Supplementary Materials: Electro-mechanical Whole-Heart Digital Twins: a Fully Coupled Multi-Physics Approach

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1. Numerical schemes

1.1. Cardiac electrical activity

Splitting algorithms for the monodomain equation is a decoupling of the parabolic diffusion problem and the ODE system as proposed for the cardiac system in Sundnes et al. [1]. The equations are devided into two subproblems, a linear PDE

$$\partial_t V_\mathrm{m} = \frac{1}{\beta C_\mathrm{m}} \nabla \cdot (\sigma \nabla V_\mathrm{m}) \,, \tag{1}$$

and a system of space independent ODEs

$$\partial_t V_\mathrm{m} = \frac{1}{C_\mathrm{m}} \big(- I_\mathrm{ion}(V_\mathrm{m}, \mathbf{w}, \mathbf{c}) + I_\mathrm{ext} \big) \,, \tag{2a}$$

$$\partial_t \mathbf{w} = \mathbf{G}_\mathbf{w}(V_\mathrm{m}, \mathbf{w}, \mathbf{c}) \,, \tag{2b}$$

$$\partial_t \mathbf{c} = \mathbf{G}_\mathbf{c}(V_\mathrm{m}, \mathbf{w}, \mathbf{c}) \,. \tag{2c}$$

Let $t_0 = 0, t_1 = \triangle t, \ldots, t_N = N \triangle t = T$ be a uniform partition of $[0, T]$ and $\mathbf{x} \in \Omega_\mathrm{EP}$. Depending on the solutions at time step t_{n-1} denoted by $V_\mathrm{m}^{n-1}(\mathbf{x})$, $\mathbf{w}^{n-1}(\mathbf{x})$ and $\mathbf{c}^{n-1}(\mathbf{x})$ we obtain the solutions $V_\mathrm{m}^{n}(\mathbf{x})$, $\mathbf{w}^{n}(\mathbf{x})$ and $\mathbf{c}^{n}(\mathbf{x})$ at t_n using the first order Godunov splitting algorithm:

1. Solve the ODE system (2) with starting value $V_\mathrm{m}^{n-1}(\mathbf{x})$, $\mathbf{w}^{n-1}(\mathbf{x})$ and $\mathbf{c}^{n-1}(\mathbf{x})$ to get the solutions $\widetilde{V}_\mathrm{m}^{n}(\mathbf{x})$, $\mathbf{w}^{n}(\mathbf{x})$ and $\mathbf{c}^{n}(\mathbf{x})$.
2. Use $\widetilde{V}_\mathrm{m}^{n}(\mathbf{x})$ to solve (1) and get $V_\mathrm{m}^{n}(\mathbf{x})$.

The two steps in the operator splitting method are defined as follows. The reaction part is independent in space so that the solution in every vertex $\{\mathbf{x}_i\}_{i=1}^M$ of the triangulation with $\mathbf{x}_i \in \overline{\Omega}_\mathrm{EP}$ for $i = 1, \ldots, M$ does not depend on information from other vertices. First the vector field $\mathbf{w}^n(\mathbf{x})$ is updated in every vertex and for every component w_i with $i = 1, \ldots, n_\mathbf{w}$ separately. The time integration method is depending on the corresponding ODE belonging to the ith component and on the specific cell model. All updates of the value $V_\mathrm{m}^{n-1}(\mathbf{x})$ at t_{n-1} are evaluated by explicit methods. For the gating mechanism we have to solve

$$\partial_t \mathbf{w} = \mathbf{G}_\mathbf{w}(V_\mathrm{m}, \mathbf{w}) \quad \text{with components} \quad G_{w,i} = \frac{w_i^\infty(V_\mathrm{m}) - w_i}{\tau_i(V_\mathrm{m})} \quad \text{for} \quad i = 1, \ldots, n_\mathbf{w} \,. \tag{3}$$

Here we use the exact integrator method, i.e., w_i^n is computed by

$$w_i^n = w_i^\infty(V_\mathrm{m}^{n-1}) - \Big(w_i^\infty(V_\mathrm{m}^{n-1}) - w_i^{n-1} \Big) \exp\Big(\frac{-\triangle t}{\tau_{w_i}(V_\mathrm{m}^{n-1})} \Big) \,. \tag{4}$$

If the gating mechanism is also depending on an ion concentration, also the old values in $\mathbf{c}^{n-1}(\mathbf{x})$ are used. Then, depending on $V_\mathrm{m}^{n-1}(\mathbf{x})$, $\mathbf{w}^{n}(\mathbf{x})$ and $\mathbf{c}^{n-1}(\mathbf{x})$, we use the explicit Euler method to solve (2a) on every vertex $\mathbf{x}_i$

$$\widetilde{V}_\mathrm{m}^{n}(\mathbf{x}_i) = V_\mathrm{m}^{n-1}(\mathbf{x}_i) + \frac{\triangle t}{C_\mathrm{m}} \big(- I_\mathrm{ion}(V_\mathrm{m}^{n-1}(\mathbf{x}_i), \mathbf{w}^n(\mathbf{x}_i), \mathbf{c}^{n-1}(\mathbf{x}_i)) + I_\mathrm{ext}(t_n, \mathbf{x}_i) \big) \,, \quad i = 1, \ldots, M \,.$$

The ODEs for the different ion concentrations are solved with an explicit Euler method and starting with $V_\mathrm{m}^{n-1}(\mathbf{x})$ and $\mathbf{c}^{n-1}(\mathbf{x})$ at the previous time step and with the updated gating components $\mathbf{w}^{n}(\mathbf{x})$. The variables for the tension model collected in the vector $\mathbf{q}$ are also updated by an explicit Euler method for every component given in Appendix 5 separately.

The diffusion part in the second step is integrated in time by a Crank-Nicolson method such that we get $V_{\mathrm{m}}^n(\mathbf{x})$ at time t_n in Ω_{EP}

$$V_{\mathrm{m}}^n = \widetilde{V}_{\mathrm{m}}^n + \frac{\triangle t}{2\beta C_{\mathrm{m}}}\left(\nabla\cdot(\sigma\nabla V_{\mathrm{m}}^n) + \nabla\cdot(\sigma\nabla\widetilde{V}_{\mathrm{m}}^n)\right),$$

where $\widetilde{V}_{\mathrm{m}}^n$ is the solution from the ODE update in the first step of the splitting algorithm.
For the discretization in space we use standard conforming linear finite elements and end up in solving a symmetric positive definite linear system of dimension M.

1.2. Mechanics

To solve the elasto-dynamic system , we use a *Newmark β-scheme* time integration scheme, where at each time step t_n we compute $\mathbf{u}(\mathbf{x}, t_n) = \mathbf{u}^n$ by approximating $\partial_t^2\mathbf{u}^n \approx \mathbf{a}^n$ with

$$\mathbf{u}^n = \mathbf{u}^{n-1} + \triangle t\mathbf{v}^{n-1} + (\triangle t)^2\left(\frac{1-2\beta_{\mathrm{N}}}{2}\mathbf{a}^{n-1} + \beta_{\mathrm{N}}\mathbf{a}^n\right), \tag{5a}$$

$$\mathbf{v}^n = \mathbf{v}^{n-1} + \triangle t\left((1-\gamma_{\mathrm{N}})\mathbf{a}^{n-1} + \gamma_{\mathrm{N}}\mathbf{a}^n\right) \tag{5b}$$

starting with $\mathbf{a}^0 = \mathbf{0}$. The Newmark system is given by

$$\varrho_0\mathbf{a}^n = \mathrm{div}\left(\mathbf{P}\big(\mathbf{I} + \mathrm{D}\mathbf{u}^n\big) + \mathbf{P}_{\mathrm{a}}(\mathbf{c}^n, \mathbf{q}^n)\right). \tag{6}$$

Solving for $\mathbf{a}^n$ in (5a) and replacing $\mathbf{a}^n$ and $\mathbf{v}^n$ in (6) yields

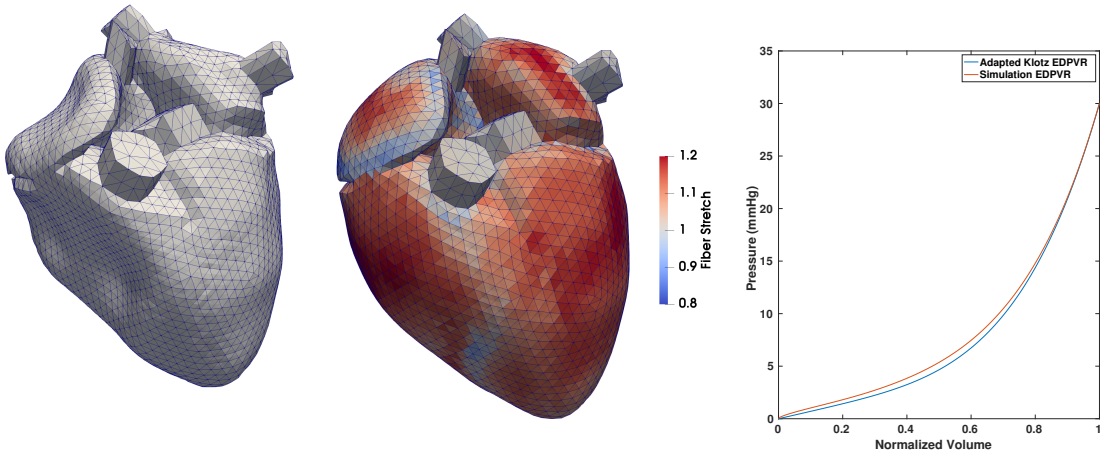
$$\varrho_0\left[\frac{1}{\beta_{\mathrm{N}}\triangle t}\left(\frac{1}{\triangle t}\left(\mathbf{u}^n - \mathbf{u}^{n-1}\right) - \mathbf{v}^{n-1}\right) - \frac{1-2\beta_{\mathrm{N}}}{2\beta_{\mathrm{N}}}\mathbf{a}^{n-1}\right] = \mathrm{div}\left(\mathbf{P}\big(\mathbf{I} + \mathrm{D}\mathbf{u}^n\big) + \mathbf{P}_{\mathrm{a}}(\mathbf{c}^n, \mathbf{q}^n)\right). \tag{7}$$

We use the Newmark parameters $\beta_{\mathrm{N}} = 0.3$ and $\gamma_{\mathrm{N}} = 0.6$.

2. Additional results

2.1. Passive material

Diastolic function of the left ventricle primarily depends on the stiffness of the heart muscle. In clinical practice, the end-diastolic pressure-volume relationship (EDPVR) is typically used to make assumptions about the stiffness of the left ventricle. While volume can be determined using non-invasive measurements such as magnetic resonance imaging (MRI), pressure inside the left ventricle can only be measured invasively by a catheter and is therefore not available in healthy subjects. If no EDPVR data is available, the empirical EDPVR by Klotz et al. [2] is a good alternative. In the simulation framework, the stiffness is determined by the parameters and type of the constitutive law that is used. We use the algorithm presented in Kovacheva et al. [3] to find an optimal parameterization for the constitutive law of Guccione et al. [4]. The algorithm finds a set of parameters that results in an EDPVR with minimal distance to the EDPVR of Klotz et al. (Figure S1c). The corresponding stress-free and pre-stressed configuration are shown in Figure S1a and S1b, respectively.



(a) Stress-free configuration **(b)** Pre-stressed configuration **(c)** EDPVR

Figure S1. (a) Stress-free configuration resulting from fixed-point iterations as described in [5]. (b) Pre-stressed configuration after inflating the stress-free configuration with $p_{\mathrm{LV}} = p_{\mathrm{LA}} = 8.25\,\mathrm{mmHg}$ and $p_{\mathrm{RV}} = p_{\mathrm{RA}} = 3.5\,\mathrm{mmHg}$. (c) End-diastolic pressure volume relationship (EDPVR) of the simulated heart compared to the adapted empirical EDPVR by Klotz et al. [2,3].

2.2. Circulatory system

A comparison of model outputs is only meaningful, if the proposed circulatory system has reached a stable limit cycle. This is the case, if the distribution of blood volume does not change across cardiac cycles. One way to ensure this, is to check, if the stroke volume of both ventricles is the same. For the simulations presented in this study, we assumed that a stable limit cycle was reached as soon as the stroke volume difference between the left and right ventricle was smaller than 1 ml. In case of the reference simulation, it took a total of nine cycles until this criterion was reached (Figure S2). Since the simulation with a scar in the left atrium was started using the stable limit cycle of the reference simulation, it took only four cardiac cycles to adapt to the changed stresses (Figure S3).

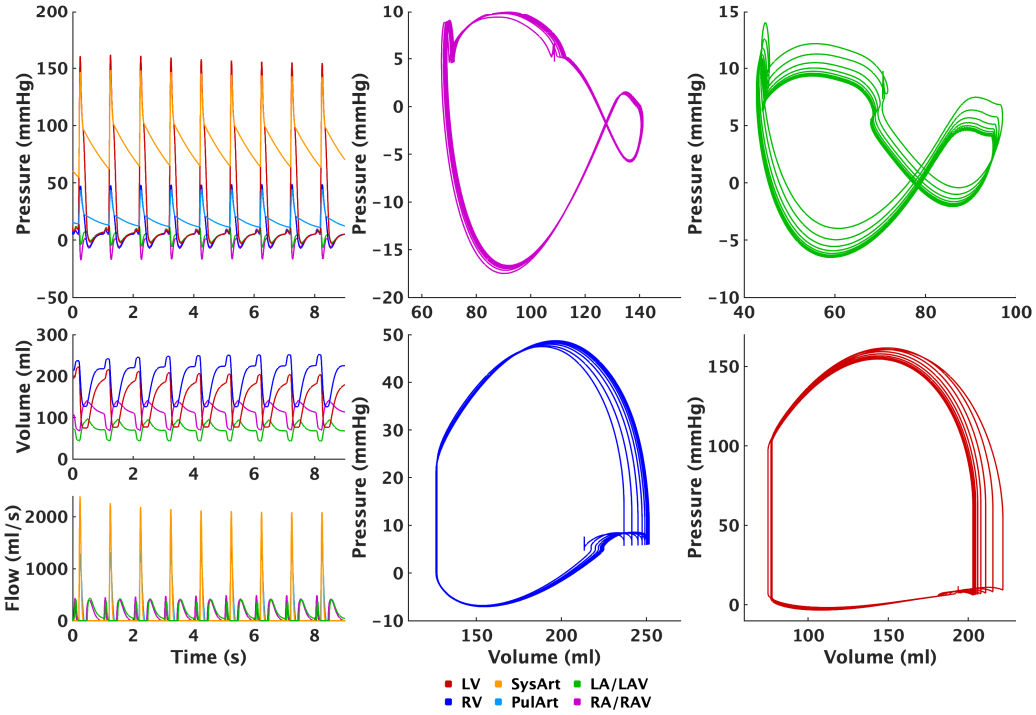


Figure S2. Pressure, volume, flow, and phase plots of the reference simulation. It took a total of nine heart beats to reach a stable limit cycle.

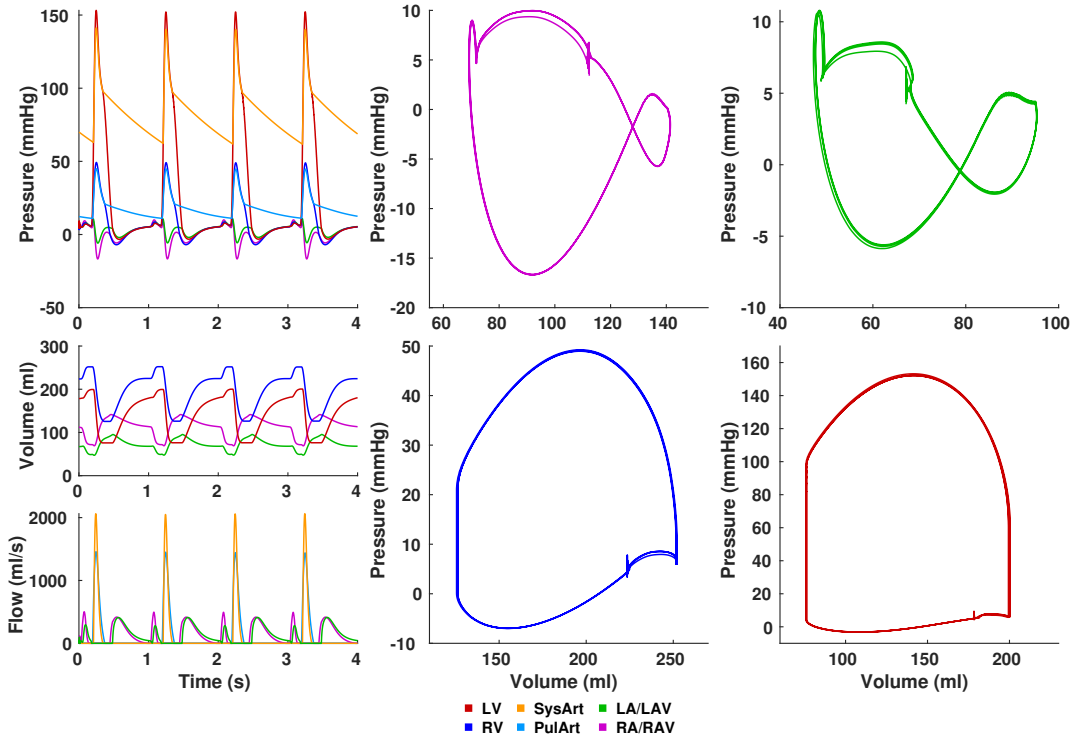


Figure S3. Pressure, volume, flow, and phase plots of the simulation with a scar in the left atrium. It took a total of four heart beats to reach a steady limit cycle.

2.3. Verification of the numerical framework

2.3.1. Electrophysiology

The framework to solve cardiac electrophysiology and more specifically the monodomain equation was verified in an N-version benchmark [6]. In the meantime, the solution method in our framework changed from finite differences to finite elements. Therefore, we include the results for different temporal and spatial resolutions using the numerical framework presented in Section 1.1. For the problem description, we refer the reader to the original publication [6]. The activation times for the different spatial and temporal discretizations are shown in Figure S4 and are in close agreement with the results in [6].

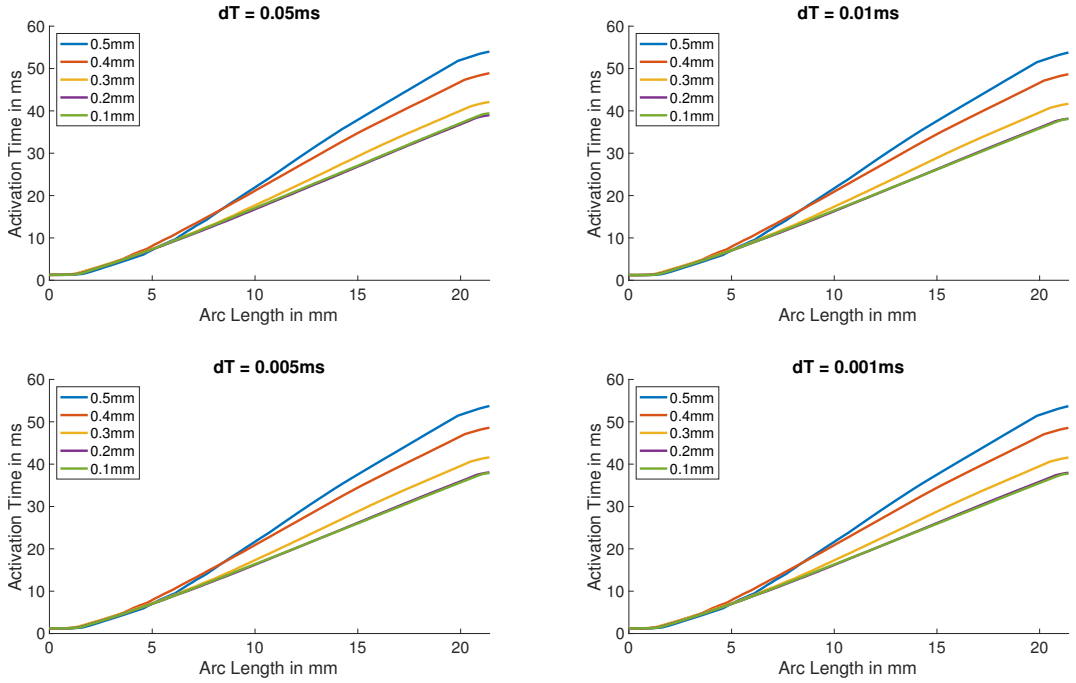


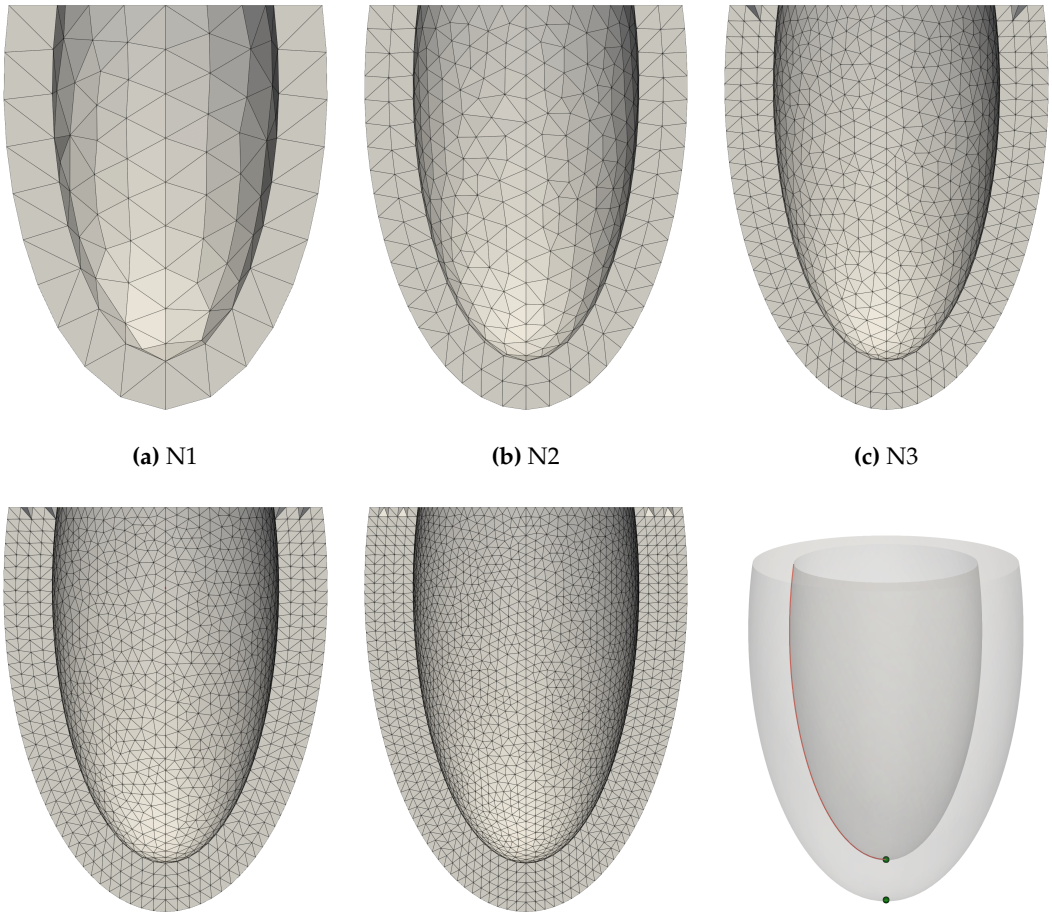
Figure S4. Activation times for the different spatial (different colors) and temporal (different subplots) discretizations on the diagonal line.

2.3.2. Mechanics

Similarly, the mechanical framework was verified in Land et al. [7]. To investigate the influence of spatial discretization and the order of the used finite element, we ran simulations based on problem description 3 in [7]. We chose problem three, since it captures several aspects important to cardiac mechanics that apply to the whole heart as well. It includes a pressure boundary condition that depends on the deformed surface orientation and area as well as the inclusion of active stress. Furthermore, it utilizes the same transverse isotropic constitutive law and a complex fiber distribution throughout the myocardial wall. We used five different discretizations (Figure S5) and simulated each discretization with linear and quadratic tetrahedral elements.
For each simulation, we evaluate three important aspects of left ventricular function:

1) apico-basal shortening in terms of the deformed apex position;
2) twist in terms of the deformation of an endocardial line;
3) cavity volume.

The results are shown in Figure S6. It becomes clear that quadratic tetrahedral elements suffer less from volumetric locking effects especially with fewer degrees of freedom (DoF). With the exception of discretization N1, all simulations with quadratic elements show the same behavior. Linear tetrahedral



(a) N1 **(b)** N2 **(c)** N3

(d) N4 **(e)** N5 **(f)**

Figure S5. Discretizations used to solve problem three with degrees of freedom (DoF) for linear and quadratic elements: (a) N1, 1098/6450 DoF; (b) N2, 5343/35460 DoF; (c) N3, 15768/110643 DoF; (d) N4, 33819/245430 DoF; (e) N5, 63357/468834 DoF. (f) line and points that were used to evaluate the deformed configuration.

elements show larger deviations in twist close to the base and are less accurate for the calculation of the cavity volume. The spatial resolution of N2 is similar to the one used in our whole-heart model.

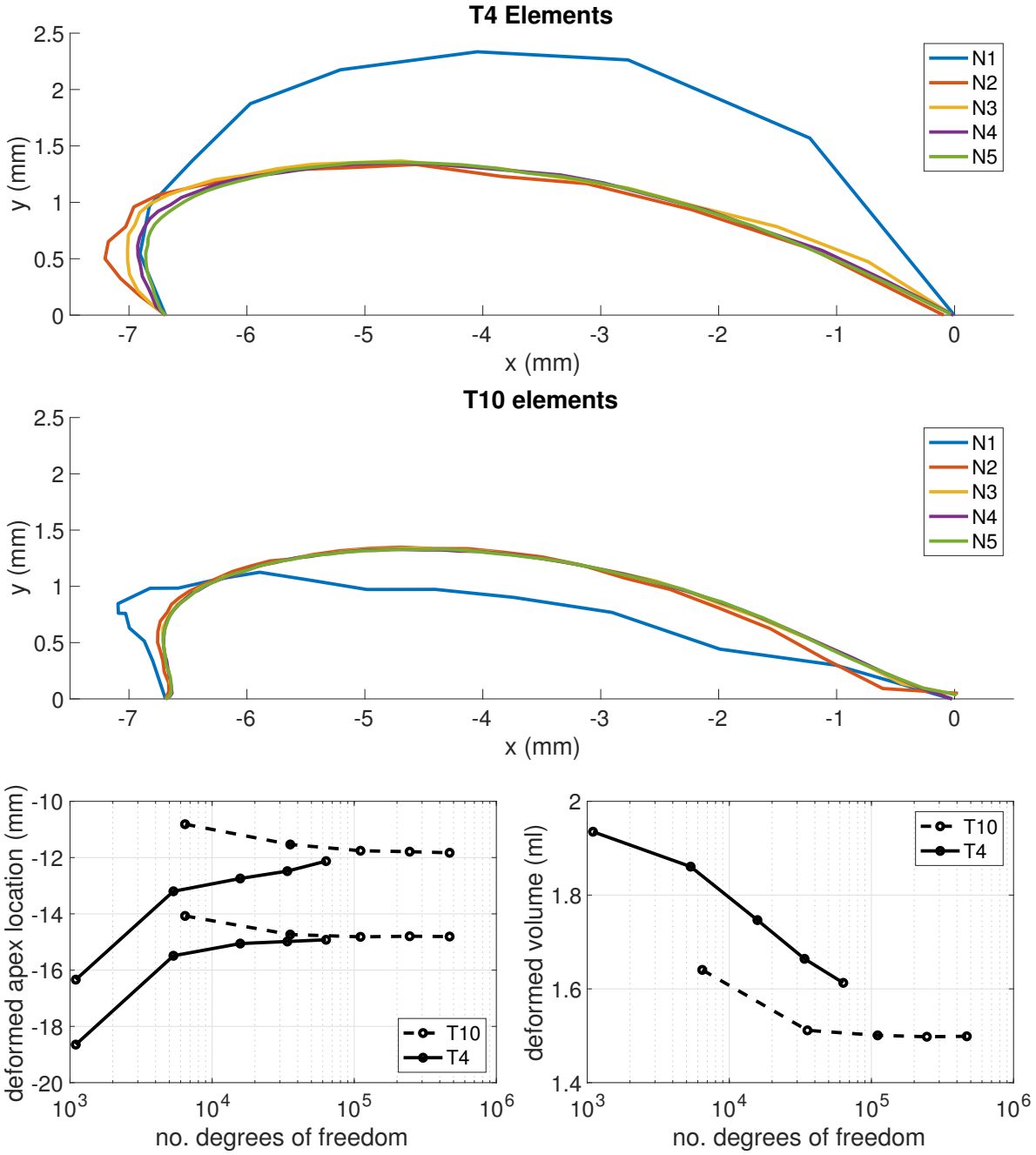


Figure S6. Top: deformation of a line that shows twist using linear elements. Middle: deformation of a line that shows twist using quadratic elements. Bottom left: endocardial and epicardial apex z-coordinate in the deformed configuration. Bottom right: cavity volume of the deformed configuration. T4: linear elements, T10: quadratic elements.

3. Courtemanche et al. model

In the atria, the Courtemanche cell model [8] is realized. It involves the intracellular ion concentration of calcium Ca, $\mathrm{Ca_{up}}$, $\mathrm{Ca_{rel}}$, sodium Na, and potassium K, and the gating variables such that

$$\begin{aligned}\mathbf{c} &= (\mathrm{Ca}, \mathrm{Ca_{up}}, \mathrm{Ca_{rel}}, \mathrm{Na}, \mathrm{K}, \mathrm{CaTRPN}) \in \mathbb{R}_+^{n_\mathbf{c}}\,,\\ \mathbf{w} &= \left(m, h, j, o_\mathrm{a}, o_\mathrm{i}, u_\mathrm{a}, u_\mathrm{i}, x_\mathrm{r}, x_\mathrm{s}, d, f, f_\mathrm{Ca}, u, v, w_\mathrm{rel}\right) \in [0,1]^{n_\mathbf{w}}\,,\end{aligned}$$

with $n_\mathbf{c} = 6$ and $n_\mathbf{w} = 15$. The evolution of the cell model is determined by I_ion and the function $\mathbf{G_w}$ and $\mathbf{G_c}$ specified in the following.

3.1. Ionic currents

The transmembrane current density in (2a) is given by $K = 12$ different ionic currents

$$\begin{aligned}I_\mathrm{ion}(V, \mathbf{w}, \mathbf{c}) = {} & I_\mathrm{Na}(V_\mathrm{m}, \mathrm{Na}, m, h, j) + I_\mathrm{CaL}(V_\mathrm{m}, d, f, f_\mathrm{Ca}) + I_\mathrm{Ks}(V_\mathrm{m}, \mathrm{K}, x_\mathrm{s}) + I_\mathrm{to}(V_\mathrm{m}, \mathrm{K}, o_\mathrm{a}, o_\mathrm{i})\\ & + I_\mathrm{Kr}(V_\mathrm{m}, \mathrm{K}, x_\mathrm{r}) + I_\mathrm{Kur}(V_\mathrm{m}, \mathrm{K}, u_\mathrm{a}, u_\mathrm{i}) + I_\mathrm{K1}(V_\mathrm{m}, \mathrm{K}) + I_\mathrm{NaCa}(V_\mathrm{m}, \mathrm{Na}, \mathrm{Ca})\\ & + I_\mathrm{NaK}(V_\mathrm{m}, \mathrm{Na}) + I_\mathrm{pCa}(\mathrm{Ca}) + I_\mathrm{bCa}(V_\mathrm{m}, \mathrm{Ca}) + I_\mathrm{bNa}(V_\mathrm{m}, \mathrm{Na})\end{aligned}$$

with

$$\begin{aligned}I_\mathrm{Na}(V_\mathrm{m}, \mathrm{Na}, m, h, j) &= 7.8 \cdot m^3 hj\big(V_\mathrm{m} - E_\mathrm{Na}(\mathrm{Na})\big)\,,\\
I_\mathrm{CaL}(V_\mathrm{m}, d, f, f_\mathrm{Ca}) &= 0.12375 \cdot dff_\mathrm{Ca}\big(V_\mathrm{m} - 65\big)\,,\\
I_\mathrm{Ks}(V_\mathrm{m}, \mathrm{K}, x_\mathrm{s}) &= 0.12941176 \cdot x_\mathrm{s}^2\big(V_\mathrm{m} - E_\mathrm{K}(\mathrm{K})\big)\,,\\
I_\mathrm{to}(V_\mathrm{m}, \mathrm{K}, o_\mathrm{a}, o_\mathrm{i}) &= 0.1652 \cdot o_\mathrm{a}^3 o_\mathrm{i}\big(V_\mathrm{m} - E_\mathrm{K}(\mathrm{K})\big)\,,\\
I_\mathrm{Kr}(V_\mathrm{m}, \mathrm{K}, x_\mathrm{r}) &= \frac{0.029411765 \cdot x_\mathrm{r}\sqrt{\frac{\mathrm{K}_e}{5.4}} \cdot \big(V_\mathrm{m} - E_\mathrm{K}(\mathrm{K})\big)}{1 + \exp\left(\frac{V_\mathrm{m}+15}{22.4}\right)}\,,\\
I_\mathrm{K1}(V_\mathrm{m}, \mathrm{K}) &= \frac{0.09\sqrt{\frac{\mathrm{K}_e}{5.4}} \cdot (V_\mathrm{m} - E_\mathrm{K}(\mathrm{K})}{1 + \exp\big(0.07(V_\mathrm{m} + 80)\big)}\,,\\
I_\mathrm{Kur}(V_\mathrm{m}, \mathrm{K}, u_\mathrm{a}, u_\mathrm{i}) &= \frac{0.055 \cdot u_\mathrm{a}^3 u_\mathrm{i}\big(V_\mathrm{m} - E_\mathrm{K}(\mathrm{K})\big)}{1 + \exp\left(\frac{-V_\mathrm{m}+15}{13}\right)}\,,\\
I_\mathrm{NaCa}(V_\mathrm{m}, \mathrm{Na}, \mathrm{Ca}) &= \frac{1600 \cdot \exp\left(\frac{0.35 V_\mathrm{m} F}{RT}\right) \cdot \left(\mathrm{Na}^3\mathrm{Ca_e} - \exp(\frac{-V_\mathrm{m}F}{RT})\mathrm{Na_e^3Ca}\right)}{\left(87.5^3 + \mathrm{Na_e^3}\right)\left(1.38 + \mathrm{Ca_e}\right)\left(1 + 0.1\exp\left(\frac{-0.65 \cdot V_\mathrm{m} F}{RT}\right)\right)}\,,\\
I_\mathrm{NaK}(V_\mathrm{m}, \mathrm{Na}) &= \frac{0.59933874 \cdot \mathrm{K}_e}{\left(1 + 0.1245\exp\left(\frac{-0.1 V_\mathrm{m} F}{RT}\right) + \frac{0.365}{7}\left(\exp(\frac{\mathrm{Na}_e}{67.3}) - 1\right)\exp\left(\frac{-V_\mathrm{m}F}{RT}\right)\right)\left(\mathrm{K}_e + 1.5\right)\left(1 + \frac{10}{\mathrm{Na}} \cdot \sqrt{\frac{10}{\mathrm{Na}}}\right)}\,,\\
I_\mathrm{pCa}(\mathrm{Ca}) &= \frac{0.275 \cdot \mathrm{Ca}}{0.0005 + \mathrm{Ca}}\,,\\
I_\mathrm{bCa}(V_\mathrm{m}, \mathrm{Ca}) &= 0.001131 \cdot \big(V_\mathrm{m} - E_\mathrm{Ca}(\mathrm{Ca})\big)\,,\\
I_\mathrm{bNa}(V_\mathrm{m}, \mathrm{Na}) &= 0.0006744375 \cdot \big(V_\mathrm{m} - E_\mathrm{Na}(\mathrm{Na})\big)\,,\end{aligned}$$

where the reverse potentials are given by

$$E_{\mathrm{Na}}(\mathrm{Na}) = \frac{RT}{F}\log\frac{[\mathrm{Na}]_e}{\mathrm{Na}},$$
$$E_{\mathrm{K}}(\mathrm{K}) = \frac{RT}{F}\log\frac{[\mathrm{K}]_e}{\mathrm{K}},$$
$$E_{\mathrm{Ca}}(\mathrm{Ca}) = \frac{RT}{2F}\log\frac{[\mathrm{Ca}]_e}{\mathrm{Ca}}.$$

3.2. Gating mechanism

For the evolution of the gating variables (2b) we specify every gating component independently. The gating variable $y \in \{m, h, j, o_{\mathrm{a}}, o_{\mathrm{i}}, u_{\mathrm{a}}, u_{\mathrm{i}}, x_{\mathrm{r}}, x_{\mathrm{s}}, d, f, w_{\mathrm{rel}}\}$ is described by a Hodgkin-Huxley type equation as given in (3). The evolution of the components $\{u, v, f_{\mathrm{Ca}}\}$ is not of Hodgkin-Huxley type and are defined at the end of this section. Starting with the Hodgkin-Huxley type gating mechanisms the functions $\tau_y(V_{\mathrm{m}})$ and $y_\infty(V_{\mathrm{m}})$ have to be defined for every y. All expressions depending on V_{m} are given in mV, i.e., in $\exp\left((V_{\mathrm{m}}+80)/6.8\right)$ the qualities 80 and 6.8 have the physical unit mV.
Here, we use for the gating variables $\{m, h, j\}$

$$\tau_y(V_{\mathrm{m}}) = \frac{1}{a_y(V_{\mathrm{m}}) + b_y(V_{\mathrm{m}})},$$
$$y_\infty(V_{\mathrm{m}}) = \frac{a_y(V_{\mathrm{m}})}{a_y(V_{\mathrm{m}}) + b_y(V_{\mathrm{m}})},$$

with

$$a_m(V_{\mathrm{m}}) = \begin{cases} 3.2 & V_{\mathrm{m}} = -47.13, \\ 0.32\frac{V_{\mathrm{m}}+47.13}{1-\exp\left(-0.1(47.13+V_{\mathrm{m}})\right)} & \text{else}, \end{cases}$$
$$b_m(V_{\mathrm{m}}) = 0.08\exp\left(-\frac{V_{\mathrm{m}}}{11}\right),$$

$$a_h(V_{\mathrm{m}}) = \begin{cases} 0 & V_{\mathrm{m}} \geq -40, \\ 0.135\exp\left(-\frac{V_{\mathrm{m}}+80}{6.8}\right) & \text{else}, \end{cases}$$
$$b_h(V_{\mathrm{m}}) = \begin{cases} \left(0.13\left(1+\exp\left(-\frac{10.66+V_{\mathrm{m}}}{11.1}\right)\right)\right)^{-1} & V_{\mathrm{m}} \geq -40, \\ 3.56\exp(0.079V_{\mathrm{m}}) + 3.1\cdot 10^5\exp(0.35V_{\mathrm{m}}) & \text{else}, \end{cases}$$

and

$$a_j(V_{\mathrm{m}}) = \begin{cases} 0 & V_{\mathrm{m}} \geq -40, \\ \frac{\left(-127140\exp(0.2444V_{\mathrm{m}}) - 3.474\cdot 10^{-5}\exp(-0.04391V_{\mathrm{m}})\right)(37.78+V_{\mathrm{m}})}{1+\exp\left(0.311(79.23+V_{\mathrm{m}})\right)} & \text{else}, \end{cases}$$
$$b_j(V_{\mathrm{m}}) = \begin{cases} 0.3\frac{\exp(-2.535\cdot 10^{-7}V_{\mathrm{m}})}{1+\exp(-0.1(V_{\mathrm{m}}+32))} & V_{\mathrm{m}} \geq -40, \\ 0.1212\frac{\exp(-0.01052V_{\mathrm{m}})}{1+\exp(-0.1378(V_{\mathrm{m}}+40.14))} & \text{else}, \end{cases}$$

For the gating variables $\{o_{\mathrm{a}}, o_{\mathrm{i}}, u_{\mathrm{a}}, u_{\mathrm{i}}, x_{\mathrm{r}}, x_{\mathrm{s}}, d, f, w_{\mathrm{rel}}\}$ we use

$$\tau_{o_{\mathrm{a}}}(V_{\mathrm{m}}) = \Big(3\big(\frac{0.65}{\exp\big(-\frac{10+V_{\mathrm{m}}}{8.5}\big)+\exp\big(-\frac{-30+V_{\mathrm{m}}}{59}\big)}+\frac{0.65}{2.5+\exp\big(\frac{82+V_{\mathrm{m}}}{17}\big)}\big)\Big)^{-1},$$
$$o_{\mathrm{a},\infty}(V_{\mathrm{m}}) = \Big(1+\exp\big(-\frac{20.47+V_{\mathrm{m}}}{17.54}\big)\Big)^{-1},$$

$$\tau_{o_{\mathrm{i}}}(V_{\mathrm{m}}) = \Big(3\big(\frac{1}{18.53+\exp\big(\frac{113.7+V_{\mathrm{m}}}{10.95}\big)}+\frac{1}{35.56+\exp\big(-\frac{1.26+V_{\mathrm{m}}}{7.44}()\big)}\big)\Big)^{-1},$$
$$o_{\mathrm{i},\infty}(V_{\mathrm{m}}) = \Big(1+\exp\big((43.1+V_{\mathrm{m}})/5.3\big)\Big)^{-1},$$

$$\tau_{u_{\mathrm{a}}}(V_{\mathrm{m}}) = \tau_{o_{\mathrm{a}}}(V_{\mathrm{m}}),$$
$$u_{\mathrm{a},\infty}(V_{\mathrm{m}}) = \Big(1+\exp\big(-\frac{30.3+V_{\mathrm{m}}}{9.6}\big)\Big)^{-1},$$

$$\tau_{u_{\mathrm{i}}}(V_{\mathrm{m}}) = \Big(3\big(\frac{1}{21+\exp\big(\frac{-185+V_{\mathrm{m}}}{28}\big)}+\exp\big(\frac{-158+V_{\mathrm{m}}}{16}\big)\big)\Big)^{-1},$$
$$u_{\mathrm{i},\infty}(V_{\mathrm{m}}) = \Big(1+\exp\big(\frac{-99.45+V_{\mathrm{m}}}{27.48}\big)\Big)^{-1},$$

$$\tau_{x_{\mathrm{r}}}(V_{\mathrm{m}}) = \Big(0.0003\frac{14.1+V_{\mathrm{m}}}{1-\exp(-(14.1+V_{\mathrm{m}})/5)}+7.3898\cdot10^{-5}\frac{-3.3328+V_{\mathrm{m}}}{\exp((-3.3328+V_{\mathrm{m}})/5.1237)-1}\Big)^{-1},$$
$$x_{\mathrm{r},\infty}(V_{\mathrm{m}}) = \Big(1+\exp\big(-\frac{14.1+V_{\mathrm{m}}}{6.5}\big)\Big)^{-1},$$

$$\tau_{x_{\mathrm{s}}}(V_{\mathrm{m}}) = 0.5\Big(4\cdot10^{-5}\frac{-19.9+V_{\mathrm{m}}}{1-\exp\big(-\frac{-19.9+V_{\mathrm{m}}}{17}\big)}+3.5\cdot10^{-5}\frac{-19.9+V_{\mathrm{m}}}{\exp\big(\frac{-19.9+V_{\mathrm{m}}}{9}\big)-1}\Big)^{-1},$$
$$x_{\mathrm{s},\infty}(V_{\mathrm{m}}) = \Big(1+\exp\big(-\frac{-19.9+V_{\mathrm{m}}}{12.7}\big)\Big)^{-\frac{1}{2}},$$

$$\tau_d(V_{\mathrm{m}}) = \begin{cases}\Big(\frac{4.579}{1+\exp\big(-\frac{10+V_{\mathrm{m}}}{6.24}\big)}\Big)^{-1} & V_{\mathrm{m}} = -10,\\ \Big(\frac{1.0-\exp\big(-\frac{10+V_{\mathrm{m}}}{6.24}\big)}{1+\exp\big(-\frac{10+V_{\mathrm{m}}}{6.24}\big)}\Big)^{-1} & \text{else},\end{cases}$$
$$d_\infty(V_{\mathrm{m}}) = \Big(1+\exp\big(-\frac{10+V_{\mathrm{m}}}{8}\big)\Big)^{-1},$$

$$\tau_f(V_{\mathrm{m}}) = \Big(\frac{9}{0.0197\exp\big(-0.0337^2(V+10.0)^2\big)+0.02}\Big)^{-1},$$
$$f_\infty(V_{\mathrm{m}}) = \frac{\exp\big(-\frac{28+V_{\mathrm{m}}}{6.9}\big)}{1+\exp\big(-\frac{28+V_{\mathrm{m}}}{6.9}\big)},$$

$$\tau_{w_{\mathrm{rel}}}(V_{\mathrm{m}}) = \begin{cases} \left(\frac{6\cdot 0.2}{1.3}\right)^{-1} & V_{\mathrm{m}} = 7.9\,, \\ 6\dfrac{1-\exp\left(-\frac{-7.9+V_{\mathrm{m}}}{5}\right)}{\left(-7.9+V_{\mathrm{m}}\right)\left(1+0.3\exp\left(-\frac{(-7.9+V_{\mathrm{m}}}{5}\right)\right)} & \text{else}\,, \end{cases}$$

$$w_{\mathrm{rel},\infty}(V_{\mathrm{m}}) = 1-\left(1+\exp\left(-\frac{-40+V_{\mathrm{m}}}{17}\right)\right)^{-1}.$$

The gating variables $\{u, v, f_{\mathrm{Ca}}\}$ are not of Hodgkin Huxley type. Here we have to solve

$$\begin{aligned} \partial_t u &= 0.125\Big(\frac{1}{1+\exp(250)\cdot F_{\mathrm{n}}(V_{\mathrm{m}},\mathrm{Ca},\mathrm{Ca}_{\mathrm{rel}},\mathrm{Na},d,f,f_{\mathrm{Ca}},u,v,w_{\mathrm{rel}})}-u\Big)\,, \\ \partial_t v &= \Big(1.0-\frac{1}{1+\exp(50)F_{\mathrm{n}}(V_{\mathrm{m}},\mathrm{Ca},\mathrm{Ca}_{\mathrm{rel}},\mathrm{Na},d,f,f_{\mathrm{Ca}},u,v,w_{\mathrm{rel}})}-v\Big) \\ &\quad\cdot\Big(1.91+2.09\cdot\frac{1}{1+\exp(250)\cdot F_{\mathrm{n}}(V_{\mathrm{m}},\mathrm{Ca},\mathrm{Ca}_{\mathrm{rel}},\mathrm{Na},d,f,f_{\mathrm{Ca}},u,v,w_{\mathrm{rel}})}\Big)^{-1}, \\ \partial_t f_{\mathrm{Ca}} &= 0.5\Big(\frac{1}{1+2857.1429\cdot\mathrm{Ca}}-f_{\mathrm{Ca}}\Big)\,, \end{aligned}$$

with

$$\begin{aligned} &I_{\mathrm{rel}}(\mathrm{Ca},\mathrm{Ca}_{\mathrm{rel}},u,v,w_{\mathrm{rel}}) = 30\cdot u^2 v w_{\mathrm{rel}}\big(\mathrm{Ca}_{\mathrm{rel}}-\mathrm{Ca}\big)\,, \\ &F_{\mathrm{n}}(V_{\mathrm{m}},\mathrm{Ca},\mathrm{Ca}_{\mathrm{rel}},\mathrm{Na},d,f,f_{\mathrm{Ca}},u,v,w_{\mathrm{rel}}) \\ &= 96.48\cdot 10^{-12} I_{\mathrm{rel}}(\mathrm{Ca},\mathrm{Ca}_{\mathrm{rel}},u,v,w_{\mathrm{rel}}) - \frac{5\cdot 10^{-13}}{F}\Big(0.5 I_{\mathrm{CaL}}(V_{\mathrm{m}},d,f,f_{\mathrm{Ca}}) - 0.2 I_{\mathrm{NaCa}}(V_{\mathrm{m}},\mathrm{Na},\mathrm{Ca})\Big)\,. \end{aligned}$$

3.3. Ion concentrations

The evolution for the different ion concentrations **c** in (2c) is given by

$$\begin{aligned} \partial_t \mathrm{Na} &= -\frac{100}{V_{\mathrm{i}}F}\Big(I_{\mathrm{Na}}(V_{\mathrm{m}},\mathrm{Na},m,h,j) + I_{\mathrm{bNa}}(V_{\mathrm{m}},\mathrm{Na}) + 3I_{\mathrm{NaK}}(V_{\mathrm{m}},\mathrm{Na}) + 3I_{\mathrm{NaCa}}(V_{\mathrm{m}},\mathrm{Na},\mathrm{Ca})\Big)\,, \\ \partial_t \mathrm{K} &= -\frac{100}{V_{\mathrm{i}}F}\Big(I_{\mathrm{K1}}(V_{\mathrm{m}},\mathrm{K}) + I_{\mathrm{to}}(V_{\mathrm{m}},\mathrm{K},o_{\mathrm{a}},o_{\mathrm{i}}) + I_{\mathrm{Kr}}(V_{\mathrm{m}},\mathrm{K},x_{\mathrm{r}}) \\ &\qquad + I_{\mathrm{Ks}}(V_{\mathrm{m}},\mathrm{K},x_{\mathrm{s}}) - 2I_{\mathrm{NaK}}(V_{\mathrm{m}},\mathrm{Na}) + I_{\mathrm{Kur}}(V_{\mathrm{m}},\mathrm{K},u_{\mathrm{a}},u_{\mathrm{i}})\Big)\,, \\ \partial_t \mathrm{Ca} &= \Big(\frac{100(-I_{\mathrm{bCa}}(V_{\mathrm{m}},\mathrm{Ca}) - I_{\mathrm{pCa}}(\mathrm{Ca}) - I_{\mathrm{CaL}}(V_{\mathrm{m}},d,f,f_{\mathrm{Ca}}) + 2I_{\mathrm{NaCa}}(V_{\mathrm{m}},\mathrm{Na},\mathrm{Ca}))}{2V_{\mathrm{i}}F} \\ &\qquad + \frac{1}{V_{\mathrm{i}}}\Big(\frac{11109.52\cdot\mathrm{Ca}_{\mathrm{up}}}{3000} - \frac{0.005\cdot\mathrm{Ca}}{1+\frac{0.00092}{\mathrm{Ca}}} + 96.48\cdot 30\cdot u^2 v w_{\mathrm{rel}}\big(\mathrm{Ca}_{\mathrm{rel}}-\mathrm{Ca}\big)\Big) \\ &\qquad - 0.07\cdot G_{\mathrm{CaTRPN}}(\mathrm{CaTRPN},\mathrm{Ca},\gamma_{\mathbf{f}})\Big) \\ &\quad\cdot\Big(1+\frac{0.070\cdot 0.0005}{(\mathrm{Ca}+0.0005)^2}+\frac{0.05\cdot 0.00238}{(\mathrm{Ca}+0.00238)^2}\Big)^{-1}, \\ \partial_t \mathrm{Ca}_{\mathrm{up}} &= \frac{0.005\cdot\mathrm{Ca}}{1+\frac{0.00092}{\mathrm{Ca}}} - \frac{\mathrm{Ca}_{\mathrm{up}}}{3000} - \frac{2(\mathrm{Ca}_{\mathrm{up}}-\mathrm{Ca}_{\mathrm{rel}})}{4140}\,, \\ \partial_t \mathrm{Ca}_{\mathrm{rel}} &= \Big(\frac{\mathrm{Ca}_{\mathrm{up}}-\mathrm{Ca}_{\mathrm{rel}}}{180} - I_{\mathrm{rel}}(\mathrm{Ca},\mathrm{Ca}_{\mathrm{rel}},u,v,w_{\mathrm{rel}})\Big)\Big(1+\frac{8}{(\mathrm{Ca}_{\mathrm{rel}}+0.8)^2}\Big)^{-1}. \end{aligned}$$

For the fraction of troponin C units with calcium bound to its regular binding site $\{\mathrm{CaTRPN}\}$ it is

$$\begin{aligned}\partial_t \mathrm{CaTRPN} &= G_{\mathrm{CaTRPN}}(\mathrm{CaTRPN}, \mathrm{Ca}, \gamma_{\mathbf{f}}) \\ &= 0.1\big(1 - \mathrm{CaTRPN}\big)\Big(\frac{1000 \cdot \mathrm{Ca}}{\mathrm{Ca}_{\mathrm{T50}}(\gamma_{\mathbf{f}})}\Big)^2 - \mathrm{CaTRPN}\,,\end{aligned}$$

where $\mathrm{Ca}_{\mathrm{T50}}(\gamma_{\mathbf{f}})$ is from the Land model described in Appendix 5.

3.4. Initial conditions and parameters

For the Courtemanche cell model, the parameters are given in Tab. S2, and initial values for the ion concentrations and gating variables at $t = 0$ and for all $\mathbf{x} \in \Omega_{\mathrm{A}} \subset \Omega_{\mathrm{EP}}$ are given in Table S1.

Table S1. Initial values in the Courtemanche cell model.

	one cell	**whole heart**
V_{m}^0	−80.8887 mV	−81.39546 mV
Ca^0	0.000112836 mM	0.0001076277 mM
$\mathrm{Ca}_{\mathrm{up}}^0$	1.52919 mM	1.466083 mM
$\mathrm{Ca}_{\mathrm{rel}}^0$	1.10817 mM	1.036785 mM
Na^0	11.183 mM	13.10342 mM
K^0	138.994 mM	134.0914 mM
m^0	0.00304588	0.002805413
h^0	0.962696	0.9665666
j^0	0.975742	0.97864
o_{a}^0	0.0309106	0.03006788
o_{i}^0	0.999163	0.9992734
u_{a}^0	0.00511314	0.00485487
u_{i}^0	0.986906	0.9916856
x_{r}^0	0.00229885	0.0007523458
x_{s}^0	0.0196603	0.0189571
d^0	0.000141583	0.0001330054
f^0	0.916064	0.949446
f_{Ca}^0	0.75607	0.7647746
u^0	0	0
v^0	0.999994	1
w_{rel}^0	0.999185	0.9992088
CaTRPN^0		0.0104203

Table S2. Parameters in the Courtemanche cell model.

gas constant	$R = 8314.472\,\mathrm{mJK^{-1}mol^{-1}}$
temperature	$T = 310$ K
Faraday constant	$F = 96.4867\,\mathrm{C/mmol}$
intracellular volume	$V_{\mathrm{i}} = 13668$
extracellular K^+ concentration	$[\mathrm{K}]_e = 5.4$ mM
extracellular Na^+ concentration	$[\mathrm{Na}]_e = 140$ mM
extracellular Ca^{2+} concentration	$[\mathrm{Ca}]_e = 1.8$ mM

4. O'Hara et al. model

For the ventricles we use the O'Hara Rudy model [9]. In this model the ion concentration of calcium Ca, sodium Na and potassium K is included and the vector is

$$\mathbf{c} = (\mathrm{Na}, \mathrm{Na_{ss}}, \mathrm{K}, \mathrm{K_{ss}}, \mathrm{Ca}, \mathrm{Ca_{ss}}, \mathrm{Ca_{nsr}}, \mathrm{Ca_{jsr}}, \mathrm{CaTRPN}), \qquad n_{\mathbf{c}} = 9.$$

The vector of gating variables is extended to $n_{\mathbf{w}} = 32$ components

$$\begin{aligned}\mathbf{w} =& (m, h_{\mathrm{fast}}, h_{\mathrm{slow}}, h_{\mathrm{CaMK,slow}}, j, j_{\mathrm{CaMK}}, m_{\mathrm{L}}, h_{\mathrm{L}}, h_{\mathrm{L,CaMK}}, d, f_{\mathrm{fast}}, f_{\mathrm{slow}}, f_{\mathrm{Ca,fast}}, f_{\mathrm{Ca,slow}}, j_{\mathrm{Ca}}, f_{\mathrm{CaMK,fast}},\\ & f_{\mathrm{Ca,CaMK,fast}}, x_{\mathrm{s1}}, x_{\mathrm{s2}}, a, a_{\mathrm{CaMK}}, i_{\mathrm{fast}}, i_{\mathrm{slow}}, i_{\mathrm{CaMK,fast}}, i_{\mathrm{CaMK,slow}}, x_{\mathrm{r,fast}}, x_{\mathrm{r,slow}}, x_{\mathrm{K1}},\\ & n, \mathrm{CaMK_{trap}}, J_{\mathrm{rel,NP}}, J_{\mathrm{rel,CaMK}}), \end{aligned} \tag{8}$$

where the first 28 variables are of Hudgkin-Huxley type and the evolution is given in 4.2, and evolution of the last four elements of **w** is given in 4.3. Initial values and parameters are collected in Tables S5 and S6.

4.1. Ionic currents

The transmembrane current density $I_{\mathrm{ion}}(V_{\mathrm{m}}, \mathbf{w})$ is defined by $K = 16$ different ionic currents

$$\begin{aligned} I_{\mathrm{ion}}(V_{\mathrm{m}}, \mathbf{w}, \mathbf{c}) = & I_{\mathrm{Na,fast}} + I_{\mathrm{Na,late}} + I_{\mathrm{CaL}} + I_{\mathrm{CaNa}} + I_{\mathrm{CaK}} + I_{\mathrm{Ks}} + 4 \cdot I_{\mathrm{to}} \\ & + 0.8 \cdot I_{\mathrm{Kr}} + I_{\mathrm{Kb}} + 1.3 \cdot I_{\mathrm{K1}} + 1.4 \cdot I_{\mathrm{NaCa}} + 1.4 \cdot I_{\mathrm{NaCass}} \\ & + 0.7 \cdot I_{\mathrm{NaK}} + I_{\mathrm{pCa}} + I_{\mathrm{Cab}} + I_{\mathrm{Nab}}. \end{aligned}$$

With the reverse potentials

$$\begin{aligned} E_{\mathrm{Na}}(\mathrm{Na}) &= \frac{RT}{F} \ln\left(\frac{[\mathrm{Na}]_e}{\mathrm{Na}}\right), \\ E_{\mathrm{K}}(\mathrm{K}) &= \frac{RT}{F} \ln\left(\frac{[\mathrm{K}]_e}{\mathrm{K}}\right), \\ E_{\mathrm{Ks}}(\mathrm{Na}, \mathrm{K}) &= \frac{RT}{F} \ln\left(\frac{[\mathrm{K}]_e + 0.01833 \cdot [\mathrm{Na}]_e}{\mathrm{K} + 0.01833\mathrm{Na}}\right), \end{aligned}$$

and simplifications

$$\begin{aligned} \mathrm{CaMK_{bound}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}) &= 0.05 \cdot \frac{1 - \mathrm{CaMK_{trap}}}{1 + \frac{0.0015}{\mathrm{Ca_{ss}}}}, \\ \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}) &= \left(1 + \frac{0.15}{\mathrm{CaMK_{bound}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}) + \mathrm{CaMK_{trap}}}\right)^{-1}, \end{aligned}$$

we set

$$I_{\mathrm{Na,fast}}(V, m, h_{\mathrm{fast}}, h_{\mathrm{slow}}, h_{\mathrm{CaMK,slow}}, j, j_{\mathrm{CaMK}}, \mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}, \mathrm{Na})$$
$$= 75 \cdot m^3 \cdot \big(V_{\mathrm{m}} - E_{\mathrm{Na}}(\mathrm{Na})\big) \cdot \Big(j \cdot \big(0.99 h_{\mathrm{fast}} + 0.01 h_{\mathrm{slow}}\big) \cdot \big(1 - \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})\big) + j_{\mathrm{CaMK}} \cdot \big(0.99 h_{\mathrm{fast}} + 0.01 h_{\mathrm{CaMK,slow}}\big) \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})\Big),$$

$$I_{\mathrm{Na,late}}(V_{\mathrm{m}}, m_{\mathrm{L}}, h_{\mathrm{L}}, h_{\mathrm{L,CaMK}}, \mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}, \mathrm{Na})$$
$$= 0.0075 \cdot m_{\mathrm{L}} \big(V_{\mathrm{m}} - E_{\mathrm{Na}}(\mathrm{Na})\big) \Big(h_{\mathrm{L}} \big(1 - \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})\big) + h_{\mathrm{L,CaMK}} \cdot \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})\Big),$$

$$I_{\mathrm{Ks}}(V_{\mathrm{m}}, x_{\mathrm{s1}}, x_{\mathrm{s2}}, \mathrm{Ca}, \mathrm{Na}, \mathrm{K}) = 0.0034 \cdot x_{\mathrm{s1}} x_{\mathrm{s2}} \big(V_{\mathrm{m}} - E_{\mathrm{Ks}}(\mathrm{Na}, \mathrm{K})\big) \Big(1 + \frac{0.6}{1 + \big(\frac{3.8 \cdot 10^{-5}}{\mathrm{Ca}}\big)^{1.4}}\Big),$$

$$I_{\mathrm{to}}(V_{\mathrm{m}}, a, a_{\mathrm{CaMK}}, i_{\mathrm{fast}}, i_{\mathrm{slow}}, i_{\mathrm{CaMK,fast}}, i_{\mathrm{CaMK,slow}}, \mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}, \mathrm{K})$$
$$= 0.02 \cdot \big(V_{\mathrm{m}} - E_{\mathrm{K}}(\mathrm{K})\big) \Big[a \Big(\frac{1}{1 + \exp\big(\frac{V_{\mathrm{m}} - 213.6}{151.2}\big)}\big(i_{\mathrm{fast}} - i_{\mathrm{slow}}\big) + i_{\mathrm{slow}}\Big) \cdot \big(1 - \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})\big) + a_{\mathrm{CaMK}} \Big(\frac{1}{1 + \exp\big(\frac{V_{\mathrm{m}} - 213.6}{151.2}\big)}\big(i_{\mathrm{CaMK,fast}} - i_{\mathrm{CaMK,slow}}\big) + i_{\mathrm{CaMK,slow}}\Big) \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})\Big],$$

$$I_{\mathrm{Kr}}(V_{\mathrm{m}}, x_{\mathrm{r,fast}}, x_{\mathrm{r,slow}}, \mathrm{K})$$
$$= 0.046 \cdot \sqrt{\frac{[\mathrm{K}]_e}{5.4}} \Big(\frac{1}{1 + \exp\big(\frac{V_{\mathrm{m}} + 54.81}{38.21}\big)}\big(x_{\mathrm{r,fast}} - x_{\mathrm{r,slow}}\big) + x_{\mathrm{r,slow}}\Big) \cdot \frac{V_{\mathrm{m}} - E_{\mathrm{K}}(\mathrm{K})}{\big(1 + \exp\big(\frac{V_{\mathrm{m}} + 55}{75}\big)\big)\big(1 + \exp\big(\frac{V_{\mathrm{m}} - 10}{30}\big)\big)},$$

$$I_{\mathrm{Kb}}(V_{\mathrm{m}}, \mathrm{K}) = 0.003 \cdot \big(V_{\mathrm{m}} - E_{\mathrm{K}}(\mathrm{K})\big) \Big(1 + \exp\Big(\frac{-V_{\mathrm{m}} + 14.48}{18.34}\Big)\Big)^{-1},$$

$$I_{\mathrm{K1}}(V_{\mathrm{m}}, x_{\mathrm{K1}}, \mathrm{K}) = 0.1908 \cdot \sqrt{[\mathrm{K}]_e} \cdot x_{\mathrm{K1}} \big(V_{\mathrm{m}} - E_{\mathrm{K}}(\mathrm{K})\big) \Big(1 + \exp\Big(\frac{V_{\mathrm{m}} + 105.8 - 2.6[\mathrm{K}]_e}{9.493}\Big)\Big)^{-1},$$

$$I_{\mathrm{pCa}}(\mathrm{Ca}) = \frac{0.0005 \cdot \mathrm{Ca}}{0.0005 + \mathrm{Ca}},$$

$$I_{\mathrm{Cab}}(V_{\mathrm{m}}, \mathrm{Ca}) = 2.5 \cdot 10^{-8} \psi_{\mathrm{Ca}}(V_{\mathrm{m}}, \mathrm{Ca}),$$

$$I_{\mathrm{Nab}}(V_{\mathrm{m}}, \mathrm{Na}) = 3.75 \cdot 10^{-10} \cdot \frac{V_{\mathrm{m}} F^2}{RT} \cdot \frac{\mathrm{Na} \exp\big(\frac{V_{\mathrm{m}} F}{RT}\big) - [\mathrm{Na}]_e}{\exp\big(\frac{V_{\mathrm{m}} F}{RT}\big) - 1}.$$

For the L-Type calcium currents we predefine

$$\psi_{\mathrm{Ca}}(V_{\mathrm{m}}, \mathrm{Ca_{ss}}) = \frac{4 V_{\mathrm{m}} F^2}{RT} \cdot \frac{\mathrm{Ca_{ss}} \exp\big(\frac{2 V_{\mathrm{m}} F}{RT}\big) - 0.341 \cdot [\mathrm{Ca}]_e}{\exp\big(\frac{2 V_{\mathrm{m}} F}{RT}\big) - 1},$$

$$\psi_{\mathrm{CaNa}}(V_{\mathrm{m}}, \mathrm{Na_{ss}}) = \frac{V_{\mathrm{m}} F^2}{RT} \cdot \frac{0.75 \cdot \mathrm{Na_{ss}} \exp\big(\frac{V_{\mathrm{m}} F}{RT}\big) - 0.75 \cdot [\mathrm{Na}]_e}{\exp\big(\frac{V_{\mathrm{m}} F}{RT}\big) - 1},$$

$$\psi_{\mathrm{CaK}}(V_{\mathrm{m}}, \mathrm{K_{ss}}) = \frac{V_{\mathrm{m}} F^2}{RT} \cdot \frac{0.75 \cdot \mathrm{K_{ss}} \exp\big(\frac{V_{\mathrm{m}} F}{RT}\big) - 0.75 \cdot [\mathrm{K}]_e}{\exp\big(\frac{V_{\mathrm{m}} F}{RT}\big) - 1},$$

$$A_{f,\mathrm{Ca,fast}}(V_{\mathrm{m}}) = 0.3 + \frac{0.6}{1 + \exp\big(\frac{V_{\mathrm{m}} - 10}{10}\big)},$$

$$\beta_{\mathrm{CaL}}(n, f_{\mathrm{fast}}, f_{\mathrm{slow}}, f_{\mathrm{Ca,fast}}, f_{\mathrm{Ca,slow}}, j_{\mathrm{Ca}}, \mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})$$
$$:= \big(1 - \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})\big) \cdot \big((0.6 f_{\mathrm{fast}} + 0.4 f_{\mathrm{slow}})(1 - n) + \big(A_{f,\mathrm{Ca,fast}} f_{\mathrm{Ca,fast}} + \big(1 - A_{f,\mathrm{Ca,fast}}(V_{\mathrm{m}})\big) f_{\mathrm{Ca,slow}}\big) n j_{\mathrm{Ca}}\big),$$

$$\beta_{\mathrm{CaMK}}(f_{\mathrm{CaMK,fast}}, f_{\mathrm{slow}}, n, j_{\mathrm{Ca}}, f_{\mathrm{Ca,CaMK,fast}}, f_{\mathrm{Ca,slow}}, \mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})$$
$$:= \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}) \cdot (0.6 f_{\mathrm{CaMK,fast}} + 0.4 f_{\mathrm{slow}})(1 - n) + n j_{\mathrm{Ca}} (0.6 f_{\mathrm{Ca,CaMK,fast}} + 0.6 f_{\mathrm{Ca,slow}}),$$

then

$$
\begin{aligned}
&I_{\mathrm{CaL}}(V_{\mathrm{m}}, \mathrm{Ca}_{\mathrm{ss}}, \mathrm{CaMK}_{\mathrm{trap}}, d, n, f_{\mathrm{fast}}, f_{\mathrm{slow}}, f_{\mathrm{Ca,fast}}, f_{\mathrm{Ca,slow}}, j_{\mathrm{Ca}}) \\
&= P_{\mathrm{Ca}} \cdot \psi_{\mathrm{Ca}}(V_{\mathrm{m}}, \mathrm{Ca}_{\mathrm{ss}}) d \Big(\beta_{\mathrm{CaL}}(n, f_{\mathrm{fast}}, f_{\mathrm{slow}}, f_{\mathrm{Ca,fast}}, f_{\mathrm{Ca,slow}}, j_{\mathrm{Ca}}, \mathrm{Ca}_{\mathrm{ss}}, \mathrm{CaMK}_{\mathrm{trap}}) \\
&\qquad + 1.1 \cdot \beta_{\mathrm{CaMK}}(f_{\mathrm{CaMK,fast}}, f_{\mathrm{slow}}, n, j_{\mathrm{Ca}}, f_{\mathrm{Ca,CaMK,fast}}, f_{\mathrm{Ca,slow}}, \mathrm{Ca}_{\mathrm{ss}}, \mathrm{CaMK}_{\mathrm{trap}}) \Big), \\
&I_{\mathrm{CaNa}}(V_{\mathrm{m}}, \mathrm{Na}_{\mathrm{ss}}, \mathrm{Ca}_{\mathrm{ss}}, \mathrm{CaMK}_{\mathrm{trap}}, d, n, f_{\mathrm{fast}}, f_{\mathrm{slow}}, f_{\mathrm{Ca,fast}}, f_{\mathrm{Ca,slow}}, j_{\mathrm{Ca}}) \\
&= P_{\mathrm{Ca}} \psi_{\mathrm{Ca,Na}}(V_{\mathrm{m}}, \mathrm{Na}_{\mathrm{ss}}) d \Big(0.00125 \cdot \beta_{\mathrm{CaL}}(n, f_{\mathrm{fast}}, f_{\mathrm{slow}}, f_{\mathrm{Ca,fast}}, f_{\mathrm{Ca,slow}}, j_{\mathrm{Ca}}, \mathrm{Ca}_{\mathrm{ss}}, \mathrm{CaMK}_{\mathrm{trap}}) \\
&\qquad + 0.001375 \cdot \beta_{\mathrm{CaMK}}(f_{\mathrm{CaMK,fast}}, f_{\mathrm{slow}}, n, j_{\mathrm{Ca}}, f_{\mathrm{Ca,CaMK,fast}}, f_{\mathrm{Ca,slow}}, \mathrm{Ca}_{\mathrm{ss}}, \mathrm{CaMK}_{\mathrm{trap}}) \Big), \\
&I_{\mathrm{CaK}}(V_{\mathrm{m}}, \mathrm{K}_{\mathrm{ss}}, \mathrm{Ca}_{\mathrm{ss}}, \mathrm{CaMK}_{\mathrm{trap}}, d, n, f_{\mathrm{fast}}, f_{\mathrm{slow}}, f_{\mathrm{Ca,fast}}, f_{\mathrm{Ca,slow}}, j_{\mathrm{Ca}}) \\
&= P_{\mathrm{Ca}} \psi_{\mathrm{Ca,K}}(V_{\mathrm{m}}, \mathrm{K}_{\mathrm{ss}}) d \Big(3.574 \cdot 10^{-4} \cdot \beta_{\mathrm{CaL}}(n, f_{\mathrm{fast}}, f_{\mathrm{slow}}, f_{\mathrm{Ca,fast}}, f_{\mathrm{Ca,slow}}, j_{\mathrm{Ca}}, \mathrm{Ca}_{\mathrm{ss}}, \mathrm{CaMK}_{\mathrm{trap}}) \\
&\qquad + 0.00039314 \cdot \beta_{\mathrm{CaMK}}(f_{\mathrm{CaMK,fast}}, f_{\mathrm{slow}}, n, j_{\mathrm{Ca}}, f_{\mathrm{Ca,CaMK,fast}}, f_{\mathrm{Ca,slow}}, \mathrm{Ca}_{\mathrm{ss}}, \mathrm{CaMK}_{\mathrm{trap}}) \Big).
\end{aligned}
$$

For the sodium calcium exchange currents I_{NaCa} and I_{NaCass} we need

$$
\begin{aligned}
h_{\mathrm{Na}}(V_{\mathrm{m}}) &= \exp\left(\frac{0.5224 V_{\mathrm{m}} F}{RT}\right), & h_{\mathrm{Ca}}(V_{\mathrm{m}}) &= \exp\left(\frac{0.1670 V_{\mathrm{m}} F}{RT}\right), \\
h_1(y, V_{\mathrm{m}}) &= 1 + \frac{y}{88.12}\left(1 + h_{\mathrm{Na}}(V_{\mathrm{m}})\right), & k_1 &= 1.5 \cdot 10^6 h_{12} [\mathrm{Ca}]_e, \\
h_2(y, V_{\mathrm{m}}) &= \frac{y \cdot h_{\mathrm{Na}}(V_{\mathrm{m}})}{88.12 \cdot h_1(y, V_{\mathrm{m}})}, & k_2 &= 5000, \\
h_3(y, V_{\mathrm{m}}) &= \frac{1}{h_1(y, V_{\mathrm{m}})}, & k_3(V_{\mathrm{m}}) &= k_9(V_{\mathrm{m}}) + k_{10}(V_{\mathrm{m}}), \\
h_4(y) &= 1 + \frac{y}{15} + \frac{y^2}{5 \cdot 15}, & k_4(y, V_{\mathrm{m}}) &= k_{11}(y, V_{\mathrm{m}}) + k_{12}(y, V_{\mathrm{m}}), \\
h_5(y) &= \frac{y^2}{5 \cdot 15 \cdot h_4(y)}, & k_5 &= 5000, \\
h_6(y) &= \frac{1}{h_4(y)}, & k_6(y, z) &= 1.5 \cdot 10^6 \cdot h_6(y) \cdot z, \\
h_7(V_{\mathrm{m}}) &= 1 + \frac{[\mathrm{Na}]_e}{88.12}\left(1 + \frac{1}{h_{\mathrm{Na}}(V_{\mathrm{m}})}\right), & k_7(y, V_{\mathrm{m}}) &= 6 \cdot 10^4 \cdot h_2(y, V_{\mathrm{m}}) \cdot h_5(y), \\
h_8(V_{\mathrm{m}}) &= \frac{[\mathrm{Na}]_e}{88.12 \cdot h_{\mathrm{Na}}(V_{\mathrm{m}}) \cdot h_7(V_{\mathrm{m}})}, & k_8(V_{\mathrm{m}}) &= 6 \cdot 10^4 \cdot h_{11} \cdot h_8(V_{\mathrm{m}}), \\
h_9(V_{\mathrm{m}}) &= \frac{1}{h_7(V_{\mathrm{m}})}, & k_9(V_{\mathrm{m}}) &= 6 \cdot 10^4 h_9(V_{\mathrm{m}}), \\
h_{10} &= 12.5 + 1 + \frac{[\mathrm{Na}]_e}{15}\left(1 + \frac{[\mathrm{Na}]_e}{5}\right), & k_{10}(V_{\mathrm{m}}) &= 5000 \cdot h_8(V_{\mathrm{m}}), \\
h_{11} &= \frac{[\mathrm{Na}]_e^2}{h_{10} \cdot 15 \cdot 5}, & k_{11}(y, V_{\mathrm{m}}) &= \frac{6 \cdot 10^6 \cdot h_3(y, V_{\mathrm{m}})}{h_{\mathrm{Ca}}(V_{\mathrm{m}})}, \\
h_{12} &= h_{10}^{-1}, & k_{12}(y, V_{\mathrm{m}}) &= 5000 \cdot h_2(y, V_{\mathrm{m}}),
\end{aligned}
$$

and

$$
\begin{aligned}
x_1(y,z,V_{\mathrm{m}}) &= k_2\cdot k_4(y,V_{\mathrm{m}})\Big(k_7(y,V_{\mathrm{m}})+k_6(y,z)\Big)+k_5\cdot k_7(y,V_{\mathrm{m}})\Big(k_2+k_3(V_{\mathrm{m}})\Big)\,,\\
x_2(y,z,V_{\mathrm{m}}) &= k_1\cdot k_7(y,V_{\mathrm{m}})\Big(k_4(y,V_{\mathrm{m}})+k_5\Big)+k_4(y,V_{\mathrm{m}})\cdot k_6(y,z)\Big(k_1+k_8(V_{\mathrm{m}})\Big)\,,\\
x_3(y,z,V_{\mathrm{m}}) &= k_1\cdot k_3(V_{\mathrm{m}})\Big(k_7(y,V_{\mathrm{m}})+k_6(y,z,V_{\mathrm{m}}\Big)+k_8(V_{\mathrm{m}})\cdot k_6(y,z)\Big(k_2+k_3(V_{\mathrm{m}})\Big)\,,\\
x_4(y,V_{\mathrm{m}}) &= k_2\cdot k_8(V_{\mathrm{m}})\big(k_4(y,V_{\mathrm{m}})+k_5\Big)+k_3(V_{\mathrm{m}})\cdot k_5\Big(k_1+k_8(V_{\mathrm{m}})\Big)\,,
\end{aligned}
$$

used for

$$
\begin{aligned}
\mathrm{allo}(y) &= \frac{1}{1+\big(\frac{15\cdot 10^{-5}}{y}\big)^2}\,,\\
E_i(y,z,V_{\mathrm{m}}) &= \frac{x_i(y,z,V_{\mathrm{m}})}{x_1(y,z,V_{\mathrm{m}})+x_2(y,z,V_{\mathrm{m}})+x_3(y,z,V_{\mathrm{m}})+x_4(y,V_{\mathrm{m}})} \qquad \text{for } i=1,\ldots,3\,,\\
E_4(y,z,V_{\mathrm{m}}) &= \frac{x_4(y,V_{\mathrm{m}})}{x_1(y,z,V_{\mathrm{m}})+x_2(y,z,V_{\mathrm{m}})+x_3(y,z,V_{\mathrm{m}})+x_4(y)}\,,\\
J_{\mathrm{NaCa,Na}}(y,z,V_{\mathrm{m}}) &= 3\big(k_7(y,V_{\mathrm{m}})\cdot E_4(y,z,V_{\mathrm{m}})-k_8(V_{\mathrm{m}})\cdot E_1(y,z,V_{\mathrm{m}})\big)\\
&\quad + k_{12(y,V_{\mathrm{m}})}\cdot E_3(y,z,V_{\mathrm{m}})-k_{10}(V_{\mathrm{m}})\cdot E_2(y,z,V_{\mathrm{m}})\,,\\
J_{\mathrm{NaCa,Ca}}(y,z,V_{\mathrm{m}}) &= k_1\cdot E_1(y,z,V_{\mathrm{m}})+k_2\cdot E_2(y,z,V_{\mathrm{m}})\,,
\end{aligned}
$$

such that

$$
\begin{aligned}
I_{\mathrm{NaCa}}(V_{\mathrm{m}},\mathrm{Na},\mathrm{Ca}) &= 1.12\cdot 10^{-3}\cdot 0.8\cdot \mathrm{allo}(\mathrm{Ca})\Big(J_{\mathrm{NaCa,Na}}(\mathrm{Na},\mathrm{Ca},V_{\mathrm{m}})+2J_{\mathrm{NaCa,Ca}}(\mathrm{Na},\mathrm{Ca},V_{\mathrm{m}})\Big)\,,\\
I_{\mathrm{NaCass}}(V_{\mathrm{m}},\mathrm{Na_{ss}},\mathrm{Ca_{ss}}) &= 1.12\cdot 10^{-3}\cdot 0.2\cdot \mathrm{allo}(\mathrm{Ca_{ss}})\Big(J_{\mathrm{NaCa,Na}}(\mathrm{Na_{ss}},\mathrm{Ca_{ss}},V_{\mathrm{m}})+2J_{\mathrm{NaCa,Ca}}(\mathrm{Na_{ss}},\mathrm{Ca_{ss}},V_{\mathrm{m}})\Big)\,.
\end{aligned}
$$

For sodium potassium ATPase current I_{NaK} we denote

$$
\begin{aligned}
P(\mathrm{Na},\mathrm{K}) &= \frac{4.2}{1+\frac{1}{1.698}+\frac{\mathrm{Na}}{224}+\frac{\mathrm{K}}{292}}\,,\\
K_{[\mathrm{Na}]_e}(V_{\mathrm{m}}) &= 27.78\exp\Big(\frac{(1+0.155)V_{\mathrm{m}}F}{3RT}\Big)\,, & K_{\mathrm{Na}}(V_{\mathrm{m}}) &= 9.073\exp\Big(\frac{-0.155V_{\mathrm{m}}F}{3RT}\Big)\,,\\
\alpha_1(V_{\mathrm{m}},\mathrm{Na},\mathrm{K}) &= \frac{949.5\big(\frac{\mathrm{Na}}{K_{\mathrm{Na}}(V_{\mathrm{m}})}\big)^3}{\big(1+\frac{\mathrm{Na}}{K_{\mathrm{Na}}(V_{\mathrm{m}})}\big)^3\big)+\big(1+\frac{\mathrm{K}}{0.5}\big)^2-1}\,, & \beta_1 &= 182.4\cdot 0.05\,,\\
\alpha_2 &= 687.2\,, & \beta_2(V_{\mathrm{m}}) &= \frac{39.4\big(\frac{[\mathrm{Na}]_e}{K_{[\mathrm{Na}]_e}(V_{\mathrm{m}})}\big)^3}{\big(1+\frac{[\mathrm{Na}]_e}{K_{[\mathrm{Na}]_e}(V_{\mathrm{m}})}\big)^3\big)+\big(1+\frac{[\mathrm{K}]_e}{0.3582}\big)^2-1}\,,\\
\alpha_3(V_{\mathrm{m}}) &= \frac{1899\big(\frac{[\mathrm{K}]_e}{0.3582}\big)^2}{\big(1+\frac{[\mathrm{Na}]_e}{K_{[\mathrm{Na}]_e}(V_{\mathrm{m}})}\big)^3\big)+\big(1+\frac{[\mathrm{K}]_e}{0.3582}\big)^2-1}\,, & \beta_3(\mathrm{Na},\mathrm{K}) &= \frac{79300\cdot 10^{-7}}{1+9.8\cdot 1.698\cdot 10^{-7}}\cdot P(\mathrm{Na},\mathrm{K})\,,\\
\alpha_4 &= 693\frac{9.8}{1.698\cdot 10^{-7}}\frac{1}{1+\frac{9.8}{1.698\cdot 10^{-7}}}=, & \beta_4(V_{\mathrm{m}},\mathrm{Na},\mathrm{K}) &= \frac{40\big(\frac{\mathrm{K}}{0.5}\big)^2}{\big(1+\frac{\mathrm{Na}}{K_{\mathrm{Na}}(V_{\mathrm{m}})}\big)^3\big)+\big(1+\frac{\mathrm{K}}{0.5}\big)^2-1}\,.
\end{aligned}
$$

We also need

$$\begin{aligned}
x_1(V_\mathrm{m},\mathrm{Na},\mathrm{K}) &= \alpha_1(V_\mathrm{m},\mathrm{Na},\mathrm{K})\alpha_2\alpha_4 + \beta_2(V_\mathrm{m})\beta_3(\mathrm{Na},\mathrm{K})\beta_4(V_\mathrm{m},\mathrm{Na},\mathrm{K}) \\
&\quad + \alpha_2\beta_3(\mathrm{Na},\mathrm{K})\beta_4(V_\mathrm{m},\mathrm{Na},\mathrm{K}) + \beta_3(\mathrm{Na},\mathrm{K})\alpha_1(V_\mathrm{m},\mathrm{Na},\mathrm{K})\alpha_2\,, \\
x_2(V_\mathrm{m},\mathrm{Na},\mathrm{K}) &= \alpha_1(V_\mathrm{m},\mathrm{Na},\mathrm{K})\alpha_2\alpha_3(V_\mathrm{m}) + \beta_1\beta_2(V_\mathrm{m})\beta_4(V_\mathrm{m},\mathrm{Na},\mathrm{K}) \\
&\quad + \alpha_3(V_\mathrm{m})\beta_1\beta_4(V_\mathrm{m},\mathrm{Na},\mathrm{K}) + \beta_4(V_\mathrm{m},\mathrm{Na},\mathrm{K})\alpha_2\alpha_3(V_\mathrm{m})\,, \\
x_3(V_\mathrm{m},\mathrm{Na},\mathrm{K}) &= \alpha_2\alpha_3(V_\mathrm{m})\alpha_4 + \beta_1\beta_2(V_\mathrm{m})\beta_3(\mathrm{Na},\mathrm{K}) + \alpha_4\beta_1\beta_2(V_\mathrm{m}) + \beta_1\alpha_3(V_\mathrm{m})\alpha_4\,, \\
x_4(V_\mathrm{m},\mathrm{Na},\mathrm{K}) &= \alpha_1(V_\mathrm{m},\mathrm{Na},\mathrm{K})\alpha_3(V_\mathrm{m})\alpha_4 + \beta_2(V_\mathrm{m})\beta_3(\mathrm{Na},\mathrm{K})\beta_4(V_\mathrm{m},\mathrm{Na},\mathrm{K}) + \alpha_2\beta_1\beta_4(V_\mathrm{m},\mathrm{Na},\mathrm{K}) + \beta_1\alpha_2\alpha_3(V_\mathrm{m})\,,
\end{aligned}$$

to define

$$E_i(V_\mathrm{m},\mathrm{Na},\mathrm{K}) = \frac{x_i(V_\mathrm{m},\mathrm{Na},\mathrm{K})}{x_1(V_\mathrm{m},\mathrm{Na},\mathrm{K}) + x_2(V_\mathrm{m},\mathrm{Na},\mathrm{K}) + x_3(V_\mathrm{m},\mathrm{Na},\mathrm{K}) + x_4(V_\mathrm{m},\mathrm{Na},\mathrm{K})}\,,$$

for $i = 1,\ldots,4$, such that

$$\begin{aligned}
I_\mathrm{NaK}(V_\mathrm{m},\mathrm{Na},\mathrm{K}) = 30\Big(&3\big(\alpha_3(V_\mathrm{m})E_1(V_\mathrm{m},\mathrm{Na},\mathrm{K}) - \beta_3(\mathrm{Na},\mathrm{K})E_2(V_\mathrm{m},\mathrm{Na},\mathrm{K})\big) \\
&+ 2\big(\beta_1 E_4(V_\mathrm{m},\mathrm{Na},\mathrm{K}) - \alpha_1(V_\mathrm{m},\mathrm{Na},\mathrm{K})E_3(V_\mathrm{m},\mathrm{Na},\mathrm{K})\big)\Big)\,.
\end{aligned}$$

4.2. Gating mechanism

The components $w_1,\ldots,w_{28}$ of the vector $\mathbf{w}$ given in (8) are described by gating mechanisms of the form (3); therefore, we specify $w_i^\infty(V_\mathrm{m})$ and $\tau_i(V_\mathrm{m})$ for $i = 1,\ldots,28$. In general they have the form

$$\begin{aligned}
w_i^\infty(V_\mathrm{m}) &= \frac{1}{1+\exp(\frac{C_1(V_\mathrm{m}+C_2)}{C_3})}\,, \\
\tau_i(V_\mathrm{m}) &= \frac{B_1}{B_2\exp(\frac{B_3(V_\mathrm{m}+B_4)}{B_5}) + B_6\exp(\frac{B_7(V_\mathrm{m}+B_8)}{B_9})} + B_{10}\,.
\end{aligned}$$

The values $C_1, C_2, C_3, B_1, \ldots, B_{10}$ are given in Table S3 and Table S4 respectively. For $i = 20, 21, 24, 25$ this is modified to (here denoted by the gate names)

$$\tau_a(V_\mathrm{m}) = 1.0515\Big(\frac{1}{1.2089\big(1+\exp(\frac{-(V_\mathrm{m}-18.41)}{29.38})\big)} + \frac{3.5}{1+\exp(\frac{V_\mathrm{m}+100}{29.38})}\Big)^{-1}, \tag{9}$$

$$\tau_{a_\mathrm{CaMK}}(V_\mathrm{m}) = \tau_a(V_\mathrm{m})\,, \tag{10}$$

$$\tau_{i_\mathrm{CaMK,fast}}(V_\mathrm{m}) = \tau_{i_\mathrm{fast}}(V_\mathrm{m})\Big(1.354 + \frac{10^{-4}}{\exp(\frac{V_\mathrm{m}-167.4}{15.89}) + \exp(\frac{-(V_\mathrm{m}-12.23)}{0.2154})}\Big)\Big(1 - \frac{0.5}{1+\exp(\frac{V_\mathrm{m}+70}{20})}\Big)\,, \tag{11}$$

$$\tau_{i_\mathrm{CaMK,slow}}(V_\mathrm{m}) = \tau_{i_\mathrm{slow}}(V_\mathrm{m})\Big(1.354 + \frac{10^{-4}}{\exp(\frac{V_\mathrm{m}-167.4}{15.89}) + \exp(\frac{-(V_\mathrm{m}-12.23)}{0.2154})}\Big)\Big(1 - \frac{0.5}{1+\exp(\frac{V_\mathrm{m}+70}{20})}\Big)\,, \tag{12}$$

Table S3. Constants for $y^{\infty}(V_{\mathrm{m}})$ with $y \in \mathbf{w}$ of the O'Hara Rudy cell model.

	C_1	C_2	C_3
m	-1	39.57	9.871
h_{fast}	1	78.5	6.22
h_{slow}	1	78.5	6.22
$h_{\mathrm{CaMK,slow}}$	1	84.7	6.22
j	1	78.5	6.22
j_{CaMK}	1	78.5	6.22
m_{L}	-1	42.85	5.264
h_{L}	1	87.61	7.488
$h_{\mathrm{L,CaMK}}$	1	93.81	7488
d	-1	3.94	4.23
f_{fast}	1	19.58	3.696
f_{slow}	1	19.58	3.696
$f_{\mathrm{Ca,fast}}$	1	19.58	3.696
$f_{\mathrm{Ca,slow}}$	1	19.58	3.696
j_{Ca}	1	19.58	3.696
$f_{\mathrm{CaMK,fast}}$	1	19.58	3.696
$f_{\mathrm{Ca,CaMK,fast}}$	1	19.58	3.696
x_{s1}	-1	11.6	8.932
x_{s2}	-1	11.6	8.932
a	-1	-14.34	14.82
a_{CaMK}	-1	-24.34	14.82
i_{fast}	1	43.94	5.711
i_{slow}	1	43.94	5.711
$i_{\mathrm{CaMK,fast}}$	1	43.94	5.711
$i_{\mathrm{CaMK,slow}}$	1	43.94	5.711
$x_{\mathrm{r,fast}}$	-1	.337	6.789
$x_{\mathrm{r,slow}}$	-1	.337	6.789
x_{K1}	-1	$2.5538[\mathrm{K}]_e + 144.59$	$1.5692[\mathrm{K}]_e + 3.8115$

Table S4. Constants for $\tau_y(V_\mathrm{m})$ with $y \in \mathbf{w}$ of the O'Hara Rudy cell model.

	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
m	1	6.765	1	11.64	34.77	8.552	-1	77.42	5.955	0
h_fast	1	$3.686 \cdot 10^{-6}$	-1	3.8875	7.8579	16	1	-0.4963	9.1843	0
h_slow	1	0.009764	-1	17.95	28.05	0.3343	1	5.73	56.66	0
$h_\mathrm{CaMK,slow}$	3	0.009764	-1	17.95	28.05	0.3343	1	5.73	56.66	0
j	1	0.8628	-1	116.7258	7.6005	1.1096	1	6.2719	9.0358	4.859
j_CaMK	1.46	0.8628	-1	116.7258	7.6005	1.1096	1	6.2719	9.0358	4.859
m_L	1	6.765	1	11.64	34.77	8.552	-1	77.42	5.955	0
h_L	0	0	0	0	1	0	0	0	1	200
$h_\mathrm{L,CaMK}$ 0	0	0	0	1	0	0	0	1	600	
d	1	1	-0.05	6	1	1	0.09	14	1	0.6
f_fast	1	0.0045	-1	20	10	0.0045	1	20	10	7
f_slow	1	0.000035	-1	5	4	0.000035	1	5	6	1000
$f_\mathrm{Ca,fast}$	1	0.04	-1	-4	7	0.04	1	-4	7	7
$f_\mathrm{Ca,slow}$	1	0.00012	-1	0	3	0.00012	1	0	7	100
j_Ca	0	0	0	0	1	0	0	0	1	75
$f_\mathrm{CaMK,fast}$	2.5	0.0045	-1	20	10	0.0045	1	20	10	$2.5 \cdot 7$
$f_\mathrm{Ca,CaMK,fast}$	2.5^2	0.0045	-1	20	10	0.0045	1	20	10	$2.5^2 \cdot 7$
x_s1	1	$2.326 \cdot 10^{-4}$	1	48.28	17.8	0.001292	-1	210	230	817.3
x_s2	1	0.01	1	-50	20	0.0193	-1	66.54	31	0
a	see (9)									
a_CaMK	see (10)									
i_fast	1	0.3933	-1	100	100	0.08004	1	50	16.59	4.562
i_slow	1	0.001416	-1	96.52	59.05	$1.7808 \cdot 10^{-8}$	1	114.1	8.079	23.62
$i_\mathrm{CaMK,fast}$	see (11)									
$i_\mathrm{CaMK,slow}$	see (12)									
$x_\mathrm{r,fast}$	1	0.3652	1	-31.66	3.869	$4.123 \cdot 10^{-5}$	-1	-47.78	20.38	12.98
$x_\mathrm{r,slow}$	1	0.06629	1	-34.7	7.355	$1.128 \cdot 10^{-5}$	-1	-29.74	25.94	1.865
x_K1	122.2	1	-1	127.2	20.36	1	1	236.8	69.33	0

4.3. Further ODE mechanisms

Now the evolution of the last four components of $\mathbf{w}$ is described. We define

$$\partial_t n = \frac{1000}{\frac{1000}{j_\mathrm{Ca}} + \left(1 + \frac{0.002}{\mathrm{Ca_{ss}}}\right)^4} - j_\mathrm{Ca} n$$

and

$$\partial_t \mathrm{CaMK_{trap}} = 0.05 \mathrm{CaMK_{bound}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}) \left(\mathrm{CaMK_{bound}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}) + \mathrm{CaMK_{trap}}\right) - 0.00068 \mathrm{CaMK_{trap}}\,.$$

For $J_\mathrm{rel,NP}$ and $J_\mathrm{rel,CaMK}$ we need

$$J^\infty_\mathrm{rel,NP}(\mathrm{Ca_{jsr}}) = 1.7 \frac{-0.5 \cdot 4.75 \cdot I_\mathrm{CaL}}{1 + \left(\frac{1.5}{\mathrm{Ca_{jsr}}}\right)^8}\,, \qquad \tau_{J_\mathrm{rel,NP}}(\mathrm{Ca_{jsr}}) = \max\left(0.001, 4.75\left(1 + \frac{0.0123}{\mathrm{Ca_{jsr}}}\right)^{-1}\right),$$

$$J^\infty_\mathrm{rel,CaMK}(\mathrm{Ca_{jsr}}) = 1.25 \cdot J^\infty_\mathrm{rel,NP}(\mathrm{Ca_{jsr}})\,, \qquad \tau_{J_\mathrm{rel,CaMK}}(\mathrm{Ca_{jsr}}) = \max\left(0.001, 1.25 \cdot \tau_{J_\mathrm{rel,NP}}(\mathrm{Ca_{jsr}})\right),$$

such that

$$\partial_t J_\mathrm{rel,NP} = \frac{J^\infty_\mathrm{rel,NP}(\mathrm{Ca_{jsr}}) - J_\mathrm{rel,NP}}{\tau_{J_\mathrm{rel,NP}}(\mathrm{Ca_{jsr}})}\,, \qquad \partial_t J_\mathrm{rel,CaMK} = \frac{J^\infty_\mathrm{rel,CaMK}(\mathrm{Ca_{jsr}}) - J_\mathrm{rel,CaMK}}{\tau_{J_\mathrm{rel,CaMK}}(\mathrm{Ca_{jsr}})}\,.$$

4.4. Ion concentrations

Here, the ion concentration modeling depends on cell geometry constants. The cell geometry is approximated by a cylinder with length $L = 0.01\,\mathrm{cm}$ and radius $r = 0.0011\,\mathrm{cm}$, and

$$\begin{aligned} v_{\mathrm{cell}} &= 1000 \cdot \pi r^2 L\,, \quad A_{\mathrm{cap}} = 4\pi(r^2 + rL)\,, \quad v_{\mathrm{myo}} = 0.68 v_{\mathrm{cell}}\,, \\ v_{\mathrm{nsr}} &= 0.0552 v_{\mathrm{cell}}\,, \quad v_{\mathrm{jsr}} = 0.0048 v_{\mathrm{cell}}\,, \quad v_{\mathrm{ss}} = 0.02 v_{\mathrm{cell}}\,. \end{aligned}$$

For sodium and potassium the evolution is given by

$$\begin{aligned} \partial_t \mathrm{Na} &= -\frac{A_{\mathrm{cap}}}{v_{\mathrm{myo}} F}\big(I_{\mathrm{Na,fast}} + I_{\mathrm{Na,late}} + 3I_{\mathrm{NaCa}} + 3I_{\mathrm{NaK}} + I_{\mathrm{Nab}}\big) + \frac{v_{\mathrm{ss}}}{2v_{\mathrm{myo}}}\big(\mathrm{Na_{ss}} - \mathrm{Na}\big)\,, \\ \partial_t \mathrm{Na_{ss}} &= -\frac{A_{\mathrm{cap}}}{v_{\mathrm{ss}} F}\big(I_{\mathrm{CaNa}} + 3I_{\mathrm{NaCass}}\big) - 0.5\big(\mathrm{Na_{ss}} - \mathrm{Na}\big)\,, \\ \partial_t \mathrm{K} &= -\frac{A_{\mathrm{cap}}}{v_{\mathrm{myo}} F}\big(I_{\mathrm{to}} + I_{\mathrm{Kr}} + I_{\mathrm{K1}} + I_{\mathrm{Kb}} + I_{\mathrm{ext}} - 2I_{\mathrm{NaK}}\big) + \frac{v_{\mathrm{ss}}}{2v_{\mathrm{myo}}}\big(\mathrm{K_{ss}} - \mathrm{K}\big)\,, \\ \partial_t \mathrm{K_{ss}} &= -\frac{A_{\mathrm{cap}}}{v_{\mathrm{ss}} F} I_{\mathrm{CaK}} - 0.5\big(\mathrm{K_{ss}} - \mathrm{K}\big)\,. \end{aligned}$$

For the evolution of the calcium ions we define the abbreviations

$$\begin{aligned} \beta_{\mathrm{Ca}}(\mathrm{Ca}) &= \Big(1 + \frac{0.05 \cdot 0.00238}{\big(0.00238 + \mathrm{Ca_{ss}}\big)^2} + \frac{0.07 \cdot 0.0005}{\big(0.0005 + \mathrm{Ca}\big)^2}\Big)^{-1}\,, \\ \beta_{\mathrm{Cass}}(\mathrm{Ca_{ss}}) &= \Big(1 + \frac{0.05 \cdot 0.00238}{\big(0.00238 + \mathrm{Ca}\big)^2} + \frac{1.124 \cdot 0.0087}{\big(0.0087 + \mathrm{Ca_{ss}}\big)^2}\Big)^{-1}\,, \\ \beta_{\mathrm{Cajsr}}(\mathrm{Ca_{jsr}}) &= \Big(1 + \frac{8}{\big(0.8 + \mathrm{Ca_{jsr}}\big)^2}\Big)^{-1}\,, \end{aligned}$$

and the fluxes

$$\begin{aligned} J_{\mathrm{up,CAMK}}(\mathrm{Ca}) &= \frac{2.75 \cdot 0.004375 \mathrm{Ca}}{0.00092 - 0.00017 + \mathrm{Ca}}\,, \\ J_{\mathrm{tr}}(\mathrm{Ca_{nsr}}, \mathrm{Ca_{jsr}}) &= 0.01\big(\mathrm{Ca_{nsr}} - \mathrm{Ca_{jsr}}\big)\,, \\ J_{\mathrm{up}}(\mathrm{Ca}, \mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}, \mathrm{Ca_{nsr}}) &= \big(1 - \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})\big)\frac{0.004375 \cdot \mathrm{Ca}}{0.00092 + \mathrm{Ca}} \\ &\quad + \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}) \cdot J_{\mathrm{up,CAMK}}(\mathrm{Ca}) - \frac{0.0039375}{15}\mathrm{Ca_{nsr}}\,, \\ J_{\mathrm{rel}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}, J_{\mathrm{rel,NP}}, J_{\mathrm{rel,CaMK}}) &= \big(1 - \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}})\big) J_{\mathrm{rel,NP}} + \phi_{\mathrm{CaMK}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}) \cdot J_{\mathrm{rel,CaMK}}\,, \end{aligned}$$

such that

$$\partial_t \mathrm{Ca} = \beta_{\mathrm{Ca}}(\mathrm{Ca}) \cdot \Big(\frac{-A_{\mathrm{cap}}}{2 v_{\mathrm{myo}} F} \left(I_{\mathrm{pCa}} + I_{\mathrm{Cab}} - 2 I_{\mathrm{NaCa}} \right) - \frac{v_{\mathrm{nsr}} J_{\mathrm{up}}(\mathrm{Ca}, \mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}, \mathrm{Ca_{nsr}})}{v_{\mathrm{myo}}} + \frac{v_{\mathrm{ss}}(\mathrm{Ca_{ss}} - \mathrm{Ca})}{0.2 \cdot v_{\mathrm{myo}}} - 0.07 \cdot G_{\mathrm{CaTRPN}}(\mathrm{CaTRPN}, \mathrm{Ca}, \gamma_{\mathbf{f}}) \Big),$$

$$\partial_t \mathrm{Ca_{ss}} = \beta_{\mathrm{Ca_{ss}}}(\mathrm{Ca_{ss}}) \cdot \Big(\frac{-A_{\mathrm{cap}}}{2 v_{\mathrm{ss}} F} \left(I_{\mathrm{CaL}} + -2 I_{\mathrm{NaCass}} \right) + \frac{v_{\mathrm{njsr}} J_{\mathrm{rel}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}, J_{\mathrm{rel,NP}}, J_{\mathrm{rel,CaMK}})}{v_{\mathrm{ss}}} - \frac{\mathrm{Ca_{ss}} - \mathrm{Ca}}{0.2} \Big),$$

$$\partial_t \mathrm{Ca_{nsr}} = J_{\mathrm{up}}(\mathrm{Ca}, \mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}, \mathrm{Ca_{nsr}}) - \frac{v_{\mathrm{jsr}}}{v_{\mathrm{nsr}}} J_{\mathrm{tr}}(\mathrm{Ca_{nsr}}, \mathrm{Ca_{jsr}}),$$

$$\partial_t \mathrm{Ca_{jsr}} = \beta_{\mathrm{Cajsr}}(\mathrm{Ca_{jsr}}) \cdot \Big(J_{\mathrm{tr}}(\mathrm{Ca_{nsr}}, \mathrm{Ca_{jsr}}) - J_{\mathrm{rel}}(\mathrm{Ca_{ss}}, \mathrm{CaMK_{trap}}, J_{\mathrm{rel,NP}}, J_{\mathrm{rel,CaMK}}) \Big).$$

We also need the fraction of troponin C units with calcium bound to its regular binding site

$$\begin{aligned} \partial_t \mathrm{CaTRPN} &= G_{\mathrm{CaTRPN}}(\mathrm{CaTRPN}, \mathrm{Ca}, \gamma_{\mathbf{f}}) \\ &= 0.1 \left(1 - \mathrm{CaTRPN}\right) \Big(\frac{1000 \cdot \mathrm{Ca}}{\mathrm{Ca_{T50}}(\gamma_{\mathbf{f}})} \Big)^2 - \mathrm{CaTRPN}, \end{aligned}$$

where $\mathrm{Ca_{T50}}(\gamma_{\mathbf{f}})$ is from the Land model described in chapter 5.

Table S5. Initial values of the O'Hara Rudy cell model.

	one cell	whole heart
V_{m}^0	−87.0 mV	−87.74551 mV
Na^0	7 mM	8.543872 mM
$\mathrm{Na_{ss}^0}$	7 mM	8.543959 mM
K^0	145 mM	143.0931 mM
$\mathrm{K_{ss}^0}$	145 mM	143.093 mM
Ca^0	0.0001 mM	$7.251537 \cdot 10^{-5}$ mM
$\mathrm{Ca_{ss}^0}$	0.0001 mM	$7.058365 \cdot 10^{-5}$ mM
$\mathrm{Ca_{nsr}^0}$	1.2 mM	1.636413 mM
$\mathrm{Ca_{jsr}^0}$	1.2 mM	1.563213 mM
m^0	0	0.007535921
h_{fast}^0	1	0.8155401
h_{slow}^0	1	0.8155401
$h_{\mathrm{CaMK,slow}}^0$	1	0.6200204
j^0	1	0.8155228
j_{CaMK}^0	1	0.8151938
m_{L}^0	0	0.0001976564
h_{L}^0	1	0.4893654
$h_{\mathrm{L,CaMK}}^0$	1	0.2533663
d^0	0	$2.487026 \cdot 10^{-9}$
$\mathrm{Ca_{Trpn}^0}$ mM	0	0.004744514 mM
CaTRPN^0		0
f_{fast}^0	1	1

	one cell	whole heart
f_{slow}^0	1	0.8834377
$f_{\mathrm{Ca,fast}}^0$	1	1
$f_{\mathrm{Ca,slow}}^0$	1	0.9996695
j_{Ca}^0	1	0.9999573
$f_{\mathrm{CaMK,fast}}^0$	1	0.001348506
$f_{\mathrm{Ca,CaMK,fast}}^0$	1	1
x_{s1}^0	0	0.3165663
x_{s2}^0	0	0.0001984007
a^0	0	
a_{CaMK}^0	0	0.0005189805
i_{fast}^0	1	0.9995338
i_{slow}^0	1	0.561649
$i_{\mathrm{CaMK,fast}}^0$	1	0.9995338
$i_{\mathrm{CaMK,slow}}^0$	1	0.6098597
$x_{\mathrm{r,fast}}^0$	0	$8.523495 \cdot 10^{-6}$
$x_{\mathrm{r,slow}}^0$	0	0.4804997
x_{K1}^0	1	0.9968261
n^0	0	1
$\mathrm{CaMK_{trap}^0}$	0	0.02069766
$J_{\mathrm{rel,NP}}^0$	0	$7.890074 \cdot 10^{-7}$
$J_{\mathrm{rel,CaMK}}^0$	0	$9.854135 \cdot 10^{-7}$

Table S6. Parameters in the O'Hara Rudy cell model.

gas constant	$R = 8314$ mJK^{-1}mol^{-1}
temperature	T= 310 K
Faraday constant	$F = 96485$ C/mol
Permeability to Ca	$P_{\mathrm{Ca}} = 0.00018$ cm s^{-1}
extracellular K^+ concentration	$[\mathrm{K}]_e$ = 5.4 mM
extracellular Na^+ concentration	$[\mathrm{Na}]_e$ = 140 mM
extracellular Ca^{2+} concentration	$[\mathrm{Ca}]_e$ = 1.8 mM

5. Land et al. model

To model the tension T_{tot} generated by activated cardiac muscle cells and used in (6), we use the model described by Land et al. [10]. It simulates cardiac myocytes using ion concentrations of calcium (Ca) and troponin (CaTRPN) as well as crossbridge binding of sarcomeres using a three-state crossbridge cycle. The evolution is determined by a system of ODEs for the vector

$$\mathbf{q} = (\text{CaTRPN}, B, W, S, \zeta_W, \zeta_S, C_d)$$

requiring the calcium concentration Ca from an electrophysiological model and the current length of the cell $\gamma_{\mathbf{f}}$ as external input.
CaTRPN represents the fraction of troponin units with calcium bond. This concentration drives the unblocking of tropomyosin. It is given by

$$\partial_t \text{CaTRPN} = 0.1 \left[\left(\frac{\text{Ca}}{\text{Ca}_{\text{T50}}} \right)^2 (1 - \text{CaTRPN}) - \text{CaTRPN} \right] . \tag{13}$$

The fraction of blocked binding sites (B) is given by

$$\partial_t B = 0.01400583 \ \min\{100, \text{CaTRPN}^{-\frac{n_{\text{Tm}}}{2}}\} U - k_{\text{u}} \text{CaTRPN}^{\frac{n_{\text{Tm}}}{2}} B \, .$$

The following crossbridge cycle containins an unbound (U), a pre-powerstroke (W) and a post-powerstroke (S) state. They are given by the relation

$$\begin{aligned} U &= (1 - B) - S - W \, , \\ \partial_t W &= 0.182 \, U - 0.17 \, W - 0.012 \, W - \gamma_{WU} W \, , \\ \partial_t S &= 0.012 \, W - 0.018 \, S - \gamma_{SU} S \, . \end{aligned}$$

The values of γ_{WU} and γ_{SU} are derived from the distortion-decay model

$$\partial_t \zeta_W = 10 \, \partial_t \gamma_{\mathbf{f}} - 0.40586 \, \zeta_W \qquad \partial_t \zeta_S = 10 \, \partial_t \gamma_{\mathbf{f}} - 0.04014 \, \zeta_S$$

$$\gamma_{WU} = 0.615 |\zeta_W| \qquad \gamma_{SU} = \begin{cases} 0.0085 \, (-\zeta_S - 1) & \text{if } \zeta_S + 1 < 0, \\ 0.0085 \zeta_S & \text{if } \zeta_S + 1 > 1, \\ 0 & \text{otherwise.} \end{cases}$$

The variables ζ_W and ζ_S represent internal distortion. Following [10] we use experimentally fitted values to get a formula for the total active tension

$$T_{\text{a}} = h(\gamma_{\mathbf{f}}) \frac{T_{\text{ref}}}{0.25} \left((\zeta_S + 1) S + \zeta_W W \right)$$

where

$$\begin{aligned} h(\gamma_{\mathbf{f}}) &= \max \left\{ 0, \tilde{h} \left(\min\{\gamma_{\mathbf{f}}, 1.2\} \right) \right\} \, , \\ \tilde{h}(\gamma_{\mathbf{f}}) &= 1 + 2.3 \left(\gamma_{\mathbf{f}} + \min\{\gamma_{\mathbf{f}}, 0.87\} - 1.87 \right) \end{aligned}$$

enforces a length-dependency on the tension. Such a dependency is also introduced for the half-activation point

$$\text{Ca}_{\text{T50}} = \text{Ca}_{\text{T50}}^{\text{ref}} + \beta_1 \left(\min\{\gamma_{\mathbf{f}}, 1.2\} - 1 \right)$$

used in equation (13).
In the coupled model, we also need to account for a passive tension within the sarcomeres. A minimal implementation of such a passive cell model is given by

$$C_s = (\gamma_{\mathbf{f}} - 1) - C_d\,, \qquad \partial_t C_d = 7\frac{C_s}{\eta}\,, \qquad \eta = \begin{cases} 200 & \text{if } C_s > 0, \\ 20 & \text{if } C_s \leq 0. \end{cases}$$

The passive tension is then calculated by

$$T_{\mathrm{p}} = 2.1\, C_s$$

and the resulting tension of the force model used to compute the macroscopic active stress is given by

$$T_{\mathrm{tot}} = T_{\mathrm{a}} + T_{\mathrm{p}}.$$

In all experiments of this paper, we use the initial conditions from Table S7.

Table S7. Initial values for the Land tension model.

	one cell	**whole heart atria**	**whole heart ventricle**
CaTRPN^0	0.0	0.01042028	0.004744517
B^0	1.0	0.9999787	0.9997976
W^0	0.0	$7.947042 \cdot 10^{-6}$	$7.576471 \cdot 10^{-5}$
S^0	0.0	$5.390739 \cdot 10^{-6}$	$5.082463 \cdot 10^{-5}$
ζ_W^0	0.0	0.0	0.0
ζ_S^0	0.0	0.0	0.0
C_d^0	0.0	0.0	0.0

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