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Cryptocurrency Portfolio Selection—A Multicriteria Approach

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Abstract: This paper proposes the PROMETHEE II based multicriteria approach for cryptocurrency portfolio selection. Such an approach allows considering a number of variables important for cryptocurrencies rather than limiting them to the commonly employed return and risk. The proposed multiobjective decision making model gives the best cryptocurrency portfolio considering the daily return, standard deviation, value-at-risk, conditional value-at-risk, volume, market capitalization and attractiveness of nine cryptocurrencies from January 2017 to February 2020. The optimal portfolios are calculated at the first of each month by taking the previous 6 months of daily data for the calculations yielding with 32 optimal portfolios in 32 successive months. The out-of-sample performances of the proposed model are compared with five commonly used optimal portfolio models, i.e., naïve portfolio, two mean-variance models (in the middle and at the end of the efficient frontier), maximum Sharpe ratio and the middle of the mean-CVaR (conditional value-at-risk) efficient frontier, based on the average return, standard deviation and VaR (value-at-risk) of the returns in the next 30 days and the return in the next trading day for all portfolios on 32 dates. The proposed model wins against all other models according to all observed indicators, with the winnings spanning from 50% up to 94%, proving the benefits of employing more criteria and the appropriate multicriteria approach in the cryptocurrency portfolio selection process.

Keywords: cryptocurrency; portfolio selection; return and risk measures; market capitalization; volume; attractiveness; PROMETHEE II; multicriteria model



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1. Introduction

As a response to the everlasting changes in the surroundings, investors adjust the structure of their portfolios in order to maximize the targeted return and risk ratio. In periods of persistently low interest rates, as exhibited in the last decade in the world, traditional investments become less interesting and investors seek alternative forms of investment in the pursuit of higher returns and possibly a lower risk obtained by diversification of the portfolio. In this regard, cryptocurrencies as an alternative form of investment, obtained increasing attention of many investors and this paper. The basic requirement that each new-alternative form of investment should meet is the contribution in terms of Markowitz diversification, i.e., contribution to a more favourable relationship between return and risk of the portfolio, which is exactly what this paper is trying to examine for cryptocurrency portfolio only.

Over the last few years, a number of papers have been published on this topic. Some consider the contribution of particular cryptocurrencies, mostly Bitcoin, to portfolios including other assets being either traditional ones or combinations of traditional and alternative investments [1–5]. Some consider cryptocurrencies' contribution through a set of cryptocurrencies, mostly represented by the cryptocurrency index (CRIX) or its subsets, regarding certain criterion like market capitalization [6–10]. All of them confirm that cryptocurrencies contribute to a better return/risk ratio of portfolios. Recognizing good features of cryptocurrencies as an asset class, some studies have evaluated pure cryptocurrency portfolios, using different approaches to portfolio selection and comparing their performances. Most

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common strategies for portfolio selection are: the 1/N equal weighted rule, so called naïve diversification, Markowitz's mean-variance optimization strategy, risk parity principle, maximum Sharpe ratio or just taking the CRIX portfolio [11–14]. There are different winning strategies depending on the observed period and the sample. The question arises about the "quality" and characteristics the cryptocurrencies should have to be included in the portfolio. It is to be expected that not all available cryptocurrencies are equally favourable for investment, due to their specificities in a number of features. In the previously mentioned papers, the prevailing strategy for selecting a cryptocurrency sample was focusing on a number of top cryptocurrencies in terms of market capitalization. Sometimes the selection was limited to, e.g., the portfolio represented by CRIX or a portfolio containing a number of "most popular" currencies. Only within a few studies, the criterion of liquidity was added in the process of cryptocurrencies selection. Trimborn et al. [8] included cryptocurrencies in their portfolios, combined with stocks from the US, German and Portuguese capital markets. Given the high volatility and a relatively low liquidity, instead of the standard mean-variance model, they propose the LIBRO (liquidity bounded risk-return optimization) method. Garcia et al. [15] extend the stochastic mean-semivariance model to a fuzzy multiobjective model. In addition to return and risk, liquidity is also considered as a portfolio performance measure. The proposed methodology is tested on a data set of assets from the Latin American integrated market, showing the effectiveness and efficiency of the model. Variations in volatility and return but also in other asset specific indicators like liquidity and attractiveness should be included in the appropriate, comprehensive manner in portfolio optimization models that include cryptocurrencies. That is why in this paper a set of different and cryptocurrency-specific criteria are considered. This is a rather novel approach in cryptocurrency portfolio selection since some papers used liquidity only as a precondition for selecting assets into the portfolio, while attractiveness has only been proved in the time-series analysis to influence the cryptocurrency prices and returns.

The traditionally applied portfolio optimization approach based on the Markowitz mean-variance model is not appropriate for cryptocurrency portfolio selection, since standard model assumptions like normal return distribution or a quadratic utility function are not met for this investment class. Thus, in the process of portfolio selection it is more suitable to consider alternative risk measures, which take into account also higher moments of distribution. This issue was partially recognized and accepted in the previously analysed papers, mostly by taking the conditional value-at-risk (CVaR) as the risk measure, i.e., by applying mean-CVaR strategy for portfolio selection. Moreover, for this class of risky assets, it is useful to include more different risk measures in the portfolio selection model.

There are numerous studies and resulting findings on including suitable alternative risk measures in models. Methods of mathematical programming and multicriteria decision making (MCDM) make it possible to incorporate all recognized specificities and constraints into the model, in order to find out which assets with the assigned weights should be selected for the optimal cryptocurrencies portfolio. Generally, MCDM methods can be classified into two categories, discrete multiattribute decision making (MADM) and continuous multiobjective decision making (MODM) methods. MODM methods are used where alternatives are non-predetermined. The aim is to design the optimal alternative by considering a set of quantifiable objectives, i.e., well-defined design constraints. Thus, MODM methods deal with the design process and the number of alternatives is infinite (continuous) [16]. Since variables other than return and risk are considered as important, the selection of the optimal portfolio becomes a multiobjective problem in which we have to design the best portfolio out of an infinite number of feasible portfolios. Sometimes before selecting the optimal portfolio, it is necessary to reduce the sample of possible constituents of the portfolio by taking exclusively those with the best properties. In that case, when security analysis is required, MADM methods are very useful.

In the last fifty years a large number of multiple criteria methods have been applied in the field of stock portfolio selection. One of the newer bibliographic reviews of papers that apply MCDM methods and procedures for stock portfolio selection was carried out by Mathematics 2021, 9, 1677 3 of 21

Aouni et al. [17] who, as a result, offered a classification of a range of MCDA techniques used in the security analysis/evaluation part and in portfolio construction/optimization parts. The review indicates analytic hierarchy process (AHP)-based techniques, ELECTRE-based approaches and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) approaches as the most popular in the security analysis phase and goal programming as the most popular in the portfolio construction phase. The main advantage of multicriteria methods over mean-risk models for portfolio selection is that they take into consideration a number of conflicting criteria, not only risk and return. It is important to emphasize that without a decision-maker there is no solution to multicriteria models. Weights of criteria are determined by the decision-makers' opinion and then the chosen multicriteria method formulates the best compromise solution for that decision-maker. Therefore, the subjectivity of the decision-maker represents the often-mentioned disadvantage of these models. An effective way of reducing subjectivity in these models is by increasing the number of participating experts in the process of decision-making.

This paper focuses on the selection of the optimal cryptocurrency portfolio, which should be observed as a multicriteria programming problem and which should be solved using the appropriate techniques. In this paper a modified and adjusted multiobjective programming model based on the PROMETHEE II approach is proposed. It will be applied and tested using a sample of cryptocurrencies for which all required data are available in the period from 2017 to 2020. Although already used for other assets, the proposed model has never been applied to cryptocurrency portfolio selection, which due to their specifics require special attention in criteria selection, and preference function types and criteria weights. The weights of the seven chosen criteria are estimated using the AHP method or, more precisely, its eigenvalue procedure, incorporating the opinion of different experts' for more objective decision-making.

Based on the problem defined though literature inspection, a research hypothesis can be defined: A multicriteria approach for cryptocurrency portfolio selection based on the PROMETHEE II model yields better out of sample performances compared to the five most commonly used portfolio optimization models in different performances aspects.

Therefore, this paper contributes to the existing literature in several ways. Firstly, by defining the appropriate model for cryptocurrency portfolio selection due to their specificities, i.e., a multicriteria approach based on the PROMETHEE II model. Secondly, by incorporating criteria that, to the best of our knowledge, have never been used before in portfolio optimization. Thirdly, by engaging different experts to obtain results that are more objective. Finally, by examining whether the proposed multicriteria model yields better out-of-sample performances compared to the five most commonly used portfolio optimization models while using different important out-of-sample performances measures.

The remainder of the paper is organized as follows. Section 2 provides a short overview of cryptocurrencies. Section 3 describes the data and offers a detailed description of the criteria used in the paper, together with descriptive statistics for the selected cryptocurrencies in the observed period. Section 4 presents the proposed multicriteria model, its implementation is presented in Section 5 and the results of the research in Section 6. Section 7 provides a discussion and interpretation of the obtained results. The last section concludes the paper. The four tables with results of out-of-sample comparisons of different models are given in the Appendix A.

2. Cryptocurrencies—A Short Overview

Cryptocurrency emerged as a byproduct of another invention. Satoshi Nakamoto, the unknown founder of Bitcoin, the most famous cryptocurrency, presented his invention as a "Peer-to-Peer Electronic Cash System" [18]. Numerous attempts made during the 1990s to establish a decentralized digital money system finally succeeded and resulted in the introduction of a new currency—cryptocurrency. While only 6 or 7 years ago most professionals in the world of finance still considered cryptocurrency as something untrustworthy and the scientific approach to studying it was scarce, we are currently witnessing a

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real surge of interest in cryptocurrencies. The Internet abounds in platforms that provide information about basic concepts, data, features, ways of trading, very interesting thoughts and analyses offered by professionals on many aspects of cryptocurrencies. Nonetheless, there is a clear need for scientific research and scientifically based analyses to provide answers to a number of issues that have recently been raised and that have drawn our attention.

Unlike money, which is tangible, the currency you can take along, cryptocurrency is digital money, a digital asset that can be exchanged. Using cryptocurrency instead of paper currency means avoiding bank intermediation and verification and transaction costs [19]. The prefix "crypto" indicates the use of cryptography for security and verification purposes in the course of creating and transferring money. The cryptocurrency transactions are processed and completed using the blockchain technology—the technology that underpins many of the innovations that are currently revolutionising the financial services sector around the world [20]. The most notable application of blockchain is in the development and operation of cryptocurrencies, but there is a space and opportunity for its application in other sectors such as international trade, taxation, supply chain management, business operations and governance. Authors demonstrate how organizations and regulators can leverage blockchain to improve business operations and efficiency while reducing operational costs.

The supply of cryptocurrency is limited, i.e., cryptocurrencies are mined and are created by decryption—complex mathematical tasks are solved by the power of computers. After finding the solution, the miner builds a block and adds it to the chain for which they are rewarded with a certain amount of cryptocurrency. For example, Bitcoin's algorithm determines the rate at which new bitcoins are created over time until they reach the maximum of 21 million bitcoins, which should be reached by the year 2140 [19]. The author discusses a possibility of introducing a Bitcoin standard and, despite some benefits it would have over the current fiat money standards, it concludes that it is unlikely that the Bitcoin standard will stem out, since the authorities will take actions to prevent it. The reason for this is twofold. The first is to protect the seigniorage revenues gained from the costless money creation, while the second is to preserve the ability to affect their domestic economies by implementing the interest rate policies.

Theoretical roots of the decentralization of money offered by today's digital currencies we can find even back in the seventies in the Friedrich von Hayek's theory of private money. Economic implications of the theory for digital currencies are investigated in the paper [21]. In the digital economy, cash can actually disappear and payments can centre on social and economic platforms, weakening traditional monetary policy channels. The article confirms that stable digital money is preferable for foresight, calculation and accounting.

However, there is no consensus among professionals dealing with cryptocurrencies about their classification and evaluation [22]. Authors propose refining the existing standards and introducing rules for classification and evaluation of cryptocurrencies. They also indicate that the best solution is to develop new international financial reporting standards for the accounting of cryptocurrencies.

Today, for Bitcoin and the rest of ever more numerous cryptocurrencies, we could say that they can fulfil their "fundamental task" of (crypto) currency, digital money as a means of payment. A continuously growing number of companies accept Bitcoins as payment for their goods and services [23]. There are Crypto ATMs—according to https://coinatmradar.com/ (accessed on 19 April 2021) there were 18,541 ATMs in 72 countries, 275,795 establishments offering other services (exchange offices, shops and various other services accepting cryptocurrency). Currently there are 270 web-based digital currency exchanges according to https://coin.market/exchanges, (accessed on 19 April 2021).

There is an opinion that cryptographic assets do not fully satisfy the conditions to be a currency and that they are more similar to an asset class [24]. White et al. [25] claim "that Bitcoin's behaviour more closely resembles..., an emerging asset class, rather than a currency...". Furthermore, [26] have shown that cryptocurrencies behave more like

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an investment instrument than a currency and [27] that Bitcoins are mostly used as a speculative investment and not as an alternative currency and medium of exchange. Bouri et al. [28] conclude that Bitcoin is a poor hedge but contributes to a well-diversified portfolio. In the focus of our interest and in that of many potential investors are cryptocurrencies as an alternative form of investment. The "infrastructure" to support cryptocurrencies as a potential special investment class exists: we can discuss the cryptocurrency market in terms of the total supply of and demand for cryptocurrencies. Furthermore, we can also talk about the primary and secondary market of cryptocurrencies: the primary market refers to newly issued cryptocurrencies that raise capital for the issuer's needs, most often start-ups based on blockchain technology, while the secondary market relates to trading in already established cryptocurrencies. Based on stock, bond and other indices, the cryptocurrency index (CRIX) has been established as a benchmark for the cryptocurrency market. The index was created as a result of the joint project of Humboldt University Berlin from Germany, SKBI School of Business, Singapore Management University and CoinGecko. A special, new methodology was required for creating the new index, given the specificity of the cryptocurrency market. The process of generating the index, the specificity of the approach and methodology is described in [29]. Besides that, the indexing methodology and information on current index composition along with other relevant data can be found at https://www.coingecko.com/en/crypto_index/crix (accessed on 5 November 2020) and/or https://thecrix.de/ (accessed on 5 November 2020).

Some papers study the characteristics of cryptocurrencies as a special class of investment by examining the relationship between return and risk, the correlation of return with that of other classes and the politicoeconomic determinants. Burniske and White [30], inspired by Greer's [31] classification of investments in superior asset classes and criteria for their identification, highlight four distinct features distinguishing between different types of investment: investability, politicoeconomic features, correlation of returns, i.e., price independence and risk-reward profile. First, for a particular type of investment there should be so called investability, which also implies a certain level of liquidity. Second, a special type of investment has a special political and economic profile that follows from the source value of investment, investment management and its primary purpose. Third, the market value of the investment should be independent of other types of investment, indicating the absence of or low correlation of their returns. The previous three characteristics should lead to differentiated risk-reward profiles, which can then be further "broken down" into the specificity of returns and volatility of each particular class. Thus, for example, ordinary shares and bonds make different types of investments: after meeting the first criterion of investability, they differ according to the other three criteria. The authors consider Bitcoin as the main representative of cryptocurrencies in the course of 5 years and conclude that Bitcoin represents a special type of investment due to the observed indicators. It should be borne in mind that cryptocurrencies are not a reliable store of value and they do not have a stable purchasing power over a long period of time, unlike fiat money. Moreover, cryptocurrencies are not able to ensure the stream of payments to the owner, unlike other assets such as real estate, stocks or bonds [32].

The risk and the return of cryptocurrencies Bitcoin, Ripple and Ethereum are considered in an extensive paper [33]. The ratio between the return and risk of cryptocurrencies differs from those found in shares, ordinary currencies and precious metals. Cryptocurrencies are exposed neither to the factors most frequently affecting the stock market nor to macroeconomic factors, their market is influenced by other specific impacts. Sajter [34] compares the returns of the same three cryptocurrencies with the returns of six major world equity indices and concludes that the observed cryptocurrencies can be considered a new, specific form of investment, since the trend of their values or returns is not related to the trend of equity index returns. Similarly, Ankenbrand and Bieri [35] concluded that cryptocurrencies can be seen as an individual asset class. Additional studies [11,36,37] offer valuable insides and facts about cryptocurrencies as a new financial asset, which makes them an effective diversification tool.

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Many studies named in the Introduction of the paper have already shown the benefits of including cryptocurrencies into portfolio optimisation processes. The task of this study is to propose appropriate methodology for selecting optimal cryptocurrency portfolio, taking into consideration a broader set of features of this new investible instrument.

3. Data and Criteria

The starting sample consisted of the top 20 cryptocurrencies ranked by market capitalization, calculated on the basis of circulating supply, according to https://coinmarketcap.com/all/views/all/ (accessed on 12 February 2020). As the first criterion we took the most common one—market capitalization. Market capitalization is a metric that indicates the market value and size of a cryptocurrency [38]. Since the cryptocurrency price can give an inaccurate measure of its total value, the market capitalization can identify the value of a cryptocurrency and accurately compare it to other cryptocurrencies. This concept is the same as the one from the stock market, where stock market capitalization is the current stock price multiplied by the total number of existing stocks. Accordingly, cryptocurrency market capitalization equals the total number of circulating coins multiplied by its current price. The most common metrics for the total number of circulating coins is the circulating supply, defined as the number of coins currently circulating in the market available to the general public. Although market capitalization is often taken as a starting criterion for the cryptocurrency sample selection, due to its importance for practitioners, it should be also taken as a separate criterion in the process of portfolio optimization.

The portfolio construction can be significantly obstructed by the problem of liquidity of assets and in this sense a special attention has to be put on cryptocurrencies, since they have far lower daily trading amounts than traditional financial assets [8]. The possibility of trading the assets on the reallocation date and selling or buying between two reallocation dates is of crucial importance in portfolio management. In a recent study [39] analyse four cryptocurrencies in order to identify the determinants of their liquidity. They have found the number of transactions to be one of the most important liquidity drivers, while the most commonly used financial market variables have not proved to have explanatory power. Since spread data for cryptocurrencies are not easily available, Trimborn et al. [8] used the turnover value as a proxy for liquidity. It is calculated for a set period, for example 24 h, as the sum of products of the number of assets and its price. That data is actually registered as the volume (24 h) on the https://coinmarketcap.com/all/views/all/ (accessed on 25 August 2020) the amount of cryptocurrency that has been traded during a certain period of time, i.e., 24 h in this case. As the second criterion we selected the trading volume, following the opinion and practice that the trading volume can also be used as a liquidity measure. It shows how easily the stock can be bought and sold. A low trading volume indicates the infrequent trading with a cryptocurrency and consequently the difficulty to purchase or to sell shares. A high trading volume means that a cryptocurrency is highly liquid and may be bought or sold easily. Besides that, among practitioners it can be seen that the volume presents one of the most valuable pieces of data, which can show the direction and movement of the cryptocurrency and predict the future price and its demand.

However, as shown by the CRIX values (according to https://thecrix.de/ (accessed on 12 February 2020)), the Bitcoin prices, market capitalization and trading volumes (https://coinmarketcap.com/currencies/bitcoin/ (accessed on 12 February 2020)), in the period from January 2015 to February 2020, the market was characterised by a sluggish movement of prices, market capitalization and volumes from 2015 to 2017. The surge in their values started in 2017, they peaked in 2018, followed by the intensive trading period characterized by high volatility with continuous fluctuations in their values until February 2020. Therefore, the period of monotonous price movements and trading from 2015 to 2017 is excluded from further calculations since it is more interesting and challenging to test the proposed model in more volatile periods.

Cryptocurrencies display high expected returns with large volatilities [11]. The mean-variance analysis is limited because of the highly non-normal return distribution of cryp-

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tocurrencies [6]. Kajtazi and Moro [2] found, in line with previous research, that bitcoin exhibits large kurtosis and is positively skewed (albeit to a much lesser extent than previously reported). Moreover, among several unique properties, [40] found that cryptocurrencies have leverage effects and Student's t error distributions. The study [41] uses the symmetric (GARCH 1,1) and asymmetric (EGARCH, TGARCH and PGARCH) models to measure the volatility of cryptocurrencies. The results prove again the high volatility of cryptocurrencies and in most cases, the asymmetric PGARCH with Student's t distribution provides a better fit. In accordance with these findings, we continued with the combination of alternative risk measures, which should be used for this class of risky assets. Besides volumes and market capitalization, daily closing prices are collected from https://coinmarketcap.com/ (accessed on 12 February 2020). Based on daily closing prices, the values of four more criteria: expected daily return, standard deviation, value-at-risk (VaR) and conditional value-at-risk (CVaR) were calculated.

From the day of its appearance within JP Morgan Bank, the value-at-risk (VaR) as a risk measure has been attracting immense attention. It has become one of the most controversial financial instruments. Despite being criticised, it became a very popular and widely used risk measure because of its simplicity, applicability and universality [42]. In addition, controlling authorities have imposed regulatory constraints on the asset allocations of financial institutions based on the estimation of VaR [43]. VaR is a statistical measure that assesses the risk of an asset or the whole portfolio, expressed with one number. It shows the worst estimated loss for a certain time horizon and a certain confidence level. VaR represents the difference between the invested amount of money and the value that is not going to be failed in $\alpha\%$ cases—the value that corresponds to the 1- α percentile of the distribution. However, VaR does not provide any information about the values from the tail of the distribution, i.e., the values that exceed the value of VaR with small probability, but high losses. The risk measure that provides such information is conditional value-at-risk (CVaR). For a given time horizon and confidence level α , CVaR is defined as the conditional expectation of losses greater than VaR. VaR has a drawback of not being a coherent risk measure, i.e., it does not fulfil the subadditivity condition: $\rho(X+Y) \leq \rho(X) + \rho(Y)$. This means that VaR of a portfolio is greater than the sum of VaRs of its constituents [44], which can discourage portfolio diversification and lead to the dangerous risk concentration. Ref. [45] compared VaR, variance and CVaR and concluded that only CVaR is a coherent risk measure. Moreover, CVaR has superior mathematical characteristics over VaR; it keeps good properties of VaR and overcomes its shortages. In agreement with experts, it is decided to proceed with both measures as criteria, VaR for estimating loss for a specific time horizon and a certain confidence level, CVaR as a coherent risk measure, which gives valuable information about losses from the tail of distribution, which exceed VaR.

Guided by opinions and findings indicating that the phenomenon of cryptocurrency created considerable interest and became an appealing investment due to its unique qualities [46], as the last, seventh criterion we took the popularity or attractiveness of cryptocurrencies. Ref. [23] studied Bitcoin attractiveness for investors and users finding its significant impact on Bitcoin price with variation over time. Sovbetov [47] concluded that attractiveness of cryptocurrencies matters, also finding that its recognition is subjected to the time factor. Positive correlation of cryptocurrency attractiveness and its price has also been confirmed by other research [46,48]. The attractiveness of cryptocurrencies is usually measured by the amount of the cryptocurrency-related posts in social media like Twitter, Google Trends, Yahoo, Wikipedia and others. Although, the fact that somebody is interested in gaining information from social media does not necessarily mean active participation in the market, many studies prove significant influences and connections between data offered by social media and trading and prices of assets. Matta et al.'s [49] study proved that the investment professionals in Bitcoin use social media activity and information extracted by a web search and found it helpful. Investors search social media when making decisions since it is proved that sentiment analysis captures information not embedded in prices [50]. News and information extracted from online social media (blogs, Twitter feeds, etc.) can be

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used to predict changes in various economic and business indicators, which is supposed to have an impact also for the Bitcoin price [51]. Stolarski et al. [52] studied cryptocurrency perception using Wikipedia and Google Trends although cryptocurrencies seem to have embraced Twitter as a major channel of communication. Park and Lee [53] investigate the Twitter-mediated communication behaviours among cryptocurrencies, finding that cryptocurrencies' active networking strategies affected their credit scores. Kaminski [54] had already shown that the microblogging platform Twitter may be interpreted as a virtual trading floor that emotionally reflects Bitcoin's market movement. Therefore, quickly recognizing and incorporating the impact of tweets on the price direction in the trading strategy can provide both a purchasing and selling advantage [55]. Kraaijeveld and De Smedt [56] study the predictive power of the Twitter sentiment for various cryptocurrencies, finding that it has predictive power for the returns of Bitcoin, Bitcoin Cash and Litecoin and for EOS and TRON. Due to the importance of Twitter in studying cryptocurrencies, its transparency and simplicity regarding data collection, we proceed with the number of Tweets, obtained from https://bitinfocharts.com/comparison/tweets-btc-eth-ltc-xrp.html (accessed on 25 August 2020), as the attractiveness measure.

Finally, the dataset contains nine cryptocurrencies: Bitcoin—BTC, Dash, Ethereum Classic—ETC, Ethereum—ETH, Litecoin—LTC, Monero—XMR, Neo, Stellar—XLM and Ripple—XRP, for which all required data are available in the period from January 2017 to February 2020. Namely, some of the top 20 cryptocurrencies by market capitalization on 12 February 2020 are not traded in the proposed period from 2017 to 2020 and for some there are no comparable and available data for the number of tweets.

Table 1 gives the overview of descriptive statistics for nine cryptocurrencies along with the Jarque–Bera (JB) test for normality. The null hypothesis of the JB test is a joint hypothesis of both the skewness and the excess kurtosis being zero, i.e., matching a normal distribution. From Table 1 it can be concluded that for all cryptocurrencies, the null hypothesis can be rejected at 1, 5 and 10% significance levels, i.e., the returns are not normally distributed. This can be corroborated by mostly all cryptocurrencies having positively skewed distribution (except for Bitcoin, which has a somewhat negatively skewed distribution, although the coefficient of skewness is roughly around zero). Moreover, all cryptocurrencies show positive excess kurtosis indicating leptokurtic distribution, meaning that the tails on this distribution is heavier than that of a normal distribution, indicating a higher degree of risk and higher probability of extreme values. For that reason, in the process of portfolio selection it is more appropriate to take into consideration alternative risk measures and other criteria, as we anticipated.

Table 1. Descriptive statistics for the selected cryptocurrencies from 1 January 2017 to 11 February 2020.

	BTC	DASH	ETC	ETH	LTC	XMR	NEO	XLM	XRP
Min	-0.2075	-0.2432	-0.4353	-0.3155	-0.3952	-0.2932	-0.4610	-0.3664	-0.6163
q1	-0.0159	-0.0275	-0.0240	-0.0213	-0.0263	-0.0262	-0.0364	-0.0325	-0.0241
Me	0.0023	-0.0012	0.0000	0.0001	-0.0008	-0.0004	-0.0017	-0.0021	-0.0026
q3	0.0215	0.0290	0.0272	0.0257	0.0268	0.0287	0.0328	0.0308	0.0205
Max	0.2251	0.4377	0.4577	0.2901	0.5103	0.4303	0.8012	0.7231	1.0274
μ	0.0020	0.0021	0.0019	0.0030	0.0025	0.0016	0.0040	0.0030	0.0033
σ	0.0426	0.0629	0.0664	0.0571	0.0625	0.0616	0.0850	0.0826	0.0778
α_3	-0.05	0.96	0.15	0.25	1.14	0.39	1.62	1.99	2.90
α_4	3.56	6.72	6.94	4.29	9.52	4.72	14.88	16.19	37.41
JB	592.47 ***	2289.7 ***	2263.1 ***	871.8 ***	4495.8 ***	1072.9 ***	10874 ***	13042 ***	67245 ***

Source: The authors' calculations in R Studio (*** indicate significance at the 0.01 level).

Values of other criteria for the whole sample are given in Table 2. VaR, measuring the level of financial risk for each cryptocurrency over the whole sample, indicates that the highest possible loss can be obtained with Monero, Neo and Stellar and the lowest possible loss with Bitcoin. The values of CVaR, measuring the mean of tail risk, is the

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lowest for Bitcoin and the highest for Neo, Stellar and Ripple. The biggest values of the mean volume (MVlm), mean market capitalization (MMC) and mean number of tweets (MoT) can be observed for Bitcoin and the lowest values for Monero, Ethereum Classic and Stellar respectively.

	BTC	DASH	ETC	ETH	LTC	XMR	NEO	XLM	XRP
VaR	0.0676	0.0921	0.0957	0.0848	0.0852	0.1026	0.1100	0.1078	0.0900
CVaR	0.1013	0.1634	0.1683	0.1475	0.1664	0.1496	0.2304	0.2212	0.2167
MVlm	9.16×10^{9}	1.86×10^{8}	3.73×10^{8}	3.64×10^{9}	1.30×10^{9}	6.76×10^{7}	2.00×10^{8}	1.33×10^{8}	8.42×10^{8}
MMC	1.11×10^{11}	1.78×10^{9}	1.14×10^{9}	2.91×10^{10}	4.30×10^{9}	1.64×10^{9}	1.59×10^{9}	2.45×10^{9}	1.55×10^{10}
MoT	37,033.30	3299.03	121.85	11,778.24	2397.39	508.05	1181.00	37.97	4296.33

Table 2. Values of other criteria for the whole sample.

Source: The authors' calculations in MATLAB and R Studio.

4. The Multicriteria Model

The multicriteria (MC) model based on the PROMETHEE II approach [57] is applied. According to the PROMETHEE II model, each alternative P, in this case the cryptocurrencies portfolio, is evaluated with two flows. The positive flow $\Phi^+(P)$ indicates how much one cryptocurrency portfolio is better than other cryptocurrency portfolios in all criteria. The higher the $\Phi^+(P)$ is, the better is the cryptocurrency portfolio. The negative flow $\Phi^-(P)$ indicates how much better the cryptocurrency portfolio is over other cryptocurrency portfolios. The lesser the $\Phi^-(P)$ is, the better the cryptocurrency portfolio is. Finally, the net flow Φ is the difference between these positive and negative flows: i.e.,

$$\Phi(P) = \Phi^+(P) - \Phi^-(P) \tag{1}$$

The higher the net flow $\Phi(P)$ is, the better the cryptocurrency portfolio is. Positive and negative flows are calculated by pairwise comparisons of all the cryptocurrency portfolios and for every criterion simultaneously.

Since the number of possible portfolios that can be made up from a sample of cryptocurrencies is infinite, it is impossible to compare all pairs of portfolios. Therefore, this study employs the procedure introduced by Khoury and Martel [58] and Zmitri et al. [59], following the applications of the procedure in [60,61].

Each cryptocurrency portfolio (its positive and negative flow) is compared to two imaginary portfolios: ideal (\overline{P}) and anti-ideal (\underline{P}) . Compared to the anti-ideal, the positive flow $\Phi^+(P)$ is obtained. The higher the $\Phi^+(P)$ is, the better is the cryptocurrency portfolio since it is more distant from the anti-ideal. The lower the $\Phi^-(P)$ is, the better the cryptocurrency portfolio is since it is closer to the ideal, Moreover, the higher the net flow Φ is, the better the cryptocurrency portfolio is.

For each criterion C_j , (j = 1, 2, ..., n), which has to be maximized, the ideal is:

$$C_j(\overline{P}) = \max_i C_j(A_i), \tag{2}$$

where $A = \{A_1, A_2, ..., A_N\}$ is the set of N alternatives, in this case nine cryptocurrencies. For the same criterion, which has to be maximized, the anti-ideal is:

$$C_j(\underline{P}) = \min_i C_j(A_i) \tag{3}$$

Without the loss of generality, we can suppose that all criteria are to be maximized.

The set of feasible solutions is the set of cryptocurrency portfolios, which can be formed from the observed cryptocurrencies. The evaluation of the cryptocurrency portfolio P according to criterion j is obtained by multiplying the share invested in each cryptocurrency A_i in the portfolio P, i.e., a_i , with the evaluation of cryptocurrency i according to criterion

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j. Obviously, the sum of all shares invested in each cryptocurrency A_i in the portfolio P equals 1.

$$C_j(P) = \sum_{i=1}^{N} a_i C_j(A_i)$$
 (4)

For each criterion C_j the preference functions are defined as in the PROMETHEE method where indifference q and preference p thresholds are predefined numbers from the interval $\left[0,C_j(\overline{P})-C_j(\underline{P})\right]$, where $0 \leq q,p \leq C_j(\overline{P})-C_j(\underline{P})$ and $q \leq p$ is always true. Moreover, q and p always have economic significance.

In this particular application we assumed that the highest value of preference threshold p cannot exceed half the span between the anti-ideal and ideal according to that criterion, i.e., $p \in \left[q, \frac{C_j(\overline{P}) - C_j(\underline{P})}{2}\right]$.

Like in [62], we used the same preference function for all criteria and in this study that is the linear preference function with the indifference threshold (type V), as it most generally displays the relations between pairs of cryptocurrency portfolios. It has the following general form:

$$\Psi(d) = \begin{cases}
0, d \le q \\
\frac{d-q}{p-q}, q < d \le p \\
1, d > p
\end{cases}$$
(5)

where d is the difference in the evaluation of the two alternatives by the same criterion.

In this application, when the cryptocurrency portfolio P is compared to the anti-ideal (\underline{P}) , i.e., when we calculate $\Phi_j^+(P)$, the difference d presents the "distance" from the anti-ideal (by j-criterion). Taking that $d_j(P) = C_j(P) - C_j(\underline{P})$ for $\Phi_j^+(P)$ we have [61]:

$$\Phi_{j}^{+}(P) = \begin{cases}
0, & C_{j}(P) \leq C_{j}(\underline{P}) + q_{j}^{-} \\
\frac{C_{j}(P) - C_{j}(\underline{P}) - q_{j}^{-}}{p_{j}^{-} - q_{j}^{-}}, & C_{j}(\underline{P}) + q_{j}^{-} < C_{j}(P) \leq C_{j}(\underline{P}) + p_{j}^{-} \\
1, & C_{j}(P) > C_{j}(\underline{P}) + p_{j}^{-}
\end{cases} (6)$$

Analogously, taking that $d_j(P) = C_j(\overline{P}) - C_j(P)$ for $\Phi_j^-(P)$ we have:

$$\Phi_{j}^{-}(P) = \begin{cases}
0, & C_{j}(P) \geq C_{j}(\overline{P}) - q_{j}^{+} \\
\frac{C_{j}(\overline{P}) - C_{j}(P) - q_{j}^{+}}{p_{j}^{+} - q_{j}^{+}}, & C_{j}(\overline{P}) - p_{j}^{+} \leq C_{j}(P) < C_{j}(\overline{P}) - q_{j}^{+} \\
1, & C_{j}(P) < C_{j}(\overline{P}) - p_{j}^{+}
\end{cases} (7)$$

Finally, for $\Phi_i(P)$ we have:

$$\Phi_{j}(P) = \begin{cases}
-1, & C_{j}(P) \leq C_{j}(\underline{P}) + q_{j}^{-} \\
\frac{C_{j}(P) - C_{j}(\underline{P}) - p_{j}^{-}}{p_{j}^{-} - q_{j}^{-}}, & C_{j}(\underline{P}) + q_{j}^{-} < C_{j}(P) \leq C_{j}(\underline{P}) + p_{j}^{-} \\
0, & C_{j}(\underline{P}) + p_{j}^{-} < C_{j}(P) \leq C_{j}(\overline{P}) - p_{j}^{+} \\
\frac{p_{j}^{+} - C_{j}(\overline{P}) + C_{j}(P)}{p_{j}^{+} - q_{j}^{+}}, & C_{j}(\overline{P}) - p_{j}^{+} \leq C_{j}(P) < C_{j}(\overline{P}) - q_{j}^{+} \\
1, & C_{j}(P) \geq C_{j}(\overline{P}) - q_{j}^{+}
\end{cases} (8)$$

or graphically as in paper [61] p. 62.

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Positive and negative flows, $\Phi_j^+(P)$ and $\Phi_j^-(P)$, have to be calculated separately for each criterion C_j , j = 1, 2, ..., n. Then, the net flow is obtained as a weighted sum of the difference between positive and negative flows, i.e.,

$$\Phi(P) = \sum_{j=1}^{n} w_j \Big(\Phi_j^+(P) - \Phi_j^-(P) \Big)$$
 (9)

where relation (10) follows for any possible cryptocurrency portfolio *P*,

$$\Phi(\underline{P}) \le \Phi(P) \le \Phi(\overline{P}) \tag{10}$$

The weights w_j of the criteria are obtained in agreement with the decision maker and/or by some of the methods for determining the weights of criteria. For obtaining the weights in this case, the AHP method and its eigenvalue procedure with pairwise comparisons obtained by a group of experts is applied.

Finally, the optimal cryptocurrency portfolio $(a_1, a_2, ..., a_N)$ is one that finds the maximum net flow, i.e.,

$$Max \Phi(P)$$
 (11)

subject to:

$$\sum_{i=1}^{N} a_i = 1 \tag{12}$$

$$0 \le a_i \le a_{M_i},\tag{13}$$

where a_i is the share invested in A_i in the cryptocurrency portfolio, a_{M_i} is the maximum proportion to invest in cryptocurrency A_i in cryptocurrency portfolio P and N is the number of cryptocurrencies, which can be included in cryptocurrency portfolio P.

5. Implementation of the Model

The presented model was used for the selection of optimal portfolios of cryptocurrencies based on the sample of nine cryptocurrencies: Bitcoin—BTC, Dash, Ethereum Classic—ETC, Ethereum—ETH, Litecoin—LTC, Monero—XMR, Neo, Stellar—XLM and Ripple—XRP and seven criteria: daily return, standard deviation, value-at-risk (VaR), conditional value-at-risk (CVaR), volume, market capitalization and attractiveness.

For criteria selection and determination of their weights, twelve experts were engaged, some of them professionals dealing with cryptocurrencies and others scientists, including the authors of the paper. By engaging twelve experts, we reduced subjectivity in the process of weights calculation. Weights of the chosen criteria were estimated using the Saaty's AHP method, its eigenvalue procedure [63,64]. After the complete Saaty matrix was obtained by the experts, the weight of criteria was calculated using Expert Choice and are given in the last row of Table 3. The reported inconsistency is 0.02.

Table 3. Heading of decision matrices.

Criterion	μ	σ	VaR	CVaR	MVlm	MMC	MoT
Min/Max	max	min	min	min	max	max	max
Type	V	V	V	V	V	V	V
Weights	0.208	0.141	0.321	0.183	0.057	0.055	0.035

We can see that the biggest accent in the evaluators' judgements for the importance of criteria is given to the possible losses by investing in cryptocurrency portfolios. Due to the generally accepted and confirmed opinion of high riskiness of cryptocurrencies, the VaR has taken the highest percentage, as much as 32%. The other risk measures, together with the expected return are also highly esteemed: expected return—E(R) 21%, CVaR 18% and standard deviation—St.dev. 14%. Volume—Vol and market capitalization—MMC have

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lower and almost equally estimated importance, 5.7% and 5.5% respectively. The lowest weight is given to the criterion of attractiveness, measured by the number of tweets (MoT), 3.5%.

For choosing the preference functions for the observed criteria, the same group of experts was consulted and it was decided to proceed with the linear preference function with the indifference threshold (type V) for all criteria, as it most generally displays the relations between the pairs of alternatives. Thresholds q^- , q^+ , p^- and p^+ were calculated in the following manner for all criteria, which is in accordance with that previously said about the intervals of the threshold's values:

$$q_{j}^{-} = (s_{j}(2) - s_{j}(1)), q_{j}^{+} = (s_{j}(9) - s_{j}(8)),$$

$$p_{j}^{-} = \frac{\left(s_{j}(9) - q_{j}^{+} - s_{j}(1) - q_{j}^{-}\right)}{2} + q_{j}^{-},$$

$$p_{j}^{+} = \frac{\left(s_{j}(9) - q_{j}^{+} - s_{j}(1) - q_{j}^{-}\right)}{2} + q_{j}^{+}, \forall j = 1, 2, \dots 7,$$

where $s_j(1), s_j(2), \dots, s_j(9)$ are sorted values of the evaluation of alternatives according to criterion j. Finally, the heading of all decision matrices is given in Table 3.

For the period from January 2017 to February 2020, using the rolling window of 6 months of daily returns, volumes and market capitalization, the mean daily returns, standard deviations, VaR, CVaR and average volumes and average market capitalization were calculated. The optimal portfolios using the described MC model were calculated at the first of each month by taking the previous 6 months of daily data for the calculations. Moreover, the attractiveness, measured by the number of tweets, was taken for each cryptocurrency on the day before the calculation of the optimal portfolio. Therefore, 32 optimal portfolios in 32 successive months were obtained. The maximum proportion to invest in cryptocurrency A_i in the optimal cryptocurrency portfolio P was limited to $a_{Mi} = 0.5 \ \forall i = 1, 2, \ldots, 9$.

6. Results

The resulting portfolios with the weights of each cryptocurrency are given in Table 4. We can see that the most favourable cryptocurrency is Bitcoin, the fact that could be discerned from the data and results given in Tables 1 and 2. It is followed by Ethereum, Litecoin, Ripple, Dash and Ethereum Classic, depending on the date and period. In general, it can be said that there was a high level of diversification, which was also supported with the constraint (13).

Table 4. Optimal	l portfolios with	the weights of	each cryptocurrency.

Date	BTC	DASH	ETC	ETH	LTC	XMR	NEO	XLM	XRP
1 July 2017	0.2322	0.0169	0.2200	0.4971	0.0068	0.0022	0.0067	0.0023	0.0158
1 August 2017	0.2890	0.0191	0.0596	0.4944	0.1186	0.0026	0.0077	0.0017	0.0073
1 September 2017	0.4893	0.0025	0.0266	0.1093	0.2581	0.0017	0.1099	0.0006	0.0020
1 Ôctober 2017	0.4842	0.0291	0.0188	0.2111	0.0338	0.0153	0.1947	0.0040	0.0089
1 November 2017	0.4656	0.0886	0.0317	0.1163	0.0553	0.0396	0.1539	0.0111	0.0379
1 December 2017	0.4990	0.4086	0.0007	0.0014	0.0594	0.0024	0.0274	0.0003	0.0007
1 January 2018	0.4783	0.1963	0.0069	0.0130	0.0936	0.1759	0.0120	0.0083	0.0156
1 February 2018	0.4545	0.0044	0.0015	0.4811	0.0037	0.0069	0.0204	0.0238	0.0038
1 March 2018	0.4709	0.0003	0.0002	0.4962	0.0082	0.0003	0.0004	0.0228	0.0007
1 April 2018	0.1742	0.0003	0.0002	0.3038	0.4956	0.0004	0.0004	0.0246	0.0005
1 May 2018	0.3200	0.0005	0.0003	0.4991	0.1567	0.0008	0.0006	0.0211	0.0008
1 June 2018	0.3893	0.0001	0.0001	0.4998	0.1017	0.0001	0.0002	0.0085	0.0004
1 July 2018	0.4686	0.0003	0.0004	0.4987	0.0009	0.0004	0.0008	0.0293	0.0005
1 August 2018	0.4721	0.0361	0.0074	0.0210	0.4140	0.0089	0.0133	0.0077	0.0195
1 September 2018	0.4938	0.0222	0.0191	0.0662	0.1051	0.0125	0.0062	0.2239	0.0510
1 October 2018	0.4935	0.0378	0.0304	0.0130	0.0367	0.0119	0.0052	0.2561	0.1155

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Table 4. Cont.

Date	ВТС	DASH	ETC	ETH	LTC	XMR	NEO	XLM	XRP
1 November 2018	0.4658	0.0098	0.0259	0.0100	0.0120	0.0168	0.0064	0.3960	0.0573
1 December 2018	0.4921	0.0008	0.0009	0.0005	0.0007	0.0011	0.0004	0.3042	0.1995
1 January 2019	0.4817	0.0113	0.0115	0.0094	0.0233	0.0169	0.0066	0.0983	0.3409
1 February 2019	0.3563	0.0415	0.0350	0.0341	0.0861	0.0461	0.0318	0.0541	0.3150
1 March 2019	0.3634	0.0008	0.0004	0.0009	0.1370	0.0006	0.0006	0.0004	0.4958
1 April 2019	0.3890	0.0541	0.0372	0.0545	0.1975	0.0505	0.0412	0.0607	0.1152
1 May 2019	0.4853	0.0050	0.0032	0.0086	0.1316	0.0048	0.0036	0.0041	0.3539
1 June 2019	0.4997	0.0003	0.0002	0.0007	0.3894	0.0002	0.0003	0.0002	0.1091
1 July 2019	0.4534	0.0936	0.0310	0.0677	0.1890	0.0560	0.0294	0.0342	0.0458
1 August 2019	0.4994	0.4289	0.0005	0.0013	0.0485	0.0066	0.0004	0.0009	0.0134
1 September 2019	0.4995	0.4596	0.0005	0.0006	0.0005	0.0030	0.0002	0.0019	0.0341
1 October 2019	0.4418	0.2611	0.0011	0.0021	0.0015	0.0015	0.0006	0.0030	0.2873
1 November 2019	0.4995	0.2797	0.0003	0.0005	0.0005	0.0004	0.0002	0.0003	0.2186
1 December 2019	0.4992	0.0006	0.0012	0.0125	0.0014	0.0025	0.0069	0.0007	0.4750
1 January 2020	0.4994	0.0004	0.0010	0.0014	0.0005	0.0029	0.0021	0.0015	0.4908
1 February 2020	0.5000	0.0000	0.4997	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003

Source: the authors' calculations in MATLAB 2017a, The MathWorks, Inc.

For the evaluation of the results, out-of-sample testing was performed. The optimal portfolios obtained by the multicriteria (MC) model are compared with the naïve portfolio (NAIVE), two mean-variance (MV) models (first one in the middle of the efficient frontier having the average variance—MV middle and the second at the end of the efficient frontier having the maximum variance and containing only one cryptocurrency—MV max). It is also compared to a portfolio obtained using the maximum Sharpe ratio (Max Sharp) and the one with mean-CVaR optimization (MCVaR), from the middle of the mean-CVaR efficient frontier. The nominated principles of portfolio selection are commonly found in related research and in portfolio optimization in general.

Brauneis and Mestel [13] compared risk and return of different MV portfolios to single cryptocurrency investments and two benchmarks, the naively diversified portfolio and the CRIX. They found that in terms of the Sharpe ratio and certainty equivalent returns, the naïvely diversified portfolio outperforms single cryptocurrencies and more than 75% of MV portfolios. The similar study by Platanakis et al. [12] concluded that naïve diversification is as good, if not better, than MV diversification. Weiyi [14] concludes that none of the observed models (minimum variance, risk parity, MV, maximum Sharpe and maximum utility) is consistently better than the 1/N rule in the Sharpe ratio. We took into consideration confirmed good features of the naïve portfolio and included the 1/N principle of portfolio selection as one of the models to be compared with the proposed MC model.

Within the group of papers considering the contribution of cryptocurrencies to portfolios with the rest of assets being either traditional ones or combinations of traditional and alternative investments, there are studies that considered the conditional value-at-risk (CVaR) as the appropriate risk measure: [2,6,8,9]. Namely, it is necessary to take into consideration the alternative risk measures. In accordance with the approach of the mentioned studies, we proceed with comparison of performances of the mean–CVaR model (Table 5) and proposed MC model.

In almost all mentioned studies, the Sharpe ratio is a standard metric for measuring risk-adjusted model performance. It is also used as a principle in the process of selecting the optimal (cryptocurrency) portfolio [9,14]. Together with NAÏVE and mean—CVaR model, in the out-of-sample testing and comparison, we used the maximum Sharpe ratio model, given in Table 5.

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	Model	Abbreviation	Objective Function	Constraints
1	1/N rule	NAIVE		
2	Markowitz model	MV middle	$Max \ E(R_P)$	$egin{array}{l} \sigma_P \leq s \ \sum\limits_{i=1}^N a_i = 1 \end{array}$
3	Markowitz model	MV max	$Max E(R_P)$	$egin{array}{l} a_i \geq 0 \ \sum\limits_{i=1}^N a_i = 1 \ a_i \geq 0 \end{array}$
4	Maximum Sharpe ratio model	Max Sharp	$Max \frac{E(R_P)}{\sigma_P}$	$\sum_{i=1}^{N} a_i = 1$
5	Mean–CVaR model	MCVaR middle	$Max E(R_P)$	$egin{aligned} a_i &\geq 0 \ CVaR_P &\leq c \ \sum\limits_{i=1}^N a_i &= 1 \ a_i &\geq 0 \end{aligned}$

Table 5. Five portfolio optimization models.

An inevitable model in the analysis is the Markowitz mean-variance (MV) model. In this study we employed two MV models, the first one in the middle of the efficient frontier having the average variance—MV middle and the second at the end of the efficient frontier having the maximum variance and containing only one cryptocurrency—MV max.

Applied portfolio optimization models for the purpose of out-of-sample testing and comparison are presented in Table 5.

The average return in the next 30 days, the standard deviation of the returns in the next 30 days, VaR of the returns in the next 30 days and return in the next trading day for 32 portfolios (on 32 dates) obtained as results of the six different optimization models are given in Appendix A, in Tables A1–A4 respectively. In Table 6 of the MC model winnings, we summarized in how many cases the MC model was better than other models, considering different indicators.

To Post on	Number of Cases When MC Model Is Better Than							
Indicator —	NAIVE	MV Middle	MV Max	Max Sharp	MCVaR			
Average return in the next 30 days	17	18	18	18	21			
Standard deviation of the returns in the next 30 days	26	22	24	22	30			
VaR of the returns in the next 30 days	25	21	23	22	25			
Return in the next trading day	20	18	19	16	17			

Table 6. MC model winnings.

7. Discussion and Interpretation of Results

Regarding the average return in the next 30 days NAÏVE portfolio performed rather close to the MC model with 15 (47%) winnings, while in all other comparisons it loses more convincingly (Table 6). This is a completely opposite finding to those of [4,12,13] that found undeniable superiority of the naïve portfolio.

When comparing performances of the mean–CVaR model and proposed MC model, only regarding the return in the next trading day the mean–CVaR model came closer to the MC model with 15 (47%) winnings, while in all other comparisons it loses more convincingly (Table 6). This proves the necessity of including more risk measures in the portfolio selection.

Looking at MC model winnings against the three models, according to all performance indicators, we could find that Max Sharp performed better than the previous two models,

s is the standard deviation of middle portfolio on the mean-variance efficient frontier; c is the CVaR of the middle portfolio on the mean-CVaR efficient frontier.

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however the MC model won in all categories, except in the return in the next trading day. There the MC model and the maximum Sharpe ratio model had the same number of winnings (Table 6).

Finally, both MV middle and MV max models were outperformed by the MC model (Table 6) as in [60].

The obtained results show that the proposed MC model functioned really well: it outperformed all other models in all four out-of-sample comparisons in a very convincing manner. It won at least in 50% of cases (MC model against Max Sharp in the comparison of returns of the portfolios in the next trading day, Table 6) up to even 94% of cases (MC model against MCVaR in the comparison of standard deviations of the returns in the next 30 days, Table 6). The best performance MC model shows in the case of comparisons of standard deviations of the returns in the next 30 days, from 22 to 30 winnings from 32 possible cases, i.e., in 69–94% of cases. It is not only the percentage of winnings that should be considered, but also the differences in the values of risk measured by standard deviation, which are mostly in favour of the MC model—they can be rather big in cases when the MC model wins and rather small in opposite cases, Table A2. This excellent performance of the MC model is followed by another comparison of risk, measured by the value-at-risk, as shown in Table A3 where the MC model had from 21 to 25 winnings, i.e., in 66-78% of cases, also with notable differences of values of VaR in favour of the MC model. A slightly lesser "success" of the MC model, but still very notable, is according to the comparisons of returns: from 17 to 21 winnings, i.e., in 53-66% of cases, when comparisons of the average return in the next 30 days were considered (Table A1), and from 16 to 20 winnings, i.e., 50-67% of cases, when comparisons of the returns of portfolios in the next trading day were observed (Table A4).

The excellent performance of the presented multicriteria model for the selection of cryptocurrencies portfolio was even stronger if we took into consideration not only the partial results from the four comparisons, but also the return–risk ratios.

Undoubtedly, the inclusion of more criteria—features of cryptocurrencies, among them more risk estimators, since simple descriptive statistics pointed out the specificities of cryptocurrencies and unfulfilled assumptions of other models—and the adoption of appropriate methodology, the appropriate multicriteria decision-making model, helped us to design the best method for cryptocurrency portfolio selection.

8. Conclusions

In the last few years numerous studies have confirmed cryptocurrencies as valuable constituents of optimal portfolios, parallel with equally remarkable amounts of research finding that cryptocurrencies have differentiated risk—reward profiles, with the absence of or very low correlation of returns with other types of investment. Depending on the sample and period, different portfolio optimization models were winners and, so, different models were recommended for cryptocurrency portfolio selection. Some of those studies recognized the need to introduce other criteria, besides return and risk. The first one is the constraint of liquidity, while market capitalization constraint is only partially adopted in the selection of the cryptocurrency sample where it is usually those with the highest market capitalization that are selected. Sometimes the observed cryptocurrency benchmark portfolio is CRIX or some number of its most weighted constituents. In that way, the criterion of market capitalization is partially introduced. Additionally, the need of applying alternative risk measures in the cryptocurrency portfolio selection process is only partially adopted. Due to the non-normal distribution of cryptocurrencies' returns, there is a clear need to overcome the mean-variance framework and to employ appropriate risk measures.

Since cryptocurrencies undoubtedly contribute to the better risk–return performances of portfolios, the issue is to recognize criteria, beside the return and risk, important for the selection of cryptocurrencies, and to propose the model for cryptocurrency portfolio selection, which will adopt the recommended criteria. These were the issues of this research.

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The seven criteria have been recognized and adopted for the cryptocurrency portfolio selection. Beside the expected return and variance as a risk measure, additional risk measures, value-at-risk and conditional value-at-risk are recognized as important for this job and employed in the process. Moreover, market capitalization and volumes of cryptocurrencies are considered as very important market indicators for many investors. The seventh criterion is attractiveness, in this case measured by number of tweets.

The proposed model is a multiobjective programming model based on the PROMETHEE II method, while weights of the seven chosen criteria are estimated using the AHP method, its eigenvalue procedure.

The model is demonstrated and tested using a sample of cryptocurrencies for which all required data were available in the period from January 2017 to February 2020.

Out-of-sample testing results and comparisons with performances of commonly used models, according to different indicators, gave a significant advantage to the proposed method for cryptocurrency portfolio selection, confirming the advantages of including a range of criteria, besides return and variance, and the use of appropriate multicriteria decision methodology.

Working on these issues, a number of questions and ideas arise. In the process of criteria evaluation, the group of consulted experts put the attention primarily to VaR and other risk measures together with the expected return. While exploring other criteria, using mostly Internet and social media as sources of information, we met practitioners' opinions where they were very much oriented towards market volume, market capitalization and/or attractiveness, together with returns, and less to the risk and appropriate risk measures. This provides an incentive to explore the importance of criteria, consulting a wider set of practitioners, not necessarily formally accepted experts, to test the model under such conditions. In the process of exploring the criteria other indicators that might be interesting as additional criteria in the model, like a hash rate and transaction costs were encountered. The importance and influence of such indicators and ways for their evaluation and inclusion in the model should be investigated. Moreover, it is worth considering other MCDM models for comparison purposes. Finally, the proposed model together with all other models studied in this paper were considering the cryptocurrency market under relatively normal conditions, before the big COVID-19 crisis. Future research should consider the behaviour and previously confirmed "independency" features of cryptocurrencies, which made them desirable portfolio constituents and the model performances under new conditions.

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Appendix A

Table A1. Mean return in the next 30 days for 6 different models.

Date	MC Model	NAIVE	MV Middle	MV Max	Max Sharp	MCVaR
1 July 2017	-0.00846	-0.00720	-0.01004	-0.00666	-0.00951	-0.00981
1 August 2017	0.01831	0.02118	0.03384	0.05164	0.02166	0.03836
1 September 2017	-0.00430	-0.00613	-0.00458	-0.00410	-0.00492	-0.00444
1 October 2017	0.00431	0.00208	-0.00140	-0.00281	0.00337	-0.00144
1 November 2017	0.01793	0.01934	0.01497	0.01278	0.01590	0.01024
1 December 2017	0.01934	0.02716	0.01697	0.01703	0.01704	0.03068
1 January 2018	-0.01059	0.00131	-0.00742	-0.00883	-0.01040	0.01044
1 February 2018	-0.00639	-0.00487	-0.01148	-0.01911	-0.00893	-0.00612
1 March 2018	-0.01426	-0.01784	-0.01557	-0.01677	-0.01630	-0.02305
1 April 2018	0.00745	0.01100	0.01384	0.01948	0.01507	0.01174
1 May 2018	-0.00612	-0.01076	-0.00860	-0.01028	-0.01018	-0.01046
1 June 2018	-0.00753	-0.00885	-0.01051	-0.01215	-0.01215	-0.01319
1 July 2018	0.00515	0.00399	0.00985	0.01724	0.01724	0.00109
1 August 2018	-0.01040	-0.01383	-0.00656	-0.00656	-0.00656	-0.01950
1 September 2018	0.00298	0.00494	-0.00005	-0.00005	-0.00005	0.00652
1 October 2018	-0.00356	-0.00543	-0.00348	-0.00501	-0.00501	-0.00562
1 November 2018	-0.01107	-0.01438	-0.01314	-0.01314	-0.01314	-0.01625
1 December 2018	-0.00679	-0.00543	-0.00706	-0.00540	-0.00540	-0.00498
1 January 2019	-0.00401	-0.00306	-0.00399	-0.00406	-0.00406	-0.00137
1 February 2019	0.00410	0.00537	0.00215	0.00123	0.00123	0.00403
1 March 2019	-0.00036	0.00105	0.00023	-0.00221	-0.00221	-0.00098
1 April 2019	0.00723	0.00619	0.00887	0.00856	0.00856	0.00314
1 May 2019	0.01003	0.00856	0.00907	0.00561	0.00561	0.00633
1 June 2019	0.00837	0.00365	0.01015	0.01308	0.01126	0.00297
1 July 2019	-0.00280	-0.00630	-0.00425	-0.00992	-0.00130	-0.00385
1 August 2019	-0.00283	-0.00490	-0.00533	-0.00880	-0.00058	-0.00745
1 September 2019	-0.00102	0.00100	-0.00119	-0.00227	-0.00227	0.00129
1 October 2019	-0.00748	-0.00963	-0.00803	-0.00820	-0.00820	-0.00898
1 November 2019	0.00047	0.00348	0.00146	0.00244	0.00244	0.00966
1 December 2019	-0.01043	-0.01316	-0.00938	-0.00853	-0.00853	-0.01360
1 January 2020	0.00825	0.01545	0.00872	0.00910	0.00910	0.01045
1 February 2020	0.00984	0.00939	0.01109	0.01365	0.01365	0.00962
MC model b	oetter	17	18	18	18	21

Source: the authors' calculations in MATLAB.

Table A2. Standard deviations of the returns in the next 30 days.

Date	MC Model	NAIVE	MV Middle	MV Max	Max Sharp	MCVaR
1 July 2017	0.07496	0.07034	0.08456	0.09640	0.07659	0.08502
1 August 2017	0.02971	0.03042	0.07466	0.13974	0.02929	0.08411
1 September 2017	0.08286	0.08559	0.10009	0.14038	0.08286	0.10368
1 October 2017	0.02294	0.02917	0.04006	0.06587	0.02291	0.04376
1 November 2017	0.02931	0.03416	0.05672	0.08712	0.03975	0.03611
1 December 2017	0.06198	0.07373	0.07455	0.10676	0.06230	0.10038
1 January 2018	0.07012	0.08032	0.07903	0.08970	0.07036	0.09869
1 February 2018	0.07227	0.08305	0.08522	0.08559	0.07905	0.08818
1 March 2018	0.04150	0.04706	0.04925	0.05854	0.05060	0.05228
1 April 2018	0.05874	0.06295	0.06776	0.07955	0.07089	0.06727
1 May 2018	0.03771	0.03840	0.04498	0.05410	0.05242	0.04872
1 June 2018	0.04323	0.04423	0.05136	0.05276	0.05276	0.04925
1 July 2018	0.03391	0.03815	0.04317	0.05796	0.05796	0.04673
1 August 2018	0.03502	0.04523	0.02895	0.02895	0.02895	0.06235
1 September 2018	0.03518	0.04872	0.02362	0.02362	0.02362	0.06286
1 October 2018	0.02748	0.03353	0.03006	0.04139	0.04139	0.03769

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 Table A2. Cont.

Date	MC Model	NAIVE	MV Middle	MV Max	Max Sharp	MCVaR
1 November 2018	0.04095	0.04527	0.03705	0.03705	0.03705	0.04776
1 December 2018	0.05492	0.06052	0.05309	0.04829	0.04829	0.06143
1 January 2019	0.03969	0.05116	0.04060	0.04527	0.04527	0.05446
1 February 2019	0.02653	0.02979	0.02501	0.02969	0.02969	0.02797
1 March 2019	0.02655	0.03019	0.02918	0.02886	0.02886	0.03839
1 April 2019	0.03636	0.03649	0.04521	0.05675	0.05675	0.03620
1 May 2019	0.04675	0.04806	0.04614	0.05193	0.05193	0.05490
1 June 2019	0.03360	0.03284	0.03676	0.04620	0.03992	0.03908
1 July 2019	0.06093	0.05895	0.06195	0.06508	0.06314	0.06872
1 August 2019	0.02973	0.02882	0.03370	0.03898	0.03230	0.03474
1 September 2019	0.02418	0.02615	0.02208	0.02248	0.02248	0.02790
1 October 2019	0.03216	0.03724	0.03032	0.02840	0.02840	0.04025
1 November 2019	0.03067	0.02952	0.03456	0.03739	0.03739	0.04333
1 December 2019	0.02573	0.03019	0.02428	0.02389	0.02389	0.03456
1 January 2020	0.03087	0.03795	0.02997	0.03032	0.03032	0.03732
1 February 2020	0.03878	0.02656	0.04639	0.06295	0.06295	0.03108
MC model b	etter	26	22	24	22	30

Source: the authors' calculations in MATLAB.

Table A3. VaR of the returns in the next 30 days.

Date	MC Model	NAIVE	MV Middle	MV Max	Max Sharp	MCVaR
1 July 2017	0.14720	0.11604	0.13227	0.14674	0.13408	0.13272
1 August 2017	0.03533	0.02392	0.06561	0.13305	0.02524	0.06911
1 September 2017	0.27211	0.28985	0.24010	0.32852	0.26675	0.24837
1 October 2017	0.03294	0.05378	0.08710	0.11948	0.03510	0.09091
1 November 2017	0.07482	0.07651	0.10852	0.12399	0.08613	0.07650
1 December 2017	0.16536	0.18522	0.20809	0.25023	0.16609	0.17318
1 January 2018	0.21151	0.25978	0.23845	0.25880	0.21708	0.29431
1 February 2018	0.17755	0.19189	0.19366	0.18522	0.18750	0.23017
1 March 2018	0.11269	0.12871	0.13160	0.15434	0.13431	0.13820
1 April 2018	0.13467	0.14447	0.14563	0.16561	0.15232	0.14870
1 May 2018	0.08546	0.09116	0.10130	0.11899	0.11694	0.09309
1 June 2018	0.11563	0.12065	0.12794	0.12948	0.12948	0.11978
1 July 2018	0.07847	0.08296	0.08595	0.10819	0.10819	0.08638
1 August 2018	0.08055	0.10988	0.06853	0.06853	0.06853	0.13807
1 September 2018	0.11266	0.16303	0.08042	0.08042	0.08042	0.19286
1 October2018	0.10553	0.14092	0.11317	0.15436	0.15436	0.16628
1 November 2018	0.12795	0.15882	0.14356	0.14356	0.14356	0.16667
1 December 2018	0.12172	0.11357	0.11108	0.08417	0.08418	0.12669
1 January 2019	0.10461	0.13097	0.10311	0.10881	0.10881	0.12652
1 February 2019	0.04699	0.05833	0.04236	0.04825	0.04825	0.05481
1 March 2019	0.09956	0.11617	0.10688	0.09842	0.09842	0.12041
1 April 2019	0.07099	0.07830	0.08055	0.10363	0.10363	0.08677
1 May 2019	0.07546	0.07203	0.07128	0.07143	0.07143	0.07445
1 June 2019	0.06173	0.07472	0.06425	0.06511	0.06457	0.09343
1 July 2019	0.14050	0.13559	0.14333	0.13756	0.14634	0.13558
1 August 2019	0.07357	0.08857	0.09535	0.10416	0.08327	0.11402
1 September 2019	0.05368	0.06360	0.05207	0.05984	0.05984	0.04936
1 October 2019	0.14063	0.16482	0.13385	0.12099	0.12099	0.16620
1 November 2019	0.06858	0.06168	0.07058	0.07231	0.07231	0.05574
1 December 2019	0.08079	0.08534	0.06071	0.04909	0.04909	0.09815
1 January 2020	0.02731	0.03345	0.02903	0.03027	0.03027	0.03496
1 February 2020	0.06595	0.04800	0.08808	0.13353	0.13353	0.05235
MC model b	etter	25	21	23	22	25

Source: the authors' calculations in MATLAB.

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		*		,		
Date	MC Model	NAIVE	MV Middle	MV Max	Max Sharp	MCVaR
1 July 2017	-0.05645	-0.06292	-0.07465	-0.09359	-0.06811	-0.07560
1 August 2017	0.03577	0.02565	0.02260	0.02486	0.03253	0.02559
1 September 2017	0.00297	0.01380	0.00572	-0.01678	0.01417	0.00383
1 October 2017	-0.01723	-0.02591	-0.03030	-0.03836	-0.01826	-0.02971
1 November 2017	0.04395	0.02462	0.03310	0.01488	0.05000	0.01074
1 December 2017	0.01040	0.10020	-0.02426	-0.05243	0.00404	0.02641
1 January 2018	-0.07168	-0.04928	-0.09462	-0.10738	-0.06419	-0.00381
1 February 2018	0.03349	0.00930	-0.00893	-0.03345	0.00466	0.02299
1 March 2018	0.04778	0.03813	0.02425	0.02016	0.01826	0.08550
1 April 2018	-0.01446	0.00911	0.00616	0.01479	0.00598	0.01621
1 May 2018	-0.03046	-0.03979	-0.00839	0.00906	0.00489	-0.04919
1 June 2018	-0.01244	-0.00694	-0.02743	-0.02966	-0.02966	0.00158
1 July 2018	-0.04448	-0.05134	-0.06004	-0.05867	-0.05867	-0.05018
1 August 2018	-0.02712	-0.02684	-0.02849	-0.02849	-0.02849	-0.02431
1 September 2018	0.00013	-0.00348	0.00641	0.00641	0.00641	-0.00126
1 October 2018	-0.05305	-0.06265	-0.05678	-0.08304	-0.08304	-0.06307
1 November 2018	-0.00680	-0.00333	0.00310	0.00310	0.00310	0.00029
1 December 2018	-0.02196	-0.01661	-0.03203	-0.04476	-0.04476	-0.00428
1 January 2019	0.01353	0.04387	0.01459	0.02443	0.02443	0.05353
1 February 2019	0.00689	0.01316	0.00035	-0.00406	-0.00406	0.00622
1 March 2019	-0.02904	-0.03510	-0.03372	-0.03543	-0.03543	-0.04990
1 April 2019	0.00806	0.00734	0.01564	0.02526	0.02526	0.01167
1 May 2019	0.00893	0.00780	0.00419	-0.00493	-0.00493	0.01672
1 June 2019	-0.03842	-0.05909	-0.03652	-0.03673	-0.03660	-0.06384
1 July 2019	0.04833	0.03980	0.04017	0.02479	0.04818	0.03693
1 August 2019	-0.00234	-0.01228	-0.01201	-0.00979	-0.01507	-0.02272
1 September 2019	-0.02812	-0.01824	-0.00708	-0.01410	-0.01410	-0.02744
1 October 2019	-0.00146	-0.01488	-0.00119	0.00666	0.00666	-0.01956

Table A4. Returns of the portfolios in the next trading day.

Source: the authors' calculations in MATLAB.

0.00050

-0.03297

0.08613

0.09245

18

0.00192

-0.03182

0.09150

0.12755

0.00192

-0.03182

0.09150

0.12755

16

-0.00002

-0.04782

0.09386

0.04793

17

References

1 November 2019

1 December 2019

1 January 2020

1 February 2020

MC model better

-0.00238

-0.03441

0.07870

0.07537

-0.00461

-0.04339

0.07880

0.04447

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