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An Evolving Partial Consensus Fuzzy Collaborative Forecasting Approach

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Received: 7 March 2020; Accepted: 8 April 2020; Published: 10 April 2020

Abstract: Current fuzzy collaborative forecasting methods have rarely considered how to determine the appropriate number of experts to optimize forecasting performance. Therefore, this study proposes an evolving partial-consensus fuzzy collaborative forecasting approach to address this issue. In the proposed approach, experts apply various fuzzy forecasting methods to forecast the same target, and the partial consensus fuzzy intersection operator, rather than the prevalent fuzzy intersection operator, is applied to aggregate the fuzzy forecasts by experts. Meaningful information can be determined by observing partial consensus fuzzy intersection changes as the number of experts varies, including the appropriate number of experts. We applied the evolving partial-consensus fuzzy collaborative forecasting approach to forecasting dynamic random access memory product yield with real data. The proposed approach forecasting performance surpassed current fuzzy collaborative forecasting that considered overall consensus, and it increased forecasting accuracy 13% in terms of mean absolute percentage error.

Keywords: fuzzy collaborative forecasting; dynamic random access memory; partial consensus; fuzzy intersection

1. Introduction

Fuzzy collaborative forecasting combines fuzzy forecasting and collaborative intelligence [1]. Multiple experts apply fuzzy forecasting methods to forecast the same target and collaborate by consulting each other's forecast, subsequently modifying fuzzy forecasting method settings or forecasts [2]. In contrast with conventional forecasting methods that focus on maximizing forecasting accuracy, fuzzy collaborative forecasting methods attempt to optimize both forecasting precision and accuracy [3,4].

This paper proposes an evolving partial consensus fuzzy collaborative forecasting approach to enhance forecasting effectiveness for dynamic random access memory (DRAM) product yield. Most current fuzzy collaborative forecasting methods apply a fuzzy intersection (FI) to aggregate expert fuzzy forecasts [5]. Though this treatment effectively elevates forecasting precision in terms of the average range of fuzzy forecasts, it has a number of drawbacks as follows.

(1) The FI result usually covers a very narrow range. Though this improves forecasting precision for training data, the probability of missing test values increases [6].
(2) The FI result becomes the null set when there is no overall consensus among experts [7].

To overcome these drawbacks, a consensus among some experts, rather than all experts, can be sought instead. Chen [5] proposed the partial consensus FI (PCFI) operator, which can be non-null set if some experts can achieve a (partial) consensus. The PCFI also usually covers a wider range than the FI, reducing the possibility of missing a test value [8]. However, determining the appropriate number of experts to optimize forecasting performance remains an issue [9]. Therefore, this paper proposes an evolving partial consensus fuzzy collaborative forecasting approach, where multiple experts apply various fuzzy forecasting methods to forecast the same target, and the PCFI operator is employed to aggregate the forecasts. The appropriate number of experts can be determined by observing PCFI changes as the number of experts varies. Therefore, we propose the concept of an evolving PCFI (EPCFI) diagram. An important EPCFI diagram function is to determine the appropriate number of experts for a fuzzy collaborative forecasting task, which is critical for fuzzy group decision making [10–12]. The proposed evolving partial-consensus fuzzy collaborative forecasting approach is based on the EPCFI diagram.

Table 1 summarizes the differences between the proposed methodology and some current methods, and the specific contributions from the proposed methodology are as follows.

1. A systematic procedure is established to determine the appropriate number of experts for fuzzy collaborative forecasting.
2. The EPCFI diagram concept is introduced to analyze aggregation changes as the number of experts varies.
3. Experts are no longer forced to modify their fuzzy forecasts when an overall consensus cannot be achieved.

Table 1. Proposed and current collaborative fuzzy forecasting methods. FLR: fuzzy linear regression; PCFI: partial consensus fuzzy intersection; evolving PCFI; and ANN: artificial neural network.

Method	Forecast Source	Number of Sources	Forecasting Method	Aggregation Mechanism	Modification Mechanism
Chen [5]	Experts	Fixed	FLR	PCFI	Subjective modification
Zarandi et al. [13]	Agents	Fixed	Fuzzy inference rules	Weighted average	Genetic algorithm
Swaroop et al. [14]	Rules	Fixed	Fuzzy inference rules	Fuzzy union	ANN
Proposed	Experts	Dynamic	FLR	EPCFI	Not required

The remainder of this paper is organized as follows. Section 2 briefly reviews relevant previous studies, and Section 3 describes some models to fit fuzzy linear regressions (FLRs). Section 4 details the proposed evolving partial consensus fuzzy collaborative forecasting approach, and Section 5 presents experimental results applying the proposed approach to forecast the DRAM product yield. Section 6 summarizes and concludes the paper, and it discusses some topics for future investigation.

2. Literature Review

Cheikhrouhou et al. [15] built an autoregressive integrated moving average (ARIMA) model to forecast polyethylene bag demand. Experts subsequently judged unexpected future event effects on demand, and these became inputs to a Mamdani's fuzzy inference system (FIS) [16] to modify demand forecasts. However, FISs, including Mamdani's, Sugeno's [17], and adaptive network based FISs (ANFISs) [18] apply fuzzy rules that cannot guarantee test value inclusion in the corresponding fuzzy forecasts [2]. Consequently, a fuzzy union (S-norm) must be applied to aggregate the forecast results by using fuzzy inference rules to avoid missing a test value, which widens the fuzzy forecast range and sacrifices forecasting precision. Swaroop et al. [14] established an FIS to forecast the load on a power system, and this FIS became an input to an artificial neural network (ANN) to tune the load forecast.

Chen [19] proposed a fuzzy collaborative forecasting method in which each expert fitted an FLR to predict the effective cost per die for DRAM product. Fuzzy parameter values for the FLR were derived by solving various nonlinear programming problems. Thus, all test values were included in the corresponding fuzzy cost forecasts, at least for the training data. A fuzzy intersection, or the minimum T-norm, was then applied to aggregate expert fuzzy cost forecasts, which optimized forecasting precision in terms of an average fuzzy cost forecast range. A back propagation network (BPN) was subsequently constructed to defuzzify the aggregation result, optimizing forecasting accuracy measured as a root mean squared error (RMSE). Similar fuzzy collaborative forecasting methods have been subsequently proposed to forecast global CO_2 concentration [5], job cycle time [20], long-term load [21], and DRAM product unit cost [22].

Zhang et al. [23] and Ostrosi et al. [24] proposed an aggregation mechanism that minimized the sum of the squared differences between expert forecasts and the aggregation. However, the aggregation mechanism assumed that a consensus existed and directly derived the aggregation result. Gao et al. [25] aggregated expert fuzzy forecasts with a fuzzy weighted average, and then they measured the distance between each fuzzy forecast and the aggregation. Experts whose fuzzy forecast distance exceeded the same threshold were asked to modify their forecasts, and those that refused to do so were downweighted until the distance between each fuzzy forecast and the aggregation result was sufficiently small. However, fuzzy forecasts with heavier weights were usually closer to the aggregation result, even when they differed from other expert's forecasts, and these were not downweighted. Furthermore, calculating the fuzzy distance and deciding the appropriate threshold required additional subjective decisions, and sometimes consensus still could not be achieved after collaboration.

Herrera-Viedma et al. [26] argued that not only did the consensus represent the group's common perception of some values, it also represented the process to reach the consensus. After collaboration, experts directly modified fuzzy forecasting parameters or fuzzy forecasts to close the gap between each other. However, it still may not be possible to achieve a consensus in this process, i.e., expert fuzzy forecasts may not overlap. Chen [27] proposed a heterogeneous fuzzy collaborative forecasting method to predict semiconductor product yield, where experts fitted the yield learning process of the product with FLR by solving mathematical programming problems or training ANNs. Zarandi et al. [13] proposed a four-layer fuzzy multiagent system to forecast next-day stock prices based on collaboration among software agents. Chen and Wang [28] and Chen and Romanowski [29] proposed software agents, rather than real experts, for fuzzy collaborative forecasting to expedite collaboration. However, software agents usually follow pre-specified rules when fuzzy parameters need to be adjusted, which may result in unrealistic fuzzy forecasts.

3. Preliminary Models for Fitting a Fuzzy Linear Regression

Table 2 summarizes the abbreviations used throughout this paper; we use the following parameters and variables in the proposed methodology.

(1) $(+)$: fuzzy addition.
(2) $\widetilde{a}_i$: $i = 0 \sim m$: FLR coefficients.
(3) d: acceptable fuzzy forecast range.
(4) $\widetilde{I}^{H/K}$: fuzzy intersection function.
(5) N: normalization function.
(6) $o \in R^+$: expert sensitivity to uncertainty in a fuzzy forecast (smaller values imply less sensitivity, and large values imply more sensitivity).
(7) $s \in [0, 1]$: required satisfaction level.
(8) s_j: satisfaction level at period j.
(9) $w \in R^+$: expert sensitivity to satisfaction level improvement (smaller values imply less sensitivity).
(10) x_{ji}; $i = 0 \sim m$, $j = 1 \sim n$: value for decision variable i forecasting y_j.
(11) y_j; $j = 1 \sim n$: test value at period j.
(12) $\widetilde{y}_j$; $j = 1 \sim n$: fuzzy forecast at period j.
(13) $\mu_{\widetilde{y}_j}$: membership function for $\widetilde{y}_j$.

Table 2. Abbreviations used in this paper.

Acronym	Meaning
ANFIS	Adaptive network based fuzzy inference system
ANN	Artificial neural network
ARIMA	Autoregressive integrated moving average
BPN	Back propagation network
COG	Center-of-gravity
DRAM	Dynamic random access memory
EPCFI	Evolving partial-consensus fuzzy intersection
FI	Fuzzy intersection
FIS	Fuzzy inference system
FLR	Fuzzy linear regression
GA	Genetic algorithm
GD	Gradient descent
LP	Linear programming
MAE	Mean absolute error
MAPE	Mean absolute percentage error
NLP	Nonlinear programming
PCFI	Partial consensus fuzzy intersection
QP	Quadratic programming
RMSE	Root mean squared error
TFN	Triangular fuzzy number

Without a loss of generality, all fuzzy parameters and variables in the proposed methodology are given as or approximated with triangular fuzzy numbers (TFNs). We also assumed that all experts apply the following FLR to forecast the same target y based on decision variable values $\{x_i\}$ [19,30]:

$$\widetilde{y}_j = \widetilde{a}_0(+)\sum_{i=1}^{m}\widetilde{a}_i x_{ji} \tag{1}$$

However, the proposed methodology could also be extended using other fuzzy forecasting methods.

Several mathematical models have been proposed to derive fuzzy parameter values in Equation (1). For example, Tanaka and Watada [31] proposed a linear programming method to minimize the fuzzy forecast sum of the ranges (or spreads), hence maximizing forecast precision. Taheri and Kelkinnama [32] solved another linear programming problem to minimize the sum of absolute errors. Peters [33] proposed a quadratic programming (QP) method to maximize average satisfaction level and, hence, improve forecast accuracy.

The simultaneous optimization of forecasting accuracy and precision has been pursued by many researchers, but it is somewhat challenging. Donoso et al. [34] proposed a compromise approach by minimizing the weighted sum of the sum of squared deviations between fuzzy forecast cores, actual values, and the sum of the squared ranges.

Chen and Lin [35] incorporated expert opinions into the Tanaka and Watada model and the Peters model, proposing two nonlinear programming (NLP) models, as follows.

NLP Model I:

$$\text{Min } Z_1 = \sum_{j=1}^{n}(y_{j3} - y_{j1})^o, \tag{2}$$

which is subject to

$$y_j \geq y_{j1} + s(y_{j2} - y_{j1}), \tag{3}$$

$$y_j \leq y_{j3} + s(y_{j2} - y_{j3}), \tag{4}$$

$$y_{j1} = a_{01} + \sum_{i=1}^{m} a_{i1}x_{ji}, \tag{5}$$

$$y_{j2} = a_{02} + \sum_{i=1}^{m} a_{i2} x_{ji}, \tag{6}$$

$$y_{j3} = a_{03} + \sum_{i=1}^{m} a_{i3} x_{ji}, \tag{7}$$

$$y_{j1} \le y_{j2} \le y_{j3}, \tag{8}$$

and

$$a_{i1} \le a_{i2} \le a_{i3}, \tag{9}$$

where $j = 1, 2, \ldots, n$ and $i = 0, 1, \ldots, m$. The objective function minimizes the high order sum of the fuzzy forecast ranges; Constraints (3) and (4) ensure that the membership of an actual value in the corresponding fuzzy forecast should be higher than s; Equations (5)–(7) are the decomposition of Equation (1); Constraints (8) and (9) define the sequence for the three TFN corners.

If o is large, it becomes difficult to optimize NLP Model I. Therefore, Chen and Wang [28] advised choosing $o \in [0, 4]$. When o is a positive integer, the model can be converted into an equivalent QP problem. Chen and Wang [36] also proposed a method to approximate the model with a QP problem. First, the y_j is normalized into [0, 1]:

$$y_j \to N(y_j) = \frac{y_j - \min\limits_k y_k}{\max\limits_k y_k - \min\limits_k y_k}. \tag{10}$$

Hence:

$$N(y_{j3}) - N(y_{j1}) \in [0, 1], \tag{11}$$

since $y_{j3} \ge y_{j1}$. Chen and Wang method approximated the objective function with a quadratic, e.g., for $o = 1.5$:

$$\begin{aligned} &\sum_{j=1}^{n} (N(y_{j3}) - N(y_{j1}))^{1.5} \\ &\cong \sum_{j=1}^{n} (0.5027(N(y_{j3}) - N(y_{j1}))^2 + 0.5308(N(y_{j3}) - N(y_{j1})) - 0.0347) \end{aligned} \tag{12}$$

NLP Model II:

$$\text{Max } Z_2 = \sum_{j=1}^{n} s_j^{\,w}, \tag{13}$$

which is subject to

$$\sum_{j=1}^{n} (y_{j3} - y_{j1})^o \le n \cdot d^o, \tag{14}$$

$$y_j \ge y_{j1} + s_j(y_{j2} - y_{j1}), \tag{15}$$

$$y_j \le y_{j3} + s_j(y_{j2} - y_{j3}), \tag{16}$$

$$y_{j1} = a_{01} + \sum_{i=1}^{m} a_{i1} x_{ji}, \tag{17}$$

$$y_{j2} = a_{02} + \sum_{i=1}^{m} a_{i2} x_{ji}, \tag{18}$$

$$y_{j3} = a_{03} + \sum_{i=1}^{m} a_{i3} x_{ji}, \tag{19}$$

$$y_{j1} \le y_{j2} \le y_{j3}, \quad (20)$$

$$a_{i1} \le a_{i2} \le a_{i3}, \quad (21)$$

and

$$0 \le s_j \le 1. \quad (22)$$

The objective function maximizes the high order sum of satisfaction levels. Constraint (14) ensures that average fuzzy forecast range is narrower than d, Constraints (15) and (16) derive membership for an actual value in the corresponding fuzzy forecast, Equations (17)–(19) are the same decomposition for Equation (1) as used for NLP Model I, Constraints (20) and (21) define the sequence for the three TFN corners, and constraint (22) defines the range of the satisfaction level. When o and w are both positive integers, the model can be converted into an equivalent QP problem. Otherwise, Chen and Wang's method can also be applied to approximate the model with a QP problem, similarly to the case for NLP Model I.

Fuzzy forecasts generated by the NLP models can be diversified by varying o, s, d, and w, which lays the basis for collaboration.

4. Proposed Methodology

Figure 1 shows the proposed methodology system diagram. The proposed approach comprises four major steps as follows.

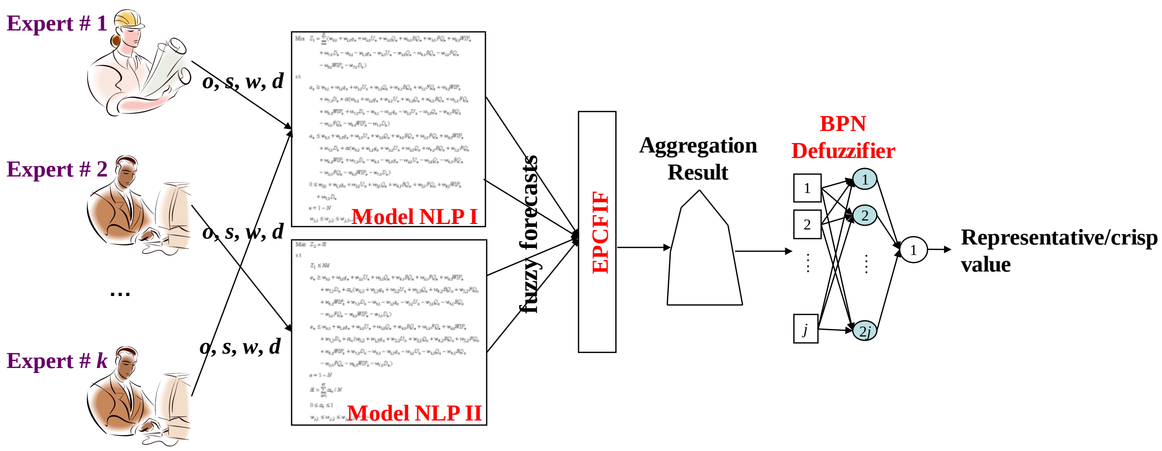


Figure 1. Proposed methodology procedure

Step 1. Experts apply either NLP model [35] to generate fuzzy forecasts with parameters o, s, w, and d, which are specified before formulating the models.

Step 2. An EPCFI is applied to aggregate expert fuzzy forecasts, where the aggregation is a polygonal fuzzy number, as shown in Figure 1.

Step 3. The appropriate number of experts is determined by observing aggregation result changes as the number of experts varies.

Step 4. A BPN is constructed to defuzzify the aggregation result, providing a representative/crisp value.

The proposed approach optimizes the NLP models to generate fuzzy forecasts and then constructs a BPN to defuzzify the aggregated result. In contrast, current methods use a BPN, an ANFIS, and other ANN types to directly generate forecasts [37–41].

4.1. EPCFI

Current fuzzy collaborative forecasting methods most commonly use an FI (i.e., minimum T-norm) to aggregate expert fuzzy forecasts:

$$\mu_{\widetilde{y}_j}(x) = \min_k(\{\mu_{\widetilde{y}_j(k)}(x) | k = 1 \sim K\}), \tag{23}$$

The prerequisite to apply an FI is that the expert fuzzy forecasts all include actual values, at least for the training (or learned) data. Otherwise, a fuzzy union (i.e., maximum T-conorm or S-norm) should be applied instead as the treatment taken in existing FISs.

A fuzzy intersection finds values common to expert fuzzy forecasts. Therefore, an FI can represent overall consensus among the experts. If each expert's fuzzy forecast is represented by a TFN, then the FI is a polygonal fuzzy number (see Figure 2) and its α cut can be expressed as:

$$\begin{aligned} \widetilde{y}_j(\alpha) &= [\widetilde{y}_j^L(\alpha), \widetilde{y}_j^R(\alpha)] \\ &= [\max_k(\widetilde{y}_j^L(k)(\alpha)), \min_k(\widetilde{y}_j^R(k)(\alpha))] \end{aligned}, \tag{24}$$

where $[\widetilde{y}_j^L(k)(\alpha), \widetilde{y}_j^R(k)(\alpha)]$ is the α cut of $\widetilde{y}_j(k)$.

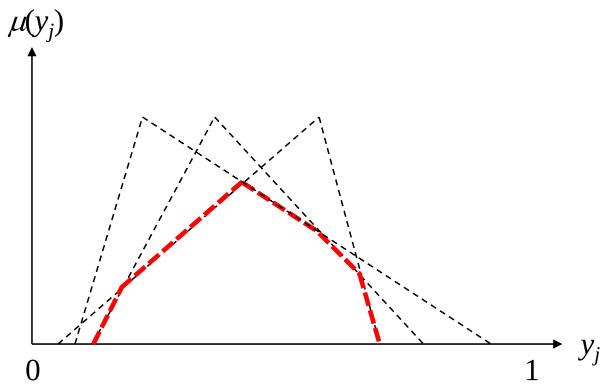


Figure 2. Fuzzy intersection (consensus).

When an overall consensus among experts cannot be achieved, the FI is the null set. In this situation, a consensus among some experts can be sought instead by using the PCFI operator [19].

Definition 1. *The H/K PCFI for fuzzy forecasts by K experts at period j, i.e.,* $\widetilde{y}_j(1)$~$\widetilde{y}_j(K)$*, is represented as* $\widetilde{I}^{H/K}(\widetilde{y}_j(1), \ldots, \widetilde{y}_j(K))$*, such that:*

$$\mu_{\widetilde{y}_j^{H/K}}(x) = \max_{\text{all}g}(\min(\mu_{\widetilde{y}_j(g(1))}(x), \ldots, \mu_{\widetilde{y}_j(g(H))}(x)))\forall x, \tag{25}$$

where $g() \in Z^+$; $1 \le g() \le K$; $g(p) \cap g(q) = \emptyset \ \forall \ p \ne q$; $H \ge 2$.

From Definition 1, each time a subset of size H is extracted from the set of K experts, membership for a value is determined by applying a minimum operator, representing a (partial) consensus among H experts. Since subsets do not overlap, the maximum operator is applied to aggregate memberships for a value.

For example, the 2/3 PCFI of $\widetilde{y}_j(1)$~$\widetilde{y}_j(K)$ can be expressed as:

$$\mu_{\widetilde{y}_j^{2/3}}(x) = \max(\min(\mu_{\widetilde{y}_j(1)}(x), \mu_{\widetilde{y}_j(2)}(x)), \min(\mu_{\widetilde{y}_j(1)}(x), \mu_{\widetilde{y}_j(3)}(x)), \min(\mu_{\widetilde{y}_j(2)}(x), \mu_{\widetilde{y}_j(3)}(x)))\forall x, \tag{26}$$

as shown in Figure 3.

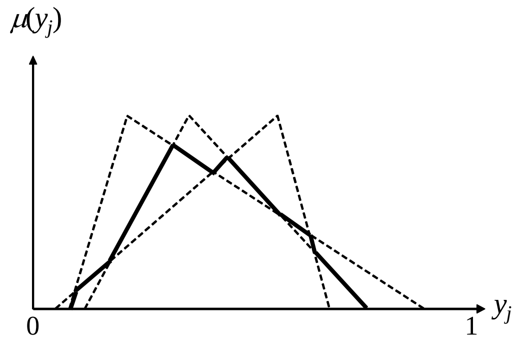


Figure 3. Partial consensus fuzzy intersection.

Property 1. *A PCFI is the hybrid of an FI and a fuzzy union for handling parts with and without consensus, respectively.*

An FI operator meets four requirements: boundary conditions, monotonicity, commutativity, and associativity. A fuzzy union operator also meets these requirements, but the boundary conditions for FI operators are contradictory to those for fuzzy union operators. Therefore, a PCFI operator meets three requirements: monotonicity, commutativity, and associativity. Thus, a PCFI eliminates the necessity to exclude fuzzy forecasts by radical experts or to force them to modify their fuzzy forecasts.

Theorem 1. *The PCFI result includes the FI result.*

Proof. The minimum of more items becomes smaller. Therefore:

$$\begin{aligned} \mu_{PCFI(\{\widetilde{y}_j(k)\})}(x) &= \mu_{\widetilde{y}_j^{H/K}}(x) \\ &= \max_{\text{all}\, g}(\min(\mu_{\widetilde{y}_j(g(1))}(x), \ldots, \mu_{\widetilde{y}_j(g(H))}(x))) \\ &\geq \max_{\text{all}\, g}(\min(\mu_{\widetilde{y}_j(g(1))}(x), \ldots, \mu_{\widetilde{y}_j(g(K))}(x))) \\ &= \mu_{\widetilde{y}_j^{K/K}}(x) \\ &= \mu_{FI(\{\widetilde{y}_j(k)\})}(x) \end{aligned} \tag{27}$$

since $H \leq K$. Thus,

$$PCFI(\{\widetilde{y}_j(k)\}) \supseteq FI(\{\widetilde{y}_j(k)\}).$$

□

Meaningful information can be determined by observing PCFI changes when the number of experts varies. To simplify this, we propose an EPCFI diagram, as shown in Figure 4 and defined as follows.

Definition 2. *An EPCFI diagram is a systematic representation of aggregation changes, i.e.,* $\widetilde{y}_j^{H/K}$*, when the number of experts, H, varies.*

In the EPCFI diagram:

(1) $\widetilde{y}_j^{2/4} \supseteq \widetilde{y}_j^{3/4} \supseteq \widetilde{y}_j^{4/4}$.

(2) If the consensus among all experts, i.e., $\widetilde{y}_j^{4/4}$, is sought, the aggregation covers a very narrow range [4.99, 5.61]. A narrower range means higher forecasting precision, which is good for the training data but may increase the possibility of missing an actual value for test data.

(3) It is easier to reach a consensus among fewer experts, e.g., $\widetilde{y}_j^{3/4}$, and the aggregation covers a much wider range [2.89, 7.71].

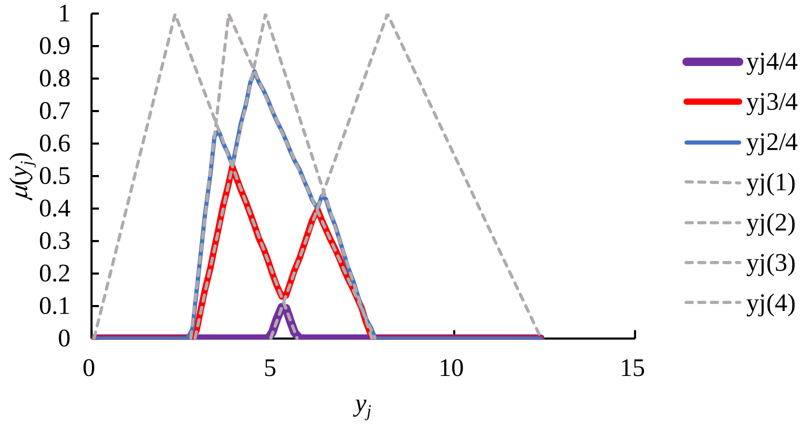


Figure 4. Typical EPCFI diagram.

A narrow PCFI can maximize the forecasting precision for training data, but the future situation may markedly differ from the past; hence, adopting a narrow PCFI (by considering consensus among all experts) is risky. Adopting a wider PCFI result by considering consensus among fewer experts provides a more robust outcome. Thus, the appropriate number of experts can be determined as follows.

(1) If $\widetilde{y}_j^{H+1/K}$ is much narrower than $\widetilde{y}_j^{H/K}$, then choosing $\widetilde{y}_j^{H/K}$ is less risky.
(2) If $\widetilde{y}_j^{H+1/K}$ is very close to $\widetilde{y}_j^{H/K}$, then $\widetilde{y}_j^{H+1/K}$ is preferable because the consensus among more experts should always be sought.

4.2. Back Propagating Network to Defuzzify the Aggregation

A BPN with the following configuration is constructed to defuzzify the aggregation.

(1) Input: BPN inputs include the value and membership for each EPCFI corner result. Deriving the representative value based on these corners is meaningful, because corner memberships are the same, which means that a consensus is achieved among experts. Consider the example shown in Figure 5. The EPCFI, in terms of $\widetilde{y}_j^{2/4}$, has seven corners; hence, there are 14 BPN inputs. However, the number of corners may differ from example to example. Therefore, the number of BPN inputs is determined by the maximum number of corners in all examples.
(2) Hidden layer: Many studies have shown that a single hidden layer is sufficient to fit complex nonlinear relationships [42]. The number of nodes in the hidden layer is twice the number of inputs [43,44].
(3) Output: o_j is compared with y_j.
(4) Learning rate: η = 0.1–1.0.
(5) Training algorithm: A gradient descent (GD) algorithm is used to prevent overfitting [45].
(6) Convergence criteria: Training terminates when the sum of squared error,

$$SSE = \sum_{j=1}^{n} (y_j - o_j)^2, \tag{28}$$

falls below a pre-specified threshold, or a maximal number of epochs have been run.

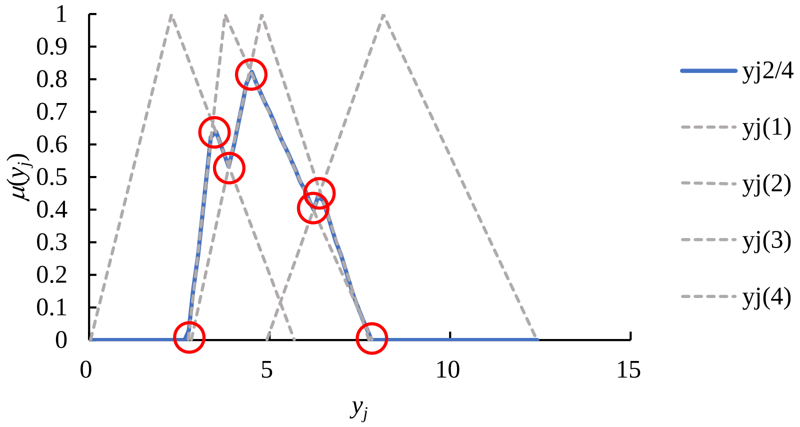


Figure 5. Determining inputs to the back propagation network (BPN).

Figure 6 shows how the BPN defuzzifier is incorporated in the proposed methodology.

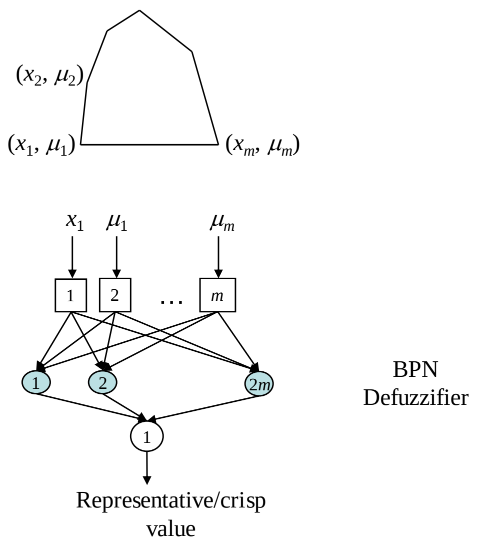


Figure 6. The role of the BPN defuzzifier.

5. Case Study: Forecasting DRAM Product Yield

5.1. Proposed Methodology Application

The proposed evolving partial-consensus fuzzy collaborative forecasting approach was applied to forecast DRAM (die) yield [36]. Product yield is the most critical performance measure for a DRAM factory [46,47], and accurately and precisely forecasting a future yield is essential to create a competitive production plan [48,49]. Improving DRAM yield can be modelled as a learning process that cannot be directly modelled with a conventional time series [50–52]:

$$\widetilde{Y}_t = \widetilde{Y}_0(\times)e^{-\frac{\widetilde{b}}{t}}, \tag{29}$$

After converting all terms on both sides to their logarithmic values:

$$\ln \widetilde{Y}_t = \ln \widetilde{Y}_0(-)\frac{\widetilde{b}}{t}, \tag{30}$$

which can be fitted as an FLR,

$$-\frac{1}{t} \to j, \tag{31}$$

$$\ln \widetilde{Y}_t \to \widetilde{y}_j, \tag{32}$$

$$\ln \widetilde{Y}_0 \to \widetilde{a}_0, \tag{33}$$

and

$$\widetilde{b} \to \widetilde{a}_1. \tag{34}$$

In the experiment, four experts applied various NLP methods to forecast DRAM yield with the following parameters:

Expert #1: NLP Model I ($o = 1$; $s = 0.5$; $w = 1$; $d = 0.4$).

Expert #2: NLP Model I ($o = 3$; $s = 0.35$; $w = 2$; $d = 0.55$).

Expert #3: NLP Model II ($o = 2$; $s = 0.4$; $w = 2$; $d = 0.35$).

Expert #4: NLP Model II ($o = 1$; $s = 0.25$; $w = 3$; $d = 0.7$).

Yield data were split into two parts, with the first six periods used to build the models and the remainder used for testing. NLP problems were solved using Lingo on a PC with a 3.6 GHz and 8 GB RAM i7-7700 CPU, which achieved an execution time of less than 3 s. Figure 7 shows the final expert forecasts.

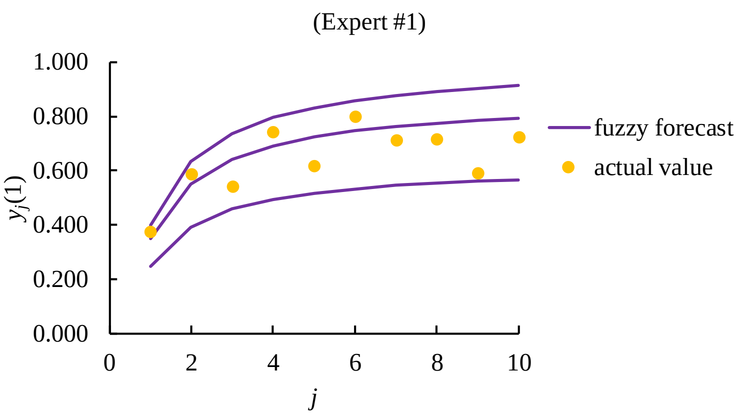
Figure 8 shows the corresponding EPCFI diagram for the expert forecasts for Period 1. All expert fuzzy yield forecasts fell within a very narrow range, and the range shrank rapidly with each additional expert included. With only two experts, the aggregation range was very wide. Thus, a partial consensus among three experts, i.e., $\widetilde{y}_j^{3/4}$, seemed to be a reasonable choice.

Figure 9 shows the aggregation for three expert fuzzy yield forecasts, i.e., $\widetilde{y}_j^{3/4}$. The actual values at all periods except for those in Period 9 fell within the corresponding aggregation results.

Subsequently, the aggregation was defuzzified with a BPN. To this end, we first derived the aggregation corners for each period, as shown in Table 3. The maximum number of corners over all periods = 6, so the number of BPN inputs = 12 and number of nodes in the hidden layer = 24. The BPN was trained with the GD algorithm to prevent overfitting. Convergence criteria were established as:

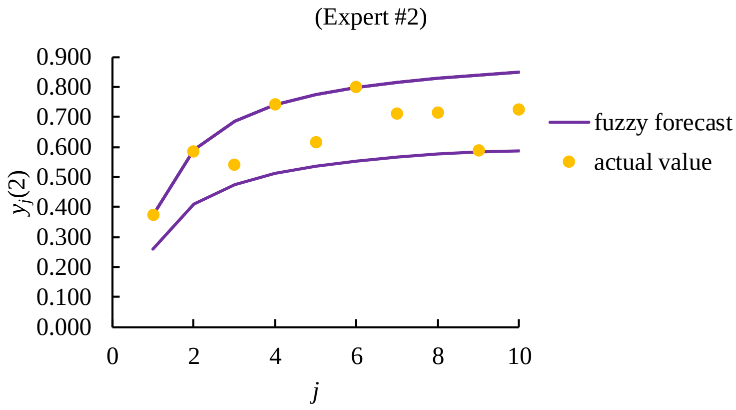
(1) SSE < 10^{-6}; or
(2) 1000 epochs.

The BPN defuzzifier was implemented with the MATLAB® 2017 neural network toolbox on the same PC, with execution times of less than 1 s. Figure 10 shows defuzzification results.

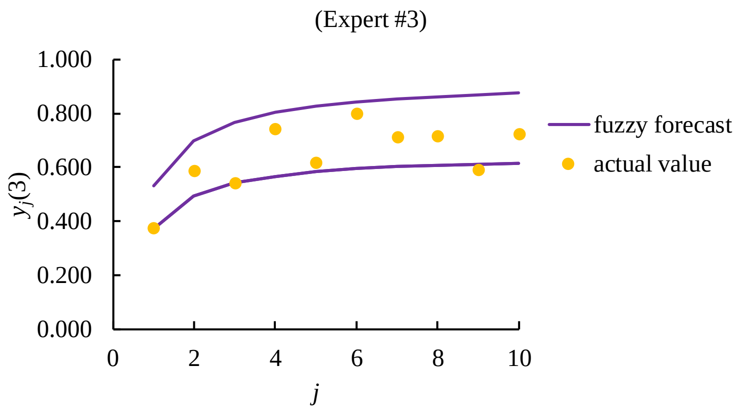


(**a**) Expert #1

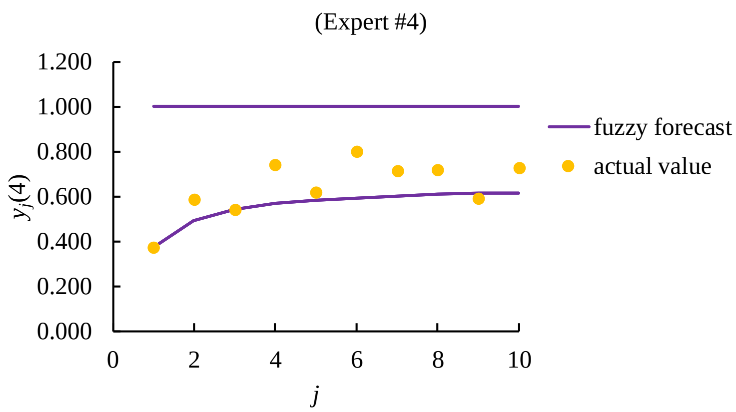
Figure 7. *Cont.*



(**b**) Expert #2



(**c**) Expert #3



(**d**) Expert #4

Figure 7. Expert dynamic random access memory (DRAM) yield forecasts.

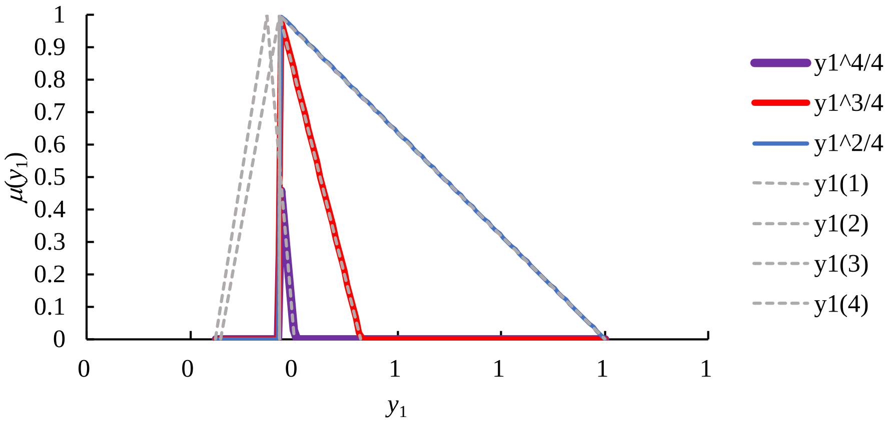


Figure 8. Example EPCFI diagram for Period 1.

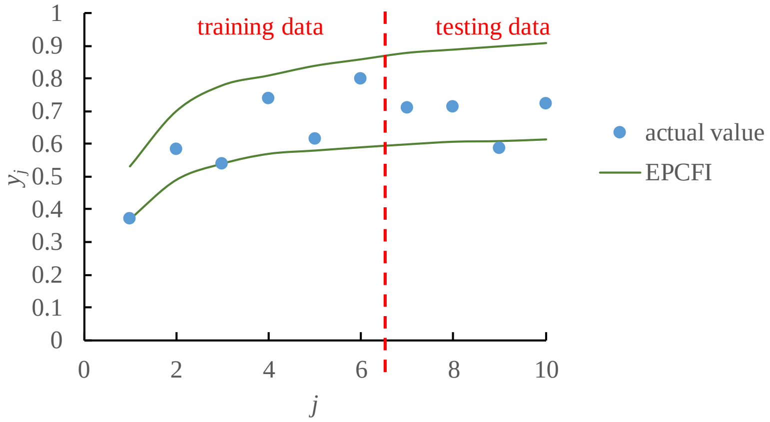


Figure 9. The aggregation results.

To validate the effectiveness of the proposed methodology, the optimized models and the trained BPN were applied to test (unlearned) data. Then, the forecasting accuracy was evaluated in terms of mean absolute error (MAE), mean absolute percentage error (MAPE), and RMSE as:

MAE = 0.04,
MAPE = 5.95%, and
RMSE = 0.06.

Table 3. Aggregation result corners

j	Corners
1	(0.37, 0.00), (0.38, 0.98), (0.53, 0.00)
2	(0.49, 0.00), (0.50, 0.66), (0.53, 0.84), (0.70, 0.00)
3	(0.54, 0.00), (0.54, 0.48), (0.60, 0.75), (0.69, 0.36), (0.69, 0.48), (0.78, 0.00)
4	(0.57, 0.00), (0.57, 0.39), (0.64, 0.71), (0.74, 0.29), (0.74, 0.51), (0.81, 0.00)
5	(0.58, 0.00), (0.58, 0.33), (0.66, 0.68), (0.77, 0.22), (0.79, 0.50), (0.84, 0.00)
6	(0.59, 0.00), (0.60, 0.31), (0.68, 0.66), (0.80, 0.18), (0.80, 0.48), (0.86, 0.00)
7	(0.60, 0.00), (0.60, 0.27), (0.69, 0.65), (0.81, 0.16), (0.82, 0.46), (0.88, 0.00)
8	(0.61, 0.00), (0.61, 0.26), (0.65, 0.70), (0.83, 0.13), (0.84, 0.42), (0.89, 0.00)
9	(0.61, 0.00), (0.61, 0.24), (0.71, 0.64), (0.84, 0.12), (0.84, 0.41), (0.90, 0.00)
10	(0.61, 0.00), (0.61, 0.23), (0.71, 0.63), (0.85, 0.10), (0.86, 0.38), (0.91, 0.00)

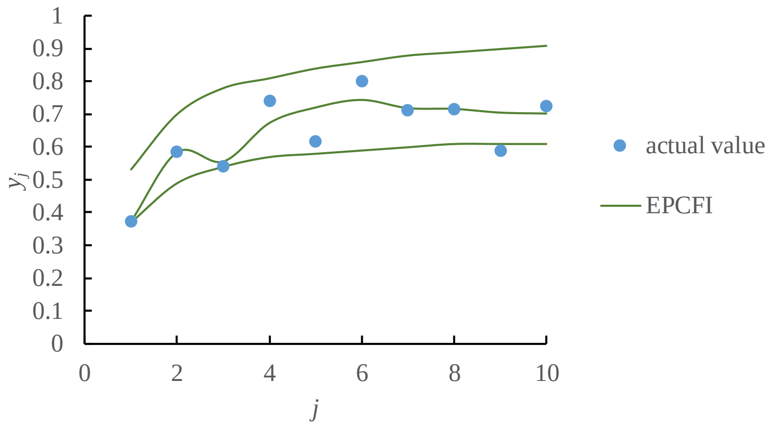


Figure 10. Defuzzification results.

5.2. Comparisons

Several current methods were applied to this case for comparison. First, we applied the Chen and Lin [35] fuzzy collaborative forecasting method based on overall consensus, employing an FI to aggregate expert fuzzy yield forecasts, as shown in Table 4. The number aggregation corners for each period was much smaller. Subsequently, we constructed a BPN to defuzzify the aggregation and provide a representative value. Figure 11 shows the final forecast results.

Table 4. Fuzzy intersection results for the Chen and Lin method.

j	Corners
1	(0.37, 0.00), (0.38, 0.46), (0.41, 0.00)
2	(0.59, 0.00), (0.59, 0.48), (0.64, 0.00)
3	(0.69, 0.00), (0.69, 0.34), (0.74, 0.00)
4	(0.74, 0.00), (0.74, 0.26), (0.80, 0.00)
5	(0.77, 0.00), (0.78, 0.20), (0.83, 0.00)
6	(0.80, 0.00), (0.80, 0.16), (0.85, 0.00)
7	(0.81, 0.00), (0.82, 0.14), (0.85, 0.00)
8	(0.83, 0.00), (0.84, 0.11), (0.87, 0.00)
9	(0.84, 0.00), (0.84, 0.11), (0.87, 0.00)
10	(0.85, 0.00), (0.85, 0.09), (0.88, 0.00)

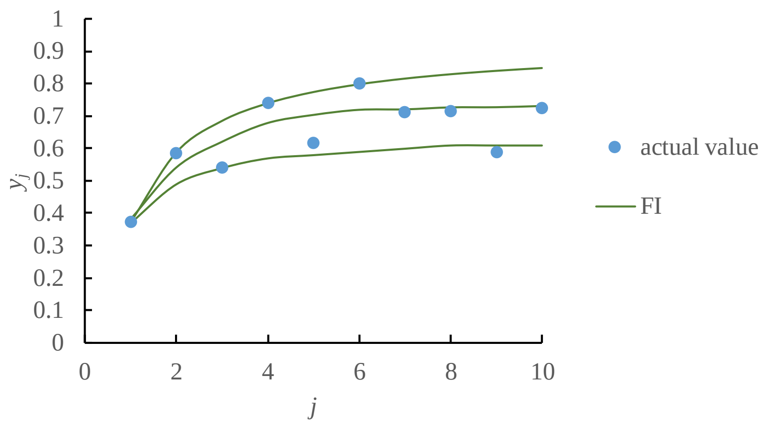


Figure 11. Chen and Lin method results based on overall consensus.

Forecast accuracy was evaluated in the same manner as for the proposed approach, with MAE = 0.04, MAPE = 6.84%, and RMSE = 0.07.

Subsequently, forecast performance was also evaluated for the case where the experts did collaborate. Expert fuzzy yield forecasts were defuzzified with the prevalent center-of-gravity (COG) method [53], as shown in Table 5.

Table 5. Forecasting performances for Chen and Lin method if experts did not collaborate.

Expert	MAE	MAPE	RMSE
1	0.06	9.26%	0.08
2	0.06	10.10%	0.09
3	0.04	7.08%	0.06
4	0.05	8.68%	0.08

Second, we applied the 6σ logistic regression method, which fitted collected yield data with the logistic regression model:

$$\log \hat{y}_j = -0.262 - \frac{0.697}{j}, \tag{35}$$

where $\sigma = 0.115$. This model achieved a coefficient of determination $R^2 = 0.87$, which was sufficiently high. Upper and lower yield forecast bounds were established by adding and subtracting 3σ to the yield forecast, respectively, as shown in in Figure 12. This method established very wide yield forecast ranges, so all actual values in the test data were included in the corresponding confidence intervals, elevating the hit rate to 100%. The forecasting accuracy when using the 6σ logistic regression method was evaluated as MAE = 0.04, MAPE = 6.36%, and RMSE = 0.06.

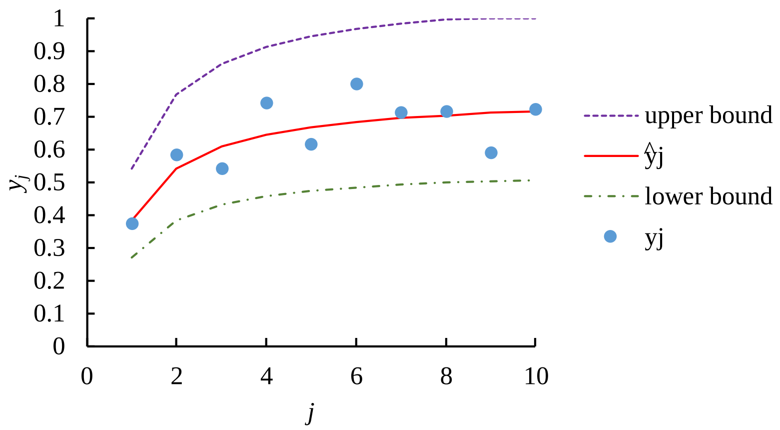


Figure 12. Forecast results found when the 6σ logistic regression method.

5.3. Discussion

Table 6 compares forecasting performances for the various considered methods. The proposed methodology achieved a superior forecast accuracy in terms of MAE, MAPE, and RMSE.

Table 6. Forecasting performance for the considered methods

Method	Hit Rate	Average Range	MAE	MAPE	RMSE
Expert #1	100%	0.34	0.06	9.26%	0.08
Expert #2	100%	0.26	0.06	10.10%	0.09
Expert #3	75%	0.26	0.04	7.08%	0.06
Expert #4	75%	0.40	0.05	8.68%	0.08
6σ logistic regression	100%	0.69	0.04	6.36%	0.06
Chen and Lin's fuzzy collaborative forecasting method [35]	75%	0.23	0.04	6.84%	0.07
The proposed methodology	75%	0.29	0.04	5.95%	0.06

Forecasting performance improved with expert collaboration. When experts achieved an overall consensus, forecasting precision improved by up to 43% in terms of the average range of the fuzzy yield forecasts. A comparable improvement was also achieved for partial consensus, with the forecasting accuracy in terms of MAPE improved by 31% after applying the proposed methodology.

The proposed evolving partial-consensus fuzzy collaborative forecasting approach surpassed Chen and Lin's fuzzy collaborative forecasting method for optimizing forecast accuracy for both fuzzy collaborative forecasting methods. This was most likely possible because $\widetilde{y}_j^{3/4}$ had more corners than $\widetilde{y}_j^{4/4}$, which gave the decision makers a higher degree of freedom in defuzzifying the aggregation. Hence, the possibility of finding actual values also improved.

Applying the common COG method to defuzzify the aggregation from the proposed methodology for each period achieved a forecast accuracy with MAE = 0.05, MAPE = 7.86%, and RMSE = 0.08, which was worse than that achieved when the BPN defuzzifier was applied, as shown in Figure 13. This confirmed the effectiveness of the BPN defuzzifier.

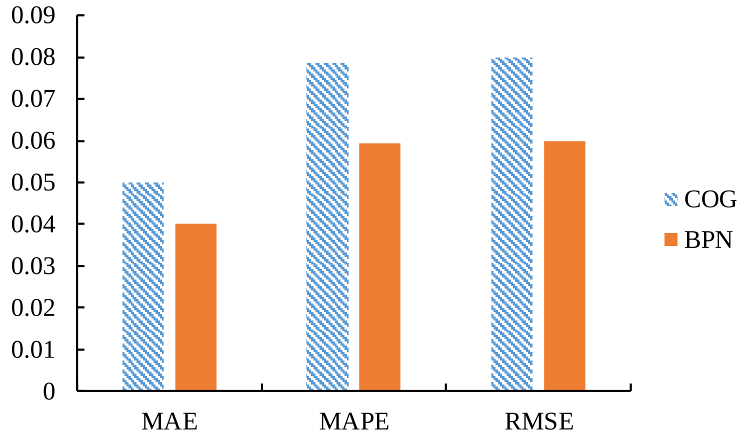


Figure 13. Forecasting performance for the proposed methodology with different defuzzifiers.

6. Conclusions

Most current fuzzy collaborative forecast methods apply an FI to aggregate expert fuzzy forecasts, a process that has several drawbacks. The PCFI operator [5] is useful to help overcome these drawbacks. However, how to determine the appropriate number of experts has not been considered—this can be assessed by observing PCFI changes when the number of experts varies. Therefore, this paper proposed EPCFI diagrams to simplify this comparison, as well as an evolving partial-consensus fuzzy collaborative forecasting approach based on the EPCFI diagrams.

The proposed evolving partial-consensus fuzzy collaborative forecasting approach was applied to forecast DRAM yield using real-world data, with the following conclusions.

(1) The proposed approach effectively improved forecast accuracy for test in terms of MAE, MAPE, and RMSE. The most significant advantage over current methods was up to 24% when MAPE was minimized.

(2) Compared with the Chen and Lin fuzzy collaborative forecasting method, the proposed methodology achieved a higher forecast accuracy at the expense of a slight increase in the average fuzzy yield forecast range due to increased degree of freedom in defuzzifying fuzzy the yield forecasts.

(3) Collaboration among experts was shown to be conducive to forecast performance. Forecast precision, in terms of average fuzzy yield forecast range, improved 43% after expert collaboration, and forecast accuracy (MAPE) also improved 31%

The proposed evolving partial-consensus fuzzy collaborative forecasting approach can be easily implemented with current data analysis software. The fuzzy forecasting method (Equation (1)) generalizes many fuzzy forecasting methods, such as the fuzzy moving average and the fuzzy ARIMA. Therefore, the proposed methodology can be easily applied to other problem types, such as fuzzy time series forecasts.

Future studies will investigate the application of the evolving partial-consensus fuzzy collaborative forecasting approach to more real cases to further investigate its effectiveness. Other mechanisms to measure partial expert consensus will also be explored. Experts often have unequal authority levels, and this situation should be incorporated when aggregating their fuzzy forecasts, such as through weighted aggregation operators like weighted FI, PCFI, and EPCFI.

Author Contributions: All authors equally contributed to the writing of this paper. All authors read and approved the final manuscript. Data curation, methodology and writing original draft: T.-C.T.C. and Y.-C.W.; writing—review and editing: T.-C.T.C., Y.-C.W., and C.-H.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflicts of interest.

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