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Recycling Pricing and Government Subsidy Strategy for End-of-Life Vehicles in a Reverse Supply Chain under Consumer Recycling Channel Preferences

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Abstract: In the existing recycling system for end-of-life vehicles (ELVs), online recycling based on the Internet platform is a useful supplement. In this paper, a Stackelberg game pricing model, which is dominated by ELV part remanufacturers and composed of remanufacturers, recyclers, and consumers, is constructed considering consumer preferences for recycling channels. The influence of different subsidy strategies on the optimal pricing, profit, and recycling volume of the reverse supply chain (RSC) of ELVs is discussed, and the effects of factors such as subsidy amount and consumer preferences on the RSC of ELVs are analyzed using numerical simulation. The results show that the increase in consumers' online recycling preferences has a positive effect on the recycling volume and profit of the RSC of ELVs. Considering the recycling volume of the RSC, when fewer subsidies are given, more recycling volume can be generated by subsidizing remanufacturers, and, on the contrary, recycling volume will be generated by subsidizing consumers. Considering the profit of the RSC, when subsidies are given at the lower-middle level, higher profits can be earned by subsidizing remanufacturers, and, on the contrary, higher profits can be earned by subsidizing consumers.

Keywords: reverse supply chain management; dual recycling channels; Stackelberg game; consumer preferences; government subsidy

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1. Introduction

The automobile industry is one of the pillar industries of the national economy. Recent years have seen a boom in the automobile industry and with it, an increase in the consumption of various natural resources required for automobile production [1,2], which has aggravated the negative impact on the environment. China is the fastest-growing country in the world in terms of car parc [3]. A vehicle typically lasts 15 years, after which it enters the scrap stage [4]. By the end of 2020, the volume of ELVs worldwide was predicted to exceed 100 million, and this number will continue to grow [3,5]. It is obvious that the volume of car parc and ELVs will inevitably increase in the future, which makes the generation of ELVs a common concern of the global automotive industry [6].

By improving the recycling level of ELVs and reducing the use of harmful substances, the environmental protection goal is achieved [7]. ELV recycling re-enters the product life cycle through the process of reuse, remanufacturing, and re-utilization, which has significant economic benefits while making full use of limited resources. Remanufacturing auto parts can save energy by 60% and raw materials by 70%. For the "five assemblies" of ELVs that have remanufacturing conditions and are dismantled in accordance with the "Measures for the Management of End-of-Life Vehicle Recycling", published by the Chinese government in 2019, they can be sold to enterprises with remanufacturing capacity for recycling under relevant state regulations [8]. Manufacturers are increasingly involved in ELV recycling in compliance with national regulations and in pursuit of economic benefits [9]. If all used automobile parts could be completely recycled and remanufactured, by 2022, it was predicted that China would reduce energy consumption by 7–9.4 billion kWh and 6.67–9.69 million tons of carbon dioxide per year [10].

The remanufacturing of ELV parts is conducive to effectively improving resource utilization and reducing environmental pollution while greatly reducing production costs, which is highly economical and practical. The policy of government subsidy is an effective way to solve the problem of ELVs [11]. To boost the recycling volume of ELVs, the government provides cash subsidies to remanufacturing enterprises and consumers who purchase remanufactured products to promote product remanufacturing [12]. Government subsidies for remanufactured products are efforts to promote remanufacturing [13]. For example, the "Implementation Rules of Subsidies for Scrapping and Upgrading of Old Cars", issued by the Shanghai Municipal Commission of Commerce and the Shanghai [14] Municipal Finance Bureau, stipulates that from 23 April 2020, individual consumers who scrap fuel vehicles with National IV and below emission standards can apply for a subsidy of RMB 4000 per vehicle. With the implementation of this subsidy policy, the government supports the development of the remanufacturing industry while increasing the recycling volume of ELVs, so as to improve resource utilization and protect the environment [8]. With the development of online business platforms, online recycling channels such as ATRenew and Alahb have started conducting commercial recycling operations. Among them, ATRenew specializes in online recycling of used electronic products [15] and is expanding into luxury goods recycling. On the other hand, Alahb focuses on electronic waste recycling [16] and is dedicated to building a closed-loop supply chain by providing online recycling services to reduce environmental impact. Recognizing the convenience value that online recycling brings to consumers, the business of end-of-life vehicle recycling is also shifting from traditional channels to online platforms. In China, Aitedaxiang, operating in online end-of-life vehicle recycling, has already commenced commercial practices. The Internet-based recycling model they implement not only provides online end-of-life vehicle recycling services to consumers but also boosts the quantity of recycling [17].

In major cities like Shanghai and Beijing, where the automotive industry thrives in China, there is not only a significant number of automobiles but also a well-developed industry for remanufacturing automotive components and a well-established formal channel for the disposal and recycling of end-of-life vehicles. However, in the developing regions of central and western China, there is a substantial risk associated with recycling end-of-life vehicles through informal channels, which can lead to severe environmental consequences and pose a serious challenge to the management of reverse supply chains [18,19]. Particularly, due to constraints on energy resources, sustainable development has become a global common goal [20], and establishing sustainable practices in the automotive supply chain contributes to achieving this goal [18,21]. Therefore, to increase the formal recycling of end-of-life vehicles, this study introduces a reverse supply chain for end-of-life vehicles that includes online recycling channels. Using game theory models, we discuss the impact of government subsidies on the pricing of end-of-life vehicle recycling. Additionally, based on equilibrium decision variable expressions for various parameters within different value ranges and the monotonicity and sensitivity relationships between these parameters and equilibrium decision variables, we attempt to explore recycling pricing and subsidy strategies applicable to the reverse supply chain of end-of-life vehicles. In particular, a two-level RSC game model, which is dominated by ELV parts remanufacturers and composed of remanufacturers, recyclers, and consumers, is constructed in this paper, taking into account consumer channel preferences. In addition to recycling ELVs through traditional offline recyclers, remanufacturers also build their own online recycling channels so that consumers can recycle ELVs through these two channels and receive government subsidies. Furthermore, the optimal strategy is derived from the model, and the sensitivity analysis of the RSC of ELVs is carried out on different subsidy methods. The following main issues are investigated in this research: the influence of different subsidy methods on the recycling pricing, recycling volume, and profit of the RSC of ELVs and the optimal subsidy strategy of the government.

The structure of this paper is as follows: Section 2 presents the literature on RSC recycling channels, pricing, and government subsidies, Section 3 expounds on the research questions and parameter settings, Section 4 explains the model construction and solution, Section 5 discusses the comparative analysis of the models, Section 6 presents a numerical example, followed by Section 7, which is the conclusion of this paper.

2. Literature Review

2.1. Recycling Channels of the Reverse Supply Chain

Remanufacturers can obtain the required ELV remanufactured parts via the RSC, which plays a vital role in recycling waste products and realizing a sustainable environment [22]. Remanufacturers collect qualified used products from consumers [23], and the recycling channel of second-hand products is the key issue for remanufacturers to make decisions [24]. A multi-channel recycling strategy can significantly improve recycling efficiency [25], and remanufacturing through manufacturers' channels or third-party channels is a common recycling channel [26]. The influencing factors of recycling channel selection for remanufacturers include the recycling cost [27], consumer preferences for remanufactured goods [28], channel competition [29] and channel cooperation [30], environmental impact [31], recovery price [32], etc. Selecting an appropriate recovery channel is an important part of operation and management in the reverse supply chain of ELVs [18].

With the continuous improvement in Internet technology and people's awareness of sustainable development, online and offline dual-channel recycling channels have begun to attract the attention of remanufacturers [33,34]. Online recycling has gradually become an effective way for remanufacturers to recycle waste products [35], and dual-channel RSC management combining online and offline recycling is of particular importance [33]. Feng et al. [36] used the RSC of dual-recycling channels including online recycling channels and discussed the advantages of dual-recycling channels over single-recycling channels. Zand et al. [37] built an online and offline closed-loop supply chain composed of manufacturers and retailers and analyzed the government's contribution to the optimal decision-making and profits of closed-loop supply chain members. Moreover, Sarkar et al. [38] studied inventory and pricing strategies using online and offline recycling channels and concluded that online and offline recycling channels increase the profit of the system.

In each of the above studies, the influencing factors of remanufacturers' selection of recycling channels and the necessity of online recycling channels are discussed. It is necessary to discuss in detail the optimal strategy of recycling channels under consumer channel preferences.

2.2. Recycling Pricing of the Reverse Supply Chain

Regarding the recycling pricing of the RSC, the optimal recycling pricing strategy in the supply chain was discussed in many studies. Fleischmann et al. [39] put forward that the price of recycled products is related to the ultimate profit of RSC members and the RSC system, and appropriate pricing can encourage enterprises to choose the Pareto optimal strategy. Savaskan et al. [40] investigated the pricing strategy in the remanufacturing RSC composed of a single manufacturer and retailer. Afterward, Savaskan and Van Wassenhove [41] considered the pricing strategy of direct and indirect RSC recycling under competition. Factors affecting RSC recycling pricing include technological innovation [42], social responsibility [43], recycling channel cooperation [44], supply chain information sharing [45], recycling price competition [46], etc. Recycling pricing directly affects the efficiency of recycling channels.

Ref. [18] proposed applying dual-recycling channels to ELV recycling management practice. Zhao et al. [47] explored the recycling pricing decision of remanufacturers under two mixed recycling channels. Taleizadeh and Sadeghi [48] discussed the recycling pricing strategies of direct recycling channels and traditional channels. Xu [49] conducted a study on the pricing decision of a closed-loop supply chain with recycling competition in dual-recycling channels. Li et al. [50] explored the revenue sharing between recyclers and retailers under the consideration of consumer preferences. Yang et al. [51] investigated the influence of online consumer reviews on the pricing strategy in a dual-recycling channel composed of manufacturers and retailers. Jin et al. [52] discussed the pricing decision and coordination of online and offline recycling channels under the influence of the power structure of RSC channels.

Previous research on RSC recycling pricing has proposed applying dual-recycling channels to the practice of RSC recycling pricing. However, for online and offline dual-channel recycling channels, more research on the influence of consumer channel preferences on recycling pricing is needed.

2.3. Consumer Behavior in the Reverse Supply Chain

In recent years, the increasing concern for environmental protection has promoted the implementation of automobile parts remanufacturing (APR) [53]. Remanufacturing plays a vital role in the field of sustainable development [54]. It realizes the unification of economic benefits and environmental benefits [55] and is regarded as one of the ways to combat global warming and natural resource depletion [56]. The government is crucial in promoting the recycling of scrapped products [57], which subsidizes remanufacturing and recycling for the purpose of maximizing social welfare [58].

The effects of government subsidies can be explored from the perspective of consumers, the environment, and supply chain members [59]. Hong, Zhang, Yu and Chu [13] explored the influence of consumers' environmental awareness on promoting remanufacturing in government subsidies for remanufactured products. Chen and Ulya [60] investigated the behavior of supply chain members under the government reward and punishment mechanism and found that the reward and punishment mechanism is a means to improve the product recycling rate. Zhang and Yu [61] explained in detail the formulation of government subsidy policy from the perspective of supply chain member pricing. Zhao and Sun [62] discussed the way to coordinate profit distribution among supply chain members using the government subsidy rate.

Incentives such as legislation, taxes, and cash subsidies have been adopted by the government to promote product remanufacturing [12]. Wu [63] explored the government's intervention in CLSC using taxation to reduce the environmental burden. A study by Yang and Chen [64] discussed the optimal level of carbon tax levied by the government in the supply chain composed of manufacturers and retailers. Chang et al. [65] explored the effect of government subsidies for recycling and remanufacturing with taxation. Afterward, Zhang et al. [66] investigated the role of strict government supervision in effectively recycling hazardous waste.

In each of the above studies, the positive effect of government subsidies on recycling is recognized, and the effects of subsidies and the role of government subsidies such as legislation, taxes, and cash are explored from the perspective of consumers, the environment, and supply chain members. However, further studies are required to discuss which end of the RSC the government should subsidize under consumers' online channel preferences.

The literature review above can be divided into three parts. Firstly, it explores reverse supply chain recycling channels, emphasizing the factors influencing the choice of recycling channels by remanufacturers. Secondly, it investigates recycling pricing strategies in reverse supply chains, particularly, the pricing practices of dual recycling channels. Furthermore, the review also discusses the impact of government subsidies on reverse supply chains. The literature review indicates, firstly, that there is limited existing research focusing on the recycling of end-of-life vehicles in the context of reverse supply chains (RSCs), while this study addresses the real-world problem of how to increase the recycling of end-of-life vehicles. Secondly, past research has paid little attention to the convenience value brought by online end-of-life car-recycling channels. Thirdly, considering the revenue generated from remanufacturing ELV components, we contemplate the impact of different government subsidy approaches on recycling pricing. In summary, this study contributes by discussing the necessity of online recycling channels and the positive role of government subsidies in product recycling, while also exploring optimal pricing strategies under different subsidy approaches.

3. Problem Description

Aiming at the problem of improving the recycling volume of ELVs, a two-level RSC game model, which is dominated by ELV parts remanufacturers and composed of remanufacturers, recyclers, and consumers, is constructed. In addition to recycling ELVs through traditional offline recyclers, remanufacturers also build their own online recycling channels so that consumers can recycle ELVs through these two channels. Remanufacturers make profits by disassembling ELVs and remanufacturing parts, and recyclers make profits by recycling activities. To improve the recycling volume of ELVs, the government conducts financial subsidy strategies for ELV recycling, including government non-subsidy (Model d), government subsidy for remanufacturers (Model G), and government subsidy for consumers (Model C). The reverse supply chain structure of ELVs is constructed as shown in Figure 1. By analyzing the influence of government subsidies on the optimal recovery pricing of the RSC, the recycling volume, and profit, this paper discusses the government's subsidy strategy from the perspective of maximizing the recycling volume and profit of the RSC.

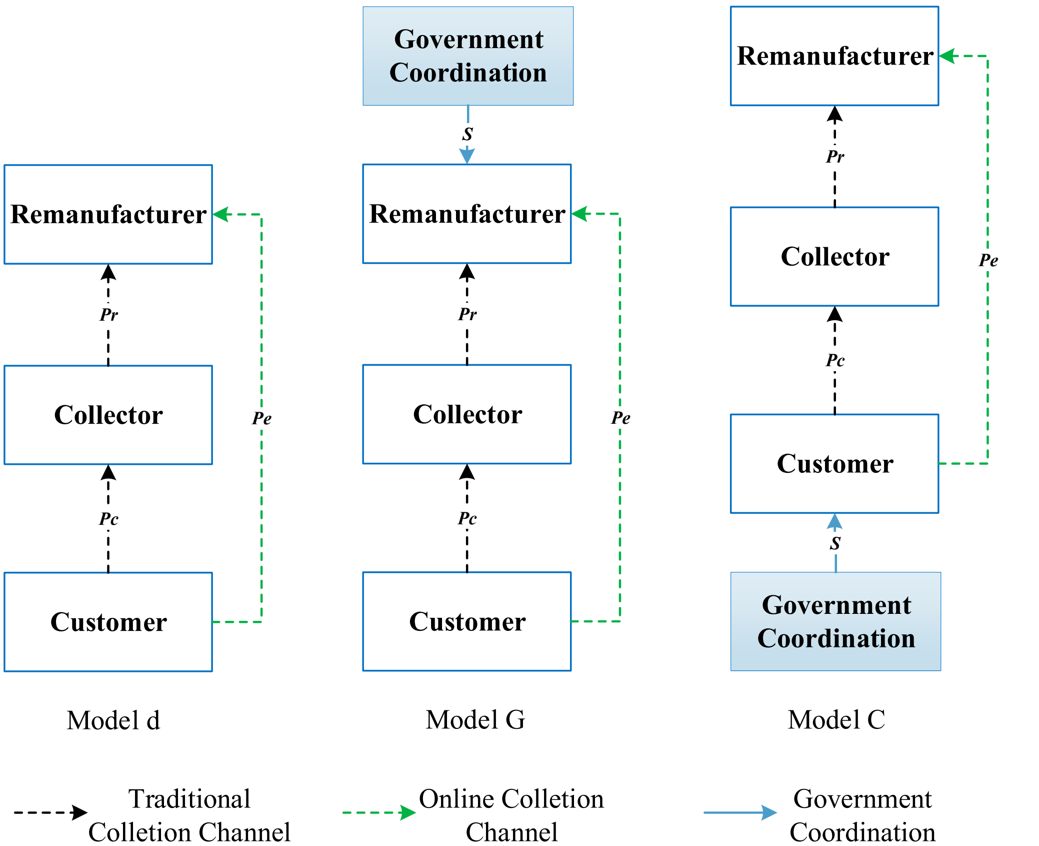

Figure 1. Reverse supply chain structure of ELVs.

3.1. Parameter Setting

The relevant parameters involved in the basic assumptions are listed in Table 1.

Table 1. Parameter definitions.

Parameter	Definition
Q_c	Recycling volume of offline recyclers
Q_e	Online recycling
I_c	Input cost of offline recycling channels
I_e	Input cost of offline recycling channels
ω	Income from dismantling ELVs and remanufacturing parts
s	per unit to remanufacturers and consumers
v	Consumer value perception of ELVs
θ	Consumer behavior preferences for online recycling channels
φ	Convenience value of online recycling channels for consumers
Decision variable	
p_r	Offline recyclers transfer payment prices to remanufacturers
p_c	Offline recyclers' recycling price
p_e	Online recycling price

This paper examines three ELV recycling models: Model d, which involves a dual-channel online and offline recycling approach; Model G, which is a government subsidy model for ELV component remanufacturers; and Model C, which is a government subsidy model for consumers. The focus is on studying the optimal pricing strategies for these three recycling models, with competition not centered around collection efforts but rather on recycling pricing. Specifically, terms like recycling price, collection price [14], and acquisition price [32] are used interchangeably and convey the same meaning. In this paper, "recycling price" is the term of choice. Apart from the parameters listed in Table 1, superscripts d, G, and C represent Models d, G, and C, while subscripts r, c, and e represent ELV component remanufacturing factories, ELV recyclers, and online ELV recyclers, respectively. An asterisk (*) is used to signify the optimal solution.

3.2. Model Gssumptions

The basic assumptions made on the recycling model in Figure 1 are as follows:

We assume that in the reverse supply chain of ELVs, consumers' recycling valuation of ELVs is v, and each consumer values the recycling of ELVs differently with uncertainty. To simplify the analysis, it is assumed that v is uniformly distributed from 0 to 1 among consumers, with a probability density of 1 [36,67].

The recycling utility of consumers in offline recycling channels is expressed by $U_c(v)$, and when the government subsidizes consumers, it is expressed by ${U_c}^C(v)$.

The recycling price of offline recyclers is set as $p_c \in [0,1]$. When the offline purchase price is higher than the consumer's valuation of the offline recycling channel, the recycling transaction is realized, i.e., when $p_c - v > 0$, consumers are willing to recycle ELVs through the offline recycling channel, namely, $U_c(v) = p_c - v$ [36,68]. When the government subsidizes consumers, the recycling effect of consumers in offline recycling channels is ${U_c}^C(v) = p_c - v + s$. Therefore, the recycling volume $Q_c = \int_0^{p_c} dv = p_c$ only through offline recycling channels, where $0 < p_c \leq 1$, which is similar to that assumed by [69]. When the government subsidizes consumers, the recycling volume through offline recycling channels is ${Q_c}^C = \int_0^{p_c+s} dv = p_c + s$.

The recycling utility of consumers in online recycling channels is expressed by $U_e(v)$, and when the government subsidizes consumers, it is expressed by ${U_e}^C(v)$.

To reflect consumer behavior preferences for online recycling channels, the parameter θ is set as consumer acceptance of online recycling channels. It is assumed that $\theta > 1$, and the closer θ is to 1, the higher the consumer behavior preferences for online recycling channels. In addition, we set the parameter φ to be $\varphi > 0$ for the convenience value brought to consumers by online recycling channels, such as the value of handling circulation procedures and saving time. Let the online recycling price be $p_e \in [0,1]$, which satisfies $p_e > p_c$. When the online purchase price and convenience value are higher than consumers' valuation of online recycling channels, the recycling transaction is realized, i.e., when

$p_e - \theta v + \varphi > 0$, consumers are willing to recycle ELVs, namely, $U_e(v) = p_e - \theta v + \varphi$. Therefore, the recycling volume $Q_e = \int_0^{\frac{p_e + \varphi}{\theta}} dv = \frac{p_e + \varphi}{\theta}$ only through online recycling channels, where $0 < p_e \leq 1$. When the government subsidizes consumers, the recycling effect of consumers in offline recycling channels is ${U_e}^C(v) = p_e - \theta v + \varphi + s$. Therefore, when the government subsidizes consumers, the recycling volume through online recycling channels is ${Q_e}^C = \int_0^{\frac{p_e + \varphi + s}{\theta}} dv = \frac{p_e + \varphi + s}{\theta}$.

When $p_c - v > 0$ and $p_c - v > p_e - \theta v + \varphi$, i.e., $p_c > v > \frac{p_e + \varphi - p_c}{\theta - 1}$, recycling will be performed through offline recycling channels. When $p_e - \theta v + \varphi > 0$ and $p_e - \theta v + \varphi > p_c - v$, i.e., $\min\left\{\frac{p_e + \varphi}{\theta}, \frac{p_e + \varphi - p_c}{\theta - 1}\right\} > v$, recycling will be performed through online channels. When $v = \frac{p_e + \varphi - p_c}{\theta - 1}$, there is no difference between traditional recycling channels and online recycling channels. When $v > \max\left\{p_c, \frac{p_e + \varphi}{\theta}\right\}$, no recycling occurs. Let $v^c = p_c$ and $v^e = \frac{p_e + \varphi}{\theta}$, where v^c and v^e are the critical values of traditional recycling channels and online recycling channels, respectively. When the consumer's perceived value $v^c > v^e$, consumers choose neither offline nor online recycling channels in $v \in [v^c, 1]$. In $v \in [v^{ce}, v^c]$, consumers prefer offline recycling. In $v \in [0, v^{ce}]$, consumers prefer online recycling. In contrast, when the consumer's perceived value $v^c < v^e$, consumers only choose online recycling $v \in [0, v^e]$. It can be concluded from the above analysis that the recycling volume of dual-recycling channels, i.e., offline channels and online channels, is:

$$ {Q_c}^d = {Q_c}^A = \begin{cases} p_c - \frac{p_e + \varphi - p_c}{\theta - 1}, & p_c \geq \frac{p_e + \varphi}{\theta} \\ 0, & p_c < \frac{p_e + \varphi}{\theta} \end{cases} \quad (1) $$

$$ {Q_e}^d = {Q_e}^A = \begin{cases} \frac{p_e + \varphi - p_c}{\theta - 1}, p_c \geq \frac{p_e + \varphi}{\theta} \\ \frac{p_e + \varphi}{\theta}, p_c < \frac{p_e + \varphi}{\theta} \end{cases} \quad (2) $$

Similar to the above analysis, we can conclude that when the government subsidizes consumers, the recycling volume of dual-recycling channels, i.e., offline channels and online channels, is:

$$ {Q_c}^C = \begin{cases} p_c + s - \frac{p_e + \varphi - p_c}{\theta - 1}, p_c \geq \frac{p_e + \varphi + s}{\theta} - s \\ 0, p_c < \frac{p_e + \varphi + s}{\theta} - s \end{cases} \quad (3) $$

$$ {Q_e}^C = \begin{cases} \frac{p_e + \varphi - p_c}{\theta - 1}, p_c \geq \frac{p_e + \varphi + s}{\theta} - s \\ \frac{p_e + \varphi + s}{\theta}, p_c < \frac{p_e + \varphi + s}{\theta} - s \end{cases} \quad (4) $$

Let us assume that the offline recyclers of ELVs have an input cost coefficient of I_c in their recycling channels and an input cost coefficient of I_e in their online recycling channels. Therefore, the input of offline recycling channels for recycling ELVs is $I_c Q_c$, and that of online recycling channels for recycling ELVs is $I_e Q_e$.

Let us assume that ELVs receive benefits from remanufacturing parts after dismantling and the average unit income of disassembled non-ferrous metals is ω, which satisfies $\omega > I_c$, $\omega > I_e$, and $I_c > I_e$, assuming ω is an exogenous variable that does not influence the quantity of recycled end-of-life vehicles. The rationale behind this assumption is that, due to the presence of other potential remanufacturers who may sell similar or identical remanufactured components, the price ω at which remanufacturers sell remanufactured components could be entirely influenced by market competition. Therefore, in the sales supply chain of remanufactured products, remanufacturers are the recipients of the price ω.

Assume that the government subsidizes s per unit to remanufacturers and consumers.

4. Model Construction and Solution

In this paper, we designed the online and offline dual-recycling channel RSC of ELVs under consumer channel preferences, explored the optimal pricing strategy of RSCs, and analyzed the influence of the government's subsidy strategy for ELV recycling on ELV recycling, in order to meet the challenge of increasing the recycling volume of the reverse supply chain of ELVs.

4.1. Government Non-Subsidy—Model d

In Model d of the government non-subsidy for the recycling of ELVs, the profit functions of remanufacturers and recyclers are as follows:

$$\Pi_{\mathrm{r}}{}^{\mathrm{d}} = \left(\omega - p_e{}^d\right)Q_e{}^d + \left(\omega - p_r{}^d\right)Q_c{}^d - I_e * Q_e{}^d \tag{5}$$

$$\Pi_{\mathrm{c}}{}^{\mathrm{d}} = \left(p_r{}^d - p_c{}^d\right)Q_c{}^d - I_c * Q_c{}^d \tag{6}$$

In the above function, $p_c{}^d \geq \frac{p_e{}^d+\varphi}{\theta}$.

$\Pi_{\mathrm{c}}{}^{\mathrm{d}}$ is a concave function of the decision variable $p_c{}^d$, while $\Pi_{\mathrm{r}}{}^{\mathrm{d}}$ is a joint concave function about the decision variables $p_e{}^d$ and $p_r{}^d$. Since there is a unique optimal solution for $\Pi_{\mathrm{c}}{}^{\mathrm{d}}$ and $\Pi_{\mathrm{r}}{}^{\mathrm{d}}$, the optimal decision of the profit function can be obtained using the reverse induction method. See the Appendix A for the proof process.

Theorem 1. *In Model d, the optimal decisions of the remanufacturer and the recycler are* $p_r{}^{d*} = \frac{I_c+\omega}{2}$, $p_e{}^{d*} = \frac{-I_e+\omega-\varphi}{2}$, *and* $p_c{}^{d*} = \frac{-I_e+\omega-I_c\theta+\omega\theta+\varphi}{4\theta}$, *respectively. See the Appendix A for the proof process.*

According to Theorem 1, we can obtain:

$$Q_c{}^{d*} = -\frac{-I_e + \omega + I_c\theta - \omega\theta + \varphi}{4(-1+\theta)} \tag{7}$$

$$Q_e{}^{d*} = \frac{I_e + \omega(-1+\theta) + I_c\theta - 2I_e\theta - \varphi + 2\theta\varphi}{4(-1+\theta)\theta} \tag{8}$$

$$\begin{aligned}\Pi_{\mathrm{r}}{}^{\mathrm{d}*} = {} & \frac{(I_e-\omega-\varphi)(\omega-I_c\theta-\omega\theta-I_e+2I_e\theta+\varphi-2\theta\varphi)}{8(-1+\theta)\theta} \\ & + \frac{(I_c-\omega)(-I_e+\omega+I_c\theta-\omega\theta+\varphi)}{8(-1+\theta)}\end{aligned} \tag{9}$$

$$\Pi_{\mathrm{c}}{}^{\mathrm{d}*} = \frac{(-I_e + \omega + I_c\theta - \omega\theta + \varphi)^2}{16(-1+\theta)\theta} \tag{10}$$

Under dual-recycling channels, the constraint condition $p_c{}^d \geq \frac{p_e{}^d+\varphi}{\theta}$ needs to be established. Equations (5) and (6) are proven to yield negative Hessian matrices, and the functions derived from these equations are established as strictly concave. This establishes the existence of a unique optimal solution, allowing the use of reverse induction to determine the optimal decision for the profit function. The detailed proof process is provided in the Appendix A. The lemma is as follows:

Lemma 1. *Under dual-recycling channels for ELVs, if* $\theta \geq -\frac{-I_e+\omega+\varphi}{I_c-\omega}$ *is established, the constraint condition* $p_c{}^d \geq \frac{p_e{}^d+\varphi}{\theta}$ *is established. See the Appendix A for the proof process.*

4.2. Government Subsidy for Remanufacturers—Model G

When subsidizing ELV parts remanufacturers, considering the role of online recycling in improving the recycling volume in the previous section, the government chooses to subsidize the construction costs of online recycling channels built by remanufacturers themselves, and the subsidy amount is $I_e * {Q_c}^A$. In Model G of the government subsidy for ELV parts remanufacturers, the profit functions of the remanufacturers and recyclers are as follows:

$$\Pi_r{}^A = \left(\omega - {p_e}^A\right){Q_e}^A + \left(\omega - {p_r}^A\right){Q_c}^A \tag{11}$$

$$\Pi_c{}^A = \left({p_r}^A - {p_c}^A\right){Q_c}^A - I_c * {Q_c}^A \tag{12}$$

In the above profit function, ${p_c}^A \geq \frac{{p_e}^A + \varphi}{\theta}$.

It is proved that $\Pi_c{}^A$ is a concave function of the decision variable ${p_c}^A$, while $\Pi_r{}^A$ is a joint concave function of the decision variables ${p_e}^A$ and ${p_r}^A$, and the proof process is the same as that of Model d and is abbreviated. Since there is a unique optimal solution for $\Pi_c{}^A$ and $\Pi_r{}^A$, the optimal decision of the profit function can be obtained using the reverse induction method. The theorem is as follows:

Theorem 2. *In Model G, the optimal decisions of the remanufacturer and the recycler are* ${p_r}^{A*} = \frac{I_c + \omega}{2}$, ${p_e}^{A*} = \frac{\omega - \varphi}{2}$, *and* ${p_c}^{A*} = \frac{\omega - I_c\theta + \omega\theta + \varphi}{4\theta}$, *respectively. See the Appendix A for the proof process.*

According to Theorem 2, we can obtain:

$$Q_c{}^{A*} = -\frac{\omega + I_c\theta - \omega\theta + \varphi}{4 - 4\theta} \tag{13}$$

$$Q_e{}^{G*} = \frac{\omega - I_c\theta - \omega\theta + \varphi - 2\theta\varphi}{4(1-\theta)\theta} \tag{14}$$

$$\Pi_r{}^{G*} = \frac{{I_c}^2\theta^2 + \omega^2\left(-1+\theta^2\right) - 2\omega(-1+\theta)(I_c\theta - \varphi) + 2I_c\theta\varphi + (-1+2\theta)\varphi^2}{8(-1+\theta)\theta} \tag{15}$$

$$\Pi_c{}^{G*} = \frac{(\omega + I_c\theta - \omega\theta + \varphi)^2}{16(-1+\theta)\theta} \tag{16}$$

Under dual-recycling channels, the constraint condition ${p_c}^A \geq \frac{{p_e}^A + \varphi}{\theta}$ needs to be established. The lemma is as follows:

Lemma 2. *Under dual-recycling channels of ELVs, if* $\theta \geq -\frac{-I_e + \omega + \varphi}{I_c - \omega}$ *is established, the constraint condition* ${p_c}^d \geq \frac{{p_e}^d + \varphi}{\theta}$ *is established. See the Appendix A for the proof process.*

4.3. Government Subsidy for Consumers—Model C

This section discusses government subsidies to consumers. Let s be the government's unit subsidy to consumers, then:

$$U_c(v) = p_c - v + s \tag{17}$$

$$U_e(v) = p_e - \theta v + \varphi + s \tag{18}$$

Similar to the analysis in the assumption section, we can conclude that the recycling volume of dual-recycling channels, i.e., offline channels and online channels, is:

$$Q_c = \begin{cases} p_c + s - \frac{p_e + \varphi - p_c}{\theta - 1}, p_c \geq \frac{p_e + \varphi + s}{\theta} - s \\ 0, p_c < \frac{p_e + \varphi + s}{\theta} - s \end{cases} \tag{19}$$

$$Q_e = \begin{cases} \frac{p_e + \varphi - p_c}{\theta - 1}, p_c \geq \frac{p_e + \varphi + s}{\theta} - s \\ \frac{p_e + \varphi + s}{\theta}, p_c < \frac{p_e + \varphi + s}{\theta} - s \end{cases} \tag{20}$$

In Model C of the government subsidy for consumers, the profit functions of remanufacturers and recyclers are as follows:

$$\Pi_r{}^C = \left(\omega - p_e{}^C\right) Q_e{}^C + \left(\omega - p_r{}^C\right) Q_c{}^C - I_e * Q_e{}^C \tag{21}$$

$$\Pi_c{}^C = \left(p_r{}^C - p_c{}^C\right) Q_c{}^C - I_c * Q_c{}^C \tag{22}$$

In the above profit function, $p_c{}^C \geq \frac{p_e{}^C + \varphi + s}{\theta} - s$.

It is proved that $\Pi_c{}^C$ is a concave function of the decision variable $p_c{}^C$, while $\Pi_r{}^C$ is a joint concave function about the decision variables $p_e{}^C$ and $p_r{}^C$, and the proof process is abbreviated. Since there is a unique optimal solution for $\Pi_c{}^C$ and $\Pi_r{}^C$, the optimal decision of the profit function can be obtained using the reverse induction method. The theorem is as follows:

Theorem 3. *In Model C, the optimal decisions of the remanufacturer and the recycler are* $p_r{}^{C*} = \frac{I_c - s + \omega}{2}$, $p_e{}^{C*} = \frac{-I_e - s + \omega - \varphi}{2}$, *and* $p_c{}^{C*} = \frac{-I_e + s + \omega - I_c\theta - 3s\theta + \omega\theta + \varphi}{4\theta}$, *respectively. See the Appendix A for the proof process.*

According to Theorem 3, we can obtain:

$$Q_c{}^{C*} = -\frac{-I_e + s + \omega + I_c\theta - s\theta - \omega\theta + \varphi}{4(-1+\theta)} \tag{23}$$

$$Q_e{}^{C*} = \frac{I_e + (s+\omega)(-1+\theta) + I_c\theta - 2I_e\theta - \varphi + 2\theta\varphi}{4(-1+\theta)\theta} \tag{24}$$

$$\begin{aligned} \Pi_r{}^{C*} &= \tfrac{(I_e - s - \omega - \varphi)(s + \omega - I_c\theta - s\theta - \omega\theta - I_e + 2I_e\theta + \varphi - 2\theta\varphi)}{8(-1+\theta)\theta} \\ &+ \tfrac{(I_e - s - \omega)(-I_e + s + \omega + I_c\theta - s\theta - \omega\theta + \varphi)}{8(-1+\theta)} \end{aligned} \tag{25}$$

$$\Pi_c{}^{C*} = \frac{(-I_e + s + \omega + I_c\theta - s\theta - \omega\theta + \varphi)^2}{16(-1+\theta)\theta} \tag{26}$$

Under dual-recycling channels, the constraint condition $p_c{}^{C*} \geq \frac{p_e{}^C + \varphi + s}{\theta} - s$ needs to be established. The lemma is as follows:

Lemma 3. *Under dual-recycling channels of ELVs, if* $\theta \geq -\frac{-I_e + s + \omega + \varphi}{I_c - s - \omega}$ *is established, the constraint condition* $p_c{}^{C*} \geq \frac{p_e{}^C + \varphi + s}{\theta} - s$ *is established. See the Appendix A for the proof process.*

5. Comparative Analysis of Models

The optimal decision of the government non-subsidy Model d, government subsidy for remanufacturers Model G, and government subsidy for consumers Model C are shown in Tables 2 and 3.

Table 2. Comparison between Model d and Model G.

Variable	Model d	Model G
${p_r}^*$	$\frac{I_c+\omega}{2}$	$\frac{I_c+\omega}{2}$
${p_e}^*$	$\frac{-I_e+\omega-\varphi}{2}$	$\frac{\omega-\varphi}{2}$
${p_c}^*$	$\frac{-I_e+\omega-I_c\theta+\omega\theta+\varphi}{4\theta}$	$\frac{\omega-I_c\theta+\omega\theta+\varphi}{4\theta}$
${Q_c}^*$	$-\frac{-I_e+\omega+I_c\theta-\omega\theta+\varphi}{4(-1+\theta)}$	$-\frac{\omega+I_c\theta-\omega\theta+\varphi}{4-4\theta}$
${Q_e}^*$	$\frac{I_e+\omega(-1+\theta)+I_c\theta-2I_e\theta-\varphi+2\theta\varphi}{4(-1+\theta)\theta}$	$\frac{\omega-I_c\theta-\omega\theta+\varphi-2\theta\varphi}{4(1-\theta)\theta}$
${\Pi_r}^*$	$\frac{(I_e-\omega-\varphi)(\omega-I_c\theta-\omega\theta-I_e+2I_e\theta+\varphi-2\theta\varphi)}{8(-1+\theta)\theta}+\frac{(I_c-\omega)(-I_e+\omega+I_c\theta-\omega\theta+\varphi)}{8(-1+\theta)}$	$\frac{{I_c}^2\theta^2+\omega^2\left(-1+\theta^2\right)-2\omega(-1+\theta)(I_c\theta-\varphi)}{8(-1+\theta)\theta}+\frac{2I_c\theta\varphi+(-1+2\theta)\varphi^2}{8(-1+\theta)\theta}$
${\Pi_c}^*$	$\frac{(-I_e+\omega+I_c\theta-\omega\theta+\varphi)^2}{16(-1+\theta)\theta}$	$\frac{(\omega+I_c\theta-\omega\theta+\varphi)^2}{16(-1+\theta)\theta}$

Table 3. Comparison between Model d and Model C.

Variable	Model d	Model G
${p_r}^*$	$\frac{I_c+\omega}{2}$	$\frac{I_c-s+\omega}{2}$
${p_e}^*$	$\frac{-I_e+\omega-\varphi}{2}$	$\frac{-I_e-s+\omega-\varphi}{2}$
${p_c}^*$	$\frac{-I_e+\omega-I_c\theta+\omega\theta+\varphi}{4\theta}$	$\frac{-I_e+s+\omega-I_c\theta-3s\theta+\omega\theta+\varphi}{4\theta}$
${Q_c}^*$	$-\frac{-I_e+\omega+I_c\theta-\omega\theta+\varphi}{4(-1+\theta)}$	$-\frac{-I_e+s+\omega+I_c\theta-s\theta-\omega\theta+\varphi}{4(-1+\theta)}$
${Q_e}^*$	$\frac{I_e+\omega(-1+\theta)+I_c\theta-2I_e\theta-\varphi+2\theta\varphi}{4(-1+\theta)\theta}$	$\frac{I_e+(s+\omega)(-1+\theta)+I_c\theta-2I_e\theta-\varphi+2\theta\varphi}{4(-1+\theta)\theta}$
${\Pi_r}^*$	$\frac{(I_e-\omega-\varphi)(\omega-I_c\theta-\omega\theta-I_e+2I_e\theta+\varphi-2\theta\varphi)}{8(-1+\theta)\theta}+\frac{(I_c-\omega)(-I_e+\omega+I_c\theta-\omega\theta+\varphi)}{8(-1+\theta)}$	$\frac{(I_e-s-\omega-\varphi)(s+\omega-I_c\theta-s\theta-\omega\theta-I_e+2I_e\theta+\varphi-2\theta\varphi)}{8(-1+\theta)\theta}+\frac{(I_e-s-\omega)(-I_e+s+\omega+I_c\theta-s\theta-\omega\theta+\varphi)}{8(-1+\theta)}$
${\Pi_c}^*$	$\frac{(-I_e+\omega+I_c\theta-\omega\theta+\varphi)^2}{16(-1+\theta)\theta}$	$\frac{(-I_e+s+\omega+I_c\theta-s\theta-\omega\theta+\varphi)^2}{16(-1+\theta)\theta}$

Conclusion 1. *In the government non-subsidy model, for any $\theta > 1$, ${Q_c}^{d*}$ and ${\Pi_c}^{d*}$ are increasing functions of θ, ${p_c}^{d*}$, ${Q_e}^{d*}$, ${\Pi_r}^{d*}$, and the total RSC recycling volume and total profit are also decreasing functions of θ. See the Appendix A for the proof process.*

Conclusion 1 shows that the change in consumer preference has a positive impact on the recycling and profit of the reverse supply chain of ELVs in the government non-subsidy Model d. The increase in consumer preferences results in an increase in the recycling volume and profit of remanufacturers. The increase in consumer preferences results in a decrease in the recycling volume and profit of offline recycling. But on the whole, the increase in consumer preferences for online channels improved the total recycling volume and profit in the RSC.

Conclusion 2. *The relationship between recycling price, recycling volume, and profit in Model d of government non-subsidy and Model G of the government subsidy for remanufacturers is as follows: ${p_e}^{A*} > {p_e}^{d*}$, $\left({Q_c}^{A*} + {Q_e}^{A*}\right) > \left({Q_c}^{d*} + {Q_e}^{d*}\right)$, $\left({\Pi_c}^{A*} + {\Pi_r}^{A*}\right) > \left({\Pi_c}^{d*} + {\Pi_r}^{d*}\right)$. See the Appendix A for the proof process.*

Conclusion 2 shows that when the government subsidizes remanufacturers, the increase in recycling price promotes the enthusiasm of consumers to recycle ELV through recycling channels and increases the recycling volume of the RSC, achieving the purpose of government subsidies. In addition, the increase in the recycling volume of RSC leads to an increase in overall profit.

Conclusion 3. *In Model C of the government subsidy for consumers, recycling prices* ${p_c}^{C*}$, ${p_e}^{C*}$, *and* ${p_r}^{C*}$ *are decreasing functions of the consumer subsidy s. The recycling volume* ${Q_c}^{C*}$ *and* ${Q_e}^{C*}$ *are increasing functions of the consumer subsidy s. Profits* ${\Pi_c}^{C*}$ *and* ${\Pi_r}^{C*}$ *are increasing functions of s. See the Appendix A for the proof process.*

Conclusion 3 shows that the total recycling volume and total profit of the RSC increase with the increase in government subsidies to consumers, achieving the purpose of government subsidies. Government subsidies to consumers will be transmitted to the RSC through consumers. With the increase in government subsidy s for remanufacturers, the recycling price will decrease and the recycling volume will increase, contributing more profits to the various participants in the RSC.

Conclusion 4. *In Model d of government non-subsidy and Model C of the government subsidy for consumers, the relationship between recycling price, recycling volume, and profit is as follows:* ${p_c}^{C*} < {p_c}^{d*}$, ${p_e}^{C*} < {p_e}^{d*}$, ${p_r}^{C*} < {p_e}^{d*}$, ${Q_c}^{C*} > {Q_c}^{d*}$, ${Q_e}^{C*} > {Q_e}^{d*}$, $\left({\Pi_c}^{C*} + {\Pi_r}^{C*}\right) > \left({\Pi_c}^{d*} + {\Pi_r}^{d*}\right)$. *See the Appendix A for the proof process.*

Conclusion 4 shows that when the government subsidizes consumers, the recycling price is reduced. However, because the government subsidies to consumers are still higher than the reduced part, consumers are willing to recycle ELVs, which increases the recycling volume of the RSC, achieving the purpose of government subsidies. In addition, the increase in the recycling volume of the RSC leads to an increase in overall profit.

Conclusion 5. *The relationship between the total recycling volume of Model G of the government subsidy for remanufacturers and Model C of the government subsidy for consumers is as follows: Under* $\theta = -\frac{\omega+\varphi}{I_c-\omega}$, when $s > \frac{I_e(I_c-\omega)}{(I_c-2\omega-\varphi)}$, $\left({Q_c}^{C*} + {Q_e}^{C*}\right) > \left({Q_c}^{A*} + {Q_e}^{A*}\right)$. *See the Appendix A for the proof process.*

Conclusion 5 shows that, under a certain consumer preference, when the government subsidizes consumers strongly, the recycling volume of the government's subsidy for consumers is greater than that of the government's subsidy for remanufacturers. However, when fewer subsidies are given by the government to consumers, the recycling volume of the government's subsidy for remanufacturers is greater than that of the government's subsidy for consumers. The strength of the government's subsidy for consumers determines whether the government subsidizes remanufacturers or consumers.

6. Numerical Analysis

In this section, a more in-depth analysis is carried out in order to verify the correctness of the conclusion. Numerical examples are used to further analyze and illustrate the influence of consumer preferences and government subsidy strategies on the reverse supply chain of ELVs. The following data are used in this numerical example: $I_e = 0.1$, $I_c = 0.2$, $\omega = 0.8$, and $\varphi = 0.1$, and let $\theta = -\frac{\omega+\varphi}{I_c-\omega}$. Since the assumption stated in Section 3.1 is that ELV-related revenue from channels like component remanufacturing is denoted as ω and satisfies $\omega > I_c$ and $\omega > I_e$, this means that the revenue must be greater than the input costs. Here, I_c and I_e represent the input costs for establishing offline and online recycling channels, respectively, with the condition $I_c > I_e$. This assumption implies that the investment cost for establishing offline recycling channels is greater than that for online channels. Additionally, θ represents consumer preference for online recycling channels, and φ represents the convenience value brought by online recycling channels. As per the assumption in Section 3.1, $\varphi > 0$, indicating that online recycling channels provide value to consumers compared with offline channels. Furthermore, Lemma 2 establishes that under the ELV dual-recycling channel, $\theta \geq -\frac{\omega+\varphi}{I_c-\omega}$ holds. Therefore, the data used in the numerical examples in this study can meet the assumptions and relevant lemmas presented in this paper.

When the government subsidizes ELV parts remanufacturers, the government subsidizes the construction costs of the remanufacturers' self-built online recycling channels, which increases the total recycling volume of the reverse supply chain, thus achieving the purpose of government subsidies. In addition, the increase in the recycling volume of ELVs leads to an increase in the overall profit of ELV recycling, as shown in Figures 2 and 3. This is consistent with the discussion in Conclusion 2.

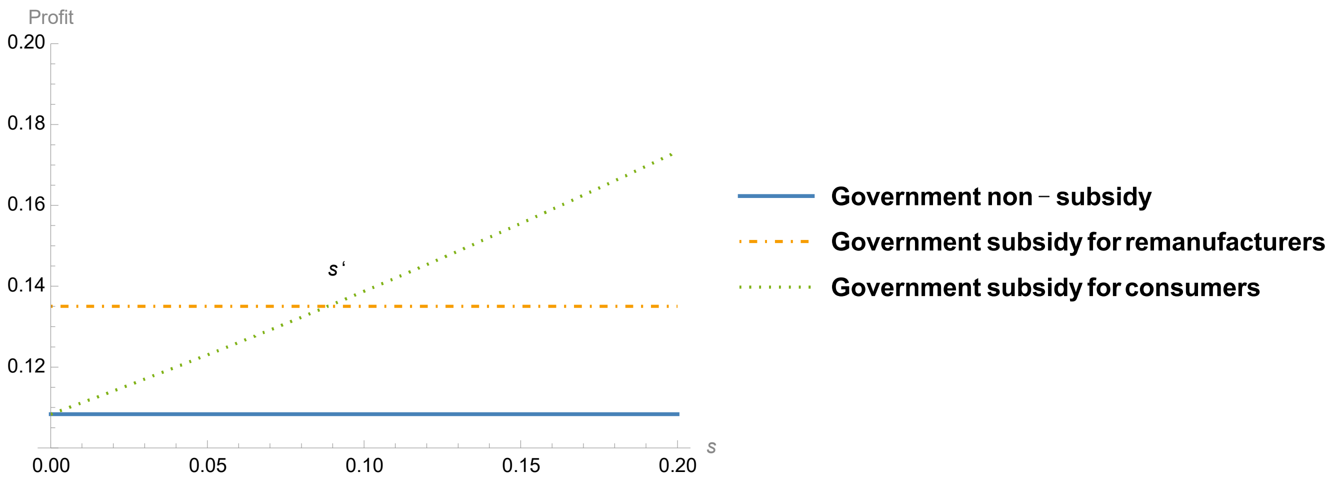

Figure 2. Influence of government subsidies on the total profit.

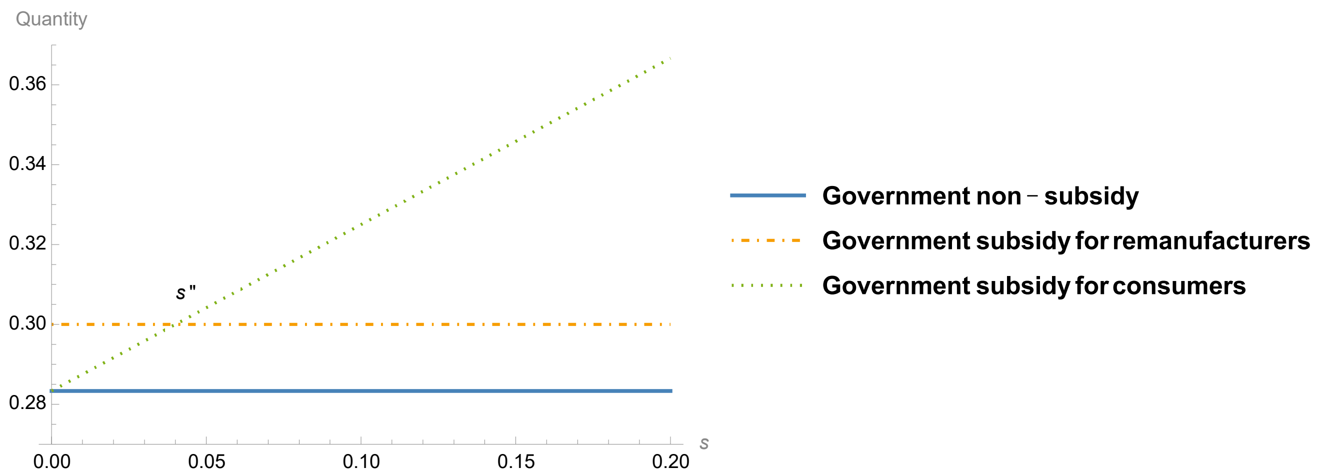

Figure 3. Influence of government subsidies on the total recycling volume.

When the government subsidizes consumers, as shown in Figures 2 and 3, the unit subsidy of the government to consumers is s. The increase in profit and recycling volume of the reverse supply chain of ELVs is related to the intensity of government subsidies to consumers. The greater the intensity of subsidies, the greater the recycling volume and profit, which is consistent with the discussion in Conclusion 3. When government subsidies to consumers and government non-subsidies are compared, government subsidies can improve recycling volume and recycling profit, which is consistent with the discussion in Conclusion 4.

Government subsidies to remanufacturers and to consumers have different effects on the profits of the RSC. When the government subsidy s for consumers is $s < s'$, the profit when the government subsidizes remanufacturers is higher than that when the government subsidizes consumers. When the government subsidy s for consumers is $s > s'$, the profit when the government subsidizes consumers is higher than that when the government subsidizes remanufacturers. As shown in Figure 2, this is consistent with the discussion of Conclusion 5. In this example, $s' = 0.083$. Government subsidies to remanufacturers and consumers not only affect the profit of the RSC but also affect its recycling volume.

When the government subsidy s for consumers is $s < s''$, the recycling volume of the government's subsidy for remanufacturers is greater than that for consumers, and when the government subsidy s for consumers is $s > s''$, the recycling volume of government's subsidy for consumers is greater than that for remanufacturers, as shown in Figure 3. In this example, $s'' = 0.04$.

Consumer preferences and government subsidies act on the reverse supply chain of ELVs simultaneously, and subsidies to remanufacturers and consumers can produce a superposition effect, as shown in Figure 4. In the model of the government subsidy for remanufacturers, the recycling volume of ELVs is greater than that when the government does not subsidize and increases with the increase in consumer preferences, which is consistent with the discussion in Conclusion 2. In the model of the government subsidy for consumers, the recycling volume of ELVs increases with the increase in consumer preferences and government subsidies. Does the government subsidize remanufacturers or consumers? It depends on consumer preferences and government subsidies. In this example, $\theta = 1.33$, and when $s > 0.39$, government subsidies to consumers may result in a greater recycling volume.

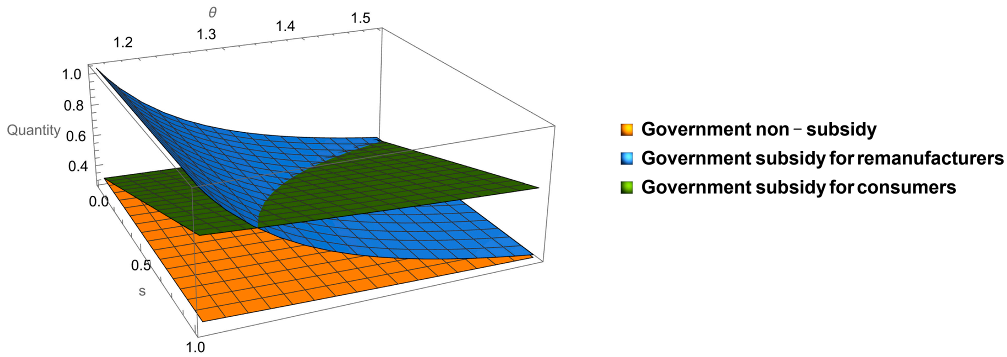

Figure 4. Influence of consumer preferences and subsidy for consumers.

7. Conclusions

To improve resource utilization and protect the environment, the government gives financial subsidies to ELV parts remanufacturers and consumers, so as to improve the recycling volume and profit of the reverse supply chain of ELVs. Based on the research of recycling channel design, this paper constructs three models of the reverse supply chain of ELVs: government non-subsidy, government subsidy for remanufacturers, and government subsidy for consumers. By comparing and analyzing these models, it discusses the optimal recycling pricing, profit, and recycling volume of the reverse supply chain of ELVs under different subsidy strategies to explore government subsidy strategies. The research results of this paper provide a reference for the government in formulating subsidy strategies:

1. The increase in consumer preferences for online recycling has a positive effect on the recycling volume and profit of the reverse supply chain of ELVs, which is reflected in three models: government non-subsidy, government subsidy for remanufacturers, and government subsidy for consumers. When formulating subsidy methods, the government should give priority to subsidizing and encouraging the productivity brought about by technological progress. In the discussion of this paper, the online recycling platform based on Internet technology provides a convenient one-stop recycling service for consumers who need to deal with ELVs, and consumer preferences for online recycling make the online recycling platform enter the recycling practice.
2. Both government subsidies for remanufacturers and consumers have an impact on the RSC. Does the government subsidize remanufacturers or consumers? It depends on the intensity of government subsidies under a certain consumer preference. From the point of view of RSC recycling, when the subsidy intensity is small, the subsidy method of subsidizing remanufacturers can bring more recycling for the RSC, while

the subsidy method of subsidizing consumers can bring more recycling when the subsidy intensity is increased. The purpose of government subsidy is not only to increase the recycling volume to increase the utilization rate of resources but also to encourage the development of the remanufacturing industry. In this case, it is necessary to consider the subsidy method from the perspective of RSC profit. When subsidies are given at the lower middle level, higher profits can be earned by subsidizing remanufacturers, and, on the contrary, higher RSC profits can be earned by subsidizing consumers.

Expanding on the foundation of this study, future investigations could explore additional avenues, such as: (1) examining the subsidy strategies the government should adopt in the context of competition among multiple third-party online recyclers; (2) investigating the varied impacts of direct subsidies, tax reductions, and penalties imposed by the government on remanufacturers; and (3) evaluating the influence of consumer preferences for remanufactured products on promoting ELV recycling.

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Data Availability Statement: All data generated during this study are included in this published article (and Appendix A). The datasets analyzed in the current study are available from the corresponding author upon reasonable request.

Conflicts of Interest: We declare that we have no financial or personal relationships with other people or organizations that can inappropriately influence our work and that there is no professional or other personal interest of any nature or kind in any product, service, and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled.

Appendix A

Proof. $\Pi_c{}^d$ is a concave function of $p_c{}^d$, and $\Pi_r{}^d$ is a joint concave function of $p_e{}^d$ and $p_r{}^d$.

Proof: According to $\frac{\partial^2 \Pi_c{}^d}{\partial {p_c{}^d}^2} = \frac{-2\theta}{-1+\theta}$, $\Pi_c{}^d$ is a concave function of $p_c{}^d$ is proved under the condition $\frac{-2\theta}{-1+\theta} < 0$. According to $H\left(\Pi_r{}^d\right) = \begin{bmatrix} \frac{\partial^2 \Pi_r{}^d}{\partial {p_e{}^d}^2} & \frac{\partial^2 \Pi_r{}^d}{\partial p_e{}^d \partial p_r{}^d} \\ \frac{\partial^2 \Pi_r{}^d}{\partial p_r{}^d \partial p_e{}^d} & \frac{\partial^2 \Pi_r{}^d}{\partial {p_r{}^d}^2} \end{bmatrix} = \begin{bmatrix} \frac{1-4\theta}{2(-1+\theta)\theta} & \frac{3}{2(-1+\theta)} \\ \frac{1}{2(-1+\theta)} & \frac{\theta}{2-2\theta} \end{bmatrix}$, the Hessian matrix of $\Pi_r{}^d$ must be negative definite under the condition $\frac{1-4\theta}{2(-1+\theta)\theta} < 0$ and $\frac{1}{-1+\theta} > 0$. □

Proof of Theorem 1. By substituting the first-order derivative $p_c{}^d = \frac{p_e{}^d - I_c\theta + p_r{}^d\theta + \varphi}{2\theta}$ of $\Pi_c{}^d$ under Model d into $\Pi_r{}^d$, the optimization problem of remanufacturers can be obtained:

$$\begin{gathered} \underset{(p_e{}^d, p_r{}^d)}{maximize} \Pi_r{}^d = \left(\omega - p_e{}^d\right) Q_e{}^d + \left(\omega - p_r{}^d\right) Q_c{}^d - I_e * Q_e{}^d \\ \text{s.t. } \frac{p_e{}^d - I_c\theta + p_r{}^d\theta + \varphi}{2\theta} \geq \frac{p_e{}^d + \varphi}{\theta} \end{gathered} \tag{A1}$$

The Kuhn–Tucker conditions for the existence of the optimal solution to the above optimization problem are as follows:

$$\begin{gathered} \frac{I_e + 2p_e{}^d + \omega(\theta-1) + I_c\theta - 2p_r{}^d\theta + \varphi}{2(\theta-1)} - \frac{\lambda}{2} = 0 \\ \frac{I_e + 2p_e{}^d - \omega - I_c\theta - 2I_e\theta + \theta\left(-4p_e{}^d + 2p_r{}^d + \omega - 2\varphi\right) + \varphi}{2(\theta-1)\theta} + \frac{\lambda}{2\theta} = 0 \\ \lambda\left(-\frac{p_e{}^d - I_c\theta + p_r{}^d\theta + \varphi}{2\theta} + \frac{p_e{}^d + \varphi}{\theta}\right) = 0 \\ \lambda \geq 0 \end{gathered} \tag{A2}$$

According to the Kuhn–Tucker conditions, when $\lambda = 0$, ${p_r}^{d*} = \frac{I_c+\omega}{2}$, ${p_e}^{d*} = \frac{-I_e+\omega-\varphi}{2}$, and ${p_c}^{d*} = \frac{-I_e+\omega-I_c\theta+\omega\theta+\varphi}{4\theta}$. Thus, Theorem 1 is proved. □

Proof of Lemma 1. According to ${p_e}^{d*} = \frac{-I_e+\omega-\varphi}{2}$ and ${p_c}^{d*} = \frac{-I_e+\omega-I_c\theta+\omega\theta+\varphi}{4\theta}$ obtained in Theorem 1, and because of the constraint condition ${p_c}^{d*} \geq \frac{{p_e}^{d*}+\varphi}{\theta}$, it is ensured that Model d implements a dual-channel recycling model. Obviously, $\theta \geq \frac{{p_e}^{d*}+\varphi}{{p_c}^{d*}}$, and by substituting ${p_e}^{d*}$ and ${p_c}^{d*}$, we obtain $\theta \geq -\frac{-I_e+\omega+\varphi}{I_c-\omega}$. Therefore, Lemma 1 is proved. □

Proof of Theorem 2. We obtain the first-order derivative ${p_c}^{A} = \frac{{p_e}^{A}-I_c\theta+{p_r}^{A}\theta+\varphi}{2\theta}$ of $\Pi_c{}^{\mathrm{A}}$ under Model G, and substitute ${p_c}^{A}$ into $\Pi_r{}^{A}$ to obtain the optimization problem for remanufacturers:

$$\begin{gathered} \max_{({p_e}^{G},{p_r}^{G})} \Pi_r{}^{A} = \left(\omega - {p_e}^{A}\right){Q_e}^{A} + \left(\omega - {p_r}^{A}\right){Q_c}^{A} \\ \text{s.t. } \frac{{p_e}^{A}-I_c\theta+{p_r}^{A}\theta+\varphi}{2\theta} \geq \frac{{p_e}^{A}+\varphi}{\theta} \end{gathered} \tag{A3}$$

The Kuhn–Tucker conditions for the existence of the optimal solution to the above optimization problem are as follows:

$$\begin{gathered} \frac{I_e+2{p_e}^{d}+\omega(\theta-1)+I_c\theta-2{p_r}^{d}\theta+\varphi}{2(\theta-1)} - \frac{\lambda}{2} = 0 \\ \frac{I_e+2{p_e}^{d}-\omega-I_c\theta-2I_e\theta+\theta\left(-4{p_e}^{d}+2{p_r}^{d}+\omega-2\varphi\right)+\varphi}{2(\theta-1)\theta} + \frac{\lambda}{2\theta} = 0 \\ \lambda\left(-\frac{{p_e}^{d}-I_c\theta+{p_r}^{d}\theta+\varphi}{2\theta} + \frac{{p_e}^{d}+\varphi}{\theta}\right) = 0 \\ \lambda \geq 0 \end{gathered} \tag{A4}$$

According to the Kuhn–Tucker conditions, when $\lambda = 0$, ${p_r}^{A*} = \frac{I_c+\omega}{2}$, ${p_e}^{A*} = \frac{\omega-\varphi}{2}$, and ${p_c}^{A*} = \frac{\omega-I_c\theta+\omega\theta+\varphi}{4\theta}$. Thus, Theorem 2 is proved. □

Proof of Lemma 2. According to ${p_e}^{A*} = \frac{\omega-\varphi}{2}$ and ${p_c}^{A*} = \frac{\omega-I_c\theta+\omega\theta+\varphi}{4\theta}$ obtained in Theorem 2, and because of the constraint condition ${p_c}^{A} \geq \frac{{p_e}^{A}+\varphi}{\theta}$, it is ensured that Model G implements a dual-channel recycling model. Obviously, $\theta \geq \frac{{p_e}^{A*}+\varphi}{{p_c}^{A*}}$, and by substituting ${p_e}^{d*}$ and ${p_c}^{d*}$, we obtain $\theta \geq -\frac{-I_e+\omega+\varphi}{I_c-\omega}$. □

Proof of Theorem 3. We obtain the first-order derivative ${p_c}^{\mathrm{C}} = \frac{{p_e}^{\mathrm{C}}+s-I_c\theta+{p_r}^{\mathrm{C}}\theta-s\theta+\varphi}{2\theta}$ of $\Pi_c{}^{\mathrm{C}}$ under Model C and substitute ${p_c}^{\mathrm{C}}$ into $\Pi_r{}^{\mathrm{C}}$ to obtain the optimization problem for remanufacturers:

$$\begin{gathered} \max_{({p_e}^{d},{p_r}^{d})} \Pi_r{}^{\mathrm{C}} = \left(\omega - {p_e}^{\mathrm{C}}\right){Q_e}^{\mathrm{C}} + \left(\omega - {p_r}^{\mathrm{C}}\right){Q_c}^{\mathrm{C}} - I_e * {Q_e}^{\mathrm{C}} \\ \text{s.t. } \frac{{p_e}^{\mathrm{C}}+s-I_c\theta+{p_r}^{\mathrm{C}}\theta-s\theta+\varphi}{2\theta} \geq \frac{{p_e}^{\mathrm{C}}+\varphi+s}{\theta} - s \end{gathered} \tag{A5}$$

The Kuhn–Tucker conditions for the existence of the optimal solution to the above optimization problem are proved in the same way as Model d and Model G, and the proof process is omitted. □

Proof of Lemma 3. According to ${p_e}^{\mathrm{C}*} = \frac{-I_e-s+\omega-\varphi}{2}$ and ${p_c}^{\mathrm{C}*} = \frac{-I_e+s+\omega-I_c\theta-3s\theta+\omega\theta+\varphi}{4\theta}$ obtained in Theorem 3, and because of the constraint condition ${p_c}^{\mathrm{C}*} \geq \frac{{p_e}^{\mathrm{C}}+\varphi+s}{\theta} - s$, it is ensured that Model C implements a dual-channel recycling model. Obviously, $\theta \geq \frac{{p_e}^{\mathrm{C}*}+\varphi+s}{{p_c}^{\mathrm{C}*}+s}$, and by substituting ${p_e}^{\mathrm{C}*}$ and ${p_c}^{\mathrm{C}*}$, we obtain $\theta \geq -\frac{-I_e+s+\omega+\varphi}{I_c-s-\omega}$. □

Proof of Conclusion 1. To find the first-order partial derivative of ${p_c}^{d*}$ with respect to θ, it is $\frac{\partial {p_c}^{d*}}{\partial \theta} = \frac{I_e-\omega-\varphi}{4\theta^2} < 0$; to find the first-order partial derivative of ${Q_c}^{d*}$ with respect to θ, it is $\frac{\partial {Q_c}^{d*}}{\partial \theta} = \frac{I_c-I_e+\varphi}{4(-1+\theta)^2} > 0$; and to find the first-order partial derivative of ${Q_e}^{d*}$ with respect to θ, it is $\frac{\partial {Q_e}^{d*}}{\partial \theta} = -\frac{\omega(-1+\theta)^2+I_c\theta^2+I_e\left(-1+2\theta-2\theta^2\right)+\varphi-2\theta\varphi+2\theta^2\varphi}{4(-1+\theta)^2\theta^2} < 0$. Let the first-order partial derivative of the total RSC recycling volume $\frac{\partial {Q_e}^{d*}}{\partial \theta} = -\frac{\omega(-1+\theta)^2+I_c\theta^2+I_e\left(-1+2\theta-2\theta^2\right)+\varphi-2\theta\varphi+2\theta^2\varphi}{4(-1+\theta)^2\theta^2} < 0$ with respect to θ be $\frac{\partial {Q_t}^{d*}}{\partial \theta} = \frac{I_e-\omega-\varphi}{4\theta^2} < 0$. Let the total profit of the RSC be ${\Pi_t}^{d*} = {\Pi_c}^{d*} + {\Pi_r}^{d*}$. We know from Lemma 1 that $\theta \geq -\frac{-I_e+\omega+\varphi}{I_c-\omega}$, that is, $\theta \geq \frac{a}{b}$, and we can obtain: $\frac{\partial {\Pi_c}^{d*}}{\partial \theta} = -\frac{(-a+b\theta)(a-2a\theta+b\theta)}{16(-1+\theta)^2\theta^2} > 0$, $\frac{\partial {\Pi_r}^{d*}}{\partial \theta} = -\frac{(a-b)^2}{8(-1+\theta)^2} - \frac{a^2}{8\theta^2} < 0$, and $\frac{\partial {\Pi_t}^{d*}}{\partial \theta} = \frac{1}{16}\left(-\frac{3(a-b)^2}{(-1+\theta)^2} - \frac{a^2}{\theta^2}\right) < 0$. Thus, Conclusion 1 is proved. □

Proof of Conclusion 2. According to Table 2, it can be obtained that ${p_e}^{A*} - {p_e}^{d*} = \frac{I_e}{2} > 0$. Let $\theta \geq -\frac{-I_e+\omega+\varphi}{I_c-\omega}$ under Model d according to Lemma 1. Let $\theta = -\frac{\omega+\varphi}{I_c-\omega}$ under Model G according to Lemma 2. We can obtain $\left({Q_c}^{A*} + {Q_e}^{A*}\right) - \left({Q_c}^{d*} + {Q_e}^{d*}\right) = \frac{-I_eI_c+I_e\omega}{4\omega+4\varphi} > 0$ and $\left({\Pi_c}^{A*} + {\Pi_r}^{A*}\right) - \left({\Pi_c}^{d*} + {\Pi_r}^{d*}\right) = \frac{I_e(I_c-\omega)(-8\varphi(\omega+\varphi)+I_e(3\omega+4\varphi)+I_c(I_e-8(\omega+\varphi)))}{16(I_c+\varphi)(\omega+\varphi)} > 0$. Thus, Conclusion 2 is proved. □

Proof of Conclusion 3. The first-order partial derivatives of ${p_c}^{C*}$, ${p_e}^{C*}$, and ${p_r}^{C*}$ and ${Q_c}^{C*}$ and ${Q_e}^{C*}$ with respect to θ are $\frac{\partial {p_c}^{C*}}{\partial s} = \frac{1}{4}\left(-3 + \frac{1}{\theta}\right) < 0$, $\frac{\partial {p_e}^{G*}}{\partial s} = -\frac{1}{2} < 0$, and $\frac{\partial {p_r}^{G*}}{\partial s} = -\frac{1}{2} < 0$ and $\frac{\partial {Q_c}^{C*}}{\partial s} = \frac{1}{4} > 0$ and $\frac{\partial {Q_e}^{C*}}{\partial s} = \frac{1}{4\theta} > 0$. Let $a = -I_e + \omega + \varphi$ and $b = -I_c + \omega$. We know from Lemma 3 that $\theta \geq -\frac{-I_e+s+\omega+\varphi}{I_c-s-\omega}$, that is, $\theta \geq \frac{a+s}{b+s}$, and we can obtain: $\frac{\partial {\Pi_c}^{C*}}{\partial s} = -\frac{a+s-(b+s)\theta}{8\theta} > 0$ and $\frac{\partial {\Pi_r}^{C*}}{\partial s} = \frac{a+s+(b+s)\theta}{4\theta} > 0$. Thus, Conclusion 3 is proved. □

Proof of Conclusion 4. According to Table 3, it can be obtained that ${p_c}^{C*} - {p_c}^{d*} = \frac{s-3s\theta}{4\theta} < 0$, ${p_e}^{C*} - {p_e}^{d*} = -\frac{s}{2} < 0$, ${p_r}^{C*} - {p_e}^{d*} = -\frac{s}{2} < 0$, ${Q_c}^{C*} - {Q_c}^{d*} = \frac{s}{4} > 0$, ${Q_e}^{C*} - {Q_e}^{d*} = \frac{s}{4\theta} > 0$. Let $\theta \geq -\frac{-I_e+\omega+\varphi}{I_c-\omega}$ under Model d according to Lemma 1, and let $\theta \geq -\frac{-I_e+s+\omega+\varphi}{I_c-s-\omega}$ under Model C according to Lemma 2. We can obtain $\left({\Pi_c}^{C*} + {\Pi_r}^{C*}\right) - \left({\Pi_c}^{d*} + {\Pi_r}^{d*}\right) = \frac{1}{4}s(-I_c - I_e + s + 2\omega + \varphi) > 0$. Thus, Conclusion 4 is proved. □

Proof of Conclusion 5. Under $\theta = -\frac{\omega+\varphi}{I_c-\omega}$, when $s > \frac{I_e(I_c-\omega)}{I_c-2\omega-\varphi}$, $({Q_c}^{C*} + {Q_e}^{C*}) - ({Q_c}^{A*} + {Q_e}^{A*}) = \frac{I_e\omega+s(2\omega+\varphi)}{4(\omega+\varphi)}) > 0$ can be obtained from Tables 2 and 3 when $s < \frac{I_e(I_c-\omega)}{I_c-2\omega-\varphi}$, $({Q_c}^{C*} + {Q_e}^{C*}) - ({Q_c}^{A*} + {Q_e}^{A*}) = \frac{I_e\omega+s(2\omega+\varphi)}{4(\omega+\varphi)}) < 0$. Thus, Conclusion 5 is proved. □

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