

Editorial

Discrete Optimization: Theory, Algorithms, and Applications

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Discrete optimization is an important area of applied mathematics that is at the intersection of several disciplines and covers both theoretical and practical aspects. In the call for papers for this issue, I asked for submissions presenting new theoretical results, structural investigations, new models, and algorithmic approaches, as well as new applications of discrete optimization problems. Among the possible subjects, integer programming, combinatorial optimization, optimization problems on graphs and networks, matroids, scheduling, and logistics were mentioned, to name a few.

In response to the call for papers, 51 submissions have been received, among which, finally, 18 papers have been accepted for this Special Issue, all of which are of high quality, reflecting the great interest in the area of discrete optimization. This corresponds to an acceptance rate of 35.3%. The authors of these publications come from 12 different countries: China, Pakistan, the United Arab Emirates, Saudi Arabia, Belarus, Germany, India, Malaysia, Russia, Taiwan, Turkey, and the USA, where the authors from the first four countries co-authored more than one paper in this issue. Although many different aspects of discrete optimization have been addressed by the submissions, among the accepted papers, there is a major part dealing with graphs and networks. A large part of these papers deals with topological indices and the domination of graphs. Several of these papers also emphasize the great importance of graph-theoretic works for practical applications. For this reason, I would like to draw the attention of the readers also to another future Special Issue planned for the journal *Mathematics* entitled “Graph-Theoretic Problems and Their New Applications” with a deadline for submissions of 31 January 2020.

All submissions have been reviewed, as a rule, by at least three experts in the discrete optimization area. Next, all published papers in this Special Issue are briefly surveyed in increasing order of their publication dates. This Special Issue contains both theoretical and practical works in the field of discrete optimization. We hope that practical operations research workers will find some interesting theoretical ideas in this Special Issue and that researchers will find new inspirations for future works.

The first accepted paper by Imran et al. [1] studies the chemical graph of an oxide network, and in particular, it deals with topological indices. The authors compute the total eccentricity, the average eccentricity, eccentricity-based Zagreb indices, the atom-bond connectivity index, and the geometric arithmetic index of such a network. In addition, arithmetically-closed formulas for these indices are given in this paper.

Imran et al [2] deal with the computation of several topological indices and polynomials for line graphs. In particular, the authors compute the first and second Zagreb indices, the hyper Zagreb index, multiple Zagreb indices, and Zagreb polynomials of the line graph of wheel and ladder graphs. To obtain their results, they used the idea of subdivision.

Yang et al. [3] deal with a problem from graph labeling that plays a role in many applications, e.g., in coding theory, radars, astronomy, or the management of databases. In particular, they discuss the total edge irregular k labeling, the total vertex irregular k labeling, and the totally irregular total k labeling of three planar graphs. The authors determine the exact value of the total irregularity strength of such graphs.

Tang and Yang [4] deal with an ordered weighted averaging aggregation problem, denoted as COWA. For the three-dimensional case, they consider two variants: the maximization variant of the COWA problem with lower bounded variables and the minimization version with upper bounded variables. They present the optimal solutions theoretically and give also empirical results. It appears that both the weights and bounds can affect the optimal solution of such three-dimensional problems. As an interesting future research subject, the authors mention the potential extension of such investigations to the case of higher dimensions.

Imran et al. [5] deal with the metric dimension of special graphs, which is defined as the minimum cardinality of a resolving set of the corresponding graph. The authors investigate the cycle, path, Harary graphs, and their rooted products, as well as their connectivity. It is shown that the metric dimension of some graphs is unbounded, while for the other investigated graphs, the metric dimension is either three or four. The paper finishes with the formulation of two open research problems.

Jiang et al. [6] investigate the double Roman domination number, which is the minimum weight of a double Roman dominating function, of so-called generalized Petersen graphs of the type $P(n, 2)$. This result is obtained by using a discharging approach. As one conclusion, it turns out that the graph $P(n, 2)$ is not double Roman for all $n \geq 5$, and the authors formulate that it is an interesting problem to find further Petersen graphs that are double Roman.

Wang et al. [7] deal with multiplicative Zagreb indices. For graphs with n vertices and k cut edges, the maximum and minimum multiplicative Zagreb indices are determined. In addition, the graphs with the smallest and largest first and second multiplicative Zagreb indices are provided.

Liu et al. [8] use the concept of the edge version of the metric dimension and doubly-resolving sets, which is based on the distances of the edges of a graph. The metric dimension can be used, e.g., in navigation, robotics, or chemistry. The authors apply this concept to a so-called necklace graph, which is a cubic Halin graph obtained by joining a cycle with all vertices of degree one with a caterpillar having n vertices of degree three and $n + 2$ vertices of degree one.

Gür and Eran [9] give an overview on goal programming applied to scheduling and planning problems arising in service systems. They discuss 143 references from the literature. First, they give some information for which activities in service systems goal programming is typically used. Then, they explain the different types of goal programming applied in the literature. The authors also briefly discuss the methods that are integrated with the goal programming method in the literature. At the time of writing this Editorial, this is the paper of this Issue with by far the most downloads.

Gao et al. [10] consider another graph-theoretic subject, namely they deal with the extremal graphs of some topological indices of a graph G with given vertex k -partiteness, which is defined as the smallest number of vertices, the deletion of which from G yields a k -partite graph. For such graphs, they characterize the extremal values of the reformulated first Zagreb index, the multiplicative-sum Zagreb index, the general Laplacian-energy-like index, the general zeroth-order Randic index, and the modified Wiener index.

Zhang et al. [11] compute the resistance distance in the H -join G of k disjoint graphs. They use the Laplacian matrix $L(G)$ and the symmetric $\{1\}$ -inverse of $L(G)$. The authors discuss also some applications of the results obtained.

Elahi et al. [12] also deal with a subject from chemical graph theory. They consider graphs containing a commutative ring, which have wide applications, e.g., in robotics, information theory, physics, or statistics. In their paper, they adopted interdisciplinary methods. They discussed vertex-based eccentric topological indices, namely the eccentric connectivity index, the total-eccentricity index, the first Zagreb eccentricity index, the connective eccentric index, the Ediz eccentric connectivity index, the eccentric connectivity polynomial, and the augmented eccentric index for zero divisor graphs of a commutative ring. These indices are useful for understanding physical structures like carbon nanostructures or those in robotics.

Liu et al. [13] consider another problem related, e.g., to chemistry. They deal with the energies of molecular graphs. In particular, they give closed forms of the distance and adjacency energies of

generalized wheel networks, also known as m -level wheels. They also illustrate the dependencies of the energies on the parameters of the wheel graphs. The results can be useful both for mathematicians and chemists in industry since generalized wheels can be considered as particular cyclic structures having a common hub.

Raza et al. [14] characterize graphs having n vertices with fault-tolerant metric dimensions n , $n - 1$, and two, which are the non-trivial extremal values of the fault-tolerant metric dimension. By means of a lemma for tracing a fault-tolerant resolving set from a given resolving set, an upper bound on the fault-tolerant metric dimension of a graph with a given resolving set is derived. The fault-tolerant resolvability is investigated for three infinite families of regular graphs.

Liu et al. [15] deal with the fractional metric dimension of a connected graph, which had been introduced only some years ago in 2012. In particular, they consider a generalized Jahingir graph $J_{m,k}$, the vertices of which can be classified into three categories, namely vertices of degree two (called minors), three (called majors), and m (called center), and there are km minor vertices, m major vertices, and one center vertex. They introduce the resolving neighborhood for any possible pair of vertices in a generalized Jahingir graph. As the main result, the authors determine the fractional metric dimension of a generalized Jahingir graph for $k \geq 0$ and $m = 5$.

Lu et al. [16] deal with the resistance-Harary index of connected graphs. In particular, they determine among the set of unicyclic graphs (these are graphs containing exactly one cycle) those graphs having the second-largest resistance-Harary index and, among the fully-loaded unicyclic graphs (these are unicyclic graphs such that no vertex with a degree less than three is in the unique cycle), those with the largest resistance-Harary index.

Gafarov and Werner [17] consider a two-machine job-shop scheduling problem, where each of n jobs consists of two operations and the processing times of all jobs on each machine are identical. The objective is to minimize the sum of the completion times. This problem arises also as a special single-track railway scheduling problem with three stations and constant travel times between any two adjacent stations. For this problem, the authors give a dynamic programming algorithm of complexity $O(n^5)$ and, in addition, a fast constructive heuristic of complexity $O(n^3)$. Computational results are presented for instances with up to 30 jobs.

Sotskov and Egorova [18] deal with a single-machine scheduling problem with uncertain processing times, where only lower and upper bounds on the processing times are known. The objective is to minimize the sum of the completion times. The authors investigate the properties of the optimality region. They introduce the quasi-perimeter of the optimality region and give a linear time algorithm for calculating it. Moreover, they present an algorithm for finding the job permutation with the largest quasi-perimeter of this region. Computational results are given for instances with up to 5000 jobs, which show that the constructed permutation is close to an optimal job sequence for the actual processing times of the jobs.

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