



A Rectifying Acceptance Sampling Plan Based on the Process Capability Index

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Abstract: The acceptance sampling plan and process capability index (PCI) are critical decision tools for quality control. Recently, numerous research papers have examined the acceptance sampling plan in combination with the PCI. However, most of these papers have not considered the aspect of rectifying inspections. In this paper, we propose a quality cost model of repetitive sampling to develop a rectifying acceptance sampling plan based on the one-sided PCI. This proposed model minimizes the total quality cost (TQC) of sentencing one lot, including inspection cost, internal failure cost, and external failure cost. In addition, sensitivity analysis is conducted to investigate the behavior of relevant parameters against TQC. To demonstrate the advantages of the proposed methodology, a comparison is implemented with the existing rectifying sampling plan in terms of TQC and average outgoing quality limit. This comparison reveals that our proposed methodology exhibits superior performance.

Keywords: acceptance sampling plan; process capability indices; rectifying inspection; average outgoing quality limit; sensitivity analysis

1. Introduction

The acceptance sampling plan and process capability index (PCI) are critical for quality control in manufacturing. The acceptance sampling plan is used to determine the acceptance or rejection of one lot based on information obtained from a sample and is applied extensively in the inspection of raw materials, semifinished products, final products, and shipments. Acceptance sampling plan can be divided into two types, attributes and variables sampling plans. In comparison with attributes sampling plan, variables sampling plans provide a less sample size to attain the same protection for producer and purchaser as well as give more information about lots. Such saving of sample size may be especially marked if inspection is destructive and the item is expensive [1–3]. By contrast, the PCI establishes the relationship between the actual manufacturing process performance and manufacturing specifications, which quantifies the effectiveness of the production process. This process capability information measures the effectiveness of the production process. Since the use of the PCI and acceptance sampling plans became popular, numerous papers have combined the two to develop variables in sampling plans for lot sentencing [4–14]. These papers are designed for different purposes



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or sampling skills. Yen et al. [4,12], Aslam et al. [6,13], Lee et al. [9], and Wu and Wang [11] used PCI to develop variables sampling plans which consider the quality of ongoing lot and preceding lots for lot sentencing. Wu and Liu [5] designed a single variables sampling plan based on process yield index. Wu et al. [7] proposed a PCI-based sampling plan for resubmitted lots which examined the situation where resampling is permitted on lots not accepted on original inspection. Yen et al. [8] and Fallah-Nezhad and Seifi [10] used PCI to design repetitive group sampling (RGS) plans for determination of lot.

Research combining acceptance sampling plans and the PCI has seldom focused on rectifying inspection, where 100% inspection will be carried out if one lot is rejected. Rectifying inspection is frequently implemented because of supplier monopoly or urgent buyer demand. Rectifying inspection means that if one lot is accepted, then only defective units detected from n units are replaced with nondefective units. However, if the lot is rejected, 100% of the remaining units are inspected and all defective units are replaced with nondefective units. After this process is completed, the lot rejected can be accepted. Three quality costs are incurred in rectifying inspection: inspection cost, internal failure cost, and external failure cost. Inspection cost occurs when products are inspected. Internal failure cost occurs when products fail to meet the minimum quality requirements prior to delivery to consumers. External cost occurs when products fail to meet the minimum quality cost is presented in the study conducted by Feigenbaum [14], Juarn [15], and Shank and Govindarajan [16].

These statements in the previous paragraph imply that the application of rectifying inspection is a practical problem for lot sentencing. Yen et al. [17] used a single sampling scheme to develop a rectifying sampling plan based on a one-sided PCI. However, other flexible sampling schemes, such as repetitive sampling [18], multiple dependent state sampling [19], and multiple dependent state repetitive sampling [20], have been presented with smaller sample sizes than that of the single sampling scheme. Therefore, we develop a rectifying sampling plan that has a lower total quality cost (TQC) than that reported by Yen et al. [17]. An exploration of the relevant literature reveals that research on rectifying sampling plans with flexible sampling schemes, such as repetitive group sampling, multiple dependent state sampling, and multiple dependent state repetitive sampling have not yet been conducted. Thus, in this paper, we apply a repetitive sampling scheme to design a rectifying sampling plan. The remainder of this paper is organized as follows. PCIs are introduced in Section 2. The design of our proposed sampling plan is presented in Section 3. Parameters for the rectifying sampling plan and the results of the sensitivity analysis are provided in Section 4. Finally, this paper's conclusions are presented in Section 5.

2. Process Capability Indices

PCIs have been proposed to provide numerical measures of process performance in the manufacturing industry. These indices establish the relationship between the actual process performance and manufacturing specifications, and they have been a focus of recent research in statistical and quality assurance. The most popular capability indices, *Cp* [21], *Cpk* [22] and *Cpm* [23], are widely used in the manufacturing industry to evaluate process performance for cases with two-sided specification limits. The index *Cp* measures the overall process variation relative to specification tolerance. The index *Cpm* measures the ability of the process to cluster around the target, which reflects the degrees of the process target. The explicit forms of indices are defined as follows.

$$C_p = \frac{USL - LSL}{6\sigma}, \ C_{pk} = \min\{C_{pu}, C_{pl}\} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\}, \text{ and } C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

where *USL* and *LSL* are the upper and lower specification limits, respectively, *T* is the target value, μ is the process mean, and σ is the process standard deviation.

For normally distributed processes with one-sided specification limits, *Cpu* and *Cpl* indices are used to evaluate process capability where characteristics are smaller-to-better and larger-to-better, respectively, defined as follows.

$$C_{pu} = \frac{USL - \mu}{6\sigma}$$
 and $C_{pl} = \frac{\mu - LSL}{6\sigma}$.

From a practical perspective, the parameter C_{pu} or C_{pl} is typically unknown. Thus, we must use sample statistics to estimate it. For the estimations of the indices Cpu and Cpl, we consider the following natural estimators.

$$\hat{C}_{pu} = \frac{USL - \overline{x}}{6s}$$
 and $\hat{C}_{pl} = \frac{\overline{x} - LSL}{6s}$

where \bar{x} is the sample mean and s is the sample standard deviation. Chou and Owen [24] revealed that both \hat{C}_{pu} and \hat{C}_{pl} are noncentral t distributions with n - 1 degrees of freedom and noncentral parameter $\delta = 3\sqrt{n}Cpu$ ($\delta = 3\sqrt{n}Cpl$). For convenience of presentation, both Cpu and Cpl are denoted as C_S in the subsequent sections.

3. Proposed Sampling Plan

In this section, we will describe the design of proposed sampling plan whose relevant contents are presented as follows.

3.1. Procedure and Probability of Acceptance of the Repetitive Sampling Plan Based on the One-Sided PCI

According to Sherman [18], the procedure of a repetitive sampling plan based on the one-sided PCI can be stated as follows.

Step 1: Determine the producer's risk α , the consumer's risk β , and the corresponding process capability requirements (C_S^{AQL} and C_S^{LTPD}).

Step 2: Obtain a random sample of size *n* (>1) and compute \hat{C}_S . Step 3: Formulate a decision on the lot as follows.

- (1) Accept the lot if $\hat{C}_s > k_a$.
- (2) Reject the lot if $\hat{C}_s < k_r$.
- (3) Otherwise, repeat Step 2 by obtaining a new sample of size *n*.

Figure 1 presents a flow chart of a repetitive sampling plan based on the PCI.



Figure 1. Flow chart of a repetitive sampling plan based on *C*_S.

According to Yen et al. [8], the probability of acceptance of repetitive sampling based on one-sided PCI can be expressed as

$$P_R = P_a + RP_a + R^2 P_a + \dots + R^k P_a + \dots = \frac{P_a}{1-R} = \frac{P_a}{P_a + P_r}$$
 (1)

where $P_a = P(t_{n-1,3\sqrt{n}C_s} > 3\sqrt{n}k_a)$, $P_r = P(t_{n-1,3\sqrt{n}C_s} < 3\sqrt{n}k_r)$, $R = 1 - P_a - P_r$, and $t_{n-1,3\sqrt{n}C_s}$ is a noncentral t distribution with n - 1 degrees of freedom and noncentral parameter $3\sqrt{n}C_s$.

3.2. Derivation of Relevant Indices for the TQC Model

With reference to Yen et al. [17], the quality cost model, $TQC = C_iATI + C_{if}D_d + C_{ef}D_n$, is adopted in this study. The TQC is a total quality cost including total inspection cost, total internal failure cost and total external failure cost while implementing the rectifying inspection, of which total inspection cost is equal to C_iATI , total internal failure cost is equal to $C_{if}D_d$ and total external failure cost is equal to $C_{ef}D_n$. Under repetitive sampling, the average total inspection (ATI), D_d , D_n , and average outgoing quality (AOQ) of rectifying inspection can be derived respectively as follows

$$ATI = \left[nP_{a} + 2nRP_{a} + 3nR^{2}P_{a} + \dots\right] + N[1 - P_{R}] = \frac{nP_{a}}{(1 - R)^{2}} + \frac{NP_{r}}{P_{a} + P_{r}} = \frac{nP_{a} + NP_{r}(P_{a} + P_{r})}{(P_{a} + P_{r})^{2}}$$
(2)

$$D_{d} = \left[nP_{a} + 2nRP_{a} + 3nR^{2}P_{a} + \cdots \right] p + N[1 - P_{R}]p = \frac{[nP_{a} + NP_{r}(P_{a} + P_{r})]p}{(P_{a} + P_{r})^{2}}$$
(3)

$$D_n = \left[P_a(N-n) + RP_a(N-2n) + R^2 P_a(N-3n) + \dots \right] p = \frac{P_a[N(P_a+P_r)-n]p}{(P_a+P_r)^2}$$
(4)

$$AOQ = \left[P_a(N-n) + RP_a(N-2n) + R^2P_a(N-3n) + \cdots \right]p = \frac{P_a[N(P_a+P_r)-n]p}{N(P_a+P_r)^2}$$
(5)

3.3. Mathematical Model of the Proposed Sampling Plan

To evaluate the performance of the sampling plan, the operating characteristic (OC) curve is used to exhibit the discriminatory power. The OC curve plots the probability of accepting one lot versus the various proportion nonconforming. A typical OC curve is depicted in Figure 2. A supplier typically focuses on a specific product quality level, which is termed acceptable quality level (AQL), with a high probability $1 - \alpha$ of accepting a lot. By contrast, a buyer typically focuses on a point at the other end of the OC curve that is termed lot tolerance percent defective (LTPD), with a low probability β of accepting a lot. Thus, a well-designed sampling plan should provide a probability of at least $1 - \alpha$ of accepting a lot if the product quality level achieves AQL and a probability of no more than β if the level of the product quality is at the LTPD level. As such, the OC curve of the acceptance sampling plan passes through the two designated points (AQL, $1 - \alpha$) and (LTPD, β). To construct a PCI-based acceptance sampling plan, the corresponding two designated points of OC would be (C_S^{AQL} , $1 - \alpha$) and (C_S^{LTPD} , β).



Figure 2. Conventional OC curve.

Consequently, the proposed sampling plan is developed based on the minimization of TQC while satisfying the two points of the OC curve. Therefore, a mathematical model of a repetitive rectifying sampling plan based on the one-sided PCI can be constructed as follows.

$$\begin{array}{l} \text{Min } TQC \ = \ C_i ATI \ + \ C_{if} D_d \ + \ C_{ef} D_n \\ st. \ P_R(C_S^{AQL}) \ \ge \ 1 \ - \ \alpha \\ P_R(C_S^{LTPD}) \ \le \ \beta \end{array}$$

Based on Equations (1)–(4), the mathematical model can be expressed as follows.

$$\operatorname{Min} TQC = C_{i} \times \frac{nP_{a} + NP_{r}(P_{a} + P_{r})}{(P_{a} + P_{r})^{2}} + C_{if} \times \frac{[nP_{a} + NP_{r}(P_{a} + P_{r})]p}{(P_{a} + P_{r})^{2}} + C_{ef} \frac{P_{a}[N(P_{a} + P_{r}) - n]p}{(P_{a} + P_{r})^{2}}$$
(6)

st.
$$\frac{P(t_{n-1,3\sqrt{n}C_{S}^{AQL}}>3\sqrt{n}k_{a})}{P\left(t_{n-1,3\sqrt{n}C_{S}^{AQL}}>3\sqrt{n}k_{a}\right)+P\left(t_{n-1,3\sqrt{n}C_{S}^{AQL}}<3\sqrt{n}k_{r}\right)} \ge 1 - \alpha$$
(7)

$$\frac{P(t_{n-1,3\sqrt{n}C_{S}^{LTPD}}>3\sqrt{n}k_{a})}{P(t_{n-1,3\sqrt{n}C_{S}^{LTPD}}>3\sqrt{n}k_{a})+P(t_{n-1,3\sqrt{n}C_{S}^{LTPD}}<3\sqrt{n}k_{r})} \geq \beta$$
(8)

4. Analysis and Discussion

In this section, we provide numerical examples to illustrate the performance of the proposed sampling plan based on Equations (6)–(8) compared with that reported by Yen et al. [17].

The parameters of sampling plan and AOQ graph

For illustration, the relevant parameters of the numerical example are set at N = 1000, C_S^{AQL} = 1.33, C_S^{LTPD} = 1.0, C_S = 1.165, C_i = 10, C_{if} = 20, and C_{ef} = 50. The sample sizes, critical values, and TQC for both sampling plans with various combinations of α and β are summarized in Table 1. From results in Table 1, we observe the following.

- (i) The parameters of the two sampling plans are determined by consumer risk β instead of producer risk α .
- (ii) For the two sampling plans, the sample size and the acceptance value both reveal a decreasing trend when consumer risk β increases along with the fixed producer risk. Also, the sample size number of the repetitive rectifying sampling plan is significantly lower than that of the single rectifying sampling plan.

(iii) When the producer risk α is fixed and the consumer risk β increases, the TQC variables of both sampling plans decrease; the TQC of the repetitive sampling plan is significantly lower than that of the single sampling plan for all combinations.

α β		Sin (Y	ngle Sampl íen et al. [1	ing 7])	Repet	itive Sampli Met	Reduction on TOC		
		п	с	TQC	n	ka	k _r	TQC	içe
	0.01	253	1.1262	4256.12	118	1.2101	1.0257	4040.99	5.05%
	0.025	223	1.1126	3588.23	96	1.2091	1.0034	3149.39	12.23%
0.01	0.05	185	1.1036	3133.8	76	1.2121	0.9894	2582.92	17.58%
	0.075	169	1.0947	2809.77	93	1.1646	0.9987	2041.59	27.34%
	0.1	153	1.0886	2595.21	99	1.1446	0.9979	1822.37	29.78%
	0.01	253	1.1262	4256.12	118	1.2101	1.0257	4040.99	5.05%
	0.025	223	1.1126	3588.23	96	1.2091	1.0034	3149.39	12.23%
0.025	0.05	185	1.1036	3133.8	76	1.2121	0.9894	2582.92	17.58%
	0.075	169	1.0947	2809.77	93	1.1646	0.9987	2041.59	27.34%
	0.1	153	1.0886	2595.21	99	1.1446	0.9979	1822.37	29.78%
	0.01	253	1.1262	4256.12	118	1.2101	1.0257	4040.99	5.05%
	0.025	223	1.1126	3588.23	96	1.2091	1.0034	3149.39	12.23%
0.05	0.05	185	1.1036	3133.8	76	1.2121	0.9894	2582.92	17.58%
	0.075	169	1.0947	2809.77	93	1.1646	0.9987	2041.59	27.34%
	0.1	153	1.0886	2595.21	99	1.1446	0.9979	1822.37	29.78%
	0.01	253	1.1262	4256.12	118	1.2101	1.0257	4040.99	5.05%
	0.025	223	1.1126	3588.23	96	1.2091	1.0034	3149.39	12.23%
0.075	0.05	185	1.1036	3133.8	76	1.2121	0.9894	2582.92	17.58%
	0.075	169	1.0947	2809.77	93	1.1646	0.9987	2041.59	27.34%
	0.1	153	1.0886	2595.21	99	1.1446	0.9979	1822.37	29.78%
	0.01	253	1.1262	4256.12	118	1.2101	1.0257	4040.99	5.05%
	0.025	223	1.1126	3588.23	96	1.2091	1.0034	3149.39	12.23%
0.1	0.05	185	1.1036	3133.8	76	1.2121	0.9894	2582.92	17.58%
	0.075	169	1.0947	2809.77	93	1.1646	0.9987	2041.59	27.34%
	0.1	153	1.0886	2595.21	99	1.1446	0.9979	1822.37	29.78%

Table 1. Parameters of the rectifying sampling plans and their corresponding total quality cost (TQC).

In addition, based on parameters in Table 1, we plot the graphs of AOQ for both of the sampling plans, as displayed in Figure 3. From Figure 3, we can see that the two sampling plans appear to be very similar, whereas the average outgoing quality limit (AOQL) of the repetitive sampling plan is slightly lower than that of the single sampling plan.

4.1. Sensitivity Analysis

To illustrate the sensitivity analysis, the values of the specific parameters are set at N = 1000, $C_S^{AQL} = 1.33$, $C_S^{LTPD} = 1.0$, $\alpha = 0.05$, and $\beta = 0.1$.



Figure 3. Graphs of average outgoing quality (AOQ) versus C_s for the two rectifying sampling plans.

To investigate the behavior of the individual C_S , C_i , C_{if} , and C_{ef} against the TQC, sensitivity analysis is executed on TQC for individual parameters. Table 2 displays the sample size, critical values, and TQC of both sampling plans for $C_S = 0.6(0.05)1.6$ with the given $C_i = 10$, $C_{if} = 20$, and $C_{ef} = 50$. The results reveal that the sample size and critical values of the individual sampling plans are constant for all values of C_S , and the TQC of the proposed methods are significantly lower than those of the single sampling plan for all cases. Table 3 displays the sample size, critical values, and TQC of the two sampling plans for $C_i = 10(10)100$, $C_{if} = 10(10)100$, and $C_{ef} = 10(10)100$ with the given $C_S = 1.165$. The results reveal that the sample size and critical values of the individual sampling plans are constant regardless of changes in the values of C_i , C_{if} , and C_{ef} , and the TQC of the proposed method is significantly lower than that of the single sampling plan for all cases. Figure 4 displays TQC vs. C_i , C_{if} , and C_{ef} based on results illustrated in Tables 2 and 3. The outline reveals that the decline in TQC for the proposed method is more significant when the process capability is very effective (i.e., greater than 1.3) or very ineffective (i.e., less than 0.7). Moreover, the TQC seems to be primarily dependent on process capability and inspection cost, whereas internal failure cost and external failure cost have very little influence.

4.3. Sensitivity Analysis on TQC for Simultaneous Changes in All Parameters

To further investigate the extent of the individual parameters' effects on TQC, sensitivity analysis is implemented on TQC for simultaneous changes in C_S , C_i , C_{if} , and C_{ef} . We establish three levels for each factor (i.e., C_S , C_i , C_{if} , and C_{ef}) and use the orthogonal array L₉(3⁴) to implement level average analysis. Table 4 exhibits the values of TQC for each combination of C_S , C_i , C_{if} , and C_{ef} , and the absolute values of the effects of each factor are presented in Table 5. From the results in Table 5, we conclude that C_S has the strongest effect on TQC, followed by C_i , whereas C_{if} and C_{ef} have the least effect on TQC.

Cs	C _i	C _{if}	C _{ef}	Single Sampling (Yen et al. [17])		Re	petitive Sa Proposed	Reduction on TOC			
				n	С	TQC	n	ka	k_r	TQC	- 2 -
0.6				153	1.0886	12,240.3	99	1.1446	0.9979	10,718.6	12.43%
0.65				153	1.0886	11,595.4	99	1.1446	0.9979	10,511.8	9.35%
0.7				153	1.0886	11,113.8	99	1.1446	0.9979	10,357.3	6.81%
0.75				153	1.0886	10,762.2	99	1.1446	0.9979	10,244.5	4.81%
0.8				153	1.0886	10,511.1	99	1.1446	0.9979	10,163.7	3.31%
0.85				153	1.0886	10,334.3	99	1.1446	0.9979	10,104.8	2.22%
0.9				153	1.0886	10,194.1	99	1.1446	0.9979	10,044.6	1.47%
0.95				153	1.0886	9957.21	99	1.1446	0.9979	9884.05	0.73%
1				153	1.0886	9230.03	99	1.1446	0.9979	9219.02	0.12%
1.05				153	1.0886	7530.42	99	1.1446	0.9979	7174.3	4.73%
1.1				153	1.0886	5103.36	99	1.1446	0.9979	4116.83	19.33%
1.15	10	20	50	153	1.0886	3024.57	99	1.1446	0.9979	2137.05	29.34%
1.2				153	1.0886	1947.55	99	1.1446	0.9979	1384.32	28.92%
1.25				153	1.0886	1606.06	99	1.1446	0.9979	1124.35	29.99%
1.3				153	1.0886	1539.02	99	1.1446	0.9979	1032.12	32.94%
1.35				153	1.0886	1530.75	99	1.1446	0.9979	1001.43	34.58%
1.4				153	1.0886	1530.07	99	1.1446	0.9979	992.722	35.12%
1.45				153	1.0886	1530.02	99	1.1446	0.9979	990.656	35.25%
1.5				153	1.0886	1530.01	99	1.1446	0.9979	990.202	35.28%
1.55				153	1.0886	1530.01	99	1.1446	0.9979	990.082	35.29%
1.6				153	1.0886	1530	99	1.1446	0.9979	990.038	35.29%
1.8				153	1.0886	1530	99	1.1446	0.9979	990.002	35.29%
2				153	1.0886	1530	99	1.1446	0.9979	990	35.29%

Table 2. Sensitivity analysis of TQC for *C*_{*S*}.

Cs	C _i	C _{if}	C _{ef}	Single Sampling (Yen et al. [17])			Re	petitive Sa Proposed	Reduction on		
				n	С	TQC	n	ka	k _r	TQC	ige
	10			153	1.0886	2595.21	99	1.1446	0.9979	1822.37	29.78%
	20			153	1.0886	5187.92	99	1.1446	0.9979	3634.17	29.95%
	30			153	1.0886	7780.64	99	1.1446	0.9979	5445.98	30.01%
	40			153	1.0886	10,373.4	99	1.1446	0.9979	7257.79	30.03%
1 165	50	20	50	153	1.0886	12,966.1	99	1.1446	0.9979	9069.59	30.05%
1.105	60	20	50	153	1.0886	15,558.8	99	1.1446	0.9979	10,881.4	30.06%
	70			153	1.0886	18,151.5	99	1.1446	0.9979	12,693.2	30.07%
	80			153	1.0886	20,744.2	99	1.1446	0.9979	14,505	30.08%
	90			153	1.0886	23,337	99	1.1446	0.9979	16,316.8	30.08%
	100			153	1.0886	25,929.7	99	1.1446	0.9979	18,128.6	30.09%
		10		153	1.0886	2594.59	99	1.1446	0.9979	1821.94	29.78%
		20		153	1.0886	2595.21	99	1.1446	0.9979	1822.37	29.78%
		30		153	1.0886	2595.82	99	1.1446	0.9979	1822.8	29.78%
		40		153	1.0886	2596.44	99	1.1446	0.9979	1823.23	29.78%
1 165	10	50	50	153	1.0886	2597.05	99	1.1446	0.9979	1823.66	29.78%
1.105	10	60	50	153	1.0886	2597.66	99	1.1446	0.9979	1824.09	29.78%
		70		153	1.0886	2598.28	99	1.1446	0.9979	1824.52	29.78%
		80		153	1.0886	2598.89	99	1.1446	0.9979	1824.95	29.78%
		90		153	1.0886	2599.51	99	1.1446	0.9979	1825.38	29.78%
		100		153	1.0886	2600.12	99	1.1446	0.9979	1825.8	29.78%
			10	153	1.0886	2594.2	99	1.1446	0.9979	1814.61	30.05%
			20	153	1.0886	2594.45	99	1.1446	0.9979	1816.55	29.98%
			30	153	1.0886	2594.7	99	1.1446	0.9979	1818.49	29.92%
			40	153	1.0886	2594.95	99	1.1446	0.9979	1820.43	29.85%
1 165	10	20	50	153	1.0886	2595.21	99	1.1446	0.9979	1822.37	29.78%
1.105	10		60	153	1.0886	2595.46	99	1.1446	0.9979	1824.31	29.71%
			70	153	1.0886	2595.71	99	1.1446	0.9979	1826.25	29.64%
			80	153	1.0886	2595.96	99	1.1446	0.9979	1828.19	29.58%
			90	153	1.0886	2596.21	99	1.1446	0.9979	1830.13	29.51%
			100	153	1.0886	2596.47	99	1.1446	0.9979	1832.07	29.44%

Table 3. Sensitivity analysis of TQC for C_i , C_{if} , and *Cef*.

 Table 4. Sensitivity analysis for simultaneous changes in parameters.

L ₉ (3 ⁴)					Single Sampling (Yen et al. [17])				epetitive Sa Proposed	ampling (Method	(The)
	C_S	C _i	C _{if}	C _{ef}	n	С	TQC	n	k _a	k _r	TQC
1	0.9	10	40	70	153	1.0886	10,321.9	99	1.1446	0.9979	10,113.9
2	0.9	20	50	80	153	1.0886	20,364.2	99	1.1446	0.9979	20,123.6
3	0.9	30	60	90	153	1.0886	30,406.6	99	1.1446	0.9979	30,133.3
4	1.2	10	50	90	153	1.0886	1948.74	99	1.1446	0.9979	1390.47
5	1.2	20	60	70	153	1.0886	3895.52	99	1.1446	0.9979	2764.97
6	1.2	30	40	80	153	1.0886	5841.57	99	1.1446	0.9979	4142.93
7	1.5	10	60	80	153	1.0886	1530.03	99	1.1446	0.9979	990.307
8	1.5	20	40	90	153	1.0886	3060.02	99	1.1446	0.9979	1980.37
9	1.5	30	50	70	153	1.0886	4590.03	99	1.1446	0.9979	2970.36

Table 5. Influence of parameters.

	Sing	le Samplin	g (Yen et al.	[17])	Repetitive	Sampling (The Propose	d Method)
	Cs	C _i	C _{if}	C _{ef}	Cs	C _i	C _{if}	C _{ef}
effect	17,304.207	9012.51	5536.22	5535.97	18,143.254	8250.6377	5883.7923	5884.97





Figure 4. Sensitivity analysis graphs for C_S, C_i, C_{if}, and C_{ef}.

5. Conclusions

In this paper, we propose a repetitive rectifying sampling plan based on the one-sided PCI by extending previous research on the single rectifying sampling plan [17]. Our proposed repetitive rectifying sampling plan minimizes TQC, including inspection cost, internal failure cost, and external failure cost. A thorough comparison with the single rectifying sampling plan is implemented to confirm the performance of the proposed sampling plan. The results of the parameters of the two sampling plans reveal that sample size and critical values seem to be only determined by consumer risk rather than producer risk, quality level, or relevant quality costs. For various risk combinations under certain parameter conditions, the TQC of the proposed sampling plan is significantly lower than that of the single rectifying sampling plan. Specifically, the TQC of the proposed sampling plan is extremely low relative to that of the single rectifying sampling plan when the process capability is very good or when it is very bad. Furthermore, the AOQL of the proposed sampling plan is lower than that of the single rectifying sampling plan despite the two sampling plans' apparently similar AOQ. Based on the results of the sensitivity analysis, we observe that C_S has the strongest effect on TQC, followed by C_i , whereas C_{if} and C_{ef} exhibit the least influence on TQC. Overall, we conclude that the proposed sampling plan exhibits superior performance to that of Yen et al. [17]. Therefore, the proposed sampling plan is recommended when a rectifying inspection must be executed. It is noted that the proposed methodology is applied in the quality characteristic of interest that follows normal distribution and has one-sided specifications.

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Nomenclature

TQC	total quality cost per lot
ATI	average total inspection
AOQ	average outgoing quality
C_i	inspection cost per unit
C_{if}	internal failure cost per unit
C_{ef}	external failure cost per item
N	lot size
С	acceptance value of single sampling
<i>k</i> _r	rejection value of repetitive sampling
C_{S}^{LTPD}	lot tolerance percent defective of one-sided PCI
D_d	defective units detected
D_n	defective units undetected
P_a	probability of accepting one lot for each sampling
P_r	probability of rejecting one lot for each sampling
R	probability of resampling after each sampling
P_R	probability of acceptance of repetitive sampling plan
п	sample size
ka	acceptance value of repetitive sampling
C_{c}^{AQL}	acceptable quality level of one-sided PCI

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