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# A Novel MADM Framework under $q$-Rung Orthopair Fuzzy Bipolar Soft Sets

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**Abstract:** In many real-life problems, decision-making is reckoned as a powerful tool to manipulate the data involving imprecise and vague information. To fix the mathematical problems containing more generalized datasets, an emerging model called $q$-rung orthopair fuzzy soft sets offers a comprehensive framework for a number of multi-attribute decision-making (MADM) situations but this model is not capable to deal effectively with situations having bipolar soft data. In this research study, a novel hybrid model under the name of $q$-rung orthopair fuzzy bipolar soft set ($q$-ROFBSS, henceforth), an efficient bipolar soft generalization of $q$-rung orthopair fuzzy set model, is introduced and illustrated by an example. The proposed model is successfully tested for several significant operations like subset, complement, extended union and intersection, restricted union and intersection, the 'AND' operation and the 'OR' operation. The De Morgan's laws are also verified for $q$-ROFBSSs regarding above-mentioned operations. Ultimately, two applications are investigated by using the proposed framework. In first real-life application, the selection of land for cropping the carrots and the lettuces is studied, while in second practical application, the selection of an eligible student for a scholarship is discussed. At last, a comparison of the initiated model with certain existing models, including Pythagorean and Fermatean fuzzy bipolar soft set models is provided.

**Keywords:** $q$-rung orthopair fuzzy soft set; bipolar soft set; score function; algorithm; decision-making



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## 1. Introduction

Nowadays, MADM is playing a vital role in dealing with the vague information having multi-attributes by offering better mathematical modeling in case of various real-life problems. Actually, such kinds of issues arise in Social Sciences, Medical Sciences, Environmental Sciences, Engineering, Ecology, Economics, and several different domains, which are highly dependent on the target of modeling uncertainties that cannot be solved using traditional mathematical theories, including probability theory. A good decision exhibits proper illustration of a dataset and helps to move further in the right direction. Zadeh [1] was credited for proposing the fuzzy set theory to deal with uncertain information and for introducing new research directions in several domains of science and technology, especially, decision-making. He assigned the objects by the belongingness (membership) grades in the real-valued interval $[0, 1]$. However, it was observed that some other extensions was also required to deal with non-belongingness (dissatisfaction) values of objects in different vague and uncertain situations. Later, Atanassov [2] generalized fuzzy sets and put forward the idea of intuitionistic fuzzy set (IFS) via belongingness and non-belongingness grades whose sum lies in $[0, 1]$. The theory of IFSs has rich potential applications in various domains ranging from medical to engineering. However, in some practical decision-making situations, IFSs cannot be employed. For example, consider that a team of senior professors is invited to provide their opinions on the performance of employees for promotion purpose. There may be a possibility that half of them express their belongingness degrees as 0.8 with respect to a particular parameter, while the remaining provide their non-belongingness degrees as 0.6 with respect to the same parameter. This situation cannot be illustrated by the IFSs because $0.8 + 0.6 = 1.4 > 1$.

After two decades of this powerful invention of IFSs, Yager [3] was the first who presented a natural extension of IFSs called Pythagorean fuzzy sets (PFSs) in 2013, because IFSs were not sufficient for handling the data involving belongingness and non-belongingness grades whose sum does not lie in $[0, 1]$. PFSs came with a major feature that the sum of squares of belongingness and non-belongingness grades bounded by 1, which actually provided more space to this model for dealing with numerous real-life problems, i.e., in the above-mentioned situation, PFSs work well because $0.8^2 + 0.6^2 = 1$. Note that PFSs are equivalent to the Atanassov's IFSs of second type [4]. With the production of PFSs, many researchers put forward their attention to this fruitful model and introduced several useful results by generalizing this concept or by its implementation to different uncertainty theories. For example, Yager and Abbasov [5] discussed about an association between Pythagorean membership values and complex numbers. In order to solve MADM situations with PFSs, Zhang and Xu [6] presented an extended TOPSIS approach based on PFSs. Yager [7] presented a number of aggregation operators based on PFSs to solve the decision-making problems. Peng and Yang [8] studied subtraction and division of PFSs and discussed their significant properties. The same authors [9] introduced interval-valued PFSs as a natural generalization of PFSs.

In the last few decades, many scholars attracted towards MADM methods to determine some more generalized novel mathematical tools for dealing with different types of uncertainties in numerous real-world problems. MADM models explain how attributes information is to be processed to compute a suitable object or ranking order of the objects to support decision-making. In the literature, MADM methods have been used in different domains, including engineering. No doubt, PFS theory is playing a significant role in solving of different real-world MADM problems but there is still a flaw in this model, that is, PFS fails to deal a situation where the sum of the square of belongingness and non-belongingness values is not bounded by one. To overcome this issue, Yager's [10] contribution came in the form of $q$-rung orthopair fuzzy sets ($q$-ROFSs) with the characteristic of the sum of the $q$th power of belongingness and non-belongingness values of elements not being more than one. The $q$-ROFSs are generally reduced to IFSs [2], PFSs [3], and Fermatean fuzzy sets (FFSs) [11] for $q = 1$, 2 and $q = 3$, respectively (see Figure 1). A number of problems have been solved by using $q$-ROFS model. For instance, Hussain et al. [12] proposed the $q$-rung orthopair fuzzy soft aggregation operators and discussed their multi-criteria decision-making applications. The $q$-ROFS model was found to be more effective when extended to range of parameterizations and used in different domains [13–16].

Pawlak [17] initiated the theory of rough sets to handle imprecise data while he was working on the problems related to intelligent systems. According to him, different parameterized values help experts in establishing an opinion during a variety of decision-making problems. The theories of both fuzzy and rough sets suffer from the drawback of not being able to describe the consideration of multiple parameters. To resolve this issue, Molodtsov [18], in 1999, proposed a new set, known as soft set (SS). This set was mainly centered on application of mathematical models for handling vague information by means of parametric perspective. Ali et al. [19], then extended the known literature and introduced a number of new notions. They further claimed that De Morgan's laws were applicable for SSs. They also paved way for a new direction in this field by linking several ideas to the notion of SSs. After that, Maji et al. [20,21] used SSs to define fuzzy soft sets (FSSs) and intuitionistic fuzzy soft sets (IFSSs). Since then, a host of research was published based on different aspects of FSSs and IFSSs and these sets were effectively used in decision-making for real-life problems [22–24]. Recently, Hamid et al. [13] proposed a new model called $q$-rung orthopair fuzzy soft sets ($q$-ROFSSs) by combining $q$-ROFSs with SSs.

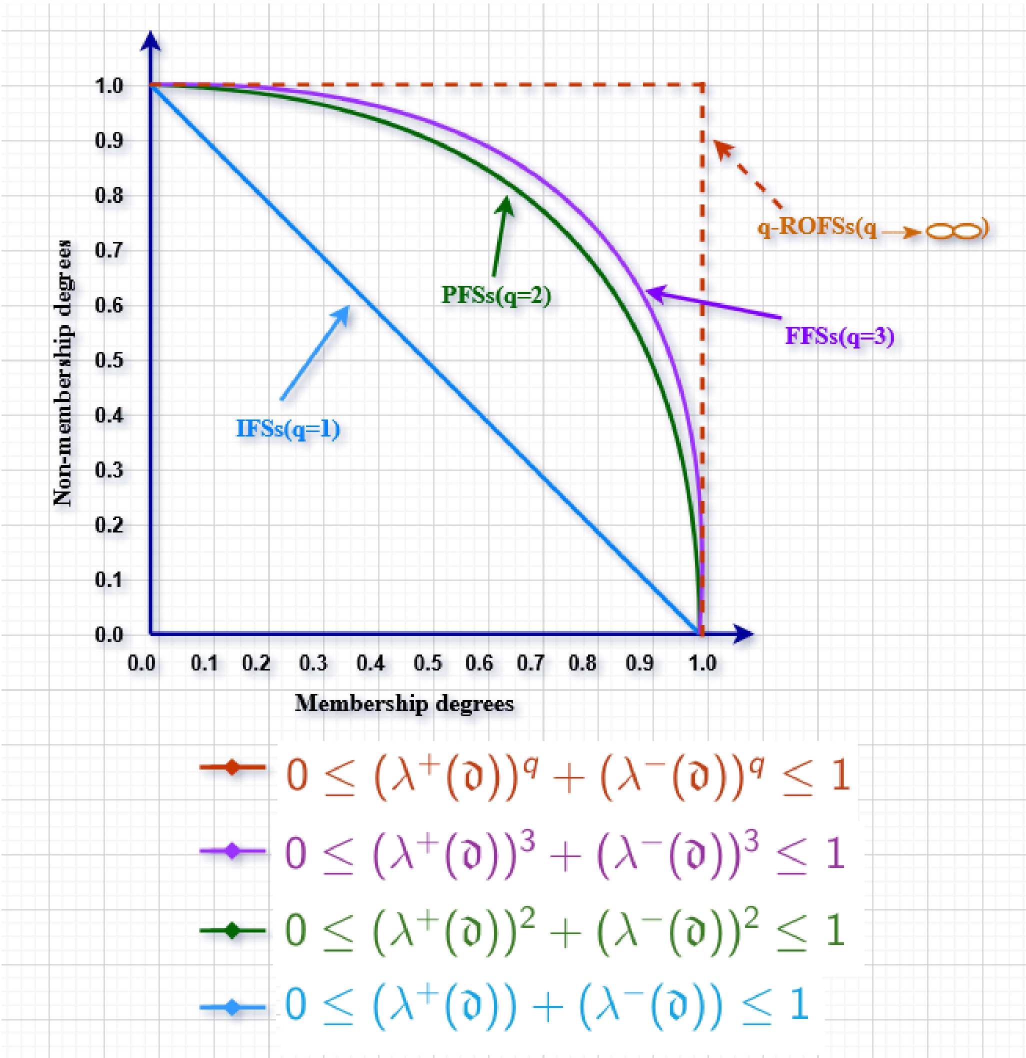


**Figure 1.** Comparison between the IFSs [2], PFSs [3], FFSs [11], and $q$-ROFSs [10].

Due to the existence of bipolar information in several practical situations, Shabir and Naz [25] extended their applications to bipolar soft sets (BSSs) and elaborated their algebraic structures. This concept was built to distinguish between preferred and adverse sides of the data. Dubois and Prade [26] introduced the role of polarity to give the reason for the positive and negative sides of alternatives. Currently, the hybrid environment of BSSs in decision-making problems has been used frequently [27–29]. Very recently, Ali and Ansari [30] presented a novel MADM model, namely, Fermatean fuzzy BSSs together with its two applications, including selection of a best surgeon robot and evaluation of the most affected country due to coronavirus disease 2019 (COVID-19). The existing models, namely, IFS, PFS, FFS, $q$-ROFS, IFSS, and $q$-ROFSS often lack in precision regarding bipolar soft knowledge when they come to decision-making with imprecise data. It can be elucidated from the above discussion that a hybrid model having the ability to depict bipolarity of soft data with $q$-rung orthopair fuzzy information is still unattended. Keeping in view the shortcomings of the existing systems, we offer a new direction for the research in the emerging era of decision-making techniques. For other useful terminologies the readers are suggested to [31–36].

The motivations of the proposed model are elaborated as follows:

1. The main feature of FFSs to handle the uncertainties in people decisions make it more cogent and efficient because FFSs deal with two dimensional (i.e., belongingness and non-belongingness) information in more wider space than IFSs and PFSs;
2. BSSs and $q$-ROFSs are two different mathematical models to address uncertain MADM situations. Therefore, there is a need of such hybrid model which have characteristics of both these models;
3. Note that for $q = 2,\ 3$, our proposed $q$-ROFBSS model can be converted into existing Pythagorean fuzzy BSSs (PFBSSs) [28] and Fermatean fuzzy BSSs (FFBSSs) [30], respectively;
4. The existing PFBSS model is inefficient to solve decision-making problems in which an expert evaluates the given information with the satisfactory and unsatisfactory degrees, whose sum of squares is not less than 1. To provide more space for evaluation values, the $q$-ROFBSS model is established, in which the sum of $q$th power of satisfactory and unsatisfactory degrees should be bounded by 1. Thus, $q$-ROFBSSs are more flexible for different vague environments as compared to certain existing models.

The major contributions of this research article are provided as below:

1. Our work focuses on the improvement of efficiency of $q$-ROFBSS model by increasing the number of acceptable orthopairs. The illustration of the proposed work comes with an example;
2. To investigate our hybrid model, we propose subset, complement, extended union and intersection, restricted union and intersection, and OR and AND operations;
3. Certain De Morgan's laws for $q$-ROFBSSs are also verified;
4. Ultimately, we combine these ideas and offer an application with algorithm regarding selection of land for cropping carrots and lettuces. We also use this model to offer another application to help in the selection of an eligible student for scholarship;
5. Furthermore, a comparison analysis with some existing models in qualitative and quantitative formats is provided;
6. At the end, some concluding remarks and future directions are given.

This paper is organized as: In Section 2, some fundamental notions are reviewed, including BSSs, $q$-ROFSs, and $q$-ROFSSs. In Section 3, $q$-ROFBSSs are discussed along with several significant operations for $q$-ROFBSSs, namely, subsets, complement, extended union and intersection, restricted union and intersection, and OR and AND operations. In Section 4, two applications are investigated by using the proposed framework. Section 5 gives comparison of the developed model with certain existing models, including Pythagorean and Fermatean fuzzy BSSs. In Section 6, some concluding remarks and future directions are given.

## 2. Preliminaries

This section recalls some fundamental notions, namely, BSS, $q$-ROFSS, and $q$-ROFS with score and accuracy functions.

**Definition 1** ([25])**.** *Let $\mathfrak{U}$ be a universal set and $P$ be a universe of parameters. For every $\mathcal{R} \subseteq P$, a triple $(f, g, \mathcal{R})$ is called a bipolar soft set or BSS on $\mathfrak{U}$, where $f$ and $g$ are functions defined as*

$$f : \mathcal{R} \to \mathcal{P}(\mathfrak{U}) \ \ and \ \ g : \neg\mathcal{R} \to \mathcal{P}(\mathfrak{U}),$$

*such that $f(\vartheta) \cap g(\neg r) = \emptyset, \forall\, r \in \mathcal{R},\ \neg r \in \neg\mathcal{R}$ where $\neg\mathcal{R}$ is the 'Not set' of parameters.*

**Definition 2** ([10])**.** *Let $\mathfrak{U}$ be a universal set. Then, a pair $G = (\lambda^+, \lambda^-)$ is called the q-rung orthopair fuzzy set or q-ROFS over $\mathfrak{U}$, where $\lambda^+$ is a belongingness function given by $\lambda^+ : \mathfrak{U} \to [0,1]$ and $\lambda^-$ is a non-belongingness function given by $\lambda^- : \mathfrak{U} \to [0,1]$ with $0 \leq (\lambda^+(\mathfrak{d}))^q + (\lambda^-(\mathfrak{d}))^q \leq 1$ where $q \geq 1,\ \lambda^+(\mathfrak{d}),\ \lambda^-(\mathfrak{d}) \in [0,1]$. In set form, a q-ROFS on $\mathfrak{U}$ is defined as*

$$G = \{\langle u, (\lambda^+(\mathfrak{d}), \lambda^-(\mathfrak{d}))\rangle \mid \mathfrak{d} \in \mathfrak{U}\},$$

*where* $\lambda^{+}(\mathfrak{d})$, $\lambda^{-}(\mathfrak{d}) \in [0,1]$ *denotes the belongingness and non-belongingness values, respectively, and satisfies* $0 \leq (\lambda^{+}(\mathfrak{d}))^{q} + (\lambda^{-}(\mathfrak{d}))^{q} \leq 1$. *Moreover,* $(\lambda^{+}(\mathfrak{d}), \lambda^{-}(\mathfrak{d}))$ *is known as a q-rung orthopair fuzzy number (q-ROFN) and denoted by* $O = (\lambda^{+}_{O}, \lambda^{-}_{O})$. *The degree of hesitance for q-ROFN* $O = (\lambda^{+}_{O}, \lambda^{-}_{O})$ *is defined as*

$$\pi_{O} = \sqrt[q]{1 - ((\lambda^{+}_{O}(\mathfrak{d}))^{q} + (\lambda^{-}_{O}(\mathfrak{d}))^{q})}.$$

*Assume that* $\overline{\mathcal{E}}(\mathfrak{U})$ *is the family of all q-ROFSs on* $\mathfrak{U}$.

**Definition 3** ([15])**.** *Let* $O = (\lambda^{+}_{O}, \lambda^{-}_{O})$ *be a q-ROFN. The score function of O is defined as*

$$s(O) = (\lambda^{+}_{O})^{q} - (\lambda^{-}_{O})^{q},\ q \geq 1. \tag{1}$$

**Definition 4** ([15])**.** *Let* $O = (\lambda^{+}_{O}, \lambda^{-}_{O})$ *be a q-ROFN. The accuracy function of O is defined as*

$$h(O) = (\lambda^{+}_{O})^{q} + (\lambda^{-}_{O})^{q},\ q \geq 1. \tag{2}$$

**Definition 5** ([15])**.** *Let* $O_1 = (\lambda^{+}_{O_1}, \lambda^{-}_{O_1})$ *and* $O_2 = (\lambda^{+}_{O_2}, \lambda^{-}_{O_2})$ *be any two q-ROFNs,* $s(O_1)$ *and* $s(O_2)$ *be the score functions of* $O_1$ *and* $O_2$*, and* $h(O_1)$ *and* $h(O_2)$ *be the accuracy functions of* $O_1$ *and* $O_2$*, then*

1. *if* $s(O_1) > s(O_2)$ *then* $O_1 > O_2$,
2. *if* $s(O_1) = s(O_2)$ *and*
   - *if* $h(O_1) > h(O_2)$ *then* $O_1 > O_2$,
   - *if* $h(O_1) = h(O_2)$ *then* $O_1 = O_2$.

**Definition 6** ([13])**.** *Let* $\mathfrak{U}$ *and P be the universal set and universe of parameters, respectively. Assume that* $\mathcal{R} \subseteq P$*, then the pair* $(A, \mathcal{R})$ *is said to be a q-rung orthopair fuzzy soft set or q-ROFSS over* $\mathfrak{U}$*, if A is a mapping given as* $A : \mathcal{R} \to \overline{\mathcal{E}}(\mathfrak{U})$. *Consider* $\mathfrak{d} \in \mathfrak{U}$ *and* $r \in \mathcal{R}$*, then* $A(r)$ *is a q-ROFS on* $\mathfrak{U}$*, which is defined as*

$$A(r) = \{\langle u, (\lambda^{+}_{A}(r)(\mathfrak{d}), \lambda^{-}_{A}(r)(\mathfrak{d}))\rangle \mid \mathfrak{d} \in \mathfrak{U}\},$$

*where* $\lambda^{+}_{A}(r)(\mathfrak{d})$, $\lambda^{-}_{A}(r)(\mathfrak{d}) \in [0,1]$ *denotes the belongingness and non-belongingness values, respectively, and satisfy* $0 \leq (\lambda^{+}_{A}(r)(\mathfrak{d}))^{q} + (\lambda^{-}_{A}(r)(\mathfrak{d}))^{q} \leq 1$.

## 3. *q*-Rung Orthopair Fuzzy Bipolar Soft Sets

This section provides a novel hybrid structure by the mixture of BSSs and q-ROFSs which is named as *q*-ROFBSSs. Here, we also present some operations and then investigate by means of numerical examples.

**Definition 7.** *Consider a universal set* $\mathfrak{U}$ *and P a universe of parameters. For any* $\mathcal{R} \subseteq P$*, a triplet* $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ *is called a q-rung orthopair fuzzy bipolar soft set or q-ROFBSS over* $\mathfrak{U}$*, if* $\mathcal{Q}$ *and* $\mathcal{S}$ *are mappings given as* $\mathcal{Q} : \mathcal{R} \to \overline{\mathcal{E}}(\mathfrak{U})$ *and* $\mathcal{S} : \neg\mathcal{R} \to \overline{\mathcal{E}}(\mathfrak{U})$*, respectively, and satisfy*

$$0 \leq (\lambda^{+}_{\mathcal{Q}}(r)(\mathfrak{d})) + (\lambda^{+}_{\mathcal{S}}(\neg r)(\mathfrak{d})) \leq 1, \tag{3}$$
$$0 \leq (\lambda^{-}_{\mathcal{Q}}(r)(\mathfrak{d})) + (\lambda^{-}_{\mathcal{S}}(\neg r)(\mathfrak{d})) \leq 1, \tag{4}$$

*for all* $r \in \mathcal{R}$, $\neg r \in \neg\mathcal{R}$*, and* $\mathfrak{d} \in \mathfrak{U}$. *Moreover,* $\lambda^{+}_{\mathcal{Q}}(r)(\mathfrak{d})$ *and* $\lambda^{+}_{\mathcal{S}}(\neg r)(\mathfrak{d})$ *are belongingness and non-belongingness values of an object '*$\mathfrak{d}$*' over* $\mathfrak{U}$ *and for any* $q \geq 1$ *satisfy*

$$0 \leq (\lambda^{+}_{\mathcal{Q}}(r)(\mathfrak{d}))^{q} + (\lambda^{+}_{\mathcal{Q}}(\neg r)(\mathfrak{d}))^{q} \leq 1, \tag{5}$$
$$0 \leq (\lambda^{-}_{\mathcal{S}}(r)(\mathfrak{d}))^{q} + (\lambda^{-}_{\mathcal{S}}(\neg r)(\mathfrak{d}))^{q} \leq 1, \tag{6}$$

*respectively.*

On the other hand, a $q$-ROFBSS over $\mathfrak{U}$ gives two parameterized $q$-rung orthopair fuzzy subsets on $\mathfrak{U}$, which satisfy the Equations (3) and (4). For any $r \in \mathcal{R}$, $\mathcal{Q}(r)$ and $\mathcal{S}(\neg r)$ are described as the sets of $r$- and $\neg r$-approximate elements of the $q$-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$, respectively.

The following example illustrate the Definition 7 precisely.

**Example 1.** *Let $\mathfrak{U} = \{\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4\}$ be the set of four refrigerators from different companies. Ahmad wants to buy a refrigerator. Consider the attribute set for the objects $\mathfrak{d}_i \in \mathfrak{U}$, $i = 1, 2, 3, 4$ is given by*

$$P = \{r_1(\text{expensive}),\ r_2(\text{big size}),\ r_3(\text{attractive}), r_4(\text{long warranty time})\}.$$

*Denote the "Not set of P" as*

$$\neg P = \{\neg r_1(\text{cheap}),\ \neg r_2(\text{small size}),\ \neg r_3(\text{ugly}), \neg r_4(\text{small warranty time})\}.$$

*For $\mathcal{R} = \{r_1(\text{expensive}),\ r_2(\text{big size}),\ r_3(\text{attractive})\} \subseteq P$, we define a q-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ with q = 5, which describe the requirements of Ahmad about the refrigerator, he wishes to buy. Then, a 5-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ is given by*

$$\begin{aligned}
\mathcal{Q}(r_1) &= \{(\mathfrak{d}_1, 0.8, 0.9), (\mathfrak{d}_2, 0.7, 0.6), (\mathfrak{d}_3, 0.4, 0.9), (\mathfrak{d}_4, 0.98, 0.3)\},\\
\mathcal{Q}(r_2) &= \{(\mathfrak{d}_1, 0.6, 0.9), (\mathfrak{d}_2, 0.5, 0.5), (\mathfrak{d}_3, 0.6, 0.3), (\mathfrak{d}_4, 0.9, 0.1)\},\\
\mathcal{Q}(r_3) &= \{(\mathfrak{d}_1, 0.6, 0.7), (\mathfrak{d}_2, 0.4, 0.5), (\mathfrak{d}_3, 0.9, 0.0), (\mathfrak{d}_4, 0.7, 0.8)\},\\
\mathcal{S}(\neg r_1) &= \{(\mathfrak{d}_1, 0.2, 0.1), (\mathfrak{d}_2, 0.3, 0.3), (\mathfrak{d}_3, 0.5, 0.1), (\mathfrak{d}_4, 0.01, 0.7)\},\\
\mathcal{S}(\neg r_2) &= \{(\mathfrak{d}_1, 0.4, 0.1), (\mathfrak{d}_2, 0.5, 0.4), (\mathfrak{d}_3, 0.4, 0.7), (\mathfrak{d}_4, 0.1, 0.8)\},\\
\mathcal{S}(\neg r_3) &= \{(\mathfrak{d}_1, 0.4, 0.3), (\mathfrak{d}_2, 0.6, 0.5), (\mathfrak{d}_3, 0.1, 0.9), (\mathfrak{d}_4, 0.3, 0.2)\}.
\end{aligned}$$

*The 5-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ can be represented in tabular form as shown in Table 1.*

**Table 1.** Table for the 5-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$.

| $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ | $r_1$ | $r_2$ | $r_3$ |
|---|---|---|---|
| $\mathfrak{d}_1$ | $\langle(0.8, 0.9), (0.2, 0.1)\rangle$ | $\langle(0.6, 0.9), (0.4, 0.1)\rangle$ | $\langle(0.6, 0.7), (0.4, 0.3)\rangle$ |
| $\mathfrak{d}_2$ | $\langle(0.7, 0.6), (0.3, 0.3)\rangle$ | $\langle(0.5, 0.5), (0.5, 0.4)\rangle$ | $\langle(0.4, 0.5), (0.6, 0.5)\rangle$ |
| $\mathfrak{d}_3$ | $\langle(0.4, 0.9), (0.5, 0.1)\rangle$ | $\langle(0.6, 0.3), (0.4, 0.7)\rangle$ | $\langle(0.9, 0.0), (0.1, 0.9)\rangle$ |
| $\mathfrak{d}_4$ | $\langle(0.98, 0.3), (0.01, 0.7)\rangle$ | $\langle(0.9, 0.1), (0.1, 0.8)\rangle$ | $\langle(0.7, 0.8), (0.3, 0.2)\rangle$ |

*Thus, $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ is a 5-ROFBSS based on expensiveness, attractiveness, and few other parameters associated with the selection of refrigerator. For instance, from Table 1, $\langle(0.8, 0.9), (0.2, 0.1)\rangle$ represents that the support of the belongingness of refrigerator $\mathfrak{d}_1$ is 0.8 and 0.9 is the support against belongingness of $\mathfrak{d}_1$ based expensiveness. In a similar manner, for the parameter 'cheap' which gives totally opposite meaning to the parameter 'expensive', 0.2 is the belongingness degree in the support of $\mathfrak{d}_1$, and 0.1 is the belongingness degree against the support of $\mathfrak{d}_1$ based cheapness.*

The 5-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ on $\mathfrak{U}$, displayed by Table 1 can also be given by Tables 2 and 3.

**Table 2.** Table for belongingness and non-belongingness values for the parameter set $\mathcal{R}$.

| $\mathcal{Q}$ | $r_1$ | $r_2$ | $r_3$ |
|---|---|---|---|
| $\mathfrak{d}_1$ | (0.8, 0.9) | (0.6, 0.9) | (0.6, 0.7) |
| $\mathfrak{d}_2$ | (0.7, 0.6) | (0.5, 0.5) | (0.4, 0.5) |
| $\mathfrak{d}_3$ | (0.4, 0.9) | (0.6, 0.3) | (0.9, 0) |
| $\mathfrak{d}_4$ | (0.98, 0.3) | (0.9, 0.1) | (0.7, 0.8) |

**Table 3.** Table for belongingness and non-belongingness values for the set $\neg\mathcal{R}$.

| $\mathcal{S}$ | $\neg r_1$ | $\neg r_2$ | $\neg r_3$ |
|---|---|---|---|
| $\mathfrak{d}_1$ | (0.2, 0.1) | (0.4, 0.1) | (0.4, 0.3) |
| $\mathfrak{d}_2$ | (0.3, 0.3) | (0.5, 0.4) | (0.6, 0.5) |
| $\mathfrak{d}_3$ | (0.5, 0.1) | (0.4, 0.7) | (0.1, 0.9) |
| $\mathfrak{d}_4$ | (0.01, 0.7) | (0.1, 0.8) | (0.3, 0.2) |

**Remark 1.** *Note that intuitionistic fuzzy BSSs, Pythagorean fuzzy BSSs [28] and Fermatean fuzzy BSSs [30] are special cases of our proposed q-ROFBSS model for $q = 1,\ 2$ and $q = 3$, respectively.*

*Basic Operations*

In this subsection, we explore some basic notions of $q$-ROFBSSs and investigate them with corresponding numerical examples.

**Definition 8.** *Let $\mathfrak{U}$ be a universal set and $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$, $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ be two q-ROFBSSs over $\mathfrak{U}$. The set $\xi_1$ is called a q-rung orthopair fuzzy bipolar soft subset of $\xi_2$, denoted as $\xi_1 \widetilde{\subset} \xi_2$, if*

1. $\mathcal{R}_1 \subseteq \mathcal{R}_2$,
2. $\mathcal{Q}_1(r) \subseteq \mathcal{Q}_2(r)$ *(that is, $\lambda^+_{\mathcal{Q}_1}(r)(\mathfrak{d}) \leq \lambda^+_{\mathcal{Q}_2}(r)(\mathfrak{d})$, $\lambda^-_{\mathcal{Q}_1}(r)(\mathfrak{d}) \geq \lambda^-_{\mathcal{Q}_2}(r)(\mathfrak{d})$) and $\mathcal{S}_1(\neg r) \supseteq \mathcal{S}_2(\neg r)$ (that is, $\lambda^+_{\mathcal{S}_2}(\neg r)(\mathfrak{d}) \leq \lambda^+_{\mathcal{S}_1}(\neg r)(\mathfrak{d})$, $\lambda^-_{\mathcal{S}_2}(\neg r)(\mathfrak{d}) \geq \lambda^-_{\mathcal{S}_1}(\neg r)(\mathfrak{d})$) for all $r \in \mathcal{R}_1$ and $\mathfrak{d} \in \mathfrak{U}$.*

**Example 2.** *Let 5-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ on $\mathfrak{U}$ be as defined in Example 1, for $\mathcal{R}_1 = \{r_1 = \text{expensive}, r_2 = \text{big size}\} \subseteq \mathcal{R}$, we give a new 5-ROFBSS $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$, which is given by Table 4:*

**Table 4.** Table for the 5-ROFBSS $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$.

| $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ | $r_1$ | $r_2$ |
|---|---|---|
| $\mathfrak{d}_1$ | $\langle(0.6, 0.9), (0.4, 0.1)\rangle$ | $\langle(0.5, 0.9), (0.5, 0.1)\rangle$ |
| $\mathfrak{d}_2$ | $\langle(0.6, 0.7), (0.4, 0.3)\rangle$ | $\langle(0.4, 0.6), (0.6, 0.4)\rangle$ |
| $\mathfrak{d}_3$ | $\langle(0.3, 0.9), (0.7, 0.1)\rangle$ | $\langle(0.5, 0.5), (0.5, 0.45)\rangle$ |
| $\mathfrak{d}_4$ | $\langle(0.9, 0.6), (0.1, 0.4)\rangle$ | $\langle(0.8, 0.4), (0.2, 0.6)\rangle$ |

*It is clear from Definition 8 that $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1) \widetilde{\subset} (\mathcal{Q}, \mathcal{S}, \mathcal{R})$.*

**Definition 9.** *Let $\mathfrak{U}$ be a universal set. Consider $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ and $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ are two q-ROFBSSs over $\mathfrak{U}$. Then, $\xi_1$ and $\xi_2$ are said to be equal, if $\xi_1 \widetilde{\subset} \xi_2$ and $\xi_2 \widetilde{\subset} \xi_1$.*

**Definition 10.** *Let $\mathfrak{U}$ be a universal set and $\xi = (\mathcal{Q}, \mathcal{S}, \mathcal{R})$ be a q-ROFBSS. Then, its complement $\xi^c = (\mathcal{Q}^c, \mathcal{S}^c, R)$ is a q-ROFBSS over $\mathfrak{U}$ with $\mathcal{Q}^c(r)(\mathfrak{d}) = (\lambda^-_{\mathcal{Q}}(r)(\mathfrak{d}), \lambda^+_{\mathcal{Q}}(r)(\mathfrak{d}))$ and $\mathcal{S}^c(\neg r)(\mathfrak{d}) = (\lambda^-_{\mathcal{S}}(\neg r)(\mathfrak{d}), \lambda^+_{\mathcal{S}}(\neg r)(\mathfrak{d}))$ for all $r \in \mathcal{R}, \neg r \in \neg\mathcal{R}$ and $\mathfrak{d} \in \mathfrak{U}$.*

**Example 3.** *Let $\xi = (\mathcal{Q}, \mathcal{S}, \mathcal{R})$ be the 5-ROFBSS over a universe $\mathfrak{U}$ as considered in Example 1. Then, by Definition 10, its complement $\xi^c = (\mathcal{Q}^c, \mathcal{S}^c, R)$ is computed in Table 5.*

**Table 5.** Table for the complement of $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$.

| $\xi^c$ | $r_1$ | $r_2$ | $r_3$ |
|---|---|---|---|
| $\mathfrak{d}_1$ | $\langle(0.9, 0.8), (0.1, 0.2)\rangle$ | $\langle(0.9, 0.6), (0.1, 0.4)\rangle$ | $\langle(0.7, 0.6), (0.3, 0.4)\rangle$ |
| $\mathfrak{d}_2$ | $\langle(0.6, 0.7), (0.3, 0.3)\rangle$ | $\langle(0.5, 0.5), (0.4, 0.5)\rangle$ | $\langle(0.5, 0.4), (0.5, 0.6)\rangle$ |
| $\mathfrak{d}_3$ | $\langle(0.9, 0.4), (0.1, 0.5)\rangle$ | $\langle(0.3, 0.6), (0.7, 0.4)\rangle$ | $\langle(0, 0.9), (0.9, 0.1)\rangle$ |
| $\mathfrak{d}_4$ | $\langle(0.3, 0.98), (0.7, 0.01)\rangle$ | $\langle(0.1, 0.9), (0.8, 0.1)\rangle$ | $\langle(0.8, 0.7), (0.2, 0.3)\rangle$ |

**Definition 11.** *A q-ROFBSS on a universe* $\mathfrak{U}$ *is refereed to as a relative null q-ROFBSS, represented by* $(\Phi, \mathfrak{U}, \mathcal{R})$, *if* $\Phi(r)(\mathfrak{d}) = (\lambda^+_\Phi(r)(\mathfrak{d}) = 0, \lambda^-_\Phi(r)(\mathfrak{d}) = 1)$ *and* $\mathfrak{U}(\neg r)(\mathfrak{d}) = (1, 0)$ $\forall\ r \in \mathcal{R},\ \neg r \in \neg\mathcal{R},\ \mathfrak{d} \in \mathfrak{U}$.

**Definition 12.** *A q-ROFBSS on a universe* $\mathfrak{U}$ *is refereed to as a relative absolute q-ROFBSS, represented by* $(\mathfrak{U}, \Phi, \mathcal{R})$, *if* $\mathfrak{U}(r)(\mathfrak{d}) = (\lambda^+_{\mathfrak{U}}(r)(\mathfrak{d}) = 1, \lambda^-_{\mathfrak{U}}(r)(\mathfrak{d}) = 0)$ *and* $\Phi(\neg r)(\mathfrak{d}) = (0, 1)$ $\forall\ r \in \mathcal{R},\ \neg r \in \neg\mathcal{R},\ \mathfrak{d} \in \mathfrak{U}$.

**Definition 13.** *Let* $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ *and* $(\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ *be two q-ROFBSSs over a universe* $\mathfrak{U}$. *Then, the "AND" operation on* $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ *and* $(\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$, *denoted by* $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1) \barwedge (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$, *is defined as* $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1) \barwedge (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2) = (\mathcal{Q}, \mathcal{S}, \mathcal{R}_1 \times \mathcal{R}_2)$ *where for all* $(r_i, r_j) \in \mathcal{R}_1 \times \mathcal{R}_2$, $(\neg r_i, \neg r_j) \in \neg\mathcal{R}_1 \times \neg\mathcal{R}_2$, *and* $\mathfrak{d} \in \mathfrak{U}$,

$$\mathcal{Q}(r_i, r_j)(\mathfrak{d}) = \left(\lambda^+_{\mathcal{Q}_1}(r_i)(\mathfrak{d}) \wedge \lambda^+_{\mathcal{Q}_2}(r_j)(\mathfrak{d}), \lambda^-_{\mathcal{Q}_1}(r_i)(\mathfrak{d}) \vee \lambda^-_{\mathcal{Q}_2}(r_j)(\mathfrak{d})\right),$$
$$\mathcal{S}(\neg r_i, \neg r_j)(\mathfrak{d}) = \left(\lambda^+_{\mathcal{S}_1}(\neg r_i)(\mathfrak{d}) \vee \lambda^+_{\mathcal{S}_2}(\neg r_j)(\mathfrak{d}), \lambda^-_{\mathcal{S}_1}(\neg r_i)(\mathfrak{d}) \wedge \lambda^-_{\mathcal{S}_2}(\neg r_j)(\mathfrak{d})\right).$$

**Definition 14.** *Let* $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ *and* $(\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ *be two q-ROFBSSs over a universe* $\mathfrak{U}$. *Then the "OR" operation on* $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ *and* $(\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$, *denoted by* $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1) \veebar (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$, *is defined as* $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1) \veebar (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2) = (\mathcal{Q}', S', \mathcal{R}_1 \times \mathcal{R}_2)$ *where for all* $(r_i, r_j) \in \mathcal{R}_1 \times \mathcal{R}_2$, $(\neg r_i, \neg r_j) \in \neg\mathcal{R}_1 \times \neg\mathcal{R}_2$, *and* $\mathfrak{d} \in \mathfrak{U}$,

$$\mathcal{Q}'(r_i, r_j)(\mathfrak{d}) = \left(\lambda^+_{\mathcal{Q}_1}(r_i)(\mathfrak{d}) \vee \lambda^+_{\mathcal{Q}_2}(r_j)(\mathfrak{d}), \lambda^-_{\mathcal{Q}_1}(r_i)(\mathfrak{d}) \wedge \lambda^-_{\mathcal{Q}_2}(r_j)(\mathfrak{d})\right),$$
$$\mathcal{S}'(\neg r_i, \neg r_j)(\mathfrak{d}) = \left(\lambda^+_{\mathcal{S}_1}(\neg r_i)(\mathfrak{d}) \wedge \lambda^+_{\mathcal{S}_2}(\neg r_j)(\mathfrak{d}), \lambda^-_{\mathcal{S}_1}(\neg r_i)(\mathfrak{d}) \vee \lambda^-_{\mathcal{S}_2}(\neg r_j)(\mathfrak{d})\right).$$

**Example 4.** *Let* $\mathfrak{U} = \{\mathfrak{d}_1, \mathfrak{d}_2, \ldots, \mathfrak{d}_6\}$ *be a set of six cars and let* $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ *and* $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ *be two q-ROFBSSs over* $\mathfrak{U}$. *Consider a universal set of parameters* $P = \{r_1 = \text{less weight}, r_2 = \text{high engine power}, r_3 = \text{large size}, r_4 = \text{high speed}, r_5 = \text{safe driving}\}$, *where* $\mathcal{R}_1 = \{r_1 = \text{less weight},\ r_2 = \text{high engine power}\}$, *and* $\mathcal{R}_2 = \{r_3 = \text{large size}\} \subseteq P$ *are the set of parameters. Then, q-ROFBSSs* $\xi_1$ *and* $\xi_2$ *for* $q = 7$ *are displayed in Tables 6 and 7, respectively.*

**Table 6.** Table for the 7-ROFBSS $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$.

| $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ | $r_1$ | $r_2$ |
|---|---|---|
| $\mathfrak{d}_1$ | $\langle(0.5, 0.9), (0.4, 0.1)\rangle$ | $\langle(0.7, 0.6), (0.3, 0.4)\rangle$ |
| $\mathfrak{d}_2$ | $\langle(0.7, 0.8), (0.3, 0.1)\rangle$ | $\langle(0.8, 0.7), (0.1, 0.2)\rangle$ |
| $\mathfrak{d}_3$ | $\langle(0.9, 0.4), (0.1, 0.5)\rangle$ | $\langle(0.6, 0.7), (0.4, 0.3)\rangle$ |
| $\mathfrak{d}_4$ | $\langle(0.5, 0.5), (0.5, 0.4)\rangle$ | $\langle(0.1, 0.2), (0.9, 0.7)\rangle$ |
| $\mathfrak{d}_5$ | $\langle(0.9, 0.9), (0.1, 0)\rangle$ | $\langle(0.5, 0.8), (0.4, 0.1)\rangle$ |
| $\mathfrak{d}_6$ | $\langle(1, 0), (0, 1)\rangle$ | $\langle(0.4, 0.5), (0.6, 0.4)\rangle$ |

**Table 7.** Table for the 7-ROFBSS $(\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$.

| $(\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ | $r_3$ |
|---|---|
| $\mathfrak{d}_1$ | $\langle(0.5, 0.7), (0.4, 0.3)\rangle$ |
| $\mathfrak{d}_2$ | $\langle(0.9, 0.8), (0.1, 0.2)\rangle$ |
| $\mathfrak{d}_3$ | $\langle(0.7, 0.5), (0.3, 0.5)\rangle$ |
| $\mathfrak{d}_4$ | $\langle(0.2, 0.8), (0.8, 0.1)\rangle$ |
| $\mathfrak{d}_5$ | $\langle(0.5, 0.7), (0.5, 0.3)\rangle$ |
| $\mathfrak{d}_6$ | $\langle(0.9, 0.5), (0.1, 0.5)\rangle$ |

*Then, the "AND" and "OR" operation between* $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ *and* $(\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ *are given by Tables 8 and 9, respectively.*

**Table 8.** Table for the 7-ROFBSS $\xi_1 \barwedge \xi_2$.

| $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1) \barwedge (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ | $(r_1, r_3)$ | $(r_2, r_3)$ |
|---|---|---|
| $\eth_1$ | $\langle(0.5, 0.9), (0.4, 0.1)\rangle$ | $\langle(0.5, 0.7), (0.4, 0.3)\rangle$ |
| $\eth_2$ | $\langle(0.7, 0.8), (0.3, 0.1)\rangle$ | $\langle(0.8, 0.8), (0.1, 0.2)\rangle$ |
| $\eth_3$ | $\langle(0.7, 0.5), (0.3, 0.5)\rangle$ | $\langle(0.6, 0.7), (0.4, 0.3)\rangle$ |
| $\eth_4$ | $\langle(0.2, 0.8), (0.8, 0.1)\rangle$ | $\langle(0.1, 0.8), (0.9, 0.1)\rangle$ |
| $\eth_5$ | $\langle(0.5, 0.9), (0.5, 0)\rangle$ | $\langle(0.5, 0.8), (0.5, 0.1)\rangle$ |
| $\eth_6$ | $\langle(0.9, 0.5), (0.1, 0.5)\rangle$ | $\langle(0.4, 0.5), (0.6, 0.4)\rangle$ |

**Table 9.** Table for the 7-ROFBSS $\xi_1 \veebar \xi_2$.

| $(\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1) \veebar (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ | $(r_1, r_3)$ | $(r_2, r_3)$ |
|---|---|---|
| $\eth_1$ | $\langle(0.5, 0.7), (0.4, 0.3)\rangle$ | $\langle(0.7, 0.6), (0.3, 0.4)\rangle$ |
| $\eth_2$ | $\langle(0.9, 0.8), (0.1, 0.2)\rangle$ | $\langle(0.9, 0.7), (0.1, 0.2)\rangle$ |
| $\eth_3$ | $\langle(0.9, 0.4), (0.1, 0.5)\rangle$ | $\langle(0.7, 0.5), (0.3, 0.5)\rangle$ |
| $\eth_4$ | $\langle(0.5, 0.5), (0.5, 0.4)\rangle$ | $\langle(0.2, 0.2), (0.8, 0.7)\rangle$ |
| $\eth_5$ | $\langle(0.9, 0.7), (0.1, 0.3)\rangle$ | $\langle(0.5, 0.7), (0.4, 0.3)\rangle$ |
| $\eth_6$ | $\langle(1, 0), (0, 1)\rangle$ | $\langle(0.9, 0.5), (0.1, 0.5)\rangle$ |

The following proposition describes that certain De Morgan's laws verify with the AND operation and the OR operation.

**Proposition 1.** *Let $\mathfrak{U}$ be a universe and let $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ and $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ be q-ROFBSSs on $\mathfrak{U}$. Then*

1. $(\xi_1 \veebar \xi_2)^c = (\xi_1)^c \barwedge (\xi_2)^c$,
2. $(\xi_1 \barwedge \xi_2)^c = (\xi_1)^c \veebar (\xi_2)^c$.

**Proof.**

1. From Definitions 10 and 14, $(\xi_1 \veebar \xi_2)^c = (\mathcal{Q}'^c, \mathcal{S}'^c, \mathcal{R}_1 \times \mathcal{R}_2)$ where $\mathcal{Q}'^c(r,s) = \mathcal{Q}_1^c(r) \barwedge \mathcal{Q}_2^c(s)$ and $\mathcal{S}'^c(\neg r, \neg s) = \mathcal{S}_1^c(\neg r) \veebar \mathcal{S}_2^c(\neg s)$ for all $(r,s) \in \mathcal{R}_1 \times \mathcal{R}_2$ and $(\neg r, \neg s) \in \neg\mathcal{R}_1 \times \neg\mathcal{R}_2$.
   Now by using Definition 10, $(\xi_1)^c = (\mathcal{Q}_1^c, \mathcal{S}_1^c, \mathcal{R}_1)$ and $(\xi_2)^c = (\mathcal{Q}_2^c, \mathcal{S}_2^c, \mathcal{R}_1)$. Therefore, $(\xi_1)^c \barwedge (\xi_2)^c = (\mathcal{Q}^c, \mathcal{S}^c, \mathcal{R}_1 \times \mathcal{R}_2)$ (by Definition 13) where $\mathcal{Q}^c(r,s) = \mathcal{Q}_1^c(r) \barwedge \mathcal{Q}_2^c(s)$ and $\mathcal{S}^c(\neg r, \neg s) = \mathcal{S}_1^c(\neg r) \veebar \mathcal{S}_2^c(\neg s)$ for all $(r,s) \in \mathcal{R}_1 \times \mathcal{R}_2$ and $(\neg r, \neg s) \in \neg\mathcal{R}_1 \times \neg\mathcal{R}_2$. Thus, $(\xi_1 \veebar \xi_2)^c = (\xi_1)^c \barwedge (\xi_2)^c$.
2. It proof is similar to part 1.

□

**Definition 15.** *Let $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ and $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ be two q-ROFBSSs on $\mathfrak{U}$. Then, the extended union of $\xi_1$ and $\xi_2$, represented by $\xi_1 \cup_{\mathcal{E}} \xi_2$, is a q-ROFBSS $((\mathcal{Q}_1 \Cup \mathcal{Q}_2), (\mathcal{S}_1 \Cap \mathcal{S}_2), \mathcal{R}_1 \cup \mathcal{R}_2)$ on $\mathfrak{U}$, defined as follows:*

$$(\mathcal{Q}_1 \Cup \mathcal{Q}_2)(r) = \begin{cases} \mathcal{Q}_1(r), & if \quad r \in \mathcal{R}_1 - \mathcal{R}_2, \\ \mathcal{Q}_2(r), & if \quad r \in \mathcal{R}_2 - \mathcal{R}_1, \\ \mathcal{Q}_1(r) \underline{\cup} \mathcal{Q}_2(r) & if \quad r \in \mathcal{R}_1 \cap \mathcal{R}_2. \end{cases}$$

$$(\mathcal{S}_1 \Cap \mathcal{S}_2)(\neg r) = \begin{cases} \mathcal{S}_1(\neg r), & if \quad \neg r \in (\neg\mathcal{R}_1) - (\neg\mathcal{R}_2), \\ \mathcal{S}_2(\neg r), & if \quad \neg r \in (\neg\mathcal{R}_2) - (\neg\mathcal{R}_1), \\ \mathcal{S}_1(\neg r) \overline{\cap} \mathcal{S}_2(\neg r) & if \quad \neg r \in (\neg\mathcal{R}_1) \cap (\neg\mathcal{R}_2), \end{cases}$$

*where*

$$\mathcal{Q}_1(r)\underline{\cup}\mathcal{Q}_2(r) = \{\langle u, (\lambda^+_{\mathcal{Q}_1}(r_i)(\eth) \vee \lambda^+_{\mathcal{Q}_2}(r_j)(\eth), \lambda^-_{\mathcal{Q}_1}(r_i)(\eth) \wedge \lambda^-_{\mathcal{Q}_2}(r_j)(\eth))\rangle \mid \eth \in \mathfrak{U}\},$$
$$\mathcal{S}_1(\neg r)\overline{\cap}\mathcal{S}_2(\neg r) = \{\langle u, (\lambda^+_{\mathcal{S}_1}(\neg r_i)(\eth) \wedge \lambda^+_{\mathcal{S}_2}(\neg r_j)(\eth), \lambda^-_{\mathcal{S}_1}(\neg r_i)(\eth) \vee \lambda^-_{\mathcal{S}_2}(\neg r_j)(\eth))\rangle \mid \eth \in \mathfrak{U}\}.$$

**Definition 16.** *Let $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ and $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ be two q-ROFBSSs over $\mathfrak{U}$. Then, the restricted union of $\xi_1$ and $\xi_2$, denoted by $\xi_1 \cup_{\mathcal{R}} \xi_2$, is a q-ROFBSS $((\mathcal{Q}_1 \Cup \mathcal{Q}_2), (\mathcal{S}_1 \Cap \mathcal{S}_2), \mathcal{R}_1 \cap \mathcal{R}_2)$ on $\mathfrak{U}$, where $(\mathcal{Q}_1 \Cup \mathcal{Q}_2)(r) = \mathcal{Q}_1(r)\underline{\cup}\mathcal{Q}_2(r)$ for all $r \in \mathcal{R}_1 \cap \mathcal{R}_2$ and $(\mathcal{S}_1 \Cap \mathcal{S}_2)(\neg r) = \mathcal{S}_1(\neg r)\overline{\cap}\mathcal{S}_2(\neg r)$ for all $\neg r \in (\neg\mathcal{R}_1) \cap (\neg\mathcal{R}_2)$, provided $\mathcal{R}_1 \cap \mathcal{R}_2 \neq \emptyset$, $(\neg\mathcal{R}_1) \cap (\neg\mathcal{R}_2) \neq \emptyset$.*

**Definition 17.** *Let $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ and $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ be two q-ROFBSSs over $\mathfrak{U}$. Then, the extended intersection of $\xi_1$ and $\xi_2$, represented by $\xi_1 \cap_{\mathcal{E}} \xi_2$, is a q-ROFBSS $((\mathcal{Q}_1 \Cap \mathcal{Q}_2), (\mathcal{S}_1 \Cup \mathcal{S}_2), \mathcal{R}_1 \cup \mathcal{R}_2)$ on $\mathfrak{U}$, defined as follows:*

$$(\mathcal{Q}_1 \Cap \mathcal{Q}_2)(r) = \begin{cases} \mathcal{Q}_1(r), & if \quad r \in \mathcal{R}_1 - \mathcal{R}_2, \\ \mathcal{Q}_2(r), & if \quad r \in \mathcal{R}_2 - \mathcal{R}_1, \\ \mathcal{Q}_1(r)\overline{\cap}\mathcal{Q}_2(r), & if \quad r \in \mathcal{R}_1 \cap \mathcal{R}_2. \end{cases}$$

$$(\mathcal{S}_1 \Cup \mathcal{S}_2)(\neg r) = \begin{cases} \mathcal{S}_1(\neg r), & if \quad \neg r \in (\neg\mathcal{R}_1) - (\neg\mathcal{R}_2), \\ \mathcal{S}_2(\neg r), & if \quad \neg r \in (\neg\mathcal{R}_2) - (\neg\mathcal{R}_1), \\ \mathcal{S}_1(\neg r)\underline{\cup}\mathcal{S}_2(\neg r), & if \ \neg r \in (\neg\mathcal{R}_1) \cap (\neg\mathcal{R}_2). \end{cases}$$

*where*

$$\mathcal{Q}_1(r)\overline{\cap}\mathcal{Q}_2(r) = \{\langle u, (\lambda^+_{\mathcal{Q}_1}(r_i)(\eth) \wedge \lambda^+_{\mathcal{Q}_2}(r_j)(\eth), \lambda^-_{\mathcal{Q}_1}(r_i)(\eth) \vee \lambda^-_{\mathcal{Q}_2}(r_j)(\eth))\rangle \mid \eth \in \mathfrak{U}\},$$
$$\mathcal{S}_1(\neg r)\underline{\cup}\mathcal{S}_2(\neg r) = \{\langle u, (\lambda^+_{\mathcal{S}_1}(\neg r_i)(\eth) \vee \lambda^+_{\mathcal{S}_2}(\neg r_j)(\eth), \lambda^-_{\mathcal{S}_1}(\neg r_i)(\eth) \wedge \lambda^-_{\mathcal{S}_2}(\neg r_j)(\eth))\rangle \mid \eth \in \mathfrak{U}\}.$$

**Definition 18.** *Let $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ and $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ be any two q-ROFBSSs over a universe $\mathfrak{U}$. Then, the restricted intersection of $\xi_1$ and $\xi_2$, represented by $\xi_1 \cap_{\mathcal{R}} \xi_2$, is a q-ROFBSS $((\mathcal{Q}_1 \Cap \mathcal{Q}_2), (\mathcal{S}_1 \Cup \mathcal{S}_2), \mathcal{R}_1 \cap \mathcal{R}_2)$ on $\mathfrak{U}$, where $(\mathcal{Q}_1 \Cap \mathcal{Q}_2)(r) = \mathcal{Q}_1(r)\overline{\cap}\mathcal{Q}_2(r)$ for all $r \in \mathcal{R}_1 \cap \mathcal{R}_2$ and $(\mathcal{S}_1 \Cup \mathcal{S}_2)(\neg r) = \mathcal{S}_1(\neg r)\underline{\cup}\mathcal{S}_2(\neg r)$ for all $\neg r \in (\neg\mathcal{R}_1) \cap (\neg\mathcal{R}_2)$, provided $\mathcal{R}_1 \cap \mathcal{R}_2 \neq \emptyset$, $(\neg\mathcal{R}_1) \cap (\neg\mathcal{R}_2) \neq \emptyset$.*

**Example 5.** *Let $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ be a 7-ROFBSS over $\mathfrak{U}$, with $\mathcal{R}_2 = \{r_1, r_2, r_3\}$ given by Table 10 and $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ be the 7-ROFBSS on the universe $\mathfrak{U}$, as considered in Example 4. Then, their extended union $\xi_1 \cup_{\mathcal{E}} \xi_2$ and the extended intersection $\xi_1 \cap_{\mathcal{E}} \xi_2$ are, respectively, displayed in Tables 11 and 12. The tabular arrangements of the restricted intersection and union are provided by Tables 13 and 14, respectively.*

**Table 10.** Table for the 7-ROFBSS $(\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$.

| $(\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ | $r_1$ | $r_2$ | $r_3$ |
|---|---|---|---|
| $\eth_1$ | $\langle(0.3, 0.9), (0.5, 0.1)\rangle$ | $\langle(0.6, 0.3), (0.4, 0.7)\rangle$ | $\langle(0.7, 0.9), (0.3, 0.1)\rangle$ |
| $\eth_2$ | $\langle(0.5, 0.8), (0.4, 0.2)\rangle$ | $\langle(0.9, 0.5), (0.1, 0.5)\rangle$ | $\langle(0.5, 0.7), (0.4, 0.2)\rangle$ |
| $\eth_3$ | $\langle(0.9, 0.8), (0.1, 0.1)\rangle$ | $\langle(0.7, 0.7), (0.05, 0.2)\rangle$ | $\langle(0.7, 0.7), (0.3, 0.3)\rangle$ |
| $\eth_4$ | $\langle(0.2, 0.7), (0.8, 0.2)\rangle$ | $\langle(0.4, 0.4), (0.6, 0.5)\rangle$ | $\langle(0.9, 0.8), (0.1, 0.1)\rangle$ |
| $\eth_5$ | $\langle(0.5, 0.6), (0.4, 0.4)\rangle$ | $\langle(0.3, 0.9), (0.7, 0.1)\rangle$ | $\langle(0.8, 0.4), (0.2, 0.5)\rangle$ |
| $\eth_6$ | $\langle(0.6, 0.9), (0.25, 0.1)\rangle$ | $\langle(0.5, 0.3), (0.4, 0.1)\rangle$ | $\langle(0.6, 0.6), (0.4, 0.2)\rangle$ |

**Table 11.** Table for the extended union $\xi_1 \cup_{\mathcal{E}} \xi_2$.

| $\xi_1 \cup_{\mathcal{E}} \xi_2$ | $r_1$ | $r_2$ | $r_3$ |
|---|---|---|---|
| $\eth_1$ | $\langle(0.5, 0.9), (0.4, 0.1)\rangle$ | $\langle(0.7, 0.3), (0.3, 0.7)\rangle$ | $\langle(0.7, 0.9), (0.3, 0.1)\rangle$ |
| $\eth_2$ | $\langle(0.7, 0.8), (0.3, 0.2)\rangle$ | $\langle(0.9, 0.5), (0.1, 0.5)\rangle$ | $\langle(0.5, 0.7), (0.4, 0.2)\rangle$ |
| $\eth_3$ | $\langle(0.9, 0.4), (0.1, 0.5)\rangle$ | $\langle(0.7, 0.7), (0.05, 0.3)\rangle$ | $\langle(0.7, 0.7), (0.3, 0.3)\rangle$ |
| $\eth_4$ | $\langle(0.5, 0.5), (0.5, 0.4)\rangle$ | $\langle(0.4, 0.2), (0.6, 0.7)\rangle$ | $\langle(0.9, 0.8), (0.1, 0.1)\rangle$ |
| $\eth_5$ | $\langle(0.9, 0.6), (0.1, 0.4)\rangle$ | $\langle(0.5, 0.8), (0.4, 0.1)\rangle$ | $\langle(0.8, 0.4), (0.2, 0.5)\rangle$ |
| $\eth_6$ | $\langle(1, 0), (0, 1)\rangle$ | $\langle(0.5, 0.3), (0.4, 0.4)\rangle$ | $\langle(0.6, 0.6), (0.4, 0.2)\rangle$ |

**Table 12.** Table for the extended intersection $\xi_1 \cap_{\mathcal{E}} \xi_2$.

| $\xi_1 \cap_{\mathcal{E}} \xi_2$ | $r_1$ | $r_2$ | $r_3$ |
|---|---|---|---|
| $\eth_1$ | $\langle(0.3, 0.9), (0.5, 0.1)\rangle$ | $\langle(0.6, 0.6), (0.4, 0.4)\rangle$ | $\langle(0.7, 0.9), (0.3, 0.1)\rangle$ |
| $\eth_2$ | $\langle(0.5, 0.8), (0.4, 0.1)\rangle$ | $\langle(0.8, 0.7), (0.1, 0.2)\rangle$ | $\langle(0.5, 0.7), (0.4, 0.2)\rangle$ |
| $\eth_3$ | $\langle(0.9, 0.8), (0.1, 0.1)\rangle$ | $\langle(0.6, 0.7), (0.4, 0.2)\rangle$ | $\langle(0.7, 0.7), (0.3, 0.3)\rangle$ |
| $\eth_4$ | $\langle(0.2, 0.7), (0.8, 0.2)\rangle$ | $\langle(0.1, 0.4), (0.6, 0.5)\rangle$ | $\langle(0.9, 0.8), (0.1, 0.1)\rangle$ |
| $\eth_5$ | $\langle(0.5, 0.9), (0.4, 0)\rangle$ | $\langle(0.3, 0.9), (0.7, 0.1)\rangle$ | $\langle(0.8, 0.4), (0.2, 0.5)\rangle$ |
| $\eth_6$ | $\langle(0.6, 0.9), (0.25, 1)\rangle$ | $\langle(0.4, 0.5), (0.6, 0.1)\rangle$ | $\langle(0.6, 0.6), (0.4, 0.2)\rangle$ |

**Table 13.** Table for the restricted intersection $\xi_1 \cap_{\mathcal{R}} \xi_2$.

| $\xi_1 \cap_{\mathcal{R}} \xi_2$ | $r_1$ | $r_2$ |
|---|---|---|
| $\eth_1$ | $\langle(0.3, 0.9), (0.5, 0.1)\rangle$ | $\langle(0.6, 0.6), (0.4, 0.4)\rangle$ |
| $\eth_2$ | $\langle(0.5, 0.8), (0.4, 0.1)\rangle$ | $\langle(0.8, 0.7), (0.1, 0.2)\rangle$ |
| $\eth_3$ | $\langle(0.9, 0.8), (0.1, 0.1)\rangle$ | $\langle(0.6, 0.7), (0.4, 0.2)\rangle$ |
| $\eth_4$ | $\langle(0.2, 0.7), (0.8, 0.2)\rangle$ | $\langle(0.1, 0.4), (0.6, 0.5)\rangle$ |
| $\eth_5$ | $\langle(0.5, 0.9), (0.4, 0)\rangle$ | $\langle(0.3, 0.9), (0.7, 0.1)\rangle$ |
| $\eth_6$ | $\langle(0.6, 0.9), (0.25, 1)\rangle$ | $\langle(0.4, 0.5), (0.6, 0.1)\rangle$ |

**Table 14.** Table for the restricted union $\xi_1 \cup_{\mathcal{R}} \xi_2$.

| $\xi_1 \cup_{\mathcal{R}} \xi_2$ | $r_1$ | $r_2$ |
|---|---|---|
| $\eth_1$ | $\langle(0.5, 0.9), (0.4, 0.1)\rangle$ | $\langle(0.7, 0.3), (0.3, 0.7)\rangle$ |
| $\eth_2$ | $\langle(0.7, 0.8), (0.3, 0.2)\rangle$ | $\langle(0.9, 0.5), (0.1, 0.5)\rangle$ |
| $\eth_3$ | $\langle(0.9, 0.4), (0.1, 0.5)\rangle$ | $\langle(0.7, 0.7), (0.05, 0.3)\rangle$ |
| $\eth_4$ | $\langle(0.5, 0.5), (0.5, 0.4)\rangle$ | $\langle(0.4, 0.2), (0.6, 0.7)\rangle$ |
| $\eth_5$ | $\langle(0.9, 0.6), (0.1, 0.4)\rangle$ | $\langle(0.5, 0.8), (0.4, 0.1)\rangle$ |
| $\eth_6$ | $\langle(1, 0), (0, 1)\rangle$ | $\langle(0.5, 0.3), (0.4, 0.4)\rangle$ |

**Lemma 1.** *Let $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ and $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ be two q-ROFBSSs on a universe $\mathfrak{U}$. Then*

1. *$\xi_1 \cup_{\mathcal{E}} \xi_2$ is the smallest q-ROFBSS over $\mathfrak{U}$ which contains both $\xi_1$ and $\xi_2$;*
2. *$\xi_1 \cap_{\mathcal{R}} \xi_2$ is the largest q-ROFBSS over $\mathfrak{U}$ which is subset of both $\xi_1$ and $\xi_2$.*

**Proof.** Straightforward. □

In the following theorem, we verify that certain De Morgan's laws hold with the extended (restricted) union and intersection.

**Theorem 1.** *Let $\xi_1 = (\mathcal{Q}_1, \mathcal{S}_1, \mathcal{R}_1)$ and $\xi_2 = (\mathcal{Q}_2, \mathcal{S}_2, \mathcal{R}_2)$ be two q-ROFBSSs on the universe $\mathfrak{U}$. Then,*

1. $(\xi_1 \cup_{\mathcal{E}} \xi_2)^c = (\xi_1)^c \cap_{\mathcal{E}} (\xi_2)^c$,
2. $(\xi_1 \cap_{\mathcal{E}} \xi_2)^c = (\xi_1)^c \cup_{\mathcal{E}} (\xi_2)^c$,

3. $(\xi_1 \cup_{\mathcal{R}} \xi_2)^c = (\xi_1)^c \cap_{\mathcal{R}} (\xi_2)^c$,
4. $(\xi_1 \cap_{\mathcal{R}} \xi_2)^c = (\xi_1)^c \cup_{\mathcal{R}} (\xi_2)^c$.

**Proof.**

1. By Definition 10 and 15, we obtain $(\xi_1 \cup_{\mathcal{E}} \xi_2)^c = ((\mathcal{Q}_1 \Cup \mathcal{Q}_2)^c, (\mathcal{S}_1 \Cap \mathcal{S}_2)^c, \mathcal{R}_1 \cup \mathcal{R}_2)$, where

$$(\mathcal{Q}_1 \Cup \mathcal{Q}_2)^c(r) = \begin{cases} \mathcal{Q}_1^c(r), & if \quad r \in \mathcal{R}_1 - \mathcal{R}_2, \\ \mathcal{Q}_2^c(r), & if \quad r \in \mathcal{R}_2 - \mathcal{R}_1, \\ \mathcal{Q}_1^c(r) \overline{\sqcap} \mathcal{Q}_2^c(r) & if \quad r \in \mathcal{R}_1 \cap \mathcal{R}_2. \end{cases}$$

$$= (\mathcal{Q}_1^c \Cap \mathcal{Q}_2^c)(r). \text{ (by Definition 17), and}$$

$$(\mathcal{S}_1 \Cap \mathcal{S}_2)^c(\neg r) = \begin{cases} \mathcal{S}_1^c(\neg r), & if \ \neg r \in (\neg \mathcal{R}_1) - (\neg \mathcal{R}_2), \\ \mathcal{S}_2^c(\neg r), & if \ \neg r \in (\neg \mathcal{R}_2) - (\neg \mathcal{R}_1), \\ \mathcal{S}_1^c(\neg r) \underline{\sqcup} \mathcal{S}_2^c(\neg r) & if \ \neg r \in (\neg \mathcal{R}_1) \cap (\neg \mathcal{R}_2). \end{cases}$$

$$= (\mathcal{S}_1^c \Cup \mathcal{S}_2^2)(\neg r). \text{ (by Definition 17)}$$

Hence, $(\xi_1 \cup_{\mathcal{E}} \xi_2)^c = (\xi_1)^c \cap_{\mathcal{E}} (\xi_2)^c$.

The remaining parts (2–4) can be easily followed. □

Here, we define a $q$-rung orthopair fuzzy weighted average operator to aggregate the $q$-rung orthopair fuzzy information.

**Definition 19.** *Let $O_1, O_2, \ldots, O_n$ be a family of q-ROFNs and every $O_i = (\lambda^+_{O_i}, \lambda^-_{O_i})$ be related with an important weight $w_i \in [0,1]$ $(i = 1, 2, \ldots, n)$, such that $\sum_{i=1}^{n} w_i = 1$, then the q-rung orthopair fuzzy weighted average (q-ROFWA) operator is given by*

$$H(O_1, O_2, \ldots, O_n) = \Big( \sum_{i=1}^{n} w_i \lambda^+_{O_i}, \sum_{i=1}^{n} w_i \lambda^-_{O_i} \Big). \tag{7}$$

## 4. Applications

MADM technique plays a significant role to handle many complicated real-life decision-making situations. Here, we describe two practical applications with MADM method based on $q$-ROFBSSs.

### *4.1. Selection of Land for Cropping Carrots and Lettuces*

With the growing world population the food demands are increasing day-by-day. Crops play a major role to fulfill these food requirements. There are various factors involve in the development of crops production, like soil condition, environment, etc. Actually, soil condition is a very important part in the production process of any crop. The fertility of land is very important for good yield of any crop. For instance, land containing clay soil is not suited for various vegetables and field crops like wheat, rice, etc. However, fruit trees, ornamental trees and shrubs can thrive on clay soil. Therefore, the land selection is very significant for the crops growth because a crop production may vary from one land piece to another. It is an uncertain problem for agriculture experts to choose best land for crops development. That is why, we investigate this daily-life problem by applying our proposed methodology.

Suppose a farmer wants to buy a land, through a land dealer company, which should be suitable for the cropping of carrots and lettuces. According to the dealer, a land with high porosity (i.e., land with many pores so that water can penetrate through soil pores easily), good soil texture, neutral pH (i.e., neither basic nor acidic), and even colour soil is suitable for farmer. There are fifteen lands which make the set of alternatives, $\mathfrak{U} = \{L_1, L_2, \ldots, L_{15}\}$. According to the company, consider $P = \{r_1, r_2, \ldots, r_5\}$ be a set of parameters. For $j = 1, 2, \ldots, 5$, the parameters $r_j$ stand for "good soil texture", "crumbly soil","high porosity", "neutral pH soil", and "single colour soil", respectively. Let the 'Not set of $P$' be $\neg P = \{\neg r_1 = \textit{bad soil texture}, \neg r_2 = \text{coarse soil}, \neg r_3 = \text{low porosity}, \neg r_4 = \text{basic pH soil}, \neg r_5 = \textit{multi} - \textit{colour soil}\}$. After a detailed discussion among the committee members of dealer company, they decide the evaluation of every piece of land will be done with a favorable subset $\mathcal{R} = \{r_1, r_2, r_3\}$ of $P$. According to the committee, the $q$-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ with $q = 4$ describes the "requirements of the lands" which is given by Table 15 below.

**Table 15.** Table for the 4-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$.

| $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ | $r_1$ | $r_2$ | $r_3$ |
|---|---|---|---|
| $L_1$ | $\langle(0.7, 0.4), (0.3, 0.5)\rangle$ | $\langle(0.9, 0.6), (0.1, 0.4)\rangle$ | $\langle(0.76, 0.3), (0.2, 0.7)\rangle$ |
| $L_2$ | $\langle(0.9, 0.7), (0.1, 0.3)\rangle$ | $\langle(0.3, 0.5), (0.7, 0.5)\rangle$ | $\langle(0.4, 0.8), (0.6, 0.1)\rangle$ |
| $L_3$ | $\langle(0.3, 0.2), (0.7, 0.8)\rangle$ | $\langle(0.4, 0.6), (0.6, 0.4)\rangle$ | $\langle(0.3, 0.8), (0.6, 0.2)\rangle$ |
| $L_4$ | $\langle(0.75, 0.6), (0.1, 0.4)\rangle$ | $\langle(0.49, 0.7), (0.51, 0.3)\rangle$ | $\langle(0.59, 0.1), (0.4, 0.9)\rangle$ |
| $L_5$ | $\langle(0.4, 0.4), (0.5, 0.6)\rangle$ | $\langle(0.3, 0.9), (0.7, 0.1)\rangle$ | $\langle(0.7, 0.3), (0.3, 0.7)\rangle$ |
| $L_6$ | $\langle(0.3, 0.9), (0.7, 0.1)\rangle$ | $\langle(0.75, 0.4), (0.25, 0.6)\rangle$ | $\langle(0.99, 0.8), (0.01, 0.2)\rangle$ |
| $L_7$ | $\langle(0.1, 0.85), (0.9, 0.1)\rangle$ | $\langle(0.4, 0.9), (0.6, 0.1)\rangle$ | $\langle(0.1, 0.7), (0.84, 0.3)\rangle$ |
| $L_8$ | $\langle(0.5, 0.7), (0.3, 0.3)\rangle$ | $\langle(0.9, 0.4), (0.1, 0.6)\rangle$ | $\langle(0.9, 0.45), (0.1, 0.55)\rangle$ |
| $L_9$ | $\langle(0.9, 0.6), (0.1, 0.3)\rangle$ | $\langle(0.8, 0.5), (0.15, 0.5)\rangle$ | $\langle(0.7, 0.7), (0.3, 0.3)\rangle$ |
| $L_{10}$ | $\langle(0.7, 0.7), (0.3, 0.25)\rangle$ | $\langle(0.2, 0.9), (0.75, 0.1)\rangle$ | $\langle(0.1, 0.2), (0.9, 0.7)\rangle$ |
| $L_{11}$ | $\langle(0.84, 0.8), (0.1, 0.15)\rangle$ | $\langle(0.9, 0.7), (0.05, 0.3)\rangle$ | $\langle(0.79, 0.4), (0.2, 0.6)\rangle$ |
| $L_{12}$ | $\langle(0.5, 0.8), (0.5, 0.2)\rangle$ | $\langle(0.7, 0.6), (0.3, 0.3)\rangle$ | $\langle(0.3, 0.9), (0.6, 0.1)\rangle$ |
| $L_{13}$ | $\langle(0.1, 0), (0.9, 0.7)\rangle$ | $\langle(0.65, 0.5), (0.35, 0.5)\rangle$ | $\langle(0.5, 0.79), (0.5, 0.2)\rangle$ |
| $L_{14}$ | $\langle(0, 1), (1, 0)\rangle$ | $\langle(0.4, 0.9), (0.55, 0.1)\rangle$ | $\langle(0.28, 0.8), (0.71, 0.2)\rangle$ |
| $L_{15}$ | $\langle(0.45, 0.7), (0.5, 0.3)\rangle$ | $\langle(0.7, 0.3), (0.3, 0.6)\rangle$ | $\langle(0.5, 0.75), (0.48, 0.25)\rangle$ |

Based on the significant of every parameter $r_j$ $(j = 1, 2, 3)$, the committee gives a specific weight to each parameter which are:

$$w_1 = 0.2, \qquad w_2 = 0.3, \qquad w_3 = 0.5.$$

By using $q$-ROFWA operator (see Definition 19), we calculate

$$\begin{aligned}
O_1 &= \Big(\sum_{j=1}^{3} w_j \lambda^{+}_{1j}, \sum_{j=1}^{3} w_j \lambda^{-}_{1j}\Big), \\
&= \big(w_1\lambda^{+}_{11} + w_2\lambda^{+}_{12} + w_3\lambda^{+}_{13},\ w_1\lambda^{-}_{11} + w_2\lambda^{-}_{12} + w_3\lambda^{-}_{13}\big), \\
&= (0.2 \times 0.7 + 0.3 \times 0.9 + 0.5 \times 0.76,\ 0.2 \times 0.4 + 0.3 \times 0.6 + 0.5 \times 0.3), \\
&= (0.79, 0.41).
\end{aligned}$$

Similarly,

$$\begin{aligned}
&O_2 = (0.47, 0.69), \quad O_3 = (0.33, 0.62), \quad O_4 = (0.592, 0.38), \\
&O_5 = (0.52, 0.5), \quad O_6 = (0.78, 0.7), \quad O_7 = (0.19, 0.79), \\
&O_8 = (0.82, 0.485), \quad O_9 = (0.77, 0.62), \quad O_{10} = (0.25, 0.51), \\
&O_{11} = (0.833, 0.57), \quad O_{12} = (0.46, 0.79), \quad O_{13} = (0.465, 0.545), \\
&O_{14} = (0.26, 0.87), \quad O_{15} = (0.55, 0.605).
\end{aligned}$$

Now

$$\begin{aligned}
\neg O_1 &= \Big(\sum_{j=1}^{3} w_j\gamma_{1j}^{+}, \sum_{j=1}^{3} w_j\gamma_{1j}^{-}\Big),\\
&= \big(w_1\gamma_{11}^{+} + w_2\gamma_{12}^{+} + w_3\gamma_{13}^{+},\ w_1\gamma_{11}^{-} + w_2\gamma_{12}^{-} + w_3\gamma_{13}^{-}\big),\\
&= (0.2\times 0.3 + 0.3\times 0.1 + 0.5\times 0.2,\ 0.2\times 0.5 + 0.3\times 0.4 + 0.5\times 0.7),\\
&= (0.19, 0.57).
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\neg O_2 = (0.53, 0.26), && \neg O_3 = (0.62, 0.38), && \neg O_4 = (0.373, 0.62),\\
&\neg O_5 = (0.46, 0.5), && \neg O_6 = (0.22, 0.3), && \neg O_7 = (0.78, 0.2),\\
&\neg O_8 = (0.14, 0.515), && \neg O_9 = (0.215, 0.36), && \neg O_{10} = (0.735, 0.43),\\
&\neg O_{11} = (0.135, 0.42), && \neg O_{12} = (0.49, 0.18), && \neg O_{13} = (0.535, 0.39),\\
&\neg O_{14} = (0.72, 0.13) && \neg O_{15} = (0.43, 0.365).
\end{aligned}$$

By Definition 3, we have

$$\begin{aligned}
&s(O_1) = -0.3612, && s(O_2) = -0.1779, && s(O_3) = -0.1359,\\
&s(O_4) = 0.1020, && s(O_5) = 0.0106, && s(O_6) = 0.1300,\\
&s(O_7) = -0.3882, && s(O_8) = 0.3968, && s(O_9) = 0.2038,\\
&s(O_{10}) = -0.0637, && s(O_{11}) = 0.3759, && s(O_{12}) = -0.3447,\\
&s(O_{13}) = -0.0414, && s(O_{14}) = -0.5683, && s(O_{15}) = -0.0425.
\end{aligned}$$

Now, the final scores are computed as:

$s(O_1) - s(\neg O_1) = 0.4655,$
$s(O_2) - s(\neg O_2) = -0.2522,$
$s(O_3) - s(\neg O_3) = -0.2628,$
$s(O_4) - s(\neg O_4) = 0.2304,$
$s(O_5) - s(\neg O_5) = 0.0283,$
$s(O_6) - s(\neg O_6) = 0.1358,$
$s(O_7) - s(\neg O_7) = -0.7567,$
$s(O_8) - s(\neg O_8) = 0.4667,$
$s(O_9) - s(\neg O_9) = 0.2184,$
$s(O_{10}) - s(\neg O_{10}) = -0.3214,$
$s(O_{11}) - s(\neg O_{11}) = 0.4067,$
$s(O_{12}) - s(\neg O_{12}) = -0.4013,$
$s(O_{13}) - s(\neg O_{13}) = -0.1003,$
$s(O_{14}) - s(\neg O_{14}) = -0.8368,$
$s(O_{15}) - s(\neg O_{15}) = -0.0589.$

Clearly, $L_8$ is the optimal alternative. Therefore, the committee will suggest the farmer to buy the land $L_8$ for cropping carrots and lettuces.

The algorithm based on $q$-ROFBSSs for the selection process of most appropriate option is given as below (see Algorithm 1):

Now we apply our developed model to another real situation under $q$-ROFBSSs.

**Algorithm 1:** Selection of a suitable object using $q$-ROFBSSs

**Input:**

(i). $\mathfrak{U} = \{\mathfrak{d}_1, \mathfrak{d}_2, \ldots, \mathfrak{d}_n\}$, a universal set of $n$ alternatives,

(ii). $\mathcal{R} \subseteq P$, a set of parameters containing $m$ elements,

(iii). a $q$-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$, where $q$-rung orthopair fuzzy bipolar soft decision matrix is provided by $o = \langle (O_{ij})_{n\times m}, (\neg O_{ij})_{n\times m} \rangle = \langle (\lambda^+_{ij}, \lambda^-_{ij})_{n\times m}, (\gamma^+_{ij}, \gamma^-_{ij})_{n\times m} \rangle$ in tabular format,

(iv). $w_j$, weights for every parameter $r_j$, where $j = 1, 2, \ldots, m$ with condition $\sum_{j=1}^{m} w_j = 1$.

**Output:** The object having maximum final score value will be the decision object.

**begin**

1. **for** $i = 1$ to $n$ **do**
2. **for** $j = 1$ to $m$ **do**
3. By applying the $q$-ROFWA operator (Definition 19), compute the $q$-ROFNs $(O_i)$ and $(\neg O_i)$ for all $\mathfrak{d}_i \in \mathfrak{U}$ as;

$$O_i = H(O_{i1}, O_{i2}, \ldots, O_{im}) = \Big( \sum_{j=1}^{m} w_j \lambda^+_{ij}, \sum_{j=1}^{m} w_j \lambda^-_{ij} \Big);$$

$$\neg O_i = H(\neg O_{i1}, \neg O_{i2}, \ldots, \neg O_{im}) = \Big( \sum_{j=1}^{m} w_j \gamma^+_{ij}, \sum_{j=1}^{m} w_j \gamma^-_{ij} \Big);$$

where each $\neg O_i$ serves as the $q$-rung orthopair fuzzy belongingness value of the alternative of the universe with respect to "Not set of parameters";

4. **end for**
5. **end for**
6. **for** $i = 1$ to $n$ **do**
7. Determine the score functions $s(O_i)$ and $s(\neg O_i)$ of every alternative $\mathfrak{d}_i$ via Equation (1);
8. **end for**
9. **for** $i = 1$ to $n$ **do**
10. Calculate final scores for each object by $\max_i\{s(O_i) - s(\neg O_i)\}$;
11. **end for**

**end**

### *4.2. Selection of Student for Scholarship*

The basic process for evaluating applicants and awarding scholarships depends on different factors which are surely uncertain. For instance, financial information about the applicant is an important factor for awarding a scholarship. It is possible that a student who is not deserving may provide incorrect financial information. Then, it is on the experts (interviewers) to decide whether he or she is deserving or not. Thus, it is an uncertain problem. Similarly, there exist many other factors which effects the selection procedure of deserving and brilliant students for scholarships, such as morality, honesty, etc.

Suppose the Higher Education Commission (HEC) of Pakistan announces a merit and need based scholarship for undergraduate students of universities. This task is given to a team of 5 senior employees. Consider there are twenty applicants from a university XYZ which make the set of alternatives, $\mathfrak{U} = \{D_1, D_2, \ldots, D_{20}\}$. According to the HEC, let $P = \{r_1, r_2, r_3, r_4, r_5\}$ be a set of parameters for the selection of candidates for scholarships. For $j = 1, 2, \ldots, 5$, the parameters $r_j$ serve as "high CGPA", "Good Character", "Poor financial state", "punctual"and "cooperative", respectively. Let the 'Not set of $P$' be $\neg P = \{\neg r_1 = \text{low CGPA}, \neg r_2 = \text{bad character}, \neg r_3 = \text{good financial state}, \neg r_4 = \text{not punctual}, \neg r_5 = \text{not cooperative}\}$. With a brief discussion among the members of selection team appointed by HEC, they decide that the evaluation of every applicant will be done with a favorable subset of parameters $\mathcal{R} = \{r_1, r_2, r_3\}$ of $P$. According to the team members, the $q$-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ with $q$ = 5 describes the "qualities of the students" which are displayed in the Table 16 below.

**Table 16.** Table for the 5-ROFBSS $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$.

| $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ | $r_1$ | $r_2$ | $r_3$ |
|---|---|---|---|
| $D_1$ | $\langle(0.3, 0.8), (0.7, 0.2)\rangle$ | $\langle(0.8, 0.9), (0.2, 0.1)\rangle$ | $\langle(0.8, 0.5), (0.2, 0.5)\rangle$ |
| $D_2$ | $\langle(0.8, 0.7), (0.2, 0.3)\rangle$ | $\langle(0.5, 0.7), (0.3, 0.3)\rangle$ | $\langle(0.5, 0.6), (0.45, 0.4)\rangle$ |
| $D_3$ | $\langle(0.4, 0.8), (0.6, 0.2)\rangle$ | $\langle(0.7, 0.8), (0.3, 0)\rangle$ | $\langle(0.9, 0.7), (0.1, 0.3)\rangle$ |
| $D_4$ | $\langle(0.2, 0.9), (0.8, 0.1)\rangle$ | $\langle(0.8, 0.5), (0.2, 0.5)\rangle$ | $\langle(0.2, 0.3), (0.8, 0.7)\rangle$ |
| $D_5$ | $\langle(0.6, 0.9), (0.4, 0.1)\rangle$ | $\langle(0.5, 0.7), (0.3, 0.2)\rangle$ | $\langle(0.56, 0.6), (0.4, 0.3)\rangle$ |
| $D_6$ | $\langle(0.1, 0.8), (0.8, 0.2)\rangle$ | $\langle(0.9, 0.6), (0.1, 0.4)\rangle$ | $\langle(0.7, 0.3), (0.3, 0.7)\rangle$ |
| $D_7$ | $\langle(0.8, 0.5), (0.1, 0.5)\rangle$ | $\langle(0.4, 0.9), (0.6, 0)\rangle$ | $\langle(0.5, 0.9), (0.5, 0.1)\rangle$ |
| $D_8$ | $\langle(0.6, 0.1), (0.4, 0.9)\rangle$ | $\langle(0.5, 0.8), (0.5, 0.2)\rangle$ | $\langle(0.7, 0.8), (0.3, 0.2)\rangle$ |
| $D_9$ | $\langle(0.9, 0.8), (0.1, 0.1)\rangle$ | $\langle(0.2, 0.3), (0.8, 0.7)\rangle$ | $\langle(0.3, 0.8), (0.7, 0.2)\rangle$ |
| $D_{10}$ | $\langle(0.1, 0.5), (0.9, 0.5)\rangle$ | $\langle(0.9, 0.8), (0.1, 0.2)\rangle$ | $\langle(0.5, 0.6), (0.5, 0.4)\rangle$ |
| $D_{11}$ | $\langle(0.3, 0.3), (0.7, 0.7)\rangle$ | $\langle(0.1, 0.9), (0.9, 0.1)\rangle$ | $\langle(0.2, 0.1), (0.8, 0.9)\rangle$ |
| $D_{12}$ | $\langle(0.7, 0.9), (0.3, 0.1)\rangle$ | $\langle(0.3, 0.8), (0.7, 0.2)\rangle$ | $\langle(0.1, 0.9), (0.9, 0.1)\rangle$ |
| $D_{13}$ | $\langle(0.4, 0.8), (0.6, 0.2)\rangle$ | $\langle(0.8, 0.7), (0.2, 0.3)\rangle$ | $\langle(0.5, 0.9), (0.5, 0)\rangle$ |
| $D_{14}$ | $\langle(0.8, 0.7), (0.2, 0.3)\rangle$ | $\langle(0.5, 0.6), (0.5, 0.4)\rangle$ | $\langle(0.3, 0.8), (0.7, 0.2)\rangle$ |
| $D_{15}$ | $\langle(0.6, 0.8), (0.4, 0.2)\rangle$ | $\langle(0.2, 0.9), (0.8, 0.1)\rangle$ | $\langle(0.8, 0.45), (0.1, 0.5)\rangle$ |
| $D_{16}$ | $\langle(0.5, 0.6), (0.5, 0.3)\rangle$ | $\langle(0.5, 0.8), (0.5, 0.2)\rangle$ | $\langle(0.9, 0.4), (0.1, 0.6)\rangle$ |
| $D_{17}$ | $\langle(0.9, 0.4), (0.1, 0.6)\rangle$ | $\langle(0.7, 0.7), (0.3, 0.3)\rangle$ | $\langle(0.8, 0.1), (0.2, 0.9)\rangle$ |
| $D_{18}$ | $\langle(0.6, 0.1), (0.4, 0.9)\rangle$ | $\langle(0.55, 0.9), (0.45, 0.1)\rangle$ | $\langle(0.9, 0.2), (0.1, 0.8)\rangle$ |
| $D_{19}$ | $\langle(0.5, 0.75), (0.5, 0.25)\rangle$ | $\langle(0.5, 0.8), (0.5, 0.2)\rangle$ | $\langle(0.5, 0.2), (0.5, 0.8)\rangle$ |
| $D_{20}$ | $\langle(0.3, 0.5), (0.7, 0.5)\rangle$ | $\langle(0, 0.9), (0.9, 0.1)\rangle$ | $\langle(0.9, 0.7), (0.1, 0.3)\rangle$ |

Based upon the significant of every parameter $r_j$ $(j = 1, 2, 3)$, the team members give a specific weight to each parameter which are:

$$w_1 = 0.3, \qquad w_2 = 0.4, \qquad w_3 = 0.3.$$

By using $q$-ROFWA operator (see Definition 19), we calculate

$$\begin{aligned} O_1 &= \Big(\sum_{j=1}^{3} w_j \lambda^+_{1j}, \sum_{j=1}^{3} w_j \lambda^-_{1j}\Big), \\ &= \big(w_1\lambda^+_{11} + w_2\lambda^+_{12} + w_3\lambda^+_{13},\ w_1\lambda^-_{11} + w_2\lambda^-_{12} + w_3\lambda^-_{13}\big), \\ &= (0.3 \times 0.3 + 0.4 \times 0.8 + 0.3 \times 0.8,\ 0.3 \times 0.8 + 0.4 \times 0.9 + 0.3 \times 0.5), \\ &= (0.65, 0.75). \end{aligned}$$

Similarly,

$$\begin{aligned} &O_2 = (0.59, 0.67), && O_3 = (0.67, 0.77), && O_4 = (0.44, 0.56), \\ &O_5 = (0.548, 0.73), && O_6 = (0.6, 0.57), && O_7 = (0.55, 0.78), \\ &O_8 = (0.59, 0.59), && O_9 = (0.44, 0.6), && O_{10} = (0.54, 0.65), \\ &O_{11} = (0.19, 0.48), && O_{12} = (0.36, 0.86), && O_{13} = (0.59, 0.79), \\ &O_{14} = (0.53, 0.69), && O_{15} = (0.5, 0.735), && O_{16} = (0.62, 0.62), \\ &O_{17} = (0.79, 0.43), && O_{18} = (0.67, 0.45), && O_{19} = (0.5, 0.605), \\ &O_{20} = (0.36, 0.72). \end{aligned}$$

Now

$$\begin{aligned}
\neg O_1 &= \Big(\sum_{j=1}^{3} w_j\gamma_{1j}^{+}, \sum_{j=1}^{3} w_j\gamma_{1j}^{-}\Big),\\
&= \big(w_1\gamma_{11}^{+} + w_2\gamma_{12}^{+} + w_3\gamma_{13}^{+},\ w_1\gamma_{11}^{-} + w_2\gamma_{12}^{-} + w_3\gamma_{13}^{-}\big),\\
&= (0.3\times 0.7 + 0.4\times 0.2 + 0.3\times 0.2,\ 0.3\times 0.2 + 0.4\times 0.1 + 0.3\times 0.5),\\
&= (0.35, 0.25).
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\neg O_2 = (0.36, 0.33), && \neg O_3 = (0.33, 0.15), && \neg O_4 = (0.56, 0.44),\\
&\neg O_5 = (0.36, 0.2), && \neg O_6 = (0.37, 0.43), && \neg O_7 = (0.42, 0.18),\\
&\neg O_8 = (0.41, 0.41), && \neg O_9 = (0.56, 0.37), && \neg O_{10} = (0.46, 0.35),\\
&\neg O_{11} = (0.81, 0.52), && \neg O_{12} = (0.64, 0.14), && \neg O_{13} = (0.41, 0.18),\\
&\neg O_{14} = (0.47, 0.31) && \neg O_{15} = (0.47, 0.25), && \neg O_{16} = (0.38, 0.35),\\
&\neg O_{17} = (0.21, 0.57), && \neg O_{18} = (0.33, 0.55), && \neg O_{19} = (0.5, 0.395),\\
&\neg O_{20} = (0.6, 0.28).
\end{aligned}$$

By Definition 3, we have

$$\begin{aligned}
&s(O_1) = -0.1213, && s(O_2) = -0.0635, && s(O_3) = -0.1357,\\
&s(O_4) = -0.0386, && s(O_5) = -0.1579, && s(O_6) = 0.0176,\\
&s(O_7) = -0.2384, && s(O_8) = 0, && s(O_9) = -0.0613,\\
&s(O_{10}) = -0.0701, && s(O_{11}) = -0.0252, && s(O_{12}) = -0.4644,\\
&s(O_{13}) = -0.2362, && s(O_{14}) = -0.1146, && s(O_{15}) = -0.1833,\\
&s(O_{16}) = 0, && s(O_{17}) = 0.2930, && s(O_{18}) = 0.1166,\\
&s(O_{19}) = -0.0498, && s(O_{20}) = -0.1874.
\end{aligned}$$

Additionally, we get

$$\begin{aligned}
&s(\neg O_1) = 0.0043, && s(\neg O_2) = 0.0022, && s(\neg O_3) = 0.0038,\\
&s(\neg O_4) = 0.0386, && s(\neg O_5) = 0.0057, && s(\neg O_6) = -0.0078,\\
&s(\neg O_7) = 0.0129, && s(\neg O_8) = 0, && s(\neg O_9) = 0.0481,\\
&s(\neg O_{10}) = 0.0153, && s(\neg O_{11}) = 0.3106, && s(\neg O_{12}) = 0.1073,\\
&s(\neg O_{13}) = 0.0114, && s(\neg O_{14}) = 0.0201, && s(\neg O_{15}) = 0.0220,\\
&s(\neg O_{16}) = -0.0027, && s(\neg O_{17}) = -0.0598, && s(\neg O_{18}) = -0.0464,\\
&s(\neg O_{19}) = 0.0216, && s(\neg O_{20}) = 0.0760.
\end{aligned}$$

Now we compute the final score values as follows:

$$\begin{aligned}
&s(O_1) - s(\neg O_1) = -0.1256,\\
&s(O_2) - s(\neg O_2) = -0.0656,\\
&s(O_3) - s(\neg O_3) = -0.1395,\\
&s(O_4) - s(\neg O_4) = -0.0772,\\
&s(O_5) - s(\neg O_5) = -0.1636,\\
&s(O_6) - s(\neg O_6) = 0.0254,\\
&s(O_7) - s(\neg O_7) = -0.2513,\\
&s(O_8) - s(\neg O_8) = 0,\\
&s(O_9) - s(\neg O_9) = -0.1094,\\
&s(O_{10}) - s(\neg O_{10}) = -0.0854,\\
&s(O_{11}) - s(\neg O_{11}) = -0.3359,
\end{aligned}$$

$s(O_{12}) - s(\neg O_{12}) = -0.5717,$
$s(O_{13}) - s(\neg O_{13}) = -0.2476,$
$s(O_{14}) - s(\neg O_{14}) = -0.1346,$
$s(O_{15}) - s(\neg O_{15}) = -0.2052,$
$s(O_{16}) - s(\neg O_{16}) = 0.0027,$
$s(O_{17}) - s(\neg O_{17}) = 0.3528,$
$s(O_{18}) - s(\neg O_{18}) = 0.1627,$
$s(O_{19}) - s(\neg O_{19}) = -0.0714,$
$s(O_{20}) - s(\neg O_{20}) = -0.2635.$

Clearly, $D_{17}$ is the most suitable student. Therefore, the HEC will select student $D_{17}$ for the merit and need based scholarship.

## 5. Sensitivity Analysis

To prove the efficiency and cogency of the developed $q$-ROFBSS model, this section discusses its advantages and comparative analysis with PFBSSs [28] and FFBSSs [30].

- **Advantages:** A quick analysis of recent years show that a rapid progress has been done for dealing with uncertain information in many MADM situations which is the evidence of this fruitful era. Due to the existence of various practical MADM situations in this universe, it is a wish of every researcher to establish a new model or its hybridized version. It is a limitless approach. Currently, BSS model and its fuzzy and Pythagorean fuzzy formats are arising as very powerful tools but a generalized fuzzy version of these models is not introduced yet. With the motivation of these facts, a new hybrid model, namely, $q$-ROFBSSs is presented which have ability to deal with many real situations involving $q$-rung orthopair fuzzy bipolar soft knowledge. The developed $q$-ROFBSS approach is more efficient and flexible to tackle vague information in different MADM problems. Particularly, if the given information involving parameters with opposite meanings. It can be easily see that existing MADM models, i.e., fuzzy BSS model is not capable to consider the non-belongingness values of alternatives in a MADM problem while PFBSS model is not able to handle the belongingness and non-belongingness degrees whose sum of their squares is not bounded by 1. Thus, developed $q$-ROFBSS method has ability to handle both fuzzy and Pythagorean fuzzy bipolar soft information.
- **Comparison:** The production of IFSs and PFSs is enough to show the importance of non-membership function in different real situations. However, there are some limitations of these models, such as they fail to handle the MADM problems in which the sum of squares of belongingness and non-belongingness values is greater than 1. In these days, to solve such critical problems, $q$-ROFSs are arising as more flexible tool as compared to IFSs and PFSs. In the literature, several soft computing models, including fuzzy BSSs [37], PFBSSs [28], FFBSSs [30] and $m$-polar fuzzy BSSs [29] have been introduced for dealing with different kinds of uncertain real-world MADM problems. Inspired by these facts, $q$-ROFBSSs are proposed to deal with different fuzzy versions of bipolar soft information. Our proposed model provided more space to belongingness and non-belongingness degrees as compared to FBSSs [37] and PFBSSs [28]. Notice that the existing MADM methods, namely, PFBSSs [28] and FFBSSs [30] fail to solve the developed applications in this study. Therefore, to check the comparison of PFBSSs [28], FFBSSs [30], and our proposed $q$-ROFBSS model (for $q = 4$), we apply them on the data-sets of Applications 1 and 2 in [28]. From the Tables 17 and 18, it can be easily see that not only optimal decision objects by applying these models are equal, that is $x_{19}$ and $y_{13}$ in Applications 1 and 2 of [28], respectively, but also ranking order are similar (for more clarification see the Figures 2 and 3). Thus, our presented MADM hybrid model is more flexible and efficient than certain existing models, including PFBSSs [28] and FFBSSs [30].

**Table 17.** Comparison table for the Application 1 (Selection of an employee) in [28].

| Models | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| PFBSSs [28] ($q = 2$) | −0.038 | 0.154 | −0.121 | −0.226 | −0.003 | 0.444 | 0.156 | 0.431 | 0.145 | 0.196 |
| FFBSSs [30] ($q = 3$) | −0.027 | 0.111 | −0.088 | −0.132 | 0.002 | 0.312 | 0.147 | 0.324 | 0.099 | 0.143 |
| Proposed $q$-ROFBSSs ($q = 4$) | −0.017 | 0.0721 | −0.059 | −0.106 | 0.004 | 0.201 | 0.127 | 0.225 | 0.060 | 0.095 |
| **Models** | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{16}$ | $x_{17}$ | $x_{18}$ | $x_{19}$ | $x_{20}$ |
| PFBSSs [28] ($q = 2$) | 0.342 | −0.060 | −0.0439 | −0.066 | 0.125 | −0.117 | −0.399 | 0.165 | 0.815 | 0.596 |
| FFBSSs [30] ($q = 3$) | 0.256 | −0.036 | −0.030 | −0.040 | 0.088 | −0.085 | −0.312 | 0.110 | 0.615 | 0.444 |
| Proposed $q$-ROFBSSs ($q = 4$) | 0.178 | −0.021 | −0.018 | −0.022 | 0.060 | −0.056 | −0.226 | 0.067 | 0.440 | 0.304 |

**Table 18.** Comparison table for the Application 2 (Selection of a house) in [28].

| Models | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ | $y_7$ | $y_8$ |
|---|---|---|---|---|---|---|---|---|
| PFBSSs [28] ($q = 2$) | −0.2244 | 0.1313 | 0.3497 | −0.3281 | 0.2221 | 0.4297 | 0.6080 | 0.6060 |
| FFBSSs [30] ($q = 3$) | −0.5022 | 0.1028 | 0.2527 | −0.2726 | 0.1719 | 0.3016 | 0.4534 | 0.4621 |
| Proposed $q$-ROFBSSs ($q = 4$) | −0.101 | 0.073 | 0.165 | −0.216 | 0.123 | 0.192 | 0.312 | 0.324 |
| **Models** | $y_9$ | $y_{10}$ | $y_{11}$ | $y_{12}$ | $y_{13}$ | $y_{14}$ | $y_{15}$ | |
| PFBSSs [28] ($q = 2$) | 0.1936 | 0.4413 | 0.0015 | −0.1756 | 0.6687 | 0.0719 | 0.4215 | |
| FFBSSs [30] ($q = 3$) | 0.1304 | 0.3318 | −0.0036 | −0.1185 | 0.4868 | 0.0563 | 0.2925 | |
| Proposed $q$-ROFBSSs ($q = 4$) | 0.079 | 0.226 | −0.005 | −0.073 | 0.331 | 0.039 | 0.186 | |

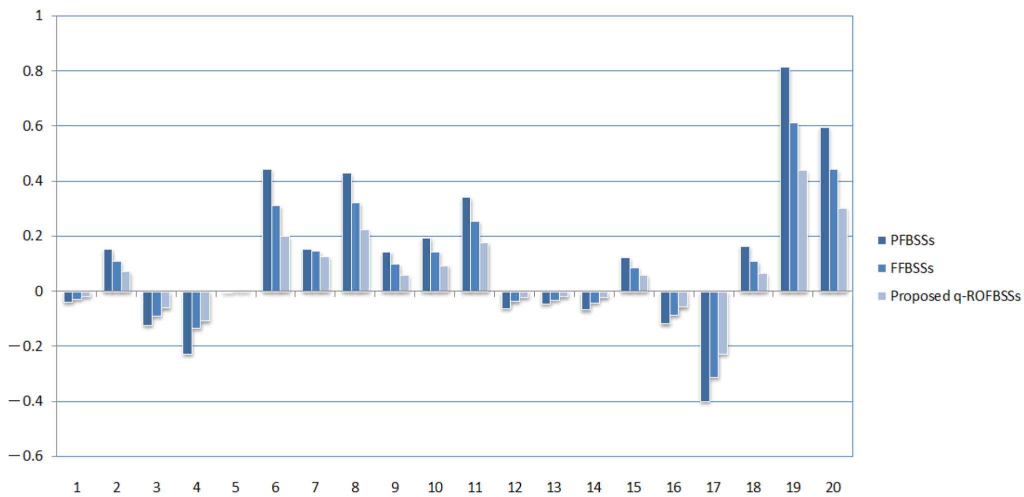


**Figure 2.** Comparison between PFBSSs [28], FFBSSs [30], and proposed $q$-ROFBSSs by applying on Application 1 (Selection of an employee) in [28].

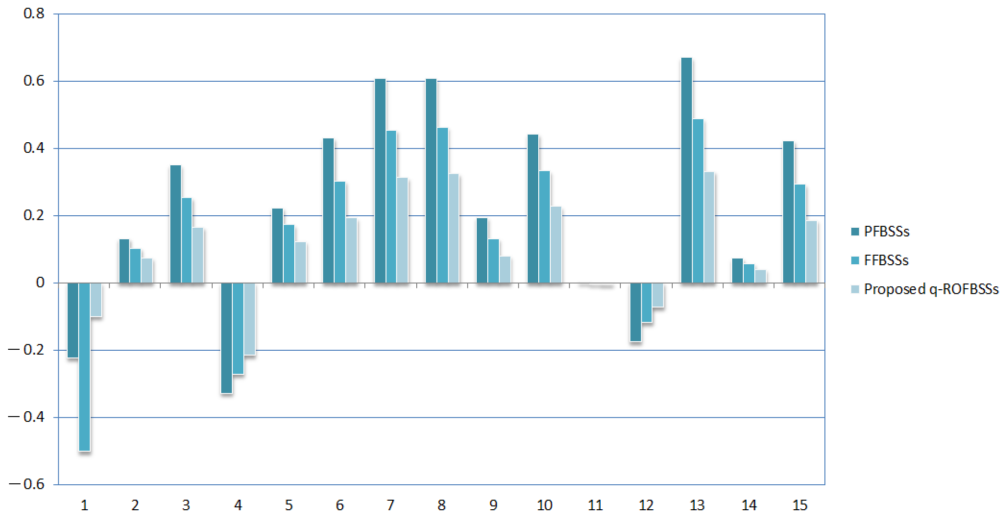


**Figure 3.** Comparison between PFBSSs [28], FFBSSs [30], and proposed $q$-ROFBSSs by applying on Application 2 (Selection of a house) in [28].

## 6. Conclusions

Decision-making performs a significant role in mathematical modeling to refine the selection of logical attributes in almost every real-life problem. In this study, we have proposed a novel hybrid model called $q$-ROFBSSs for MADM by combining $q$-ROFSs and BSSs. The developed model leads us to use parametrization tool regarding bipolarity during problem-solving as compared to existing mathematical tools for dealing with uncertain information. Furthermore, we have described some fundamental operations defined on $q$-ROFBSSs and investigated them with examples. We have also developed decision-making methods by the means of novel constructions in Section 3. Moreover, we have provided justification of our proposed work by solving two real-world problems involving uncertain information, which are: (a) selection of land for cropping the carrots and the lettuces; (b) selection of eligible student for scholarship. Hence, it is observed that hybridization of different models make us able to get more accurate and best information than other existing models. In last, we have studied a comparison analysis of developed approach with certain existing models, including Pythagorean and Fermatean fuzzy BSS models. The presented work can therefore be extended in the following lines:

- $q$-rung orthopair fuzzy bipolar soft sets can be generalized to interval-valued $q$-rung orthopair fuzzy bipolar soft sets to evaluate different MADM problems more effectively;
- A novel hybrid model, namely, $q$-rung orthopair picture fuzzy bipolar soft sets can be established by combining $q$-rung orthopair fuzzy bipolar soft sets and picture fuzzy sets;
- $q$-rung orthopair fuzzy bipolar soft sets can be extended to $q$-rung orthopair fuzzy bipolar soft expert sets to solve different group decision-making problems.

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