mathematics

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Reduction of Bundles, Connection, Curvature, and Torsion of the Centered Planes Space at Normalization

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Received: 20 August 2019; Accepted: 23 September 2019; Published: 26 September 2019

Abstract: The space Π of centered m-planes is considered in projective space P_n. A principal bundle is associated with the space Π and a group connection is given on the principal bundle. The connection is not uniquely induced at the normalization of the space Π. Semi-normalized spaces Π^1, Π^2 and normalized space $\Pi^{1,2}$ are investigated. By virtue of the Cartan–Laptev method, the dynamics of changes of corresponding bundles, group connection objects, curvature and torsion of the connections are discovered at a transition from the space Π to the normalized space $\Pi^{1,2}$.

Keywords: differentiable manifold; Cartan–Laptev method; space of centered planes; normalization; reduction; connection

1. Introduction

This paper refers to the field of differential geometry, or, more precisely, to the theory of differentiable manifolds equipped with various "geometric structures" [1], such as connection, curvature and torsion. We use the following methods: the moving frame method (the E. Cartan method [2] of differential-geometric research) and the G. F. Laptev method of extensions and envelopments, which includes the Cartan moving frame method and gives a universal character to the first one. Universality and efficiency of the Cartan method were shown in many papers. Upon use of this method, a research of geometry of a manifold with geometric structures fixed on it is reduced to study of geometry of other manifolds (total space of frames above the given manifold or subbundles of the bundle). Thus, automatically there is an analytical apparatus that is most adapted to research of the initial structure [3]. The method of extensions and envelopments is based on the invariant differential-algebraic apparatus of structure differential forms of considered bundles [4]. In this paper, the Cartan–Laptev method is applied to research of the centered m-planes space in projective space P_n.

The connection theory (see, e.g., [5,6]) has an important place in differential geometry. A lot of research devoted to the geometry of planes manifolds in classical spaces includes studying connections. The connection theory also plays a fundamental role in physics.

Normalization of a manifold [7] of centered planes in projective space can be defined by an analogy with the Norden normalization [8]. A. P. Norden has described the normalization of a surface. A surface X_m can be considered as an m-dimensional manifold of m-dimensional centered planes T_m [9]. Yu. G. Lumiste [10] has entered a similar normalization of a manifold of m-planes in projective space. An analogue of this normalization is used in this paper. Thus, in our case, the normal of the first kind (the first normal) is a subspace N_{n-m} of P_n having only one common point with a centered m-plane T_m; and the normal of the second kind (the second normal) is a subspace N_{m-1} of the centered m-plane T_m not passing through its centre [11]. Moreover, we will also use a reduction that is frequently applied in geometry (see, e.g., [4,12–17]).

This paper is a continuation of author's research [18,19]. Our purpose is to give the full analysis of the dynamics of changes of bundles, connection, curvature and torsion at transition from the space of centered planes to the normalized space.

The timeliness of the present paper is caused by the facts that the space of centered planes is a set of all m-dimensional centered planes, which we may say about a communication with the Grassmann manifold (the set of all m-planes) [20]. It is important to emphasize that the Grassmann manifold plays a key role in topology and geometry as the base space of an universal vector bundle. Moreover, $Gr(1, n+1)$ is projective space P_n.

2. Analytical Apparatus

Projective space P_n can be presented as the quotient space $L_{n+1}/\sim$ of a vector space L_{n+1} by equivalence relation (collinearity) $\sim$ of nonzero vectors, i.e., $P_n = (L_{n+1} \setminus \{0\})/\sim$ (see, e.g., [21]). Thus, we can set the quotient map by

$$L_{n+1} \setminus \{0\} \;\to\; P_n.$$

As is known [11], a projective frame in the space P_n is a system consisting of points $A_{I'}$, $I' = 0, ..., n$, and the unity point E. In the vector space L_{n+1}, linearly independent vectors $e_{I'}$ correspond to the points $A_{I'}$ and the vector $e = \sum_{I'=0}^{n} e_{I'}$ corresponds to the point E. These vectors are defined up to a common factor in L_{n+1}. The unity point E is given along with the basis points $A_{I'}$, though we might not mention it each time.

It is supposed that a frame in the vector space L_{n+1} is normalized, i.e., $e_0 \wedge e_1 \wedge ... \wedge e_n = 1$, where $\wedge$ sets an exterior product.

The equations of infinitesimal displacements of the moving frame in P_n can be written in the following way:

$$dA_{I'} = \theta_{I'}^{J'} A_{J'},$$

$I' = 0, ..., n$, with the condition of normalization $A_0 \wedge A_1 \wedge ... \wedge A_n = 1$. Here, d denotes ordinary differentiation in P_n. The forms $\theta_{I'}^{J'}$ are linear differential forms; they depend on parameters u (defining a location of the frame) and their differentials du.

The forms $\theta_{I'}^{J'}$ are connected by the relation $\theta_0^0 + \theta_1^1 + ... + \theta_n^n = 0$. This condition is also necessary for the number of linearly independent forms $\theta_{I'}^{J'}$ that became equal to the number of parameters on which the group of projective transformations of space P_n depends.

The structure equations of projective space P_n have the form

$$D\theta_{I'}^{J'} = \theta_{I'}^{K'} \wedge \theta_{k'}^{J'},$$

where D is a symbol of exterior derivative.

By the condition $\theta_{I'}^{I'} = 0$ from the linear group $GL(n+1)$, it is possible to determine the special linear group $SGL(n+1)$ [21] acting effectively in P_n.

Introducing the following new forms (see, e.g., [9,10]) $\omega_{J'}^{I'} = \theta_{J'}^{I'} - \delta_{J'}^{I'}\theta_0^0$ and fixing the index $I' = \{0, I\}$, $I = 1, ..., n$, we can expand the forms $\omega_{J'}^{I'}$ as

$$\omega_0^I = \theta_0^I, \quad \omega_J^I = \theta_J^I - \delta_J^I \theta_0^0, \quad \omega_I^0 = \theta_I^0 \qquad (\omega_0^0 = 0).$$

The formulas of infinitesimal displacements can be written in more detail:

$$dA = \theta A + \omega^I A_I, \quad dA_I = \theta A_I + \omega_I^J A_J + \omega_I A, \tag{1}$$

where $A = A_0$, $\omega^I = \omega_0^I$, $\omega_I = \omega_I^0$, and the form $\theta = \theta_0^0$ plays the role of the proportionality coefficient.

Introducing the basis forms ω and omitting, for simplicity, the index 0 in the notation of the forms ω_0^I and ω_I^0, the Cartan equations can be defined

$$D\omega^I = \omega^J \wedge \omega_J^I, \quad D\omega_J^I = \omega_J^K \wedge \omega_K^I + \delta_J^I \omega_K \wedge \omega^K + \omega_J \wedge \omega^I, \quad D\omega_I = \omega_I^J \wedge \omega_J, \tag{2}$$

where ω^I, ω_J^I, ω_I are the basis forms of the projective group $GP(n)$ acting effectively on P_n.

Remark 1. *We employ the inhomogeneous analytic apparatus with the derivation formulas* (1) *and the structure equations* (2)*. By contrast to the homogeneous case, this apparatus is more convenient for investigation of centered planes; and it was used in the previous author's papers [22–25].*

3. The Space of Centered Planes

For the purpose of this paper, the term "space of centered planes", denoted by Π, will be taken to mean a space of all m-dimensional centered planes P_m^0 in projective space P_n. The space Π is a differentiable manifold and its points are m-dimensional centered planes.

Putting the vertice A of the moving frame on a m-plane P_m and fixing it as a centre, we get a centered plane P_m^0. Putting the vertices A_a of the frame on the plane P_m^0, we fix index ranges $1 \leq a, b, ... \leq m$ and $m+1 \leq \alpha, \beta, ... \leq n$. From the derivation formulas (1), we immediately get stationarity equations for the centered plane. These equations have the form $\omega^\alpha = 0$, $\omega_a^\alpha = 0$, $\omega^a = 0$. The forms ω^α, ω_a^α, ω^a are the basis forms of the space Π; the rest forms ω_b^a, ω_a, ω_β^α, ω_α^a, ω_α are secondary.

Remark 2. *The dimension of the space* Π *of centered planes differs from the dimension of the Grassmann manifold* $Gr(m,n)$ *[26] by the size m [22], i.e.,* $\dim \Pi = \dim Gr(m,n) + m = n + m(n-m)$.

4. Principal Bundle of the Space Π

The specification of the moving frame to the space Π yields the principal bundle $G(\Pi)$, its typical fiber is a stationary subgroup G of the centered plane P_m^0 and base space is the space Π; in addition, thereto, $\dim G = n^2 - mn + m^2 + n$. Total space of the bundle $G(\Pi)$ [3] is the projective group $GP(n)$ and the projection $\pi : GP(n) \to \Pi$ associates with each element of the group $GP(n)$ the plane P_m^0 in Π, which is invariant under the action of this element.

The basis forms ω^α, ω^a, ω_a^α satisfy the Cartan structure equations

$$\begin{gathered} D\omega^\alpha = \omega^a \wedge \omega_a^\alpha - \omega_\beta^\alpha \wedge \omega^\beta, \quad D\omega^a = \omega^b \wedge \omega_b^a - \omega_\alpha^a \wedge \omega^\alpha, \\ D\omega_a^\alpha = (\delta_\beta^\alpha \omega_a^b - \delta_a^b \omega_\beta^\alpha) \wedge \omega_b^\beta + \omega_a \wedge \omega^\alpha. \end{gathered} \tag{3}$$

The exterior differentials of the secondary forms are as follows:

$$D\omega_b^a = \omega_b^c \wedge \omega_c^a + (\delta_c^a \omega_b + \delta_b^a \omega_c) \wedge \omega^c + \delta_b^a \omega_\alpha \wedge \omega^\alpha - \omega_\alpha^a \wedge \omega_b^\alpha; \tag{4}$$

$$D\omega_a = \omega_a^b \wedge \omega_b - \omega_\alpha \wedge \omega_a^\alpha; \tag{5}$$

$$D\omega_\beta^\alpha = \omega_\beta^\gamma \wedge \omega_\gamma^\alpha + \delta_\beta^\alpha \omega_a \wedge \omega^a + (\delta_\gamma^\alpha \omega_\beta + \delta_\beta^\alpha \omega_\gamma) \wedge \omega^\gamma + \omega_\beta^a \wedge \omega_a^\alpha; \tag{6}$$

$$D\omega_\alpha^a = \omega_\alpha^b \wedge \omega_b^a + \omega_\alpha^\beta \wedge \omega_\beta^a + \omega_\alpha \wedge \omega^a; \tag{7}$$

$$D\omega_\alpha = \omega_\alpha^a \wedge \omega_a + \omega_\alpha^\beta \wedge \omega_\beta.$$

Remark 3. *The principal bundle* $G(\Pi)$ *of the space* Π *contains the following five quotient bundles [22]:*

1. $L_{m^2}(P_n)$ *is the quotient bundle of linear plane frames belonging to the planes* P_m^0*, its typical fiber is the linear quotient group* $L_{m^2} = GL(m)$ *acting on the pencil of lines on the plane* P_m^0 *with the structure Equations* (3) *and* (4)*;*

2. *$L_{(n-m)^2}(P_n)$ is the quotient bundle of normal linear frames; it is dual to the quotient bundle of linear plane frames; the typical fiber is the linear quotient group $L_{(n-m)^2} = GL(n-m)$ acting on the quotient space $P_{n-m-1} = P_n/P_m^0$ with the structure Equations (3) and (6);*
3. *$C_{m(m+1)}(P_n)$ is the quotient bundle of plane co-affine frames belonging to the plane P_m^0; its typical fiber is the co-affine quotient group $C_{m(m+1)} = GA^*(m)$ acting on the plane P_m^0 and $GL(m) \subset GA^*(m) \subset G$. This quotient bundle has the structure Equations (3)–(5);*
4. *$H_k(P_n)$ is the affine quotient bundle whose typical fiber H_k ($k = n(n-m) + m^2$) is an affine quotient group [27] of the group $Gr \subset GP(n)$ acting on the pencil of lines through A with the structure Equations (3), (4), (6) and (7);*
5. *the maximal quotient bundle is made from the quotient bundle of plane co-affine frames and the affine quotient bundle with the structure Equations (3)–(7).*

Normalization of the space Π is made by the fields of the following geometric patterns: the first kind normal, i.e., an $(n-m)$-plane N_{n-m} intersecting the plane P_m^0 only at the point A and the second kind normal, i.e., an $(m-1)$-plane N_{m-1} contained in the centered plane P_m^0 and not passing through its centre A (see, e.g., [11,28]).

Let us now analyze the dynamics of changes of the bundle $G(\Pi)$ at the consecutive canonizations:

1. by placing the vertices A_α on the first normal N_{n-m} (the 1st canonization);
2. by placing the vertices A_a on the second normal N_{m-1} (the 2nd canonization);
3. by simultaneous placing the vertices on the corresponding normals (full canonization).

Remark 4. *The space Π^1 or Π^2 is said to be a semi-normalized space in the first or second case, respectively, and the space $\Pi^{1,2}$ is a normalized space in the third case.*

4.1. The Bundle $G^1(\Pi)$

We put the vertices A_α on the first normal N_{n-m}. Then, the following relations must hold:

$$\omega_\alpha^a = g_{\alpha\beta}^a \omega^\beta + g_{\alpha b}^a \omega^b + g_{\alpha\beta}^{ab} \omega_b^\beta, \tag{8}$$

with the differential congruences

$$\Delta g_{\alpha\beta}^a + g_{\alpha\beta}^{ab} \omega_b \equiv 0, \;\; \Delta g_{\alpha b}^a - \delta_b^a \omega_\alpha \equiv 0, \;\; \Delta g_{\alpha\beta}^{ab} \equiv 0 \quad (mod \quad \omega^\alpha,\, \omega_a^\alpha,\, \omega^a). \tag{9}$$

Here, and subsequently, the differential operator Δ acts in the standard way (see, e.g., [29])

$$\Delta g_{\alpha\beta}^a = d g_{\alpha\beta}^a + g_{\alpha\beta}^c \omega_c^a - g_{\gamma\beta}^a \omega_\alpha^\gamma - g_{\alpha\gamma}^a \omega_\beta^\gamma.$$

Taking into account (8), from the structure Equation (2), we have

$$\begin{aligned} D\omega_b^a = \omega_b^c \wedge \omega_c^a + (\delta_c^a \omega_b + \delta_b^a \omega_c) \wedge \omega^c + \delta_b^a \omega_\alpha \wedge \omega^\alpha + \\ (\ldots)_{\alpha\beta}^a \omega_b^\alpha \wedge \omega^\beta + (\ldots)_{\alpha c}^a \omega_b^\alpha \wedge \omega^c + (\ldots)_{\alpha\beta}^{ac} \omega_b^\alpha \wedge \omega_c^\beta; \end{aligned} \tag{10}$$

$$D\omega_a = \omega_a^b \wedge \omega_b - \omega_\alpha \wedge \omega_a^\alpha; \tag{11}$$

$$\begin{aligned} D\omega_\beta^\alpha = \omega_\beta^\gamma \wedge \omega_\gamma^\alpha + \delta_\beta^\alpha \omega_a \wedge \omega^a + (\delta_\gamma^\alpha \omega_\beta + \delta_\beta^\alpha \omega_\gamma) \wedge \omega^\gamma + \\ (\ldots)_{\beta\gamma}^a \omega^\gamma \wedge \omega_a^\alpha + (\ldots)_{\beta b}^a \omega^b \wedge \omega_a^\alpha + (\ldots)_{\beta\gamma}^{ab} \omega_b^\gamma \wedge \omega_a^\alpha; \end{aligned} \tag{12}$$

$$D\omega_\alpha = \omega_\alpha^\beta \wedge \omega_\beta + \omega^\beta \wedge g_{\alpha\beta}^a \omega_a + \omega^b \wedge g_{\alpha b}^a \omega_a + \omega_b^\beta \wedge g_{\alpha\beta}^{ab} \omega_a. \tag{13}$$

From Equations (10)–(13), it can be argued that, at the first canonization, the principal bundle $G(\Pi)$ is narrowed to the principal bundle $G^1(\Pi)$; its typical fiber is the stationary subgroup $G^1 \subset G$ of a pair of the affine additional planes $\{P_m, N_{n-m}\}$. There are four quotient bundles in the subbundle $G^1(\Pi)$:

1. the quotient bundle of plane linear frames with the structure Equations (3) and (10);
2. the quotient bundle of normal linear frames (3) and (12);
3. the quotient bundle of plane co-affine frames (3), (10), and (11);
4. the quotient bundle of normal co-affine frames (3), (12), and (13).

4.2. The Bundle $G^2(\Pi)$

If we do not use the previous canonization and place the vertices A_a on the second normal N_{m-1}, then

$$\omega_a = g_{a\alpha}\omega^\alpha + g_{ab}\omega^b + g^b_{a\alpha}\omega^\alpha_b \tag{14}$$

and

$$\Delta g_{a\alpha} - g_{ab}\omega^b_\alpha \equiv 0, \quad \Delta g_{ab} \equiv 0, \quad \Delta g^b_{a\alpha} + \delta^b_a\omega_\alpha \equiv 0 \quad (mod \quad \omega^\alpha,\ \omega^\alpha_a,\ \omega^a). \tag{15}$$

Then, from the structure Equation (2), we have

$$\begin{gathered} D\omega^a_b = \omega^c_b \wedge \omega^a_c + \omega^\alpha_b \wedge \omega^a_\alpha + \delta^a_b\omega_\alpha \wedge \omega^\alpha + \\ (\ldots)^a_{b\alpha c}\omega^\alpha \wedge \omega^c + (\ldots)^a_{bce}\omega^c \wedge \omega^e + (\ldots)^{ae}_{bc\alpha}\omega^\alpha_e \wedge \omega^c; \end{gathered} \tag{16}$$

$$\begin{gathered} D\omega^\alpha_\beta = \omega^\gamma_\beta \wedge \omega^\alpha_\gamma + (\delta^\alpha_\gamma\omega_\beta + \delta^\alpha_\beta\omega_\gamma) \wedge \omega^\gamma - \omega^\alpha_a \wedge \omega^a_\beta + \\ (\ldots)^\alpha_{\beta\gamma a}\omega^\gamma \wedge \omega^a + (\ldots)^\alpha_{\beta ba}\omega^b \wedge \omega^a + (\ldots)^{\alpha b}_{\beta\gamma a}\omega^\gamma_b \wedge \omega^a; \end{gathered} \tag{17}$$

$$D\omega^a_\alpha = \omega^b_\alpha \wedge \omega^a_b + \omega^\beta_\alpha \wedge \omega^a_\beta + \omega_\alpha \wedge \omega^a; \tag{18}$$

$$D\omega_\alpha = \omega^\beta_\alpha \wedge \omega_\beta - \omega^\beta \wedge g_{a\beta}\omega^a_\alpha - \omega^b \wedge g_{ab}\omega^a_\alpha - \omega^\beta_b \wedge g^b_{a\beta}\omega^a_\alpha. \tag{19}$$

In fact, according to Equations (16)–(19), we can make a conclusion that, at the second canonization, the principal bundle $G(\Pi)$ is narrowed to the principal bundle $G^2(\Pi)$; its typical fiber is the stationary subgroup $G^2 \subset G$ of the pair $\{A, N_{m-1}\}$. There are four quotient bundles in the subbundle $G^2(\Pi)$:

1. the quotient bundle of plane linear frames with the structure Equations (3) and (16);
2. the quotient bundle of normal linear frames (3) and (17);
3. the bundle $H(\Pi)$ (3), (16)–(18) whose typical fiber H is an affine quotient group of G_2;
4. the bundle of normal co-affine frames (3), (17), and (19).

4.3. The Bundle $G^{1,2}(\Pi)$

Now, suppose that we have already made canonizations considered in items 4.1 and 4.2 simultaneously, that is, $A_\alpha \in N_{n-m}$ and $A_a \in N_{m-1}$. In this case, conditions (8) and (14) are satisfied and the structure Equation (2) will become

$$\begin{gathered} D\omega^a_b = \omega^c_b \wedge \omega^a_c + \delta^a_b\omega_\alpha \wedge \omega^\alpha + (\ldots)^a_{bc\alpha}\omega^\alpha \wedge \omega^c + (\ldots)^a_{bce}\omega^c \wedge \omega^e + \\ (\ldots)^{ae}_{b\alpha c}\omega^\alpha_e \wedge \omega^c + (\ldots)^a_{\alpha\beta}\omega^\alpha_b \wedge \omega^\beta + (\ldots)^{ac}_{\alpha\beta}\omega^\alpha_b \wedge \omega^\beta_c; \end{gathered} \tag{20}$$

$$\begin{gathered} D\omega^\alpha_\beta = \omega^\gamma_\beta \wedge \omega^\alpha_\gamma + (\delta^\alpha_\gamma\omega_\beta + \delta^\alpha_\beta\omega_\gamma) \wedge \omega^\gamma + (\ldots)^\alpha_{\beta\gamma a}\omega^\gamma \wedge \omega^a + \\ +(\ldots)^\alpha_{\beta ba}\omega^b \wedge \omega^a + (\ldots)^{\alpha b}_{\beta\gamma a}\omega^\gamma_b \wedge \omega^a + (\ldots)^a_{\beta\gamma}\omega^\gamma \wedge \omega^\alpha_a + (\ldots)^{ab}_{\beta\gamma}\omega^\gamma_b \wedge \omega^\alpha_a; \end{gathered} \tag{21}$$

$$\begin{gathered} D\omega_\alpha = \omega^\beta_\alpha \wedge \omega_\beta + (\ldots)_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma + (\ldots)_{\alpha\beta b}\omega^\beta \wedge \omega^b + (\ldots)^b_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma_b + \\ (\ldots)_{abc}\omega^b \wedge \omega^c + (\ldots)^c_{\alpha b\beta}\omega^b \wedge \omega^\beta_c + (\ldots)^{bc}_{\alpha\beta\gamma}\omega^\beta_b \wedge \omega^\gamma_c. \end{gathered} \tag{22}$$

Obviously, $G^{1,2}$ is the stationary subgroup of the centered $(n-m)$ pair $\{N^*_{n-m}, N_{m-1}\}$ [30]—then at the full canonization from narrowing $G^{1,2}(\Pi)$ of the principal bundle $G(\Pi)$. The following three quotient bundles are allocated:

1. the bundle of plane linear frames (3) and (20);
2. the bundle of normal linear frames (3) and (21);
3. the bundle of normal co-affine frames (3), (21), and (22).

5. A Connection on the Bundle Associated with the Space Π

Using the Laptev–Lumiste method (see, e.g., [4,31]), on the principal bundle $G(\Pi)$, we define a fundamental-group connection by the forms

$$\begin{gathered}\tilde{\omega}^a_b = \omega^a_b - L^a_{b\alpha}\omega^\alpha - L^a_{bc}\omega^c - \Gamma^{ac}_{b\alpha}\omega^\alpha_c, \quad \tilde{\omega}^\alpha_\beta = \omega^\alpha_\beta - L^\alpha_{\beta\gamma}\omega^\gamma - L^\alpha_{\beta a}\omega^a - \Gamma^{\alpha a}_{\beta\gamma}\omega^\gamma_a, \\ \tilde{\omega}^a_\alpha = \omega^a_\alpha - L^a_{\alpha\beta}\omega^\beta - L^a_{\alpha b}\omega^b - \Gamma^{ab}_{\alpha\beta}\omega^\beta_b, \quad \tilde{\omega}_a = \omega_a - \Gamma_{a\alpha}\omega^\alpha - \Gamma_{ab}\omega^b - \Pi^b_{a\alpha}\omega^\alpha_b, \\ \tilde{\omega}_\alpha = \omega_\alpha - \Gamma_{\alpha\beta}\omega^\beta - \Gamma_{\alpha a}\omega^a - \Pi^a_{\alpha\beta}\omega^\beta_a.\end{gathered} \tag{23}$$

The components of the connection object [4]

$$\Gamma = \{L^a_{b\alpha}, L^a_{bc}, \Gamma^{ac}_{b\alpha}, L^\alpha_{\beta\gamma}, L^\alpha_{\beta a}, \Gamma^{\alpha a}_{\beta\gamma}, L^a_{\alpha\beta}, L^a_{\alpha b}, \Gamma^{ab}_{\alpha\beta}, \Gamma_{a\alpha}, \Gamma_{ab}, \Pi^b_{a\alpha}, \Gamma_{\alpha\beta}, \Gamma_{\alpha a}, \Pi^a_{\alpha\beta}\}$$

satisfy the following differential congruences modulo the basis forms ω^α, ω^α_a, ω^a:

$$\begin{gathered}\Delta L^a_{b\alpha} - L^a_{bc}\omega^c_\alpha + \Gamma^{ac}_{b\alpha}\omega_c - \delta^a_b\omega_\alpha \equiv 0, \quad \Delta L^a_{bc} - \delta^a_c\omega_b - \delta^a_b\omega_c \equiv 0, \quad \Delta\Gamma^{ac}_{b\alpha} + \delta^c_b\omega^a_\alpha \equiv 0, \\ \Delta L^\alpha_{\beta\gamma} - L^\alpha_{\beta a}\omega^a_\gamma + \Gamma^{\alpha a}_{\beta\gamma}\omega_a - \delta^\alpha_\beta\omega_\gamma - \delta^\alpha_\gamma\omega_\beta \equiv 0, \quad \Delta L^\alpha_{\beta a} - \delta^\alpha_\beta\omega_a \equiv 0, \quad \Delta\Gamma^{\alpha a}_{\beta\gamma} - \delta^\alpha_\gamma\omega^a_\beta \equiv 0, \\ \Delta L^a_{\alpha\beta} + \Gamma^{ab}_{\alpha\beta}\omega_b - L^a_{b\beta}\omega^b_\alpha + L^\gamma_{\alpha\beta}\omega^a_\gamma - L^a_{\alpha b}\omega^b_\beta \equiv 0, \\ \Delta L^a_{\alpha b} - L^a_{cb}\omega^c_\alpha + L^\beta_{\alpha b}\omega^a_\beta - \delta^a_b\omega_\alpha \equiv 0, \quad \Delta\Gamma^{ab}_{\alpha\beta} - \Gamma^{ab}_{d\beta}\omega^d_\alpha + \Gamma^{\gamma b}_{\alpha\beta}\omega^a_\gamma \equiv 0, \\ \Delta\Gamma_{a\alpha} - \Gamma_{ab}\omega^b_\alpha + (\Pi^b_{a\alpha} + L^b_{a\alpha})\omega_b \equiv 0, \\ \Delta\Gamma_{ab} + L^c_{ab}\omega_c \equiv 0, \quad \Delta\Pi^b_{a\alpha} + \Gamma^{db}_{a\alpha}\omega_d + \delta^b_a\omega_\alpha \equiv 0, \\ \Delta\Gamma_{\alpha\beta} - \Gamma_{\alpha a}\omega^a_\beta + (\Pi^a_{\alpha\beta} + L^a_{\alpha\beta})\omega_a - \Gamma_{a\beta}\omega^a_\alpha + L^\gamma_{\alpha\beta}\omega_\gamma \equiv 0, \\ \Delta\Gamma_{\alpha a} - \Gamma_{ba}\omega^b_\alpha + L^b_{\alpha a}\omega_b + L^\beta_{\alpha a}\omega_\beta \equiv 0, \quad \Delta\Pi^a_{\alpha\beta} + \Gamma^{ba}_{\alpha\beta}\omega_b - \Pi^a_{c\beta}\omega^c_\alpha + \Gamma^{\gamma a}_{\alpha\beta}\omega_\gamma \equiv 0.\end{gathered} \tag{24}$$

Remark 5. *The connection object* Γ *contains the following five geometric subobjects* $\Gamma_1 = \{L^a_{b\alpha}, L^a_{bc}, \Gamma^{ac}_{b\alpha}\}$, $\Gamma_2 = \{L^\alpha_{\beta\gamma}, L^\alpha_{\beta a}, \Gamma^{\alpha a}_{\beta\gamma}\}$, $\Gamma_3 = \{\Gamma_1, \Gamma_{a\alpha}, \Gamma_{ab}, \Pi^b_{a\alpha}\}$, $\Gamma_4 = \{\Gamma_1, \Gamma_2, L^a_{\alpha\beta}, L^a_{\alpha b}, \Gamma^{ab}_{\alpha\beta}\}$, *and* $\Gamma_5 = \{\Gamma_3 \setminus \Gamma_1, \Gamma_4\}$. *These subobjects determine connections on the corresponding (see Remark 3) quotient bundles.*

Let us consider the dynamics of changes of the connection Γ at consecutive canonizations and we will be convinced that the connection Γ is not uniquely induced at the normalization of the space Π.

5.1. The Connection Object at Adaptation of the Moving Frame to the First Normal

By placing the vertices A_α on the first normal N_{n-m}, condition (8) is satisfied, that is, the forms ω^a_α become principal and the connection object Γ is narrowed to the object $\Gamma^1 = \Gamma \setminus (\Gamma_4 \setminus (\Gamma_1 \cup \Gamma_2))$; and differential congruences for its components have the form

$$\begin{gathered}\Delta L^a_{b\alpha}+\Gamma^{ac}_{b\alpha}\omega_c-\delta^a_b\omega_\alpha\equiv 0,\quad \Delta L^a_{bc}-\delta^a_c\omega_b-\delta^a_b\omega_c\equiv 0,\quad \Delta\Gamma^{ac}_{b\alpha}\equiv 0,\\ \Delta L^\alpha_{\beta\gamma}+\Gamma^{\alpha a}_{\beta\gamma}\omega_a-\delta^\alpha_\beta\omega_\gamma-\delta^\alpha_\gamma\omega_\beta\equiv 0,\quad \Delta L^\alpha_{\beta a}-\delta^\alpha_\beta\omega_a\equiv 0,\quad \Delta\Gamma^{\alpha a}_{\beta\gamma}\equiv 0,\\ \Delta\Gamma_{a\alpha}+(\Pi^b_{a\alpha}+L^b_{a\alpha})\omega_b\equiv 0,\quad \Delta\Gamma_{ab}+L^c_{ab}\omega_c\equiv 0,\\ \Delta\Pi^b_{a\alpha}+\Gamma^{db}_{a\alpha}\omega_d+\delta^b_a\omega_\alpha\equiv 0,\quad \Delta\Gamma_{\alpha\beta}+(\Pi^a_{\alpha\beta}+g^a_{\alpha\beta})\omega_a+L^\gamma_{\alpha\beta}\omega_\gamma\equiv 0,\\ \Delta\Gamma_{\alpha a}+L^\beta_{\alpha a}\omega_\beta+g^b_{\alpha a}\omega_b\equiv 0,\quad \Delta\Pi^a_{\alpha\beta}+\Gamma^{\gamma a}_{\alpha\beta}\omega_\gamma+g^{ba}_{\alpha\beta}\omega_b\equiv 0.\end{gathered}\tag{25}$$

All of this points to the fact that the following theorem holds.

Theorem 1. *At an adaptation of the moving frame to a field of the first normals the connection object Γ is reduced to the object Γ^1. The object Γ^1 contains three subobjects Γ^1_1, Γ^1_2, Γ^1_3 that set connections on the quotient bundles of plane linear frames, normal linear frames, and plane co-affine frames, respectively.*

5.2. Connection Object at Adaptation of the Moving Frame to the Second Normal

Without using the previous canonization and placing the vertices A_a on the second normal N_{m-1}, we get condition (14). The connection object Γ is narrowed to the object $\Gamma^2=\Gamma\setminus(\Gamma_3\setminus\Gamma_1)$ with the following congruences for its components:

$$\begin{gathered}\Delta L^a_{b\alpha}-L^a_{bc}\omega^c_\alpha-\delta^a_b\omega_\alpha\equiv 0,\quad \Delta L^a_{bc}\equiv 0,\quad \Delta\Gamma^{ac}_{b\alpha}+\delta^c_b\omega^a_\alpha\equiv 0,\\ \Delta L^\alpha_{\beta\gamma}-L^\alpha_{\beta a}\omega^a_\gamma-\delta^\alpha_\beta\omega_\gamma-\delta^\alpha_\gamma\omega_\beta\equiv 0,\quad \Delta L^\alpha_{\beta a}\equiv 0,\quad \Delta\Gamma^{\alpha a}_{\beta\gamma}-\delta^\alpha_\gamma\omega^a_\beta\equiv 0,\\ \Delta L^a_{\alpha\beta}-L^a_{b\beta}\omega^b_\alpha-L^a_{\alpha b}\omega^b_\beta+L^\gamma_{\alpha\beta}\omega^a_\gamma\equiv 0,\quad \Delta L^a_{\alpha b}-L^a_{cb}\omega^c_\alpha+L^\beta_{\alpha b}\omega^a_\beta-\delta^a_b\omega_\alpha\equiv 0,\\ \Delta\Gamma^{ab}_{\alpha\beta}-\Gamma^{ab}_{d\beta}\omega^d_\alpha+\Gamma^{\gamma b}_{\alpha\beta}\omega^a_\gamma\equiv 0,\quad \Delta\Gamma_{\alpha\beta}-\Gamma_{\alpha a}\omega^a_\beta+L^\gamma_{\alpha\beta}\omega_\gamma-g_{a\beta}\omega^a_\alpha\equiv 0,\\ \Delta\Gamma_{\alpha a}+L^\beta_{\alpha a}\omega_\beta-g_{ba}\omega^b_\alpha\equiv 0,\quad \Delta\Pi^a_{\alpha\beta}+\Gamma^{\gamma a}_{\alpha\beta}\omega_\gamma-g^a_{b\beta}\omega^b_\alpha\equiv 0.\end{gathered}\tag{26}$$

The arguments given above prove Theorem 2.

Theorem 2. *At an adaptation of the moving frame to a field of the second normals, the connection object Γ is reduced to the object Γ^2. The object Γ^2 contains three subobjects Γ^2_1, Γ^2_2, Γ^2_3 that set connections on the quotient bundles of plane linear frames, normal linear frames, and affine quotient bundle, respectively.*

5.3. Connection Object at Normalization

By making both canonizations simultaneously, that is, placing the vertices A_α on the first normal N_{n-m} and the vertices A_a on the second normal N_{m-1}, the differential congruences for object's $\Gamma^{1,2}=\Gamma\setminus((\Gamma_4\setminus(\Gamma_1\cup\Gamma_2))\cup(\Gamma_3\setminus\Gamma_1))$ components will become

$$\begin{gathered}\Delta L^a_{b\alpha}-\delta^a_b\omega_\alpha\equiv 0,\quad \Delta L^a_{bc}\equiv 0,\quad \Delta\Gamma^{ac}_{b\alpha}\equiv 0,\quad \Delta L^\alpha_{\beta\gamma}-\delta^\alpha_\beta\omega_\gamma-\delta^\alpha_\gamma\omega_\beta\equiv 0,\quad \Delta L^\alpha_{\beta a}\equiv 0,\\ \Delta\Gamma^{\alpha a}_{\beta\gamma}\equiv 0,\quad \Delta\Gamma_{\alpha\beta}+L^\gamma_{\alpha\beta}\omega_\gamma\equiv 0,\quad \Delta\Gamma_{\alpha a}+L^\beta_{\alpha a}\omega_\beta\equiv 0,\quad \Delta\Pi^a_{\alpha\beta}+\Gamma^{\gamma a}_{\alpha\beta}\omega_\gamma\equiv 0.\end{gathered}\tag{27}$$

In addition, we have the following theorem.

Theorem 3. *At an adaptation of the moving frame to normalization of the space Π, the connection object Γ is reduced to the object $\Gamma^{1,2}$. The object $\Gamma^{1,2}$ contains two subobjects $\Gamma^{1,2}_1$, $\Gamma^{1,2}_2$ that set connections on the quotient bundles of plane and normal linear frames.*

5.4. Reduced Connection Objects

With the help of conditions (8), the forms ω^a_α become principal, and, therefore, congruences for the components $L^a_{\alpha\beta}$, $L^a_{\alpha b}$, $\Gamma^{ab}_{\alpha\beta}$ in (24) will be carried out identically and they can be omitted.

Using conditions (8) in the rest of the differential congruences (24), the components of the reduced connection object Γ^I will satisfy (25) if the following conditions hold:

$$g^a_{\alpha\beta} = L^a_{\alpha\beta}, \quad g^b_{\alpha a} = L^b_{\alpha a}, \quad g^{ab}_{\alpha\beta} = \Gamma^{ab}_{\alpha\beta}. \tag{28}$$

Theorem 4. *The reduced object Γ^I coincides with the object Γ^1 only if conditions (28) hold, where the object Γ^1 gives a connection on the reduced bundle that arises at the adaptation of the moving frame to a field of the first normals.*

By substituting conditions (14) into the differential congruences for components of the connection object Γ, we get congruences (26) with conditions

$$g_{a\beta} = \Gamma_{a\beta}, \quad g_{ab} = \Gamma_{ab}, \quad g^a_{b\alpha} = \Pi^a_{b\alpha} \tag{29}$$

for the components of the reduced connection object Γ^{II}.

Theorem 5. *The reduced object Γ^{II} coincides with the object Γ^2 only in the case (29), where the object Γ^2 gives a connection on the reduced bundle that arises at the adaptation of the moving frame to a field of the second normals.*

Taking into account conditions (8) and (14) in the first six and last three differential congruences (24), we have that the components of the reduced connection object $\Gamma^{I,II}$ satisfy the differential congruences (27).

Theorem 6. *The reduced connection object $\Gamma^{I,II}$ coincides with the object $\Gamma^{1,2}$, which gives a connection on the reduced bundle at the adaptation of the moving frame to the normalization of the space Π.*

Remark 6. *Adaptations of the moving frame cause the reductions of associated bundle and differential congruences for components of group connection object. Semi-canonizations (the first and second canonizations) lead to reductions of bundle and connection object, but, according to Theorems 4 and 5, the reduced connection objects can differ from the objects specifying connections on the reduced bundles.*

6. Curvature and Torsion Objects

Let us now consider curvature and torsion objects [4] of group connection on the bundle associated with the space Π of centered planes at a transition to the normalized space $\Pi^{1,2}$.

6.1. Curvature Objects

Generic curvature R of the connection Γ of the space Π was studied in [22]. Denote by R^1 and R^2 the objects of reduced curvature on the bundle associated with the space Π.

6.1.1. Curvature Object at the First Canonization

At the adaptation of the moving frame to a field of the first normals, we have

$$\begin{aligned}
D\tilde{\omega}_a &= \tilde{\omega}^b_a \wedge \tilde{\omega}_b + R_{a\alpha\beta}\omega^\alpha \wedge \omega^\beta + R_{abc}\omega^b \wedge \omega^c + R_{a\alpha b}\omega^\alpha \wedge \omega^b + \\
&\quad K^b_{a\alpha\beta}\omega^\alpha \wedge \omega^\beta_b + R^c_{ab\alpha}\omega^b \wedge \omega^\alpha_c + K^{bc}_{a\alpha\beta}\omega^\alpha_b \wedge \omega^\beta_c, \\
D\tilde{\omega}^a_b &= \tilde{\omega}^c_b \wedge \tilde{\omega}^a_c + R^a_{b\alpha\beta}\omega^\alpha \wedge \omega^\beta + R^a_{bce}\omega^c \wedge \omega^e + R^a_{b\alpha c}\omega^\alpha \wedge \omega^c + \\
&\quad R^{ac}_{b\alpha\beta}\omega^\alpha \wedge \omega^\beta_c + R^{ad}_{bc\alpha}\omega^c \wedge \omega^\alpha_d + R^{acd}_{b\alpha\beta}\omega^\alpha_c \wedge \omega^\beta_d, \\
D\tilde{\omega}^\alpha_\beta &= \tilde{\omega}^\gamma_\beta \wedge \tilde{\omega}^\alpha_\gamma + R^\alpha_{\beta\gamma\mu}\omega^\gamma \wedge \omega^\mu + R^\alpha_{\beta a\ b}\omega^a \wedge \omega^b + R^\alpha_{\beta\gamma a}\omega^\gamma \wedge \omega^a + \\
&\quad R^{\alpha a}_{\beta\gamma\mu}\omega^\gamma \wedge \omega^\mu_a + R^{\alpha b}_{\beta a\gamma}\omega^a \wedge \omega^\gamma_b + R^{\alpha a\ b}_{\beta\gamma\mu}\omega^\gamma_a \wedge \omega^\mu_b,
\end{aligned}$$

$$D\tilde{\omega}_\alpha = \tilde{\omega}^a_\alpha \wedge \tilde{\omega}_a + \tilde{\omega}^\beta_\alpha \wedge \tilde{\omega}_\beta + R_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma + R_{\alpha a\ b}\omega^a \wedge \omega^b + R_{\alpha\beta a}\omega^\beta \wedge \omega^a +$$
$$K^a_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma_a + R^b_{\alpha a\beta}\omega^a \wedge \omega^\beta_b + K^{ab}_{\alpha\beta\gamma}\omega^\beta_a \wedge \omega^\gamma_b.$$

The components of the curvature object R are determined by the following relations:

$$R_{a\alpha\beta} = \Gamma_{a[\alpha\beta]} - \Gamma_{ab}g^b_{[\alpha\beta]} - L^b_{a[\alpha}\Gamma_{b\beta]}, \quad R_{a\alpha b} = \Gamma_{a\alpha b} - \Gamma_{ab\alpha} - \Gamma_{ac}g^c_{\alpha b} - L^c_{a\alpha}\Gamma_{cb} + L^c_{ab}\Gamma_{c\alpha},$$
$$R_{abc} = \Gamma_{a[bc]} - L^e_{a[b}\Gamma_{ec]}, \quad K^b_{a\alpha\beta} = \Gamma^b_{a\alpha\beta} - \Pi^b_{a\beta\alpha} - \Gamma_{ac}g^{cb}_{\alpha\beta} - L^c_{a\alpha}\Pi^b_{c\beta} + \Gamma^{cb}_{a\beta}\Gamma_{c\alpha},$$
$$R^c_{ab\alpha} = \Gamma^c_{ab\alpha} - \Pi^c_{a\alpha b} + \Gamma^{ec}_{a\alpha}\Gamma_{eb} - L^e_{ab}\Pi^c_{e\alpha} - \delta^c_b\Gamma_{a\alpha}, \quad K^{bc}_{a\alpha\beta} = \Pi_a{}^{[bc}_{[\alpha\beta]} - \Gamma^e_a{}^{[b}_{[\alpha}\Pi^{c]}_{e\beta]},$$
$$R^a_{b\alpha\beta} = L^a_{b[\alpha\beta]} - L^c_{b[\alpha}L^a_{c\beta]} - L^a_{bc}g^c_{[\alpha\beta]}, \quad R^a_{bce} = L^a_{b[ce]} - L^d_{b[c}L^a_{de]},$$
$$R^a_{b\alpha c} = L^a_{b\alpha c} - L^a_{bc\alpha} - L^e_{b\alpha}L^a_{ec} + L^e_{bc}L^a_{e\alpha} - L^a_{be}g^e_{\alpha c},$$

$$R^{ac}_{b\alpha\beta} = L^{ac}_{b\alpha\beta} - \Gamma^{ac}_{b\beta\alpha} - L^e_{b\alpha}\Gamma^{ac}_{e\beta} + \Gamma^{ec}_{b\beta}L^a_{e\alpha} - L^a_{be}g^{ec}_{\alpha\beta} - \delta^c_b g^a_{\beta\alpha},$$
$$R^{ad}_{bc\alpha} = L^{ad}_{bc\alpha} - \Gamma^{ad}_{b\alpha c} - \delta^d_c L^a_{b\alpha} - L^e_{bc}\Gamma^{ad}_{e\alpha} + \Gamma^{ed}_{b\alpha}L^a_{ec} - \delta^d_b g^a_{\alpha c},$$
$$R^{acd}_{b\alpha\beta} = \Gamma^a_b{}^{[cd]}_{[\alpha\beta]} - \Gamma^e_b{}^{[c}_{[\alpha}\Gamma^{ad]}_{e\beta]} + \delta^{[c}_b g^{ad]}_{[\alpha\beta]},$$
$$R^\alpha_{\beta\gamma\mu} = L^\alpha_{\beta[\gamma\mu]} - L^\eta_{\beta[\gamma}L^\alpha_{\eta\mu]} - L^\alpha_{\beta a}g^a_{[\gamma\mu]}, \quad R^\alpha_{\beta ab} = L^\alpha_{\beta[ab]} - L^\gamma_{\beta[a}L^\alpha_{\gamma b]},$$
$$R^\alpha_{\beta\gamma a} = L^\alpha_{\beta\gamma a} - L^\alpha_{\beta a\gamma} - L^\mu_{\beta\gamma}L^\alpha_{\mu a} + L^\mu_{\beta a}L^\alpha_{\mu\gamma} - L^\alpha_{\beta b}g^b_{\gamma a},$$
$$R^{\alpha a}_{\beta\gamma\mu} = L^{\alpha a}_{\beta\gamma\mu} - \Gamma^{\alpha a}_{\beta\mu\gamma} - L^\eta_{\beta\gamma}\Gamma^{\alpha a}_{\eta\mu} + \Gamma^{\eta a}_{\beta\mu}L^\alpha_{\eta\gamma} - L^\alpha_{\beta b}g^{ba}_{\gamma\mu} + \delta^\alpha_\mu g^a_{\beta\gamma},$$
$$R^{\alpha b}_{\beta a\gamma} = L^{\alpha b}_{\beta a\gamma} - \Gamma^{\alpha b}_{\beta\gamma a} - L^\mu_{\beta a}\Gamma^{\alpha b}_{\mu\gamma} + \Gamma^{\mu b}_{\beta\gamma}L^\alpha_{\mu a} - \delta^b_a L^\alpha_{\beta\gamma} + \delta^\alpha_\gamma g^b_{\beta a},$$
$$R^{\alpha ab}_{\beta\gamma\mu} = \Gamma^\alpha_\beta{}^{[ab]}_{[\gamma\mu]} - \Gamma^\eta_\beta{}^{[a}_{[\gamma}\Gamma^{\alpha b]}_{\eta\mu]} - \delta^\alpha_{[\gamma}g^{[ab]}_{\beta\mu]}, \quad R_{\alpha\beta\gamma} = \Gamma_{\alpha[\beta\gamma]} - L^\mu_{\alpha[\beta}\Gamma_{\mu\gamma]} - \Gamma_{\alpha a}g^a_{[\beta\gamma]}, \quad R_{\alpha ab} = \Gamma_{\alpha[ab]} - L^\beta_{\alpha[a}\Gamma_{\beta b]},$$
$$R_{\alpha\beta a} = \Gamma_{\alpha\beta a} - \Gamma_{\alpha a\beta} - L^\gamma_{\alpha\beta}\Gamma_{\gamma a} + L^\gamma_{\alpha a}\Gamma_{\gamma\beta} - \Gamma_{\alpha b}g^b_{\beta a},$$
$$K^a_{\alpha\beta\gamma} = \Gamma^a_{\alpha\beta\gamma} - \Pi^a_{\alpha\gamma\beta} - L^\mu_{\alpha\beta}\Pi^a_{\mu\gamma} + \Gamma^{\mu a}_{\alpha\gamma}\Gamma_{\mu\beta} - \Gamma_{\alpha b}g^{ba}_{\beta\gamma},$$
$$R^b_{\alpha a\beta} = \Gamma^b_{\alpha a\beta} - \Pi^b_{\alpha\beta a} - L^\gamma_{\alpha a}\Pi^b_{\gamma\beta} + \Gamma^{\gamma b}_{\alpha\beta}\Gamma_{\gamma a} - \delta^b_a\Gamma_{\alpha\beta}, \quad K^{ab}_{\alpha\beta\gamma} = \Pi_\alpha{}^{[ab]}_{[\beta\gamma]} - \Gamma^\mu_\alpha{}^{[a}_{[\beta}\Pi^{b]}_{\mu\gamma]}.$$

Here, and in what follows, square brackets mean alternation over extreme indices.

Theorem 7. *The curvature R is reduced to the object $R^1 = \{R_{a\alpha\beta}, R_{abc}, R_{a\alpha b}, K^b_{a\alpha\beta}, R^c_{ab\alpha}, K^{bc}_{a\alpha\beta}, R^a_{b\alpha\beta}, R^a_{bce}, R^a_{b\alpha c}, R^{ac}_{b\alpha\beta}, R^{ad}_{bc\alpha}, R^{acd}_{b\alpha\beta}, R^\alpha_{\beta\gamma\mu}, R^\alpha_{\beta a\ b}, R^\alpha_{\beta\gamma a}, R^{\alpha a}_{\beta\gamma\mu}, R^{ab}_{\beta a\gamma}, R^{\alpha a\ b}_{\beta\gamma\mu}, R_{\alpha\beta\gamma}, R_{\alpha a\ b}, R_{\alpha\beta a}, K^a_{\alpha\beta\gamma}, R^b_{\alpha a\beta}, K^{ab}_{\alpha\beta\gamma}\}$; and the reduced curvature object R^1 of the semi-normalized space Π^1 is a quasi-tensor together with the quasi-tensor $\{g^a_{\alpha\beta}, g^a_{\alpha b}, g^{ab}_{\alpha\beta}\}$. The object R^1 contains three subtensors that are curvature objects of subconnections on the bundles of plane linear frames, normal linear frames, and plane co-affine frames.*

Proof of Theorem 7. Extending the differential equations for components of the connection object Γ and using (9), the differential congruences of curvature object components may be written as

$$\Delta R_{a\alpha\beta} + K^b_{a[\alpha\beta]}\omega_b + R^b_{a\alpha\beta}\omega_b \equiv 0, \quad \Delta R_{a\alpha b} - (R^c_{ab\alpha} - R^c_{a\alpha b})\omega_c \equiv 0, \quad \Delta R_{abc} + R^e_{abc}\omega_e \equiv 0,$$
$$\Delta K^b_{a\alpha\beta} + (2K^{cb}_{a\alpha\beta} + R^{cb}_{a\alpha\beta})\omega_c \equiv 0, \quad \Delta R^c_{ab\alpha} + R^{ec}_{ab\alpha}\omega_e \equiv 0, \quad \Delta K^{bc}_{a\alpha\beta} + R^{ebc}_{a\alpha\beta}\omega_e \equiv 0,$$
$$\Delta R^a_{b\alpha\beta} + R^{ac}_{b[\alpha\beta]}\omega_c \equiv 0, \quad \Delta R^a_{bce} \equiv 0, \quad \Delta R^a_{b\alpha c} - R^{ae}_{bc\alpha}\omega_e \equiv 0, \quad \Delta R^{ac}_{b\alpha\beta} + 2R^{aec}_{b\alpha\beta}\omega_e \equiv 0,$$
$$\Delta R^{ad}_{bc\alpha} \equiv 0, \quad \Delta R^{ace}_{b\alpha\beta} \equiv 0, \quad \Delta R^\alpha_{\beta\gamma\mu} + R^{\alpha a}_{\beta[\gamma\mu]}\omega_a \equiv 0, \quad \Delta R^\alpha_{\beta ab} \equiv 0, \quad \Delta R^\alpha_{\beta\gamma a} - R^{\alpha b}_{\beta a\gamma}\omega_b \equiv 0,$$
$$\Delta R^{\alpha a}_{\beta\gamma\mu} - 2R^{\alpha ab}_{\beta\mu\gamma}\omega_b \equiv 0, \quad \Delta R^{\alpha b}_{\beta a\gamma} \equiv 0, \quad \Delta R^{\alpha ab}_{\beta\gamma\mu} \equiv 0,$$
$$\Delta R_{\alpha\beta\gamma} + R^\mu_{\alpha\beta\gamma}\omega_\mu + (K^a_{\alpha[\beta\gamma]} + g^a_{\alpha[\beta\gamma]} - g^a_{\alpha b}g^b_{[\beta\gamma]})\omega_a \equiv 0, \quad \Delta R_{\alpha ab} + R^\beta_{\alpha ab}\omega_\beta + g^c_{\alpha[ab]}\omega_c \equiv 0,$$
$$\Delta R_{\alpha\beta a} + (g^b_{\alpha\beta a} - g^b_{\alpha a\beta} - g^b_{\alpha c}g^c_{\beta a} - R^b_{\alpha a\beta})\omega_b + R^\gamma_{\alpha\beta a}\omega_\gamma \equiv 0,$$

$$\Delta K^a_{\alpha\beta\gamma} + (2K^{ba}_{\alpha\beta\gamma} + g^{ba}_{\alpha\beta\gamma} - g^{ba}_{\alpha\gamma\beta} - g^b_{\alpha c}g^{ca}_{\beta\gamma})\omega_b + R^{\mu a}_{\alpha\beta\gamma}\omega_\mu \equiv 0,$$
$$\Delta R^b_{\alpha a\beta} + R^{\gamma b}_{\alpha a\beta}\omega_\gamma + (g^{cb}_{\alpha a\beta} - g^{cb}_{\alpha\beta a} - \delta^b_a g^c_{\alpha\beta})\omega_c \equiv 0, \quad \Delta K^{ab}_{\alpha\beta\gamma} + R^{\mu ab}_{\alpha\beta\gamma}\omega_\mu + g^c_\alpha{}^{[ab}_{\beta\gamma]}\omega_c \equiv 0,$$

which proves the theorem. □

6.1.2. Curvature Object at the Second Canonization

At the adaptation of the moving frame to a field of the second normals, we get

$$D\tilde{\omega}^a_b = \tilde{\omega}^c_b \wedge \tilde{\omega}^a_c + R^a_{b\alpha\beta}\omega^\alpha \wedge \omega^\beta + R^a_{bce}\omega^c \wedge \omega^e + R^a_{b\alpha c}\omega^\alpha \wedge \omega^c +$$
$$R^{ac}_{b\alpha\beta}\omega^\alpha \wedge \omega^\beta_c + R^{ad}_{bc\alpha}\omega^c \wedge \omega^\alpha_d + R^{acd}_{b\alpha\beta}\omega^\alpha_c \wedge \omega^\beta_d,$$
$$D\tilde{\omega}^\alpha_\beta = \tilde{\omega}^\gamma_\beta \wedge \tilde{\omega}^\alpha_\gamma + R^\alpha_{\beta\gamma\mu}\omega^\gamma \wedge \omega^\mu + R^\alpha_{\beta a\ b}\omega^a \wedge \omega^b + R^\alpha_{\beta\gamma a}\omega^\gamma \wedge \omega^a +$$
$$R^{\alpha a}_{\beta\gamma\mu}\omega^\gamma \wedge \omega^\mu_a + R^{\alpha b}_{\beta a\gamma}\omega^a \wedge \omega^\gamma_b + R^{\alpha a\ b}_{\beta\gamma\mu}\omega^\gamma_a \wedge \omega^\mu_b,$$
$$D\tilde{\omega}^a_\alpha = \tilde{\omega}^b_\alpha \wedge \tilde{\omega}^a_b + \tilde{\omega}^\beta_\alpha \wedge \tilde{\omega}^a_\beta + R^a_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma + R^a_{\alpha bc}\omega^b \wedge \omega^c + R^a_{\alpha\beta b}\omega^\beta \wedge \omega^b +$$
$$R^{ac}_{\alpha b\beta}\omega^b \wedge \omega^\beta_c + R^{ab}_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma_b + R^{abc}_{\alpha\beta\gamma}\omega^\beta_b \wedge \omega^\gamma_c,$$
$$D\tilde{\omega}_\alpha = \tilde{\omega}^a_\alpha \wedge \tilde{\omega}_a + \tilde{\omega}^\beta_\alpha \wedge \tilde{\omega}_\beta + R_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma + R_{\alpha a\ b}\omega^a \wedge \omega^b + R_{\alpha\beta a}\omega^\beta \wedge \omega^a +$$
$$K^a_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma_a + R^b_{\alpha a\beta}\omega^a \wedge \omega^\beta_b + K^{ab}_{\alpha\beta\gamma}\omega^\beta_a \wedge \omega^\gamma_b.$$

The components of the curvature object have the form

$$R^a_{b\alpha\beta} = L^a_{b[\alpha\beta]} - L^c_{b[\alpha}L^a_{c\beta]} + \Gamma^{ac}_{b[\alpha}g_{c\beta]}, \quad R^a_{bce} = L^a_{b[ce]} - L^d_{b[c}L^a_{de]} - \delta^a_{[c}g_{be]} - \delta^a_b g_{[ce]},$$
$$R^a_{b\alpha c} = L^a_{b\alpha c} - L^a_{bc\alpha} - L^e_{b\alpha}L^a_{ec} + L^e_{bc}L^a_{e\alpha} + \Gamma^{ae}_{b\alpha}g_{ec} + \delta^a_c g_{b\alpha} + \delta^a_b g_{c\alpha},$$
$$R^{ac}_{b\alpha\beta} = L^{ac}_{b\alpha\beta} - \Gamma^{ac}_{b\beta\alpha} - L^e_{b\alpha}\Gamma^{ac}_{e\beta} + \Gamma^{ec}_{b\beta}L^a_{e\alpha} + \Gamma^{ae}_{b\alpha}g^c_{e\beta},$$
$$R^{ad}_{bc\alpha} = L^{ad}_{bc\alpha} - \Gamma^{ad}_{b\alpha c} - \delta^d_c L^a_{b\alpha} - L^e_{bc}\Gamma^{ad}_{e\alpha} + \Gamma^{ed}_{b\alpha}L^a_{ec} - \delta^a_c g^d_{b\alpha} - \delta^a_b g^d_{c\alpha}, \quad R^{acd}_{b\alpha\beta} = \Gamma^a_b{}^{[cd}_{\alpha\beta]} - \Gamma^e_b{}^{[c}_{\alpha}\Gamma^{ad]}_{e\beta]},$$
$$R^\alpha_{\beta\gamma\mu} = L^\alpha_{\beta[\gamma\mu]} - L^\eta_{\beta[\gamma}L^\alpha_{\eta\mu]} + \Gamma^{\alpha a}_{\beta[\gamma}g_{a\mu]}, \quad R^\alpha_{\beta ab} = L^\alpha_{\beta[ab]} - L^\gamma_{\beta[a}L^\alpha_{\gamma b]} - \delta^\alpha_\beta g_{[ab]},$$
$$R^\alpha_{\beta\gamma a} = L^\alpha_{\beta\gamma a} - L^\alpha_{\beta a\gamma} - L^\mu_{\beta\gamma}L^\alpha_{\mu a} + L^\mu_{\beta a}L^\alpha_{\mu\gamma} + \Gamma^{\alpha b}_{\beta\gamma}g_{ba} + \delta^\alpha_\beta g_{a\gamma},$$
$$R^{\alpha a}_{\beta\gamma\mu} = L^{\alpha a}_{\beta\gamma\mu} - \Gamma^{\alpha a}_{\beta\mu\gamma} - L^\eta_{\beta\gamma}\Gamma^{\alpha a}_{\eta\mu} + \Gamma^{\eta a}_{\beta\mu}L^\alpha_{\eta\gamma} + \Gamma^{\alpha b}_{\beta\gamma}g^a_{b\mu},$$
$$R^{\alpha b}_{\beta a\gamma} = L^{\alpha b}_{\beta a\gamma} - \Gamma^{\alpha b}_{\beta\gamma a} - L^\mu_{\beta a}\Gamma^{\alpha b}_{\mu\gamma} + \Gamma^{\mu b}_{\beta\gamma}L^\alpha_{\mu a} - \delta^b_a L^\alpha_{\beta\gamma} - \delta^\alpha_\beta g^b_{a\gamma}, \quad R^{\alpha ab}_{\beta\gamma\mu} = \Gamma^\alpha_\beta{}^{[ab}_{\gamma\mu]} - \Gamma^\eta_\beta{}^{[a}_{\gamma}\Gamma^{\alpha b]}_{\eta\mu]},$$
$$R^a_{\alpha\beta\gamma} = L^a_{\alpha[\beta\gamma]} - L^b_{\alpha[\beta}L^a_{b\gamma]} - L^\mu_{\alpha[\beta}L^a_{\mu\gamma]} + \Gamma^{ab}_{\alpha[\beta}g_{b\gamma]}, \quad R^a_{\alpha bc} = L^a_{\alpha[bc]} - L^e_{\alpha[b}L^a_{ec]} - L^\beta_{\alpha[b}L^a_{\beta c]},$$
$$R^a_{\alpha\beta b} = L^a_{\alpha\beta b} - L^a_{\alpha b\beta} - L^e_{\alpha\beta}L^a_{eb} + L^c_{\alpha b}L^a_{c\beta} - L^\gamma_{\alpha\beta}L^a_{\gamma b} + L^\mu_{\alpha b}L^a_{\mu\beta} + \Gamma^{ac}_{\alpha\beta}g_{cb},$$
$$R^{ac}_{\alpha b\beta} = L^{ac}_{\alpha b\beta} - \Gamma^{ac}_{\alpha\beta b} - L^e_{\alpha b}\Gamma^{ac}_{e\beta} + \Gamma^{ec}_{\alpha\beta}L^a_{eb} - L^\gamma_{\alpha b}\Gamma^{ac}_{\gamma\beta} + \Gamma^{\gamma c}_{\alpha\beta}L^a_{\gamma b} - \delta^c_b L^a_{\alpha\beta},$$
$$R^{ab}_{\alpha\beta\gamma} = L^{ab}_{\alpha\beta\gamma} - \Gamma^{ab}_{\alpha\gamma\beta} - L^c_{\alpha\beta}\Gamma^{ab}_{c\gamma} + \Gamma^{cb}_{\alpha\gamma}L^a_{c\beta} - L^\mu_{\alpha\beta}\Gamma^{ab}_{\mu\gamma} + \Gamma^{\mu b}_{\alpha\gamma}L^a_{\mu\beta} + \Gamma^{ac}_{\alpha\beta}g^b_{c\gamma},$$
$$R^{abc}_{\alpha\beta\gamma} = \Gamma^a_\alpha{}^{[bc}_{\beta\gamma]} - \Gamma^e_\alpha{}^{[b}_{\beta}\Gamma^{ac]}_{e\gamma]} - \Gamma^\mu_\alpha{}^{[b}_{\beta}\Gamma^{ac]}_{\mu\gamma]}, \quad R_{\alpha\beta\gamma} = \Gamma_{\alpha[\beta\gamma]} - L^\mu_{\alpha[\beta}\Gamma_{\mu\gamma]} + \Pi^a_{\alpha[\beta}g_{a\gamma]},$$
$$R_{\alpha ab} = \Gamma_{\alpha[ab]} - L^\beta_{\alpha[a}\Gamma_{\beta b]}, \quad R_{\alpha\beta a} = \Gamma_{\alpha\beta a} - \Gamma_{\alpha a\beta} - L^\gamma_{\alpha\beta}\Gamma_{\gamma a} + L^\gamma_{\alpha a}\Gamma_{\gamma\beta} + \Pi^b_{\alpha\beta}g_{ba},$$
$$K^a_{\alpha\beta\gamma} = \Gamma^a_{\alpha\beta\gamma} - \Pi^a_{\alpha\gamma\beta} - L^\mu_{\alpha\beta}\Pi^a_{\mu\gamma} + \Gamma^{\mu a}_{\alpha\gamma}\Gamma_{\mu\beta} + \Pi^b_{\alpha\beta}g^a_{b\gamma},$$
$$R^b_{\alpha a\beta} = \Gamma^b_{\alpha a\beta} - \Pi^b_{\alpha\beta a} - L^\gamma_{\alpha a}\Pi^b_{\gamma\beta} + \Gamma^{\gamma b}_{\alpha\beta}\Gamma_{\gamma a} - \delta^b_a\Gamma_{\alpha\beta}, \quad K^{ab}_{\alpha\beta\gamma} = \Pi_\alpha{}^{[ab}_{\beta\gamma]} - \Gamma^\mu_\alpha{}^{[a}_{\beta}\Pi^{b]}_{\mu\gamma]}.$$

Theorem 8. *The curvature R is reduced to the object* $R^2 = \{R^a_{b\alpha\beta}, R^a_{bce}, R^a_{b\alpha c}, R^{ac}_{b\alpha\beta}, R^{ad}_{bc\alpha}, R^{acd}_{b\alpha\beta}, R^\alpha_{\beta\gamma\mu}, R^\alpha_{\beta a\ b}, R^\alpha_{\beta\gamma a}, R^{\alpha a}_{\beta\gamma\mu}, R^{\alpha b}_{\beta a\gamma}, R^{\alpha a\ b}_{\beta\gamma\mu}, R^a_{\alpha\beta\gamma}, R^a_{\alpha bc}, R^a_{\alpha\beta b}, R^{ac}_{\alpha b\beta}, R^{ab}_{\alpha\beta\gamma}, R^{abc}_{\alpha\beta\gamma}, R_{\alpha\beta\gamma}, R_{\alpha a\ b}, R_{\alpha\beta a}, K^a_{\alpha\beta\gamma}, R^b_{\alpha a\beta}, K^{ab}_{\alpha\beta\gamma}\}$*; and the reduced curvature object* R^2 *of the semi-normalized space* Π^2 *is a quasi-tensor together with the quasi-tensor* $\{g_{a\alpha}, g_{ab}, g^b_{a\alpha}\}$*. The object* R^2 *contains three subtensors that are curvature objects of subconnections on the* *bundles of plane and normal linear frames, and also on the bundle $H(\Pi^2)$, where H is an affine quotent group of the stationary group of the pair $\{A, N_{m-1}\}$.*

Proof of Theorem 8. The proof of this theorem is not fundamentally different from the proof of Theorem 7, but now we use conditions (15). We obtain

$$
\begin{gathered}
\Delta R^a_{b\alpha\beta} - R^a_{b[\alpha c}\omega^c_{\beta]} \equiv 0, \quad \Delta R^a_{bce} \equiv 0, \quad \Delta R^a_{b\alpha c} + 2R^a_{bce}\omega^e_\alpha \equiv 0, \quad \Delta R^{ac}_{b\alpha\beta} - R^{ac}_{be\beta}\omega^e_\alpha \equiv 0, \\
\Delta R^{ad}_{bc\alpha} \equiv 0, \quad \Delta R^{ace}_{b\alpha\beta} \equiv 0, \quad \Delta R^\alpha_{\beta\gamma\mu} - R^\alpha_{\beta[\gamma a}\omega^a_{\mu]} \equiv 0, \quad \Delta R^\alpha_{\beta ab} \equiv 0, \\
\Delta R^\alpha_{\beta\gamma a} + 2R^\alpha_{\beta ab}\omega^b_\gamma \equiv 0, \quad \Delta R^{\alpha a}_{\beta\gamma\mu} - R^{\alpha a}_{\beta b\mu}\omega^b_\gamma \equiv 0, \quad \Delta R^{\alpha b}_{\beta a\gamma} \equiv 0, \quad \Delta R^{\alpha ab}_{\beta\gamma\mu} \equiv 0, \\
\Delta R^a_{\alpha\beta\gamma} - R^a_{\alpha[\beta b}\omega^b_{\gamma]} - R^a_{b\beta\gamma}\omega^b_\alpha + R^\mu_{\alpha\beta\gamma}\omega^a_\mu \equiv 0, \quad \Delta R^a_{\alpha bc} - R^a_{ebc}\omega^e_\alpha + R^\beta_{\alpha bc}\omega^a_\beta \equiv 0, \\
\Delta R^a_{\alpha\beta b} - 2R^a_{\alpha cb}\omega^c_\beta - R^a_{c\beta b}\omega^c_\alpha + R^\gamma_{\alpha\beta b}\omega^a_\gamma \equiv 0, \quad \Delta R^{ac}_{\alpha b\beta} - R^{ac}_{eb\beta}\omega^e_\alpha + R^{\gamma c}_{\alpha b\beta}\omega^a_\gamma \equiv 0, \\
\Delta R^{ab}_{\alpha\beta\gamma} - R^{ab}_{\alpha c\gamma}\omega^c_\beta + R^{\mu b}_{\alpha\beta\gamma}\omega^a_\mu - R^{ab}_{c\beta\gamma}\omega^c_\alpha \equiv 0, \quad \Delta R^{abc}_{\alpha\beta\gamma} - R^{abc}_{e\beta\gamma}\omega^e_\alpha + R^{\mu bc}_{\alpha\beta\gamma}\omega^a_\mu \equiv 0, \\
\Delta R_{\alpha\beta\gamma} + R^\mu_{\alpha\beta\gamma}\omega_\mu - R_{\alpha[\beta a}\omega^a_{\gamma]} - g_{a[\beta\gamma]}\omega^a_\alpha - g^a_{b[\beta}g_{a\gamma]}\omega^b_\alpha \equiv 0, \quad \Delta R_{\alpha ab} + R^\beta_{\alpha ab}\omega_\beta - g_{c[ab]}\omega^c_\alpha \equiv 0, \\
\Delta R_{\alpha\beta a} + 2R_{\alpha ab}\omega^b_\beta + R^\gamma_{\alpha\beta a}\omega_\gamma + (g_{ba\beta} - g_{b\beta a} - g^c_{b\beta}g_{ca})\omega^b_\alpha \equiv 0, \\
\Delta K^a_{\alpha\beta\gamma} - R^a_{\alpha b\gamma}\omega^b_\beta + R^{\mu a}_{\alpha\beta\gamma}\omega_\mu + (g^a_{b\gamma\beta} - g^a_{b\beta\gamma} - g^c_{b\beta}g^a_{c\gamma})\omega^b_\alpha \equiv 0, \\
\Delta R^b_{\alpha a\beta} + R^{\gamma b}_{\alpha a\beta}\omega_\gamma + (g^b_{c\beta a} - g^b_{ca\beta} + \delta^b_a g_{c\beta})\omega^c_\alpha \equiv 0, \quad \Delta K^{ab}_{\alpha\beta\gamma} + R^{\mu ab}_{\alpha\beta\gamma}\omega_\mu - g_{c\left[{ab \atop \beta\gamma}\right]}\omega^c_\alpha \equiv 0.
\end{gathered}
$$

These differential congruences prove the theorem. □

6.1.3. Curvature Object at the Full Canonization

At the normalization of the space Π, that is, at the conditions $A_\alpha \in N_{n-m}$ and $A_a \in N_{m-1}$, we have

$$
\begin{gathered}
D\tilde{\omega}^a_b = \tilde{\omega}^c_b \wedge \tilde{\omega}^a_c + R^a_{b\alpha\beta}\omega^\alpha \wedge \omega^\beta + R^a_{bce}\omega^c \wedge \omega^e + R^a_{b\alpha c}\omega^\alpha \wedge \omega^c + \\
R^{ac}_{b\alpha\beta}\omega^\alpha \wedge \omega^\beta_c + R^{ad}_{bc\alpha}\omega^c \wedge \omega^\alpha_d + R^{acd}_{b\alpha\beta}\omega^\alpha_c \wedge \omega^\beta_d, \\
D\tilde{\omega}^\alpha_\beta = \tilde{\omega}^\gamma_\beta \wedge \tilde{\omega}^\alpha_\gamma + R^\alpha_{\beta\gamma\mu}\omega^\gamma \wedge \omega^\mu + R^\alpha_{\beta a\ b}\omega^a \wedge \omega^b + R^\alpha_{\beta\gamma a}\omega^\gamma \wedge \omega^a + \\
R^{\alpha a}_{\beta\gamma\mu}\omega^\gamma \wedge \omega^\mu_a + R^{\alpha b}_{\beta a\gamma}\omega^a \wedge \omega^\gamma_b + R^{\alpha a\ b}_{\beta\gamma\mu}\omega^\gamma_a \wedge \omega^\mu_b, \\
D\tilde{\omega}_\alpha = \tilde{\omega}^a_\alpha \wedge \tilde{\omega}_a + \tilde{\omega}^\beta_\alpha \wedge \tilde{\omega}_\beta + R_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma + R_{\alpha a\ b}\omega^a \wedge \omega^b + R_{\alpha\beta a}\omega^\beta \wedge \omega^a + \\
K^a_{\alpha\beta\gamma}\omega^\beta \wedge \omega^\gamma_a + R^b_{\alpha a\beta}\omega^a \wedge \omega^\beta_b + K^{ab}_{\alpha\beta\gamma}\omega^\beta_a \wedge \omega^\gamma_b.
\end{gathered}
$$

The components of the curvature object are defined by the following relations:

$$
\begin{gathered}
R^a_{b\alpha\beta} = L^a_{b[\alpha\beta]} - L^c_{b[\alpha}L^a_{c\beta]} - L^a_{bc}g^c_{\alpha\beta]} + \Gamma^{ac}_{b[\alpha}g_{c\beta]}, \quad R^a_{bce} = L^a_{b[ce]} - L^d_{b[c}L^a_{de]} - \delta^a_{[c}g_{be]} - \delta^a_b g_{[ce]}, \\
R^a_{b\alpha c} = L^a_{b\alpha c} - L^a_{bc\alpha} - L^e_{b\alpha}L^a_{ec} + L^e_{bc}L^a_{e\alpha} - L^a_{be}g^e_{\alpha c} + \Gamma^{ae}_{b\alpha}g_{ec} + \delta^a_c g_{b\alpha} + \delta^a_b g_{c\alpha}, \\
R^{ac}_{b\alpha\beta} = L^{ac}_{b\alpha\beta} - \Gamma^{ac}_{b\beta\alpha} - L^e_{b\alpha}\Gamma^{ac}_{e\beta} + \Gamma^{ec}_{b\beta}L^a_{e\alpha} - L^a_{be}g^{ec}_{\alpha\beta} - \delta^c_b g^a_{\beta\alpha} + \Gamma^{ae}_{b\alpha}g^c_{e\beta}, \\
R^{ad}_{bc\alpha} = L^{ad}_{bc\alpha} - \Gamma^{ad}_{b\alpha c} - \delta^d_c L^a_{b\alpha} - L^e_{bc}\Gamma^{ad}_{e\alpha} + \Gamma^{ed}_{b\alpha}L^a_{ec} - \delta^d_b g^a_{\alpha c} - \delta^a_c g^d_{b\alpha} - \delta^a_b g^d_{c\alpha}, \\
R^{acd}_{b\alpha\beta} = \Gamma^a_{b\left[{cd \atop \alpha\beta}\right]} - \Gamma^e_{b\left[{c \atop \alpha}\right.}\Gamma^{ad}_{\left.e\beta\right]} + \delta^{\lceil c}_b g^{ad}_{\lfloor\alpha\beta]}, \\
R^\alpha_{\beta\gamma\mu} = L^\alpha_{\beta[\gamma\mu]} - L^\eta_{\beta[\gamma}L^\alpha_{\eta\mu]} - L^\alpha_{\beta a}g^a_{[\gamma\mu]} + \Gamma^{\alpha a}_{\beta[\gamma}g_{a\mu]}, \quad R^\alpha_{\beta ab} = L^\alpha_{\beta[ab]} - L^\gamma_{\beta[a}L^\alpha_{\gamma b]} - \delta^\alpha_\beta g_{[ab]}, \\
R^\alpha_{\beta\gamma a} = L^\alpha_{\beta\gamma a} - L^\alpha_{\beta a\gamma} - L^\mu_{\beta\gamma}L^\alpha_{\mu a} + L^\mu_{\beta a}L^\alpha_{\mu\gamma} - L^\alpha_{\beta b}g^b_{\gamma a} + \Gamma^{\alpha b}_{\beta\gamma}g_{ba} + \delta^\alpha_\beta g_{a\gamma},
\end{gathered}
$$

$$R^{\alpha a}_{\beta\gamma\mu} = L^{\alpha a}_{\beta\gamma\mu} - \Gamma^{\alpha a}_{\beta\mu\gamma} - L^{\eta}_{\beta\gamma}\Gamma^{\alpha a}_{\eta\mu} + \Gamma^{\eta a}_{\beta\mu}L^{\alpha}_{\eta\gamma} - L^{\alpha}_{\beta b}g^{ba}_{\gamma\mu} + \delta^{\alpha}_{\mu}g^{a}_{\beta\gamma} + \Gamma^{\alpha b}_{\beta\gamma}g^{a}_{b\mu},$$
$$R^{\alpha b}_{\beta a\gamma} = L^{\alpha b}_{\beta a\gamma} - \Gamma^{\alpha b}_{\beta\gamma a} - L^{\mu}_{\beta a}\Gamma^{\alpha b}_{\mu\gamma} + \Gamma^{\mu b}_{\beta\gamma}L^{\alpha}_{\mu a} - \delta^{b}_{a}L^{\alpha}_{\beta\gamma} + \delta^{\alpha}_{\gamma}g^{b}_{\beta a} - \delta^{\alpha}_{\beta}g^{b}_{a\gamma},$$
$$R^{\alpha ab}_{\beta\gamma\mu} = \Gamma^{\alpha}_{\beta}{}^{[ab}_{[\gamma\mu]} - \Gamma^{\eta}_{\beta}{}^{[a}_{[\gamma}\Gamma^{\alpha b]}_{\eta\mu]} - \delta^{\alpha}_{[\gamma}g^{[ab]}_{\beta\mu]},$$
$$R_{\alpha\beta\gamma} = \Gamma_{\alpha[\beta\gamma]} - L^{\mu}_{\alpha[\beta}\Gamma_{\mu\gamma]} - \Gamma_{\alpha a}g^{a}_{[\beta\gamma]} + g^{a}_{\alpha[\beta}g_{a\gamma]} + \Pi^{a}_{\alpha[\beta}g_{a\gamma]}, \quad R_{\alpha ab} = \Gamma_{\alpha[ab]} - L^{\beta}_{\alpha[a}\Gamma_{\beta b]} + g^{c}_{\alpha[a}g_{cb]},$$
$$R_{\alpha\beta a} = \Gamma_{\alpha\beta a} - \Gamma_{\alpha a\beta} - L^{\gamma}_{\alpha\beta}\Gamma_{\gamma a} + L^{\gamma}_{\alpha a}\Gamma_{\gamma\beta} - \Gamma_{\alpha b}g^{b}_{\beta a} + g^{b}_{\alpha\beta}g_{ba} - g^{b}_{\alpha a}g_{b\beta} + \Pi^{b}_{\alpha\beta}g_{ba},$$
$$K^{a}_{\alpha\beta\gamma} = \Gamma^{a}_{\alpha\beta\gamma} - \Pi^{a}_{\alpha\gamma\beta} - L^{\mu}_{\alpha\beta}\Pi^{a}_{\mu\gamma} + \Gamma^{\mu a}_{\alpha\gamma}\Gamma_{\mu\beta} - \Gamma_{\alpha b}g^{ba}_{\beta\gamma} + g^{b}_{\alpha\beta}g^{a}_{b\gamma} - g^{ba}_{\alpha\gamma}g_{b\beta} + \Pi^{b}_{\alpha\beta}g^{a}_{b\gamma},$$
$$R^{b}_{\alpha a\beta} = \Gamma^{b}_{\alpha a\beta} - \Pi^{b}_{\alpha\beta a} - L^{\gamma}_{\alpha a}\Pi^{b}_{\gamma\beta} + \Gamma^{\gamma b}_{\alpha\beta}\Gamma_{\gamma a} - \delta^{b}_{a}\Gamma_{\alpha\beta} + g^{c}_{\alpha a}g^{b}_{c\beta} - g^{cb}_{\alpha\beta}g_{ca},$$
$$K^{ab}_{\alpha\beta\gamma} = \Pi_{\alpha}{}^{[ab}_{[\beta\gamma]} - \Gamma^{\mu}_{\alpha}{}^{[a}_{[\beta}\Pi^{b]}_{\mu\gamma]} + g^{c}_{\alpha}{}^{[a}_{[\beta}g^{b]}_{c\gamma]}.$$

Theorem 9. *The curvature R is reduced to the object $R^{1,2} = \{R^{a}_{b\alpha\beta}, R^{a}_{bce}, R^{a}_{b\alpha c}, R^{ac}_{b\alpha\beta}, R^{ad}_{bc\alpha}, R^{acd}_{b\alpha\beta}, R^{\alpha}_{\beta\gamma\mu}, R^{\alpha}_{\beta a\ b}, R^{\alpha}_{\beta\gamma a}, R^{\alpha a}_{\beta\gamma\mu}, R^{ab}_{\beta a\gamma}, R^{\alpha a\ b}_{\beta\gamma\mu}, R_{\alpha\beta\gamma}, R_{\alpha a\ b}, R_{\alpha\beta a}, K^{a}_{\alpha\beta\gamma}, R^{b}_{\alpha a\beta}, K^{ab}_{\alpha\beta\gamma}\}$. The curvature object $R^{1,2}$ of the normalized space $\Pi^{1,2}$ is a tensor. The tensor $R^{1,2}$ contains two subtensors on the bundles of plane and normal linear frames, and also one curvature subtensor on the bundle of normal co-affine frames.*

Proof of Theorem 9. Indeed,

$$\Delta R^{a}_{b\alpha\beta} \equiv 0, \quad \Delta R^{a}_{bce} \equiv 0, \quad \Delta R^{a}_{b\alpha c} \equiv 0, \quad \Delta R^{ac}_{b\alpha\beta} \equiv 0, \quad \Delta R^{ad}_{bc\alpha} \equiv 0, \quad \Delta R^{acd}_{b\alpha\beta} \equiv 0,$$
$$\Delta R^{\alpha}_{\beta\gamma\mu} \equiv 0, \quad \Delta R^{\alpha}_{\beta ab} \equiv 0, \quad \Delta R^{\alpha}_{\beta\gamma a} \equiv 0, \quad \Delta R^{\alpha a}_{\beta\gamma\mu} \equiv 0, \quad \Delta R^{\alpha b}_{\beta a\gamma} \equiv 0, \quad \Delta R^{\alpha ab}_{\beta\gamma\mu} \equiv 0,$$
$$\Delta R_{\alpha\beta\gamma} + R^{\mu}_{\alpha\beta\gamma}\omega_{\mu} \equiv 0, \quad \Delta R_{\alpha ab} + R^{\beta}_{\alpha ab}\omega_{\beta} \equiv 0, \quad \Delta R_{\alpha\beta a} + R^{\gamma}_{\alpha\beta a}\omega_{\gamma} \equiv 0,$$
$$\Delta K^{a}_{\alpha\beta\gamma} + R^{\mu a}_{\alpha\beta\gamma}\omega_{\mu} \equiv 0, \quad \Delta R^{b}_{\alpha a\beta} + R^{\gamma b}_{\alpha a\beta}\omega_{\gamma} \equiv 0, \quad \Delta K^{ab}_{\alpha\beta\gamma} + R^{\mu ab}_{\alpha\beta\gamma}\omega_{\mu} \equiv 0.$$

These congruences conclude the proof. □

6.2. Torsion Objects

The following equations are a result of substituting the connection forms (23) into the structure equations of basis forms of the space Π.

$$D\omega^{\alpha} = \omega^{a} \wedge \omega^{\alpha}_{a} - \tilde{\omega}^{\alpha}_{\beta} \wedge \omega^{\beta} + S^{\alpha}_{\beta\gamma}\omega^{\beta} \wedge \omega^{\gamma} + S^{\alpha}_{\beta a}\omega^{\beta} \wedge \omega^{a} + S^{\alpha a}_{\beta\gamma}\omega^{\beta} \wedge \omega^{\gamma}_{a},$$
$$D\omega^{a} = \omega^{b} \wedge \tilde{\omega}^{a}_{b} + \omega^{\alpha} \wedge \tilde{\omega}^{a}_{\alpha} + S^{a}_{bc}\omega^{b} \wedge \omega^{c} + S^{a}_{b\alpha}\omega^{b} \wedge \omega^{\alpha} + S^{a}_{\alpha\beta}\omega^{\alpha} \wedge \omega^{\beta} + S^{ac}_{b\alpha}\omega^{b} \wedge \omega^{\alpha}_{c} + S^{ab}_{\alpha\beta}\omega^{\alpha} \wedge \omega^{\beta}_{b},$$
$$D\omega^{\alpha}_{a} = \tilde{\omega}^{b}_{a} \wedge \omega^{\alpha}_{b} - \tilde{\omega}^{\alpha}_{\beta} \wedge \omega^{\beta}_{a} + \tilde{\omega}_{a} \wedge \omega^{\alpha} + S^{\alpha}_{a\beta\gamma}\omega^{\beta} \wedge \omega^{\gamma} + S^{\alpha}_{ab\beta}\omega^{b} \wedge \omega^{\beta} + S^{\alpha b}_{a\beta\gamma}\omega^{\beta} \wedge \omega^{\gamma}_{b} + S^{\alpha b}_{a\beta c}\omega^{c} \wedge \omega^{\beta}_{b} + S^{\alpha bc}_{a\beta\gamma}\omega^{\beta}_{b} \wedge \omega^{\gamma}_{c},$$

where the object's S components are defined as follows:

$$S^{\alpha}_{\beta\gamma} = L^{\alpha}_{[\beta\gamma]}, \quad S^{\alpha}_{\beta a} = L^{\alpha}_{\beta a}, \quad S^{\alpha a}_{\beta\gamma} = \Gamma^{\alpha a}_{\beta\gamma}, \quad S^{a}_{bc} = L^{a}_{[bc]}, \quad S^{a}_{b\alpha} = L^{a}_{b\alpha} - L^{a}_{\alpha b}, \quad S^{a}_{\alpha\beta} = L^{a}_{[\alpha\beta]},$$
$$S^{ac}_{b\alpha} = \Gamma^{ac}_{b\alpha}, \quad S^{ab}_{\alpha\beta} = \Gamma^{ab}_{\alpha\beta}, \quad S^{\alpha}_{a\beta\gamma} = \delta^{\alpha}_{[\gamma}\Gamma_{a\beta]}, \quad S^{\alpha}_{ab\gamma} = \delta^{\alpha}_{\gamma}\Gamma_{ab}, \tag{30}$$
$$S^{\alpha b}_{a\beta\gamma} = \delta^{\alpha}_{\gamma}L^{b}_{a\beta} - \delta^{b}_{a}L^{\alpha}_{\gamma\beta} - \delta^{\alpha}_{\beta}\Pi^{b}_{a\gamma}, \quad S^{\alpha b}_{a\beta c} = \delta^{\alpha}_{\beta}L^{b}_{ac} - \delta^{b}_{a}L^{\alpha}_{\beta c}, \quad S^{\alpha bc}_{a\beta\gamma} = -\delta^{\alpha}_{[\beta}\Gamma^{[bc]}_{a\gamma]} + \delta^{[b}_{a}\Gamma^{\alpha c]}_{[\beta\gamma]}.$$

The right-hand sides of equalities (30) contain only components of the subobject Γ_1, and, therefore, let object S be a torsion object of linear subconnection Γ_1 of the group connection Γ in the space Π of centered planes P^{0}_{m}.

Taking into account congruences (24) for the components of subobject Γ_1, the congruences modulo the basis forms

$$
\begin{gathered}
\Delta S^{\alpha}_{\beta\gamma} + S^{\alpha a}_{[\beta\gamma]}\omega_a - S^{\alpha}_{[\beta a}\omega^a_{\gamma]} \equiv 0, \quad \Delta S^{\alpha}_{\beta a} - \delta^{\alpha}_{\beta}\omega_a \equiv 0, \quad \Delta S^{\alpha a}_{\beta\gamma} - \delta^{\alpha}_{\gamma}\omega^a_{\beta} \equiv 0, \quad \Delta S^{a}_{bc} \equiv 0, \\
\Delta S^{a}_{b\alpha} - 2S^{a}_{bc}\omega^c_{\alpha} + S^{ac}_{b\alpha}\omega_c - S^{\beta}_{\alpha b}\omega^a_{\beta} \equiv 0, \quad \Delta S^{a}_{\alpha\beta} + S^{a}_{b[\alpha}\omega^b_{\beta]} + S^{\gamma}_{\alpha\beta}\omega^a_{\gamma} + S^{ab}_{[\alpha\beta]}\omega_b \equiv 0, \\
\Delta S^{ac}_{b\alpha} + \delta^{c}_{b}\omega^a_{\alpha} \equiv 0, \quad \Delta S^{ab}_{\alpha\beta} - S^{ab}_{c\beta}\omega^c_{\alpha} + S^{\gamma b}_{\alpha\beta}\omega^a_{\gamma} \equiv 0, \quad \Delta S^{\alpha}_{a\beta\gamma} + S^{\alpha}_{ab[\beta}\omega^b_{\gamma]} + S^{\alpha b}_{a[\beta\gamma]}\omega_b - S^{\alpha}_{\beta\gamma}\omega_a \equiv 0, \\
\Delta S^{\alpha}_{ab\beta} + S^{\alpha c}_{a\beta b}\omega_c + S^{\alpha}_{\beta b}\omega_a \equiv 0, \quad \Delta S^{\alpha b}_{a\beta c} - \delta^{b}_{c}\delta^{\alpha}_{\beta}\omega_a \equiv 0, \quad \Delta S^{\alpha bc}_{a\beta\gamma} \equiv 0, \\
\Delta S^{\alpha b}_{a\beta\gamma} - S^{\alpha b}_{a\gamma c}\omega^c_{\beta} - 2\delta^{\alpha}_{[\beta}S^{[cb}_{a\gamma}]\omega_c - \delta^{b}_{a}S^{\alpha c}_{\gamma\beta}\omega_c \equiv 0
\end{gathered} \tag{31}
$$

are obtained. Now, it is evident that the following theorem holds.

Theorem 10. *The torsion object S of the subconnection Γ_1 is a quasi-tensor containing the tensors S^{a}_{bc}, $S^{\alpha bc}_{a\beta\gamma}$ and the quasi-tensors $S^{\alpha}_{\beta a}$, $S^{\alpha a}_{\beta\gamma}$, $S^{ac}_{b\alpha}$, $S^{\alpha a}_{b\beta c}$, $\{S^{\alpha}_{\beta a}, S^{\alpha a}_{\beta\gamma}, S^{\alpha}_{\beta\gamma}\}$, $\{S^{a}_{bc}, S^{ac}_{b\alpha}, S^{\beta}_{\alpha b}, S^{a}_{b\alpha}\}$, $\{S^{ab}_{c\beta}, S^{\gamma b}_{\alpha\beta}, S^{ab}_{\alpha\beta}\}$, $\{S^{\alpha c}_{a\beta b}, S^{\alpha}_{\beta b}, S^{\alpha}_{ab\beta}\}$, $\{S^{\alpha b}_{a\gamma c}, S^{cb}_{a\gamma}, S^{\alpha c}_{\gamma\beta}, S^{\alpha b}_{a\beta\gamma}\}$.*

Remark 7. *Because of Theorem 10, the connection Γ_1 is always with torsion (see [32], cf. [33]) as the torsion object S of connection Γ_1 is a quasi-tensor.*

It remains to consider the dynamics of changes of the torsion quasi-tensor S at consecutive canonizations.

6.2.1. The First Canonization

We put the vertices A_α on the first normal N_{n-m}; then, conditions (8) can be written as

$$
\omega^a_\alpha \equiv 0 \quad (mod \quad \omega^\alpha, \, \omega^\alpha_a, \, \omega^a). \tag{32}
$$

Using congruences (32), the differential congruences (31) will be written in the form:

$$
\begin{gathered}
\Delta S^{\alpha}_{\beta\gamma} + S^{\alpha a}_{[\beta\gamma]}\omega_a \equiv 0, \quad \Delta S^{\alpha}_{\beta a} - \delta^{\alpha}_{\beta}\omega_a \equiv 0, \quad \Delta S^{\alpha a}_{\beta\gamma} \equiv 0, \quad \Delta S^{a}_{bc} \equiv 0, \\
\Delta S^{a}_{b\alpha} + S^{ac}_{b\alpha}\omega_c \equiv 0, \quad \Delta S^{a}_{\alpha\beta} + S^{ab}_{[\alpha\beta]}\omega_b \equiv 0, \quad \Delta S^{ac}_{b\alpha} \equiv 0, \quad \Delta S^{ab}_{\alpha\beta} \equiv 0, \\
\Delta S^{\alpha}_{a\beta\gamma} + S^{\alpha b}_{a[\beta\gamma]}\omega_b - S^{\alpha}_{\beta\gamma}\omega_a \equiv 0, \quad \Delta S^{\alpha}_{ab\beta} + S^{\alpha c}_{a\beta b}\omega_c + S^{\alpha}_{\beta b}\omega_a \equiv 0, \\
\Delta S^{\alpha b}_{a\beta c} - \delta^{b}_{c}\delta^{\alpha}_{\beta}\omega_a \equiv 0, \quad \Delta S^{\alpha bc}_{a\beta\gamma} \equiv 0, \quad \Delta S^{\alpha b}_{a\beta\gamma} - 2\delta^{\alpha}_{[\beta}S^{[cb}_{a\gamma}]\omega_c - \delta^{b}_{a}S^{\alpha c}_{\gamma\beta}\omega_c \equiv 0.
\end{gathered} \tag{33}
$$

Theorem 11. *Suppose that the components of the torsion quasi-tensor S satisfy congruences (31); then, S is reduced to the quasi-tensor S^1 with congruences (33) at the adaptation of the moving frame to a field of the first normals N_{n-m}. The quasi-tensor S^1 contains the tensors $S^{\alpha a}_{\beta\gamma}$, S^{a}_{bc}, $S^{ac}_{b\alpha}$, $S^{ab}_{\alpha\beta}$, $S^{\alpha bc}_{a\beta\gamma}$, $\{S^{\alpha a}_{\beta\gamma}, S^{\alpha}_{\beta\gamma}\}$, $\{S^{ab}_{\alpha\beta}, S^{a}_{\alpha\beta}\}$, $\{S^{ac}_{b\alpha}, S^{a}_{b\alpha}\}$, $\{S^{cb}_{a\gamma}, S^{\alpha c}_{\gamma\beta}, S^{\alpha b}_{a\beta\gamma}\}$ and the quasi-tensors $S^{\alpha}_{\beta a}$, $S^{\alpha b}_{a\beta c}$, $\{S^{\alpha c}_{a\gamma b}, S^{\alpha}_{\gamma b}, S^{\alpha}_{ab\gamma}\}$.*

6.2.2. The Second Canonization

Without using the previous canonization, we put the vertices A_a on the second normal N_{m-1}, then conditions (8) can be written as

$$
\omega_a \equiv 0 \quad (mod \quad \omega^\alpha, \, \omega^\alpha_a, \, \omega^a). \tag{34}
$$

Taking into account condition (34), the differential congruences for the components of the torsion object S have the form

$$
\begin{gathered}
\Delta S^{\alpha}_{\beta\gamma} - S^{\alpha}_{[\beta a}\omega^{a}_{\gamma]} \equiv 0, \quad \Delta S^{\alpha}_{\beta a} \equiv 0, \quad \Delta S^{\alpha a}_{\beta\gamma} - \delta^{\alpha}_{\gamma}\omega^{a}_{\beta} \equiv 0, \quad \Delta S^{a}_{bc} \equiv 0, \\
\Delta S^{a}_{b\alpha} - 2S^{a}_{bc}\omega^{c}_{\alpha} - S^{\beta}_{\alpha b}\omega^{a}_{\beta} \equiv 0, \quad \Delta S^{a}_{\alpha\beta} + S^{a}_{b[\alpha}\omega^{b}_{\beta]} + S^{\gamma}_{\alpha\beta}\omega^{a}_{\gamma} \equiv 0, \\
\Delta S^{ac}_{b\alpha} + \delta^{c}_{b}\omega^{a}_{\alpha} \equiv 0, \quad \Delta S^{ab}_{\alpha\beta} - S^{ab}_{c\beta}\omega^{c}_{\alpha} + S^{\gamma b}_{\alpha\beta}\omega^{a}_{\gamma} \equiv 0, \quad \Delta S^{\alpha}_{a\beta\gamma} + S^{\alpha}_{ab[\beta}\omega^{b}_{\gamma]} \equiv 0, \\
\Delta S^{\alpha}_{ab\beta} \equiv 0, \quad \Delta S^{\alpha b}_{a\beta\gamma} - S^{\alpha b}_{a\gamma c}\omega^{c}_{\beta} \equiv 0, \quad \Delta S^{\alpha b}_{a\beta c} \equiv 0, \quad \Delta S^{\alpha bc}_{a\beta\gamma} \equiv 0.
\end{gathered}
\tag{35}
$$

Thus, Theorem 12 follows from these congruences.

Theorem 12. *Suppose that the components of the torsion quasi-tensor S satisfy congruences (31); then S is reduced to the quasi-tensor S^2 with congruences (35) at the adaptation of the moving frame to a field of the second normals N_{m-1}. The quasi-tensor S^2 contains the tensors $S^{\alpha}_{\beta a}$, S^{a}_{bc}, $S^{\alpha bc}_{a\beta\gamma}$, $S^{\alpha b}_{a\beta c}$, $S^{\alpha}_{ab\gamma}$, $\{S^{\alpha}_{\beta a}, S^{\alpha}_{\beta\gamma}\}$, $\{S^{a}_{bc}, S^{\beta}_{\alpha b}, S^{a}_{b\alpha}\}$, $\{S^{\alpha b}_{a\gamma c}, S^{\alpha b}_{a\beta\gamma}\}$, $\{S^{\alpha}_{ab\beta}, S^{\alpha}_{a\beta\gamma}\}$ and the quasi-tensors $S^{\alpha a}_{\beta\gamma}$, $S^{ac}_{b\alpha}$, $\{S^{ab}_{c\beta}, S^{\gamma b}_{\alpha\beta}, S^{ab}_{\alpha\beta}\}$.*

6.2.3. The Full Canonization

After all, if $A_\alpha \in P_{n-m}$ and $A_a \in P_{m-1}$, then conditions (32) and (34) hold and the differential congruences of the torsion object components have the form

$$
\begin{gathered}
\Delta S^{\alpha}_{\beta\gamma} \equiv 0, \quad \Delta S^{\alpha}_{\beta a} \equiv 0, \quad \Delta S^{\alpha a}_{\beta\gamma} \equiv 0, \quad \Delta S^{a}_{bc} \equiv 0, \quad \Delta S^{a}_{b\alpha} \equiv 0, \quad \Delta S^{a}_{\alpha\beta} \equiv 0, \quad \Delta S^{ac}_{b\alpha} \equiv 0, \\
\Delta S^{ab}_{\alpha\beta} \equiv 0, \quad \Delta S^{\alpha}_{a\beta\gamma} \equiv 0, \quad \Delta S^{\alpha}_{ab\beta} \equiv 0, \quad \Delta S^{\alpha b}_{a\beta\gamma} \equiv 0, \quad \Delta S^{\alpha b}_{a\beta c} \equiv 0, \quad \Delta S^{\alpha bc}_{a\beta\gamma} \equiv 0.
\end{gathered}
$$

Theorem 13. *At the adaptation of the moving frame to the normalization of the space Π, the torsion quasi-tensor S becomes a tensor; furthermore, all its components are the one-component tensors (cf. in Theorems 11 and 12, the quasi-tensors S^1 and S^2 contain also multi-component tensors and quasi-tensors).*

Remark 8. *It is true that, at the considered canonizations, the number of components of the torsion object S does not change, in contrast to the reduced objects of curvature and connection, yet the fact remains that differential congruences of its components differ from congruences (31). This testifies to possible change of properties of torsion for semi-normalized spaces Π^1, Π^2 and also for the normalized space $\Pi^{1,2}$.*

Funding: This research received no external funding.

Conflicts of Interest: The author declares no conflict of interest.

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