able to self-adapt its configuration. This is reached by means of machine learning, where a cluster analysis is used. Experimental results are presented for the set covering problem. A comparison with other hybrid bio-inspired algorithms is also performed. The authors mention some possible future works, e.g., improving the criterion of population

increase and decrease by using clusterization strategies or implementing further machine learning techniques.

Paper [13] develops a novel hybrid optimization framework, entitled learning-based linear balancer. The authors also design a regression model to predict better movements for the approach and to improve the performance. The approach is based on balancing the intensification and diversification performed by the hybrid approach in an online form. To test the suggested approach, 15 benchmark functions are considered. The authors also compare their approach against a spotted hyena optimizer and a neural network approach.

In the fourteenth paper [14], a subject from queueing theory is considered, where the suggested approach consists of two stages. In the first stage, Little's law in Multiphase Systems is analyzed. In particular, Strong Law of Large Numbers-type theorems are proven. Then, the results obtained in this stage are verified by means of simulation. Here, the Python concept is used to test the results obtained in the first stage.

The last paper [15] deals with the university course timetabling problem. It presents a new integer programming model for generating a timetable of an academic department considering basic workload and course overload and, also, the profile and area of each professor. A real-world case is considered. By analyzing different strategies, the efficiency of the new model is shown.

Finally, as the guest editor, it is my pleasure to thank the editorial staff of the journal *Mathematics* for the pleasant cooperation, not only during the preparation of this, but also for the previous four Special Issues which I handled as editor for the journal *Mathematics*. I would also to thank all referees for their thorough and timely reports on the submitted works and also the authors for submitting many interesting works from a broad spectrum in the Operations Research area. For potential authors who missed the deadline for this Special Issue, I remind that there is another future Special Issue in *Mathematics* entitled 'Recent Advances of Discrete Optimization and Scheduling' edited by Alexander Lazarev, Bertrand Lin and myself, which deals with similar subjects.

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