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Complex Characteristics and Control of Output Game in Cross-Border Supply Chains: A Perspective of Inter-Chain Competition

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Abstract: In this paper, an output dynamic game model of intertwined supply chains operating in two different countries is established. The Nash equilibrium point of the model and its stable region are obtained using nonlinear dynamic principles. The complex properties of the system, such as stability, period-doubling bifurcations, and chaos, are investigated using numerical simulations. Our results suggest that the level of output and the system's profits undergo bifurcation and chaos with an increase in the output adjustment speed. An interesting phenomenon occurs in that higher tariffs lead to the expansion of the stable range of the supply chain in the product-exporting country. The chaotic behavior of the system is sensitive to the value of the initial level of output. In supply chain competition, each supply chain firm should make suitable adjustments to the speed of output. To maintain the stability of domestic markets, excessive tariffs should be avoided. It is essential that each supply chain firm evaluates the potential impacts of different initial output values when making initial decisions. Using the method of delayed feedback control, the chaotic behavior of the system can effectively be controlled. These findings offer valuable and novel insight into inter-chain competition in supply chain networks.

Keywords: output game; inter-chain competition; chaos control; dynamic game; complexity analysis

MSC: 91A25

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1. Introduction

As competition among supply chain firms continues to intensify, supply chain competition has become an important focus of research in the field of management [1–3]. From the perspective of supply chain networks, companies not only face competition from other companies in a vertical chain but must also take into account horizontal competition with other supply chains at the same level. Meanwhile, in operational practice, competition among enterprises is gradually evolving into inter-chain competition with the goal of achieving win–win outcomes [4]. Therefore, the study of inter-supply chain competition can not only enrich and develop the theoretical literature on supply chain system management but also has important significance for the practical guidance of supply chain management. Currently, with the advancements in information technology and economics, globalization has led to a new era of international competition. Supply chains now expand across many countries and are intricately intertwined [5]. Against this backdrop, one firm can participate in multiple supply chains, which leads to increased uncertainty and complexity in inter-chain competition [6]. Any issues that arise in one supply chain may potentially trigger a chain reaction affecting the entire supply chain system. Therefore, studying dynamic changes, game relationships, and decision strategies in global supply chains has become a research hotspot in academia [7,8]. However, the complex dynamics of interactions, especially in competition within cross-border supply chains, has not been clearly explained in previous research.

Researchers have extensively developed and examined the concept of supply chain competition. Initially, scholars concentrated on the phenomenon of competition within a single supply chain with a single channel, including evolutionary games and strategic decisions among supply chain participants [1,2]; supply chain decisions considering supply uncertainty, demand uncertainty, random yield, and consumer preferences [3,9–12]; and supply chain coordination under various types of contracts [13–15]. Subsequently, dual-channel supply chains, consisting of both a direct online channel and a traditional retail channel, have been discussed from different perspectives by researchers. The existing research on dual-channel supply chains can be divided into two streams: the first stream in the literature focuses on the price and coordination choices made in dual-channel supply chains [16–19], and the second stream of research focuses on contract selection in dual-channel environments [20,21].

These studies mainly focus on competition among several supply chain members. Considering the characteristics of multiple players in a real-world supply chain, Nagurney et al. [22,23] introduced a supply chain network competition equilibrium model, developing a finite-dimensional variational inequality formulation that includes many decision makers and numerous supply chains. A unified variational inequality framework was also constructed by Nagurney et al. [24] for the global spatial pricing network equilibrium model under tariff quota constraints. On the basis of this model, Yang et al. [25] and Zhou et al. [26] investigated competitive equilibrium issues in closed-loop supply chain networks under various carbon tax policies and in global supply chain networks under various trade policies, respectively. Xiao et al. [27] created a supply chain network equilibrium model with capacity restrictions and demand unpredictability.

The above literature is limited to the analysis of competition among different members (i.e., firms) at various tiers of a supply chain and does not consider competition among supply chains. However, a firm's competitive advantage largely depends on its supply chain's competitive advantage. Competition among firms has gradually evolved into inter-chain competition [4]. Recently, studies on inter-chain competition have concentrated on the analysis of the structure of channels (integrated or decentralized) between two parallel supply chains with exclusive suppliers [28–30], and contract options between two dual-channel supply chains [31]. Furthermore, Lou et al. [7] developed a model of two parallel supply chains and analyzed the complex dynamics of the model using numerical simulations. The results show that both supply chain systems will enter a chaotic state when the adjustment speed of the price exceeds the domain of attraction. Ma et al. [8] constructed dynamic decision-making models for low-carbon apparel supply chains and explored the models' complex dynamical phenomena of chaos.

There are some real-world supply chain competition scenarios that can be accurately described by inter-chain competition in parallel supply chains. However, as the global division of labor continues to be refined, supply chains increasingly exhibit the following traits: a growing number of participants, multiple tiers, and firms that are able to participate in multiple supply chains. On this basis, Zhang et al. [32] developed a supply chain competition model with intertwined structures in which numerous products compete in multiple marketplaces from the perspective of inter-chain competition. Feng et al. [4] built an output competition model among global supply chains and examined how trade policies affect the equilibrium of global supply chain networks. The above two studies both regard inter-chain competition as a static process: competition is perceived as a single-stage process or a one-shot game. It is clear that this does not correspond to supply chain competition in reality. In a real-world scenario, the output decisions made by each supply chain are modified over time. For instance, the global smartphone shipments of electronic manufacturers, such as Apple and Samsung, are dynamically adjusted every quarter. Global supply chain shipments of the iPhone, in particular, reached 70 million units in Q4 2022 before declining to 58 million units in Q1 2023 [33]. Therefore, the inter-chain output game is a multi-stage dynamic game process, rather than a single-stage static game.

In this work, we focused on the following research questions: (1) How can we develop a model that describes the complex dynamics of competition among cross-border supply chains? (2) With the provided model, how can we obtain the equilibrium solution of the problem? (3) How do the parameters of the model affect the dynamic evolution of the output, and how can we control effects that are already occurring? Firstly, we establish a discrete dynamical model of inter-chain competition. Secondly, using nonlinear dynamic principles, we obtain the model's Nash equilibrium solution and its stable region. Thirdly, the influence of the parameters is analyzed using numerical simulations. To control chaos, we apply the delayed feedback control method. This study offers insight into the output decisions made by supply chain firms and serves as a reference for government organizations conducting policy analysis.

The paper is organized as follows: the model is provided in Section 2, and Section 3 discusses the model analytically. In Section 4, we present bifurcation diagrams, the maximum Lyapunov exponent, and time series diagrams using numerical simulations. Also, we control the chaotic behavior of the model. Lastly, Section 5 is the conclusion of the paper.

2. Model Description and Formulation

2.1. Model Description

We adopt the concept of the product–market chain (PM chain), as proposed by Feng et al. [4] to describe the inter-chain competition. A PM chain comprises the firms and activities needed for a product's procurement, production, transportation, and marketing, ultimately delivering the final product to a market. We focus on a three-tier PM chain consisting of raw material suppliers, upstream manufacturers, downstream manufacturers, and demand markets.

To study the dynamic evolution of inter-chain competition in cross-border supply chains, we consider a scenario in which there are three representative PM chains between two countries of product $P*$ (country 1 and country 2), as shown in Figure 1. Among these three PM chains, two represent nonmultinational PM chains within each country, and one represents a multinational supply chain symbolizing the trade relationship between the two countries. Without a loss of generality, these three supply chains succinctly describe the interactive relationships among cross-border supply chains.

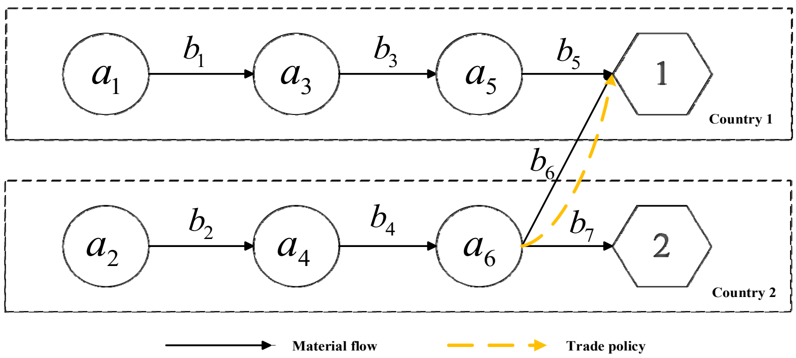


Figure 1. Cross-border supply chain of countries 1 and 2 for product $P*$.

The three-tier PM chain $s_1 = \{a_1, a_3, a_5; b_1, b_3, b_5; 1\}$ is shown in Figure 1, where a_1, a_3, and a_5 are the suppliers of raw materials, upstream manufacturers, and downstream manufacturers, respectively; b_1, b_3, and b_5 denote the links where the flow of material occurs; and 1 denotes that the target market of the PM chain is country 1.

Let $S = \{s_1, s_2, s_3\}$ represent the set of all PM chains as shown in Figure 1, where $s_2 = \{a_2, a_4, a_6; b_2, b_4, b_7; 2\}$ and $s_3 = \{a_2, a_4, a_6; b_2, b_4, b_6; 1\}$. Let $G = [A, B, J]$ represent the cross-border supply chain as a whole, where A, B, and J denote the set of all firms (circles), all links (black arrow lines), and all consumer markets (hexagons), respectively. Thus, $A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$, $B = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$, and $J = \{1, 2\}$.

We can conclude that every PM chain is directed toward a certain market, and the product cannot be completed if any firm or link is removed from the PM chain. Thus, the consideration of one PM chain as a decision-making unit in a supply chain game is reasonable.

To increase the number of local jobs and to support local industries, countries usually implement trade policies related to imported and exported products. Therefore, subsidies and tariffs are imposed on the cross-border links, which are represented as yellow, dashed lines in Figure 1. $s_3 = \{a_2, a_4, a_6; b_2, b_4, b_6; 1\}$ is a cross-border PM chain, where b_6 is a cross-border link.

Each firm faces production costs. At the same time, transaction costs occur at the links with positive material flow, including distribution and transportation costs [4]. In addition to transaction costs, trade costs occur at the cross-border links, which include subsidies and tariff costs.

2.2. Model Formulation

The main symbols used in the establishment of the discrete dynamical model for cross-border supply chains are provided in Table 1.

In a cross-border supply chain, if two PM chains are oriented toward the same market or contain the same firms, they will directly compete in that market or share the resources offered by those firms. That is, there exists a competitive relationship between two PM chains when $s_i \cap s_{i-} \neq \varnothing$. Taking Figure 1 as an example, we can see $s_1 \cap s_3 = \{1\}$, which indicates that PM chains s_1 and s_3 compete directly in country 1, and there is a direct competitive relationship between PM Chains s_1 and s_3. However, $s_1 \cap s_2 = \varnothing$, which indicates that there is no direct competition between PM chains s_1 and s_2.

Thus, the inter-chain relationships in Figure 1 result in direct competition among the three PM chains. The competitive relationships among the three players are depicted in Figure 2, where a PM chain represents a game player.

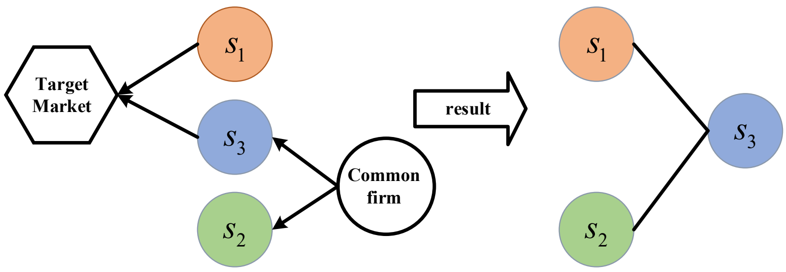


Figure 2. The competitive relationship between the cross-border supply chains.

Three PM chains compete in a Nash–Cournot pattern to meet the consumer demand in the market. They produce product $P*$, and their strategy set involves selecting the optimal output level, with decisions occurring within a discrete period of time $T = 1, 2, \ldots$. Let X denote the set of output quantities for $P*$, where $X = \{x_{s_1}, x_{s_2}, x_{s_3}\}$. We assume that $P*$ is sold at the market-clearing price. Then, the market's demand equals the output quantity of $P*$ by PM chains oriented to the market. When the output quantity of s_i is $x_{s_i} (x_{s_i} \geq 0)$, the total output quantity of the final product delivered to market $j (j \in J)$ is $\sum\limits_{s_i \in S_j} x_{s_i}$, where S_j represents the set of all PM chains oriented to market J. Thus, in Figure 1, the demand of the market 1 (i.e., country 1) is $x_{s_1} + x_{s_3}$ and that of market 2 (country 2) is x_{s_2}.

We assume that the sensitivity of the price to demand equals 1. The inverse demand functions for the three PM chains are defined as follows:

$$P_{s_1} = u_1 - x_{s_1} - \beta x_{s_3} \tag{1}$$

$$P_{s_2} = u_2 - x_{s_2} \tag{2}$$

$$P_{s_3} = u_1 - \beta x_{s_1} - x_{s_3} \tag{3}$$

where $u_j > 0 (j \in J)$ is interpreted as the maximum price of the final products $P*$, for market j; $\beta (0 < \beta < 1)$ represents the substitutability of the final products produced by two countries.

Table 1. Key notations.

Notations	Meaning
$P*$	The final product
j	The end market in Figure 1, $j \in \{1, 2\}$
s_i	The PM chain in Figure 1, $i \in \{1, 2, 3\}$
a_n	The firm on the PM chain, $n \in \{1, 2, \ldots, 6\}$
b_m	The link on the PM chain, $m \in \{1, 2, 3, \ldots, 7\}$
Sets	
S	The set of all PM chains in Figure 1
A	The set of all firms in Figure 1
B	The set of all links in Figure 1
J	The set of all end markets in Figure 1
As_i	The set of all firms on PM chain s_i
Bs_i	The set of all links on PM chain s_i
Functions	
P_{s_i}	The price of $P*$ for PM chain s_i
$f_{a_n s_i}$	The output quantity produced by firm a_n for PM chain s_i
f_{a_n}	The total output quantity of firm a_n
C_{a_n}	The total production cost of firm a_n
$l_{b_m s_i}$	The material flow on link b_m of chain s_i
l_{b_m}	The total material flow on link b_m
$C^1_{b_m}$	The transaction cost on link b_m
$C^2_{b_m}$	The trade cost on link b_m
C_{b_m}	The total cost on link b_m
C_{s_i}	The total cost of PM chain s_i
π_{s_i}	The profits of the PM chain s_i
$\Phi_{s_i,T}$	The marginal profit of the PM chain s_i in T period
Decision variables	
$x_{s_i,T+1}$	The output quantity of $P*$ on PM chain s_i in T period
Parameters	
ω_{s_i}	The speed of production adjustment for PM chain s_i
u_j	The maximum price of $P*$ in market j
β	The substitution rate between $P*$ produced in the two countries
ρ_{a_n}	The output quantity produced by firm a_n to make one unit of $P*$
$\delta_{a_n s_i}$	The binary variable (indicating whether a_n is involved in PM chain s_i)
r_{b_m}	The marginal transaction cost of link b_m
$\sigma_{a_n b_m}$	The binary variable (indicating whether b_m is involved in PM chain s_i)
r_{a_n}	The marginal cost of production of firm an
h_{b_m}	The value of the subsidy imposed on link b_m
t_{b_m}	Unit tariff imposed on link b_m
$\eta_{\mathrm{b_m}}$	The binary variable (indicating whether trade policy acts on the link b_m)
k	Controlling factor

In the supply chain for product $P*$, firm a_n is responsible for providing the corresponding intermediate material. Let ρ_{a_n} represent the quantity of intermediate material provided by firm a_n for 1 unit of final product $P*$. For example, a screw producer, firm a_1, participates in a PM chain for an automobile tire. Four screws are needed in one tire ($\rho_{a_1} = 4$), and firm a_1 is required to deliver $f_{a_1 s_1} = 40$ screws for the PM chain when the output quantity of the tire is $x_{s_1} = 10$. Thus, the output quantity of each firm (a_n) in PM chain s_i can be represented as $f_{a_n s_i}$.

$$f_{a_n s_i} = \delta_{a_n s_i} \rho_{a_n} x_{s_i}, n = 1, 2, \ldots, 6; i = 1, 2, 3 \tag{4}$$

Here, $\delta_{a_n s_i}$ is a binary variable. If the firm a_n participates in PM chain s_i, then $\delta_{a_n s_i} = 1$, otherwise $\delta_{a_n s_i} = 0$.

Firm a_n may participate in various PM chains. Thus, the total output quantity of firm a_n equals the sum of its own output quantities for all PM chains in which it participates.

$$f_{a_n} = \sum_{s_i \in S} \delta_{a_n s_i} \rho_{a_n} x_{s_i}, n = 1, 2, 3, \ldots, 6; i = 1, 2, 3 \tag{5}$$

In a complex supply chain system, a firm's increase in output quantity per unit tends to result in a nonlinear increase in production costs. In addition, there may be diseconomies of scale phenomena that occur when the scale of each firm a_n expands to a certain extent. Thus, it is appropriate to adopt the quadratic cost function as the production cost function [34]. We can determine the production cost function of firm a_n in the PM chain using the following:

$$C_{a_n} = r_{a_n} {f_{a_n}}^2, n = 1, 2, 3, \ldots, 6 \tag{6}$$

where $r_{a_n} > 0$ represents the marginal cost of production of firm a_n.

The output quantity of the front-end firm a_n of link b_m determines the material flow of the link. However, the output quantity of firm a_n is in turn determined by the output quantity of the PM chain to which the firm belongs. Therefore, the material flow of each link can be determined by tracing back the output quantity of $P*$. For example, the output quantity of $P*$ for PM chain s_1, which is x_{s_1}, determines the output of firm a_5 as x_{s_1} and the output of firm a_3 as $\rho_{a_3} x_{s_1}$. Then, the front-end firm a_5 for link b_5 determines the material flow for link b_5 as x_{s_1}; the front-end firm a_3 determines the material flow for link b_3 as $\rho_{a_3} x_{s_1}$. To ascertain the relationships between firms and links, we use the binary variable $\sigma_{a_n b_m} = 1$ to indicate that firm a_n is the front-end firm of link b_m and $\sigma_{a_n b_m} = 0$ to indicate that firm a_n is not the front-end firm of link b_m. Then, the material flow of link b_m in PM chain s_i, $l_{b_m s_i}$, can be expressed as follows:

$$l_{b_m s_i} = \sigma_{a_n b_m} f_{a_n s_i}, n = 1, 2, 3, \ldots, 6; m = 1, 2, 3, \ldots, 7; i = 1, 2, 3 \tag{7}$$

The total material flow of each link (b_m) may be diverted and distributed to multiple end markets of downstream manufacturers. Therefore, the overall flow of material at each link (l_{b_m}) can be denoted as follows:

$$l_{b_m} = \sum_{s_i \in S} \sigma_{a_n b_m} f_{a_n s_i}, n = 1, 2, 3, \ldots, 6; m = 1, 2, 3, \ldots, 7; i = 1, 2, 3 \tag{8}$$

Transaction costs occur at the links with a positive material flow. There may be diseconomies of scale when the transaction scale expands to a certain extent. Therefore, the transaction cost function $C^1_{b_m}$ of link b_m can be assumed to be quadratic in form.

$$C^1_{b_m} = r_{b_m} {l_{b_m}}^2, m = 1, 2, 3, \ldots, 7 \tag{9}$$

Here, $r_{b_m} > 0$ is the marginal transaction cost for link b_m.

Under the subsidy regime, we assume that country 2 imposes a direct subsidy, h_{b_m}, on the final product exported to country 1. For instance, in Beijing, China, there is a program to encourage the export of electromechanical products and enhance the international competitiveness of related enterprises. It involves the provision of uncompensated financial assistance, not exceeding RMB 500,000, for the export of electromechanical products. Similarly, the European Union's agricultural subsidy policy directly subsidizes agricultural exports within the EU region [35]. Under the tariff regime, a unit tariff of t_{b_m} occurs on the final product imported from country 2. For the cross-border links, we assume that the tariff cost is proportional to the material flow of the links [4]. Thus, the trade cost $C_{b_m}^2$ on link b_m can be assumed as follows:

$$C_{b_m}^2 = \eta_{\mathrm{b_m}}(t_{b_m} l_{b_m} - h_{b_m}), m = 1, 2, 3, \ldots, 7 \tag{10}$$

where $\eta_{\mathrm{b_m}}$ is a binary variable indicating whether a trade policy impacts the link. If yes, $\eta_{\mathrm{b_m}} = 1$; otherwise, $\eta_{\mathrm{b_m}} = 0$.

On the basis of the above analysis, the total cost of link b_m can be described as follows:

$$C_{b_m} = C_{b_m}^1 + C_{b_m}^2, m = 1, 2, 3, \ldots, 7 \tag{11}$$

The total cost of PM chain s_i can be defined as follows:

$$C_{s_i} = \sum_{a_n \in As_i} C_{a_n} + \sum_{\mathrm{b}_m \in Bs_i} C_{b_m}, n = 1, 2, 3, \ldots, 6; m = 1, 2, 3, \ldots, 7; i = 1, 2, 3 \tag{12}$$

where As_i is the set of all firms that participate in PM chain s_i, and Bs_i is the set of all links participating in PM chain s_i.

According to (12), the total cost of the three PM chains (s_1, s_2, and s_3) can be given, respectively, as follows:

$$\begin{aligned} C_{s_1} &= C_{a_1} + C_{a_3} + C_{a_5} + C_{b_1} + C_{b_3} + C_{b_5} \\ &= [\rho_{a_1}{}^2(r_{a_1} + r_{b_1}) + \rho_{a_3}{}^2(r_{a_3} + r_{b_3}) + \rho_{a_5}{}^2(r_{a_5} + r_{b_5})]x_{s_1}{}^2 \end{aligned} \tag{13}$$

$$\begin{aligned} C_{s_2} &= C_{a_2} + C_{a_4} + C_{a_6} + C_{b_2} + C_{b_4} + C_{b_7} \\ &= [\rho_{a_2}{}^2(r_{a_2} + r_{b_2}) + \rho_{a_4}{}^2(r_{a_4} + r_{b_4}) + \rho_{a_6}{}^2 r_{a_6}](x_{s_2} + x_{s_3})^2 + \rho_{a_6}{}^2 r_{b_7} x_{s_2}{}^2 \end{aligned} \tag{14}$$

$$\begin{aligned} C_{s_3} &= C_{a_2} + C_{a_4} + C_{a_6} + C_{b_2} + C_{b_4} + C_{b_6} \\ &= [\rho_{a_2}{}^2(r_{a_2} + r_{b_2}) + \rho_{a_4}{}^2(r_{a_4} + r_{b_4}) + \rho_{a_6}{}^2 r_{a_6}](x_{s_2} + x_{s_3})^2 + \rho_{a_6}{}^2 r_{b_6} x_{s_3}{}^2 + t_{b_{a_6}} x_{s_3} - h_{b_6} \end{aligned} \tag{15}$$

Accordingly, the profits of the three PM chains (s_1, s_2, and s_3) can be given as follows:

$$\pi_{s_1} = P_{s_1} x_{s_1} - C_{s_1} = (u_1 - x_{s_1} - \beta x_{s_3}) x_{s_1} - \gamma_{s_1} x_{s_1}{}^2 \tag{16}$$

$$\pi_{s_2} = P_{s_2} x_{s_2} - C_{s_2} = (u_2 - x_{s_2}) x_{s_2} - \gamma_{s_{246}} (x_{s_2} + x_{s_3})^2 - \rho_{a_6}{}^2 r_{b_7} x_{s_2}{}^2 \tag{17}$$

$$\pi_{s_3} = P_{s_3} x_{s_3} - C_{s_3} = (u_1 - \beta x_{s_1} - x_{s_3}) x_{s_3} - \gamma_{s_{246}} (x_{s_2} + x_{s_3})^2 - \rho_{a_6}{}^2 r_{b_6} x_{s_3}{}^2 - t_{b_{a_6}} x_{s_3} + h_{b_6} \tag{18}$$

where

$$\gamma_{s_1} = \rho_{a_1}{}^2(r_{a_1} + r_{b_1}) + \rho_{a_3}{}^2(r_{a_3} + r_{b_3}) + \rho_{a_5}{}^2(r_{a_5} + r_{b_5}),$$

$$\gamma_{s_{246}} = \rho_{a_2}{}^2(r_{a_2} + r_{b_2}) + \rho_{a_4}{}^2(r_{a_4} + r_{b_4}) + \rho_{a_6}{}^2 r_{a_6}.$$

In the Cournot model, each PM chain is aware of the output level of the other PM chains. When making output decisions, they can observe the total output of the market, as well as the output of other supply chains. This allows each PM chain to make optimal production decisions based on complete information. However, in a real-world market, the game among PM chains is ongoing. Therefore, decision-making by PM chains is a long-term repetitive dynamic process characterized not only by adaptability but also by long-term memory. We assume that all firms are bounded rational. That is, if the estimated marginal profit is negative (positive) in period T, the supply chain firm will reduce (increase) its output quantity in period T + 1. Such adjustments will be reflected in the output of final product $P*$ in the PM chain s_i. Therefore, all PM chains in Figure 1 are bounded rational. Let $\omega_{s_i}(\omega_{s_i} \geq 0)$ represent the output adjustment speed of PM chain s_i. Then, the output adjustment process for PM chain s_i can be expressed as follows:

$$x_{s_i,T+1} = x_{s_i,T} + \omega_{s_i} x_{s_i,T} \Phi_{s_i,T}, i = 1, 2, 3 \tag{19}$$

where $\Phi_{s_i,T}$ denotes the marginal profit function of the PM chain in period T.

$$\Phi_{s_i,T} = \frac{\partial \pi_{s_i,T}}{\partial x_{s_i,T}} = P_{s_i} + \frac{\partial P_{s_i}}{\partial x_{s_i}} x_{s_i,T} - \sum_{a \in As_i} \frac{\partial C_{a_n}}{\partial x_{s_i}} - \sum_{a \in Bs_i} \frac{\partial C_{b_m}}{\partial x_{s_i}}, \quad n = 1,2,3,\ldots,6; m = 1,2,3,\ldots,7; i = 1,2,3 \tag{20}$$

By substituting Equation (20) into Equation (19), the discrete dynamical model for the inter-chain output game of the studied supply chain can be obtained as follows:

$$\begin{cases} x_{s_1,T+1} = x_{s_1,T} + \omega_{s_1} x_{s_1,T}[u_1 - 2(1+\gamma_{s_1})x_{s_1,T} - \beta x_{s_3,T}] \\ x_{s_2,T+1} = x_{s_2,T} + \omega_{s_2} x_{s_2,T}[u_2 - 2(1+\gamma_{s_2})x_{s_2,T} - 2\gamma_{s_{246}} x_{s_3,T}] \\ x_{s_3,T+1} = x_{s_3,T} + \omega_{s_3} x_{s_3,T}[u_1 - t_{b_6} - 2(1+\gamma_{s_3})x_{s_3,T} - \beta x_{s_1,T} - 2\gamma_{s_{246}} x_{s_2,T}] \end{cases} \tag{21}$$

where

$$\gamma_{s_2} = {\rho_{a_2}}^2(r_{a_2} + r_{b_2}) + {\rho_{a_4}}^2(r_{a_4} + r_{b_4}) + {\rho_{a_6}}^2(r_{a_6} + r_{b_7}),$$
$$\gamma_{s_3} = {\rho_{a_2}}^2(r_{a_2} + r_{b_2}) + {\rho_{a_4}}^2(r_{a_4} + r_{b_4}) + {\rho_{a_6}}^2(r_{a_6} + r_{b_6}).$$

3. Dynamical Analysis for the Model

In the discrete dynamical model (21), by setting $x_{s_i,T+1} = x_{s_i,T}$, we can obtain the eight equilibrium solutions:

$$E_1 = (0,0,0),\ E_2 = (\frac{u_1}{2\gamma_{s_1}+2}, \frac{u_2}{2\gamma_{s_2}+2}, 0),\ E_3 = (\frac{u_1}{2\gamma_{s_1}+2}, 0, 0),\ E_4 = (0, \frac{u_2}{2\gamma_{s_2}+2}, 0)$$

$$E_5 = (0,0,\frac{u_1 - t_{b_6}}{2\gamma_{s_3}+2}),\ E_6 = (0, \frac{(1+\gamma_{s_3})u_2 - \gamma_{s_{246}}(u_1 - t_{b_6})}{2(-{\gamma_{s_{246}}}^2 + \gamma_{s_2} + \gamma_{s_3} + 1 + \gamma_{s_2}\gamma_{s_3})}, \frac{(1+\gamma_{s_2})(u_1 - t_{b_6}) - \gamma_{s_{246}} u_2}{2(-{\gamma_{s_{246}}}^2 + \gamma_{s_2} + \gamma_{s_3} + 1 + \gamma_{s_2}\gamma_{s_3})})$$

$$E_7 = (\frac{(2-\beta+2\gamma_{s_3})u_1 + \beta t_{b_6}}{4\gamma_{s_3} + 4\beta\gamma_{s_1} + 4\gamma_{s_1}\gamma_{s_3} + 4 - \beta^2}, 0, \frac{(2-\beta+2\gamma_{s_1})u_1 - (2+2\gamma_{s_1})t_{b_6}}{4\gamma_{s_3} + 4\beta\gamma_{s_1} + 4\gamma_{s_1}\gamma_{s_3} + 4 - \beta^2})$$

$$\begin{aligned} E_8 = (&\frac{(-2{\gamma_{s_{246}}}^2 + 2 - \beta + 2\gamma_{s_2} + 2\gamma_{s_3} - \beta\gamma_{s_2} + 2\gamma_{s_2}\gamma_{s_3})u_1 + \beta\gamma_{s_{246}} u_2 + \beta(1+\gamma_{s_2})t_{b_6}}{A}, \\ &\frac{(2\beta - 4 - 4\gamma_{s_1})\gamma_{s_{246}} u_1 + (4 - \beta^2 + 4\gamma_{s_1} + 4\gamma_{s_3} + 4\gamma_{s_1}\gamma_{s_3})u_2 + 4\gamma_{s_{246}}(1+\gamma_{s_1})t_{b_6}}{2A}, \\ &\frac{(2 + 2\gamma_{s_1} + 2\gamma_{s_2} - \beta\gamma_{s_2} + 2\gamma_{s_1}\gamma_{s_2} - \beta)u_1 - 2\gamma_{s_{246}}(1+\gamma_{s_1})u_2 + 2(1 - \gamma_{s_1} - \gamma_{s_2} - \gamma_{s_1}\gamma_{s_2})t_{b_6}}{A}) \end{aligned}$$

where

$$A = 4\gamma_{s_1} + 4\gamma_{s_2} + 4\gamma_{s_3} + 4\gamma_{s_1}\gamma_{s_2} + 4\gamma_{s_2}\gamma_{s_3} + 4\gamma_{s_1}\gamma_{s_3} - 4\gamma_{s_1}{\gamma_{s_{246}}}^2 - 4{\gamma_{s_{246}}}^2 + 4\gamma_{s_1}\gamma_{s_2}\gamma_{s_3} - (1+\gamma_{s_2})\beta^2 + 4.$$

E_2, E_3, and E_4 are boundary equilibrium points and non-negative; when $u_1 > t_{b_6}$, E_5 is a boundary equilibrium point and non-negative; when $\frac{\gamma_{s_{246}}}{1+\gamma_{s_2}} < \frac{u_1 - t_{b_6}}{u_2} < \frac{1+\gamma_{s_3}}{\gamma_{s_{246}}}$, E_6 is a boundary equilibrium point and non-negative; when $u_1 > t_{b_6}$ and $\frac{u_1 - t_{b_6}}{u_1} < \frac{\beta}{2+2\gamma_{s_1}}$, E_7 is a boundary equilibrium point and non-negative; when $0 < \frac{u_1 - t_{b_6}}{u_2} < \frac{4\beta^2 + 4\gamma_{s_1} + 4\gamma_{s_3} + 4\gamma_{s_1}\gamma_{s_3}}{4(1+\gamma_{s_1})}$ and $\frac{u_1}{u_2} > \frac{2\gamma_{s_{246}}(1+\gamma_{s_1})}{2 - \beta\gamma_{s_2} - \beta}$, E_8 is an interior equilibrium point and non-negative.

The Jacobian matrix of the complex dynamical model (21) is as follows:

$$\mathbf{J} = \begin{bmatrix} D_{11} & 0 & -\beta\omega_{s_1} x_{s_1} \\ 0 & D_{22} & -2\gamma_{s_{246}}\omega_{s_2} x_{s_2} \\ -\beta\omega_{s_3} x_{s_3} & -2\gamma_{s_{246}}\omega_{s_3} x_{s_3} & D_{33} \end{bmatrix} \tag{22}$$

where $D_{ii} = 1 + \omega_{s_i}[\Phi_{s_i} - 2(1+\gamma_{s_i})x_{s_i}], i = 1,2,3.$

By analyzing the characteristic roots of the Jacobian matrix (22), the stability of the system can be judged. When the absolute values of all characteristic roots are strictly less than 1, the equilibrium point tends to be stable, and the equilibrium point is the Nash equilibrium point of the discrete dynamical model (21) [7].

To discuss the stable region of the discrete dynamical model (21), we substitute the Nash equilibrium points into the Jacobian matrix (22). The characteristic polynomial is obtained as follows:

$$g(\lambda) = \lambda^3 + B_1\lambda^2 + B_2\lambda + B_3 \tag{23}$$

The discrete dynamical model (21) is also a three-dimensional discrete dynamical system. According to the Jury's criterion [36], the discrete dynamical system (21) will stabilize asymptotically if the following conditions are satisfied.

$$\begin{cases} g(1) = 1 + B_1 + B_2 + B_3 > 0 \\ -g(-1) = 1 - B_1 + B_2 + B_3 > 0 \\ {B_3}^2 - 1 < 0 \\ \left({B_3}^2 - 1\right)^2 > (B_1B_3 - B_2)^2 \end{cases} \tag{24}$$

4. Numerical Simulations

We assume that in terms of production costs, the marginal production cost in the importing country (country 1) is higher than that in the exporting country (country 2). In terms of transaction costs, the marginal transaction cost of the cross-border links is higher than that of domestic links. For the purpose of comparative analysis, we examine an initial case in which country 1 does not impose tariffs on product $P*$ imported from country 2. On the basis of these considerations and with reference to Zhang et al. [32] and Feng et al. [4], the following simulation parameters are assumed:

$$\beta = 0.3,\ u_1 = 100,\ u_2 = 100,\ t_{b_6} = 0,\ h_{b_6} = 28.5,\ r_{a_1} = r_{a_3} = 1,$$

$$r_{a_2} = r_{a_4} = 0.5,\ r_{a_5} = 2,\ r_{a_6} = 1,\ r_{b_1} = r_{b_2} = r_{b_3} = r_{b_4} = r_{b_5} = r_{b_7} = 0.1,\ r_{b_6} = 0.2$$

We assume that the suppliers of raw material and the upstream manufacturers require 2 units of materials to produce 1 unit of final product $P*$, as shown in Table 2. If a firm participates in chain s_i, a value is provided; otherwise, nonparticipation is represented by "-".

Table 2. List of materials required by each firm.

PM Chain	The Number of Materials Required by the Firm a_n					
	ρ_{a_1}	ρ_{a_2}	ρ_{a_3}	ρ_{a_4}	ρ_{a_5}	ρ_{a_6}
s_1	2	-	2	-	1	-
s_2	-	2	-	2	-	1
s_3	-	2	-	2	-	1

The equilibrium solutions of the complex dynamical system (21) are $E_1 = (0, 0, 0)$, $E_2 = (0, 0, 7.143)$, $E_3 = (0, 7.246, 0)$, $E_4 = (4.202, 0, 0)$, $E_5 = (4.202, 7.246, 0)$, $E_6 = (0, 4.093, 3.752)$, $E_7 = (4.113, 0, 7.055)$, and $E_8 = (4.158, 4.340, 3.458)$.

By substituting the above solution into the Jacobian matrix (22), we can observe that only the characteristic roots of E_8 are less than 1. Therefore, E_8 is the unique Nash equilibrium point of the complex dynamical system (21).

The Jacobian matrix at point E_8 is given by the following:

$$\mathbf{J} = \begin{bmatrix} 1 - 98.958\omega_{s_1} & 0 & -1.247\omega_{s_1} \\ 0 & 1 - 59.897\omega_{s_2} & -50.344\omega_{s_2} \\ -1.037\omega_{s_3} & -40.113\omega_{s_3} & 1 - 48.415\omega_{s_3} \end{bmatrix} \tag{25}$$

The characteristic polynomial can be calculated as Equation (23). Where

$$B_1 = 98.958\omega_{s_1} + 59.897\omega_{s_2} + 48.415\omega_{s_3} - 3,$$
$$B_2 = 5927.287\omega_{s_1}\omega_{s_2} + 4789.758\omega_{s_1}\omega_{s_3} + 880.464\omega_{s_2}\omega_{s_3} - 197.916\omega_{s_1} - 119.794\omega_{s_2} - 96.830\omega_{s_3} + 3,$$
$$B_3 = 98.958\omega_{s_1} + 59.897\omega_{s_2} + 48.415\omega_{s_3} - 5927.287\omega_{s_1}\omega_{s_2} - 4789.758\omega_{s_1}\omega_{s_3} - 880.464\omega_{s_2}\omega_{s_3} + 87051.539\omega_{s_1}\omega_{s_2}\omega_{s_3} - 1.$$

According to the Jury criterion (24), the stable region of E_8 is shown in Figure 3.

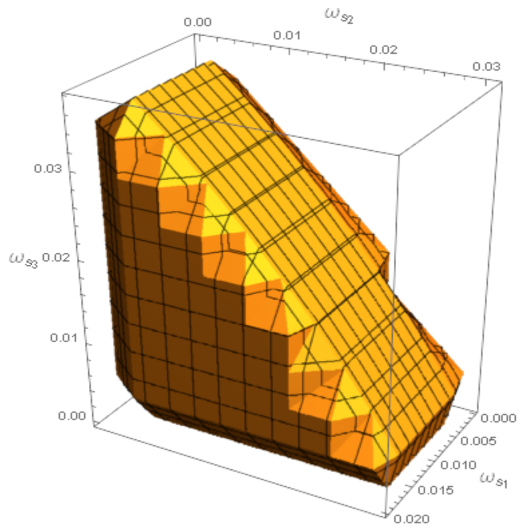


Figure 3. The system's stable region of output adjustment speeds ω_{s_1}, ω_{s_2}, and ω_{s_3}.

No matter the initial output level of the PM chain, when the values of the output adjustment speed $(\omega_{s_1}, \omega_{s_2}, \omega_{s_3})$ are within the yellow region shown in Figure 3, the output level will eventually stabilize at E_8. If not, complex behaviors like bifurcation and chaos will occur.

4.1. Bifurcation and Chaos Simulations

4.1.1. The Influence of Output Adjustment on the System

To obtain the bifurcation diagrams of the output decisions of PM chain s_i with respect to its own adjustment speed ω_{s_i}, we fix the output adjustment speed of the other PM chains at $\omega_{s_i} = 0.01$. The results are shown in Figure 4. Additionally, we calculate the maximum Lyapunov exponent, which is an indicator of the system's stability, and plot its variation in Figure 5.

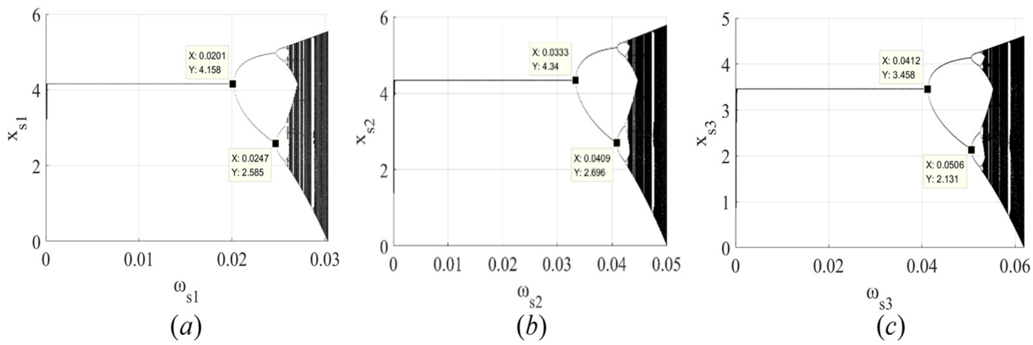


Figure 4. (**a**) Bifurcation diagram of the output of s_1 with respect to ω_{s_1}; (**b**) bifurcation diagram of the output of s_2 with respect to ω_{s_2}; (**c**) bifurcation diagram of the output of s_3 with respect to ω_{s_3}.

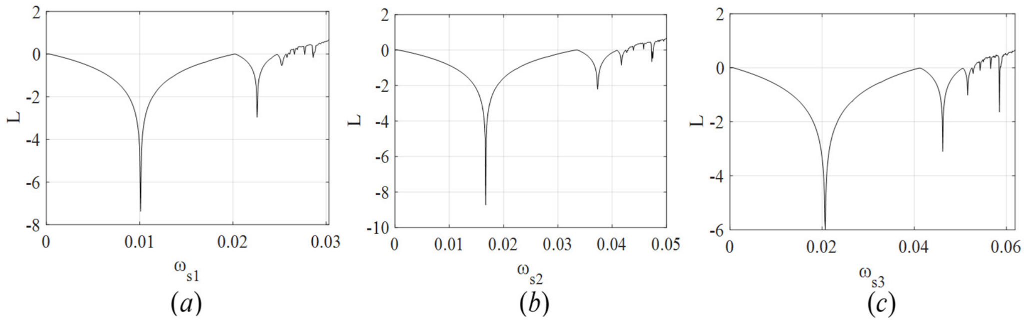


Figure 5. (**a**) Maximum Lyapunov exponent of the output of s_1 with respect to ω_{s_1}; (**b**) maximum Lyapunov exponent of the output of s_2 with respect to ω_{s_2}; (**c**) maximum Lyapunov exponent of the output of s_3 with respect to ω_{s_3}.

In Figures 4a and 5a, when $0 < \omega_{s_1} < 0.0201$ (i.e., the Lyapunov exponent is negative), the output level of the PM chain s_1 is stable; when $\omega_{s_1} = 0.0201$ (i.e., the Lyapunov exponent is zero), the output level of s_1 occurs at the bifurcation; when $\omega_{s_1} = 0.0247$ and $\omega_{s_1} = 0.0257$ (i.e., the Lyapunov exponent is zero), the output level of s_1 occurs at the bifurcation again. Therefore, $0.0202 < \omega_{s_1} < 0.0247$ is the two-periodic orbit, and $0.0247 < \omega_{s_1} < 0.0257$ is the four-periodic orbit. In addition, when $\omega_{s_1} > 0.0257$ (i.e., the Lyapunov exponent is positive), there is an emergence of the chaotic state of the output of s_1. Similar phenomena can also be observed in Figures 4b,c and 5b,c.

Therefore, as the output adjustment of each PM chain is continuously increasing, the output levels of the three PM chains show stability, period-doubling bifurcations, and eventually reach a chaotic state.

To explore the influence of output adjustments on the system (21), we fix $\omega_{s_2} = 0.011$ and $\omega_{s_3} = 0.026$, and the bifurcation diagram of the system is shown in Figure 6a. The bifurcation diagrams created by fixing other parameter combinations are shown in Figure 6b–d and show the chaotic attractor of the system.

Figure 6a shows that when $0 < \omega_{s_1} < 0.02016$, the system is in a stable state. When $\omega_{s_1} = 0.02015$, bifurcation occurs in the system for the first time and enters a two-period orbit. When ω_{s_1} further increases to 0.02468, bifurcation occurs in the system a second time and enters a four-period orbit. Similarly, when $\omega_{s_1} = 0.02568$, bifurcation in the system occurs for the third time. As ω_{s_1} continues to increase, the system eventually enters a chaotic state.

Figure 6b and Figure 6c show that as ω_{s_2} and ω_{s_3} increase, the system reaches a stable state and a period-doubling bifurcation and chaotic state, respectively, similar to Figure 6a.

Figure 7 shows the bifurcation and chaos phenomena in three-dimensional space with respect to ω_{s_2} and ω_{s_3}. With the increase in the output adjustment speed of ω_{s_2}, the stable range of s_2 will decrease, and s_3 will enter a chaotic state. Similarly, an increase in ω_{s_3} will reduce the stable range of s_3 and cause the output level of s_2 to eventually enter a state of bifurcation or even chaos.

We can conclude that the system (21) will lose stability as the output adjustment speed increases. When the adjustment speed of any PM chains in the studied supply chain surpasses a specific threshold, the system will eventually enter a chaotic state. This phenomenon leads to fluctuations not only in the output of one PM chain but also in the output of the entire system. This causes further fluctuations in the market. Therefore, in order to avoid market fluctuations as much as possible, it is crucial that each PM chain makes an effort to control the adjustment speed of the output.

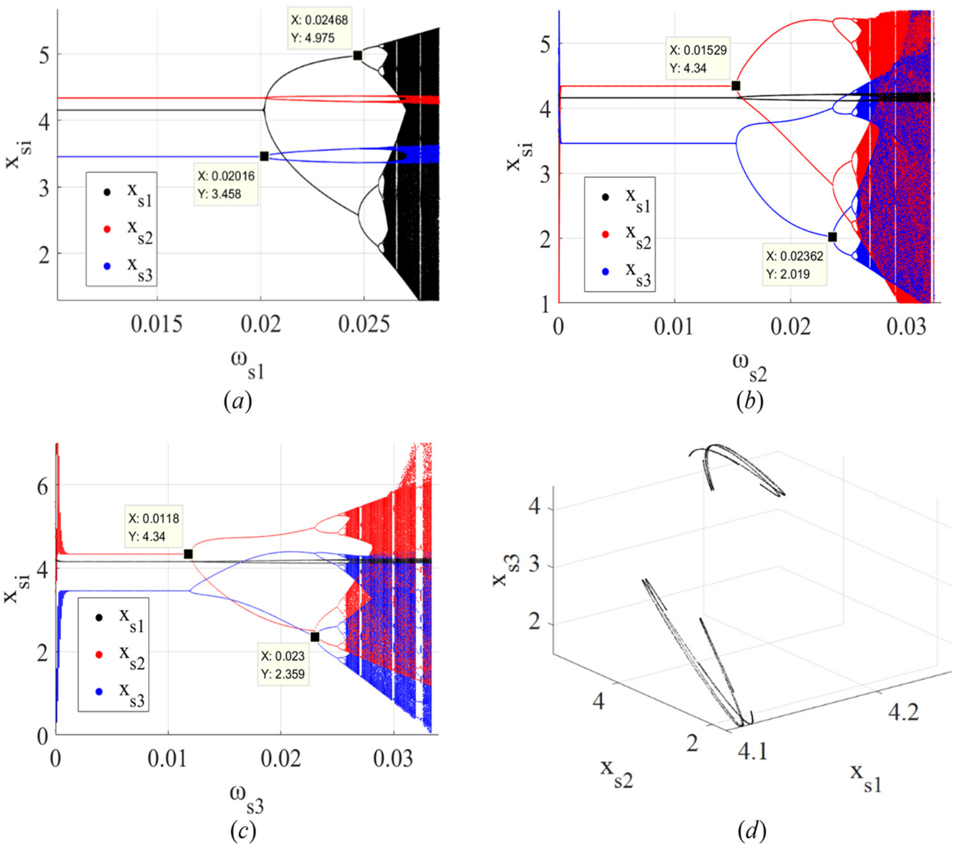


Figure 6. (**a**) Bifurcation diagram of the outputs of s_1, s_2, and s_3 with respect to ω_{s_1} when $\omega_{s_2} = 0.011$ and $\omega_{s_3} = 0.026$; (**b**) bifurcation diagram of the outputs of s_1, s_2, and s_3 with respect to ω_{s_2} when $\omega_{s_1} = 0.016$ and $\omega_{s_3} = 0.026$; (**c**) bifurcation diagram of the outputs of s_1, s_2, and s_3 with respect to ω_{s_3} when $\omega_{s_1} = 0.016$ and $\omega_{s_2} = 0.026$; (**d**) chaotic attractor of the system at $\omega_{s_1} = 0.016$, $\omega_{s_2} = 0.027$, and $\omega_{s_3} = 0.026$.

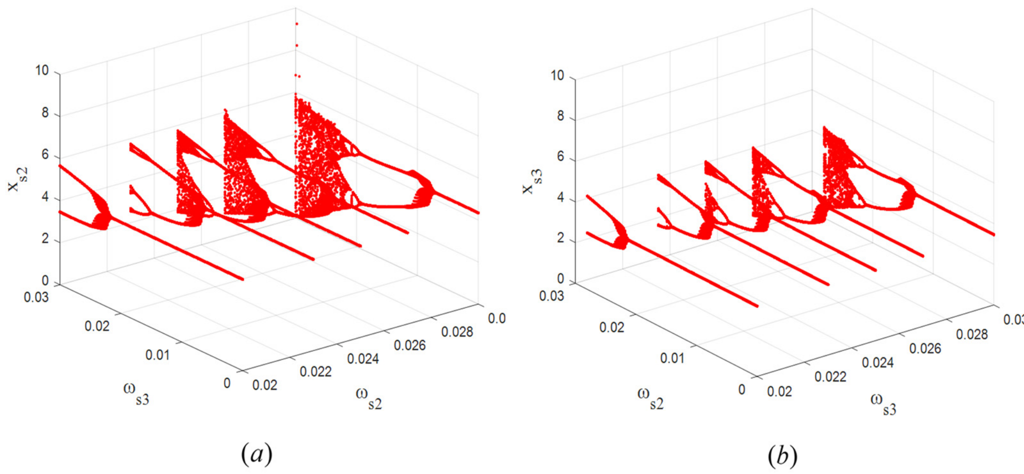
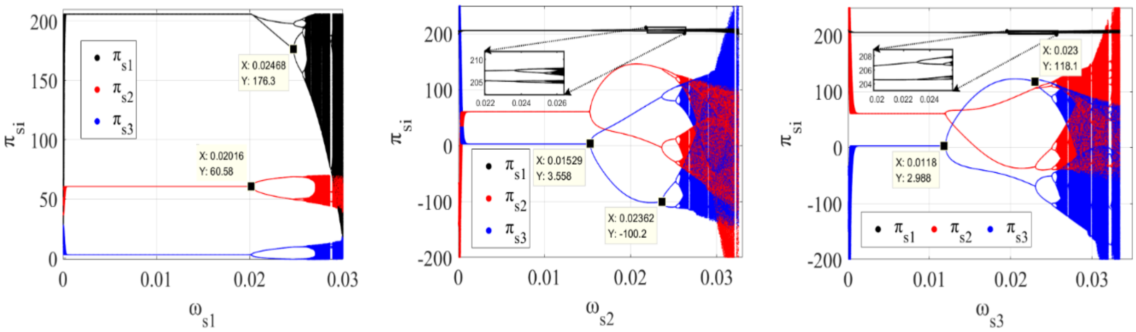


Figure 7. (**a**) Three-dimensional bifurcation diagram of the output of s_2 with respect to ω_{s_2} and ω_{s_3}; (**b**) three-dimensional bifurcation diagram of the output of s_3 with respect to ω_{s_2} and ω_{s_3}.

4.1.2. Influence of Output Adjustment on the System

When the adjustment speed of the output surpasses a specific threshold, the output level of the system (21) will enter a chaotic state, which also affects the profits of each PM chain. Thus, to explore the influence of the output adjustment on the system's profits, we fixed the same parameters as in Figure 6a, Figure 6b, and Figure 6c, and the results are shown in Figure 8a, Figure 8b, and Figure 8c, respectively. The intervals of the stable, period-doubling bifurcation, and chaotic states shown in Figure 8 are consistent with Figure 6.

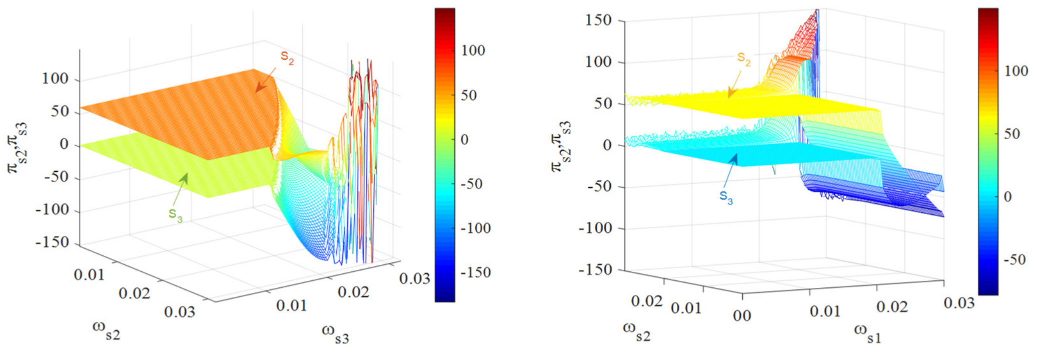


(**a**) (**b**) (**c**)

Figure 8. (**a**) Bifurcation diagram of the profits of s_1, s_2, and s_3 with respect to ω_{s_1} when $\omega_{s_2} = 0.011$ and $\omega_{s_3} = 0.026$; (**b**) bifurcation diagram of the profits of s_1, s_2, and s_3 with respect to ω_{s_2} when $\omega_{s_1} = 0.016$ and $\omega_{s_3} = 0.026$; (**c**) bifurcation diagram of the profits of s_1, s_2, and s_3 with respect to ω_{s_3} when $\omega_{s_1} = 0.015$ and $\omega_{s_2} = 0.026$.

Figure 8 shows that the system's profits change from a stable to period-doubling bifurcation state and eventually enter a chaotic state as the adjustment speeds of ω_{s_1}, ω_{s_2}, and ω_{s_3} increase, respectively. The long-term development of the three PM chains will be severely hampered by the fluctuation in profits. Furthermore, the profits of PM chain s_1, as shown in Figure 8a, fall after entering a period-doubling bifurcation state, which suggests that an excessive output adjustment speed not only destabilizes but also decreases the profits of PM chain s_1. To protect the profits of the entire system, each PM chain firm should make suitable adjustments to the speed of output.

Figure 9 shows the three-dimensional profits diagrams of s_2 and s_3 with respect to ω_{s_2}, ω_{s_3} and ω_{s_1}, ω_{s_2}. In Figure 9a, when ω_{s_2} is small, the increase in ω_{s_3} does not destabilize the profits of s_2 and s_3. However, when ω_{s_2} is large, the increase in ω_{s_3} will cause the profits of both s_2 and s_3 to decline and eventually enter a chaotic state. This shows that for an exporting country, when the output adjustment speed of export products is too high, the profits of the supply chain in the exporting country will decline. This is unfavorable for exporting countries, so they should opt for a smaller adjustment speed for the quantity of exports. In Figure 9b, when the value of ω_{s_2} is relatively small, the increase in ω_{s_1} results in an overall rise in the profits of s_2 and s_3. However, when ω_{s_2} is large, the increase in ω_{s_1} will cause the profits of both s_2 and s_3 to decline. By comprehensively analyzing Figure 9a,b, we can conclude that, whether making decisions on the export quantity or the level of domestic output, the exporting country should adopt a relatively smaller output adjustment speed.



(*a*) (*b*)

Figure 9. (**a**) Profits of s_2 and s_3 with respect to ω_{s_2} and ω_{s_3} when $\omega_{s_1} = 0.01$; (**b**) profits of s_2 and s_3 with respect to ω_{s_1} and ω_{s_2} when $\omega_{s_3} = 0.01$.

4.1.3. Influence of Tariffs on the System

The analysis above was based on the assumption that country 1 does not impose tariffs on country 2. To investigate the influence of tariffs on system dynamics, we conducted simulations with different tariff levels, which are represented by the parameter t_{b_6}. The results are presented in Figure 10a,b.

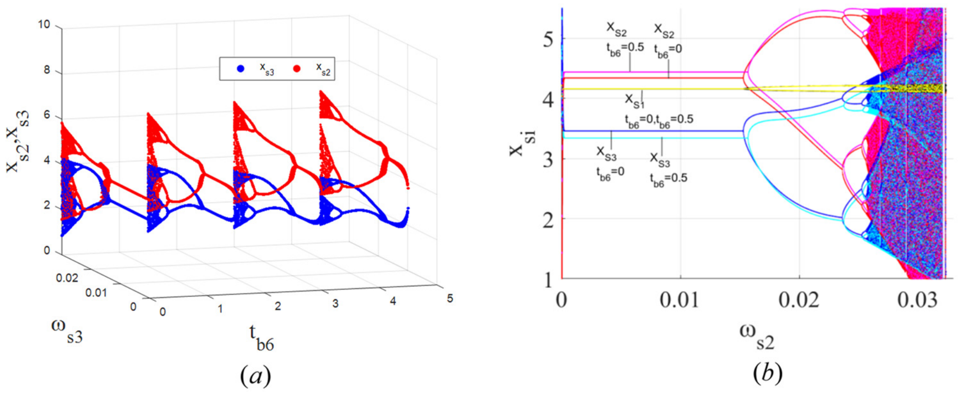


Figure 10. (**a**) Three-dimensional bifurcation diagram of the outputs of s_2 and s_3 with respect to ω_{s_3} and t_{b_6} when $\omega_{s_1} = 0.015$ and $\omega_{s_2} = 0.026$; (**b**) bifurcation diagram of the outputs of the system with respect to ω_{s_2} and t_{b_6} when $\omega_{s_1} = 0.016$ and $\omega_{s_3} = 0.026$.

Figure 10a shows a three-dimensional bifurcation diagram of s_2 and s_3 with respect to ω_{s_3} and t_{b_6}. When the unit tariff cost increases, the equilibrium of the output level of PM chain s_3 declines, and the stable ranges of s_2 and s_3 are reduced. It is more challenging for products to enter overseas markets, and the supply chain of imported goods is more unstable, when higher tariffs are imposed on the products.

Figure 10b shows that when the adjustment speed ω_{s_2} rises, changes in the unit tariff cost t_{b_6} have no effect on the dynamic evolution of the PM chain s_1, but they influence s_2 and s_3. As the unit tariff cost t_{b_6} increases from 0 to 0.5, the equilibrium output level of PM chain s_2 increases, while that of the PM chain s_3 declines. Additionally, as the unit tariff cost t_{b_6} rises, there is an expansion in the stable range of PM chains s_2 and s_3. This indicates that for the tariff-imposing country (country 1), although the increase in tariffs can protect domestic producers, it can also lead to the expansion of the stable range of the PM chain in the product-exporting country (country 2). Thus, when the output adjustment speed surpasses a specific threshold, the PM chains of country 2 enter a bifurcation state or even a chaotic state, whereas the PM chains of country 1 exhibit sustained stability. This is unfavorable for the tariff-imposing country (country 1). Moreover, the likelihood of such an adverse situation increases as tariffs increase. To prevent fluctuations in its own market, the tariff-imposing country should impose appropriate tariffs.

4.1.4. Influence of Initial Values on the System

In Figure 3, we can see that when the adjustment speeds of s_1, s_2, and s_3 are 0.029, 0.049, and 0.059, respectively, the system is in a chaotic state. This means that at these adjustment speeds, the output level of each PM chain will fluctuate unpredictably over time. To explore the influence of the initial values on the chaotic state of the system, we made slight adjustments to the equilibrium output level E_8 of the three PM chains denoted as $F_1 = (4.159, 4.340, 3.458)$, $F_2 = (4.158, 4.341, 3.458)$, and $F_3 = (4.158, 4.340, 3.459)$, and plotted a time series graph, as shown in Figure 11.

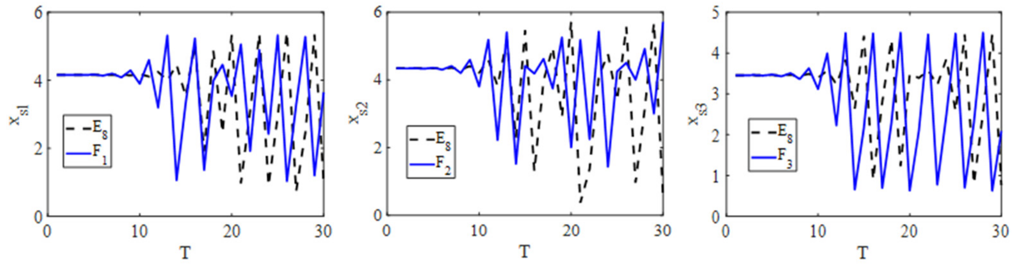


Figure 11. Time series graph of the output levels of s_1, s_2, and s_3 under different initial values.

Figure 11 shows that during the early stages of the system's evolution, the output levels based on equilibrium point E_8, as the initial value, exhibited relatively stable changes over time. However, when F_1, F_2, and F_3 were used as initial values, the trajectory of the output levels showed significant fluctuations. As the system continuously evolves, the trajectories of the output levels based on equilibrium point E_8, as the initial value, differ significantly from the trajectories based on F_1, F_2, and F_3 as initial values. Overall, we can conclude that the chaotic state of the system (21) is sensitive to initial values, and even slight differences in the initial conditions can lead to substantial variations in the evolution of the system (21).

4.2. Chaos Control

On the basis of the above analysis, we can conclude that chaos will cause damage to the system, which causes profits to fall and fluctuate irregularly. We applied the delayed feedback control method to control the system's chaotic behavior [37]. In our study, a PM chain is a game player, and the firms in it all make efforts to achieve an output of x_{s_i} for the final product. Therefore, firms in the PM chain have the same output adjustment speed and control factor k for delayed feedback decision-making. When the time delay of PM chain s_2 is chosen as 1, the control system can be described as follows:

$$\begin{cases} x_{s_1,T+1} = x_{s_1,T} + \omega_{s_1} x_{s_1,T}[u_1 - 2(1+\gamma_{s_1})x_{s_1,T} - \beta x_{s_3,T}] \\ x_{s_2,T+1} = x_{s_2,T} + \omega_{s_2} x_{s_2,T}[u_2 - 2(1+\gamma_{s_2})x_{s_2,T} - \gamma_{s_{246}} x_{s_3,T}] + Z_2 \\ x_{s_3,T+1} = x_{s_3,T} + \omega_{s_3} x_{s_3,T}[u_1 - t_{b_6 1} - 2(1+\gamma_{s_3})x_{s_3,T} - \beta x_{s_1,T} - 2\gamma_{s_{246}} x_{s_2,T}] \end{cases} \tag{26}$$

where $Z_2 = k(x_{s_2,T} - x_{s_2,T+1})$, k is a controlling factor.

When the time delay of PM chains s_1, s_2, and s_3 are all chosen as 1, the control system can be represented as follows:

$$\begin{cases} x_{s_1,T+1} = x_{s_1,T} + \omega_{s_1} x_{s_1,T}[u_1 - 2(1+\gamma_{s_1})x_{s_1,T} - \beta x_{s_3,T}] + Z_1 \\ x_{s_2,T+1} = x_{s_2,T} + \omega_{s_2} x_{s_2,T}[u_2 - 2(1+\gamma_{s_2})x_{s_2,T} - \gamma_{s_{246}} x_{s_3,T}] + Z_2 \\ x_{s_3,T+1} = x_{s_3,T} + \omega_{s_3} x_{s_3,T}[u_1 - t_{b_6 1} - 2(1+\gamma_{s_3})x_{s_3,T} - \beta x_{s_1,T} - 2\gamma_{s_{246}} x_{s_2,T}] + Z_3 \end{cases} \tag{27}$$

where $Z_1 = k(x_{s_1,T} - x_{s_1,T+1})$, $Z_3 = k(x_{s_3,T} - x_{s_3,T+1})$.

Figure 12 shows that the system's chaotic state stabilized with an increase in k, which indicates that the method of delayed feedback control is effective for controlling chaos in the system (21). This further indicates that a PM chain firm should adjust its output level by considering not only the last period's profits as a benchmark but also the profits of further previous periods as a reference to improve the stability when making decisions. Comparing Figure 12a and Figure 12b, we also notice that the transition from a chaotic state to stable state in Figure 12b occurred faster than Figure 12a, which indicates that each PM chain firm in the system should adopt the delayed feedback control method to adjust its level of output.

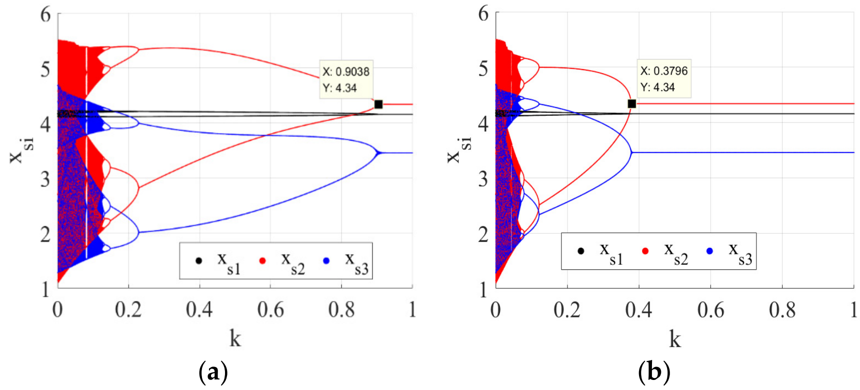


Figure 12. (**a**) Effect of control factor k on the output with a delayed feedback control of s_2; (**b**) effect of control factor k on the output with delayed feedback controls of s_1, s_2, and s_3.

5. Conclusions

We established a discrete dynamical model for an output game from the perspective of inter-chain competition and obtained the Nash equilibrium point and its stable region. From the numerical simulations, bifurcation diagrams, maximum Lyapunov exponents, and time series graphs are presented to illustrate the complex dynamic behaviors of the dynamical model.

Our research showed that the output adjustment speed has a significant impact on the stability of the whole supply chain system. It can magnify fluctuations in the level of output in a supply chain system and can eventually lead to a chaotic state when the output adjustment speed surpasses a specific threshold. Supply chain firms need to pay attention to whether the output adjustment speed is excessive, as this will not only cause instability in the supply chain in which it participates but also reduce the profits of the supply chain in which it participates. To protect their own profits, each supply chain firm should adjust its output level to a lower speed. The increase in tariffs will expand the stable range of the PM chain in the product-exporting country. Therefore, to prevent fluctuations in its own market, governments of tariff-imposing countries should impose appropriate tariffs. Additionally, the chaotic behavior of the system is sensitive to the initial level of output. Each PM chain should evaluate the potential impacts of different initial output values carefully. When the market is in a chaotic state, applying the method of delayed feedback control would have an effect on the system stability. To maintain system stability and prevent profit loss, it is particularly important for each PM chain firm to adjust its output level by considering not only the profits of the last period but also the profits of further previous periods as references. Government organizations should also actively introduce relevant policies to regulate the phenomena of chaos.

To better explore the nature of solutions of inter-chain competition and the complexity of dynamic evolution, our study selectively focused on three representative supply chains within a global supply chain network. Therefore, there are some limitations of this study, and a great deal of work is yet to be completed. More complex scenarios, such as the export of raw materials or intermediate products, as well as special cases like tariff exemptions between countries with technological disparities, were not considered in our model. Interestingly, if a company engages in the export of raw materials or intermediate materials, a new PM chain would be added to the game system based on our model. Subsequently, this new PM chain must be incorporated into the nonlinear dynamical system model, leading to an increase in model's dimensionality and analytical complexity. Thus, enriching our model is an interesting subject for future study.

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