mathematics

Article

Experimental Design for Progressive Type I Interval Censoring on the Lifetime Performance Index of Chen Lifetime Distribution

Shu-Fei Wu * and Meng-Zong Song

Department of Statistics, Tamkang University, Tamsui District, New Taipei City 251037, Taiwan
* Correspondence: 100665@mail.tku.edu.tw

Abstract: The lifetime performance index is commonly utilized to assess the lifetime performance of products. Based on the testing procedure for the lifetime of products following Chen distribution, an experimental design for progressive type I interval censoring is determined to achieve the desired power level while minimizing total experimental cost. For fixed inspection interval lengths and an unfixed number of inspection intervals, the required number of inspection intervals and sample sizes to achieve the minimum experimental costs are computed and presented in a table format. For unfixed termination times, the required number of inspection intervals, minimum sample sizes, and equal interval lengths are obtained and presented in a table format, while the minimum experimental costs are achieved. Finally, a practical example is presented to demonstrate the utilization of this experimental design for collecting samples and conducting a testing procedure to evaluate the lifetime performance of products.

Keywords: Chen distribution; progressive type I interval censoring; maximum likelihood estimator; lifetime performance index; testing procedure; experimental design

MSC: 62P30

Citation: Wu, S.-F.; Song, M.-Z. Experimental Design for Progressive Type I Interval Censoring on the Lifetime Performance Index of Chen Lifetime Distribution. *Mathematics* **2023**, *11*, 1554. https://doi.org/10.3390/math11061554

Academic Editors: Juan Francisco Sánchez-Pérez, Gonzalo García Ros and Manuel Conesa

Received: 27 February 2023
Revised: 19 March 2023
Accepted: 21 March 2023
Published: 22 March 2023

1. Introduction

During the pandemic, there was an increase in demand for advanced technological devices, such as laptops, desktops, and mobile phones. The high lifespan of high-tech products can be a key factor in attracting more consumers and enhancing the brand's market value. The process capability index C_L was introduced by Montgomery [1] to assess the quality of larger-the-better characteristics, such as lifespan, the hardness of smart phone cases, battery capacity, and more. In many cases, the experimenters may not have access to complete data, resulting in the need to handle censored data. The two most common types of censoring are type I censoring and type II censoring. Type I censoring occurs when a life test is terminated at a fixed time point, and the number of failure units is random. Type II censoring occurs when a study is terminated when a predetermined fixed number of failure units is observed so that the termination time is random. Progressive censoring possesses the characteristic of permitting the removal of units at certain time points that may not necessarily be the ultimate termination point. More inferences about the progressive censored data can be seen in Balakrishnan and Aggarwala [2], Aggarwala [3], Balakrishnan [4], and Balakrishnan and Cramer [5]. For the progressive type II censored sample, Laumen and Cramer [6] studied the inferences for the lifetime performance index following the gamma distributions. Bdair et al. [7] studied the estimation and prediction for flexible Weibull distribution based on the progressive type II censored sample. Panahi [8] investigated the interval estimation of Kumaraswamy parameters based on progressively type II censored data and record values. EL-Sagheer [9] studied the estimation of parameters of Weibull–Gamma distribution for the progressively censored sample. Lee et al. [10] assessed the lifetime performance index for the exponential distribution model. Wu et al. [11] tested the lifetime performance index based on the Bayesian approach. The advantage of progressive type I interval censoring is that it is very convenient for quality personnel to conduct the life test and collect the censored data in practical situations. Under this type of censoring, Wu and Lin [12] used the maximum likelihood estimator for the lifetime performance index to develop a testing procedure for exponential lifetime distribution. Wu et al. [13] conducted an experimental analysis for the sampling design of Gompertz life time distribution based on progressive type I interval sampled data. For products following the Chen lifetime distribution, Wu [14] developed a testing procedure for the lifetime performance index under progressive type I interval censoring. The research goal of this study is to investigate the experimental plan for the progressive type I interval censoring design for products following the Chen distribution based on the testing procedure proposed in Wu [14]. In Section 2, we introduce and summarize a testing procedure along with the test power to assess whether the lifetime performance of a production process achieves the desired target index for the lifetime of products following the Chen distribution. Section 3 determines the minimum number of inspection intervals required to minimize the total cost under a pre-specified power level and level of significance for either a fixed or unfixed total experimental time, with the aim of achieving the lowest total cost. Additionally, one real-life example is presented to demonstrate the testing procedure. Finally, we conclude the study by summarizing all relevant findings in Section 4.

2. The Introduction of the Testing Procedure for the Lifetime Performance Index in Wu [14]

Chen [15] presented a new two-parameter lifetime distribution with a bathtub shape or increasing failure rate function called Chen distribution. Let U be the lifetime of products following a Chen distribution with the probability density function (pdf) and the failure rate function defined as:

$$f_U(u) = k\beta u^{\beta-1} e^{u^\beta} \exp\left\{k(1-e^{u^\beta})\right\}, 0 \leq u \leq \infty, k > 0, \beta > 0 \tag{1}$$

and

$$r_U(u) = k\beta u^{\beta-1} e^{u^\beta}. \tag{2}$$

Chen indicated that this distribution has an increasing failure rate function when $\beta \geq 1$ and a bathtub shape failure rate function when $\beta < 1$. After the transformation from U to Y by $Y = e^{U^\beta} - 1,\ \beta > 0$, the pdf of the new variable Y is an exponential distribution with failure rate k. The mean and standard deviation of Y are $\mu = 1/k$ and $\sigma = 1/k$. If L_U is the specified lower specification limit for U, then $L = e^{L_U{}^\beta} - 1$ is the specified lower specification limit for Y. The lifetime performance index proposed by Montgomery [11] is defined as:

$$C_L = \frac{\mu - L}{\sigma}, \tag{3}$$

where μ is denoted as the process mean, σ is regarded as the process standard deviation, and L is the specified lower specification limit. Substituting μ and σ with the mean and standard deviation of Y into Equation (3), we obtain the lifetime performance index $C_L = 1 - kL$.

Subsequently, the calculation of the conforming rate is performed as $P_r = P(U \geq L_U) = P(Y \geq L) = \exp(-kL) = \exp(C_L - 1) \quad , \quad -\infty < C_L < 1$ and the value of P_r increases as C_L increases. The relationship between P_r and C_L is displayed in Figure 1.

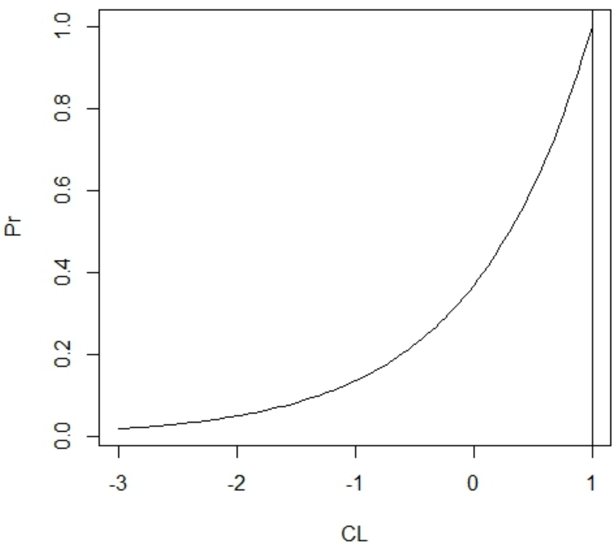


Figure 1. The relationship between P_r and C_L.

To obtain the sample using the progressive type I interval censoring scheme, the following steps are followed:

Step 1: Put n products on a life test at the starting time 0. Set the termination time as T and the number of inspections as m. Then, we decide the observation time points $t_1, \ldots,$ t_m, where $t_m = T$.

Step 2: Observe the number of failure units X_i and removed R_i units with the removing rate of p_i, where R_i follows a binomial distribution denoted as bin($n - \sum_{j=1}^{i} X_j - \sum_{j=1}^{i-1} R_j, p_i$), $I = 1, \ldots, m$. Wu [14] obtained the maximum likelihood of k as the numerical solution of the following log-likelihood equation

$$\frac{d}{dk}\ln L(k) = \sum_{i=1}^{m} X_i \frac{(y_i - y_{i-1})\, e^{-k(y_i - y_{i-1})}}{1 - e^{-k(y_i - y_{i-1})}} - \sum_{i=1}^{m}(R_i\, y_i + X_i\, y_{i-1})$$

$$= \sum_{i=1}^{m} X_i \frac{\left(e^{t_i{}^{\beta}} - e^{t_{i-1}{}^{\beta}}\right) e^{-k(e^{t_i{}^{\beta}} - e^{t_{i-1}{}^{\beta}})}}{1 - e^{-k(e^{t_i{}^{\beta}} - e^{t_{i-1}{}^{\beta}})}} - \sum_{i=1}^{m}\left(R_i\,(e^{t_i{}^{\beta}} - 1) + X_i\,(e^{t_{i-1}{}^{\beta}} - 1)\right) = 0 \quad (4)$$

Its asymptotic variance is the reciprocal of the Fisher's information, given by

$$I(k) = -E[\frac{d^2 \ln L(k)}{dk^2}] = \frac{n}{k^2}\sum_{i=1}^{m} \frac{\ln^2(1 - q_i)}{q_i} \prod_{j=1}^{i-1}(1 - p_j) \prod_{l=1}^{i}(1 - q) \quad (5)$$

where $q_i = 1 - \exp(-\ k(e^{t_i{}^{\beta}} - e^{t_{i-1}{}^{\beta}}))$.

To facilitate data collection, we considered the case of equal interval lengths $t_i - t_{i-1} = t$ and $p_1 = \ldots = p_{m-1} = p$ $i = 1, \ldots, m$. That is, $t_i = it$, $i = 1, \ldots, m$, and $q_i = 1 - \exp(-\ k(e^{(it)^{\beta}} - e^{((i-1)t)^{\beta}}))$. Then, Equation (4) is reduced to

$$\frac{d}{dk}\ln L(k) = \sum_{i=1}^{m} X_i \frac{\ln(1 - q_i)(1 - q_i)}{q_i} - \sum_{i=1}^{m}\left(R_i\left(e^{(it)^{\beta}} - 1\right) + X_i\left(e^{((i-1)t)^{\beta}} - 1\right)\right) \equiv 0 \quad (6)$$

The information number in (5) is reduced to

$$I(k) = \frac{n}{k^2}\sum_{i=1}^{m} \frac{\ln^2(1 - q_i)}{q_i}(1 - p)^{i-1} \prod_{l=1}^{i}(1 - q) \quad (7)$$

Furthermore, we have $\hat{k} \underset{n\to\infty}{\overset{d}{\to}} \mathrm{N}(k, g(k))$ where g(k) = $I^{-1}(k)$ is the asymptotic variance of $\hat{k}$.

Due to the property of the invariance of MLE, the MLE of C_L can be acquired as

$$\hat{C}_L = 1 - \hat{k}L \tag{8}$$

Let c_0 be the desired level of the lifetime performance index to make the process capable. Then, we want to test $H_0 : C_L \le c_0$ (the process is not capable) vs. $H_a : C_L > c_0$ (the process is capable). Under the level of significance α, the MLE of C_L found as $\hat{C}_L = 1 - \hat{k}L$ is utilized as the test statistic. The critical region for this right-sided test is $\{\hat{C}_L | \hat{C}_L > C_L^0\}$, where the critical value C_L^0 is determined as $C_L^0 = 1 - L\left(k_0 + Z_{1-\alpha}\sqrt{g(k_0)}\right)$ and Z_α represents the $1 - \alpha$ percentile of the standard normal distribution. Moreover, the power function denoted by $h(c_1)$ of this test at the point of $C_L = c_1 > c_0$ is obtained as

$$h(c_1) = \Phi\left(\frac{k_0 - k_1 + Z_{1-\alpha}\sqrt{g(k_0)}}{\sqrt{g(k_1)}}\right) \tag{9}$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution, $k_0 = \frac{1-c_0}{L}$ and $k_1 = \frac{1-c_1}{L}$.

3. Reliability Sampling Design

The objective of this section is to identify the optimal sampling design for progressive type I interval sampling for products' lifetime following the Chen distribution, given that the parameters of the Chen distribution may have different structures. In Section 3.1, for the fixed experimental time T, we determine the required minimal number of inspection intervals based on the criterion of minimum total cost so that the required sample size can be calculated to reach the specified test power of the level α testing procedure. In Section 3.2, for the unfixed experimental time T, we determine the required minimal number of inspection intervals and the equal length of intervals to minimize the total experimental cost so that the required sample size can be calculated under the specified test power of the level α testing procedure.

Consider the following function $w(k) = I(k)/n = \frac{1}{k^2}\sum_{i=1}^{m} \frac{\ln^2(1-q_i)}{q_i}(1-p)^{i-1}\prod_{l=1}^{i}(1-q)$, which is not a function of sample size n. The power function can be rewritten as $h(c_1) = \Phi\left(\frac{k_0-k_1+Z_{1-\alpha}\sqrt{g(k_0)}}{\sqrt{g(k_1)}}\right) = \Phi\left(\frac{k_0-k_1+Z_{1-\alpha}\sqrt{w^{-1}(k_0)/n}}{\sqrt{w^{-1}(k_1)/n}}\right)$.

In order to attain the pre-specified power 1-β or the probability of type II error β at c_1 under the level of significance α, assign the above power function to 1-β at c_1, and then the sample size is determined as

$$n = ceiling\left(\frac{Z_\beta\sqrt{w^{-1}(k_1)} + Z_\alpha\sqrt{w^{-1}(k_0)}}{k_1 - k_0}\right)^2 \tag{10}$$

where *ceiling*(x) is a ceiling function mapping x to the smallest integer, which is greater than or equal to x.

3.1. The Minimal Required m for Fixed T

In numerous practical situations, the experimenters aim to minimize the number of inspection intervals m so that they do not need to frequently gather data for the progressive type I interval sampling. Suppose that the upper limit of m is m_0 for experimenters, the value of m must satisfy $m \le m_0$. If the value for m_0 is not predetermined, the default value of 30 is utilized for m. In this subsection, our aim is to find the optimal value of m, denoted as m*, that minimizes the total cost incurred during the progressive type I interval censoring procedure. Similar to Huang and Wu [16], we consider the following costs:

1. Inspection cost C_I: the cost for operating a single inspection station;
2. Sample cost C_s: the cost for obtaining one unit of sample;
3. Operation cost C_o: the cost incurred for conducting the experiment per unit of time, which encompasses expenses such as the personnel cost, the depreciation of test equipment, and other related costs.

Taking into account all of these expenses, the overall cost of conducting this experiment is

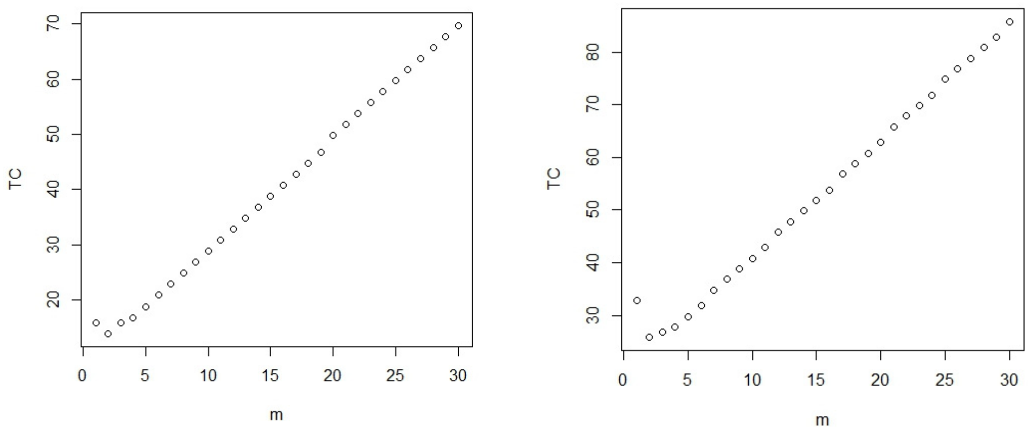
$$TC(m) \; = \; mC_I + ceiling\left(\frac{Z_\beta\sqrt{w^{-1}(k_1)} + Z_\alpha\sqrt{w^{-1}(k_0)}}{k_1 - k_0}\right)^2 C_s \; + \; TC_0 \tag{11}$$

Here is the Algorithm 1 that utilizes the numeration method to search for the optimal (m, n):

Algorithm 1: Utilize the numeration method to search for the optimal (m, n)

Step 1: Specify the pre-assigned values of m = 1, c_0, c_1, α, β, p, T, L, and m_0 (the default value is 30) and $C_I = aC_o$, $C_s = bC_o$, C_o.
Step 2: Compute the sample size n in Equation (11) first and then compute the related total cost $TC(m)$ in Equation (12).
Step 3: If $m < m_0$, then $m = m + 1$ and go to Step 2; otherwise go to Step 4.
Step 4: For a array of total costs $TC(1), \ldots, TC(m_0)$, The optimal solution of m^* is the minimum m value, such that $TC(m^*) = TC^* = \min\limits_{m \le m_0} TC(m)$, and then the related sample size n^* in Equation (11) is computed.
Step 5: Calculate the value of $k_0 = \frac{1-c_0}{L}$ followed by determining the critical value $C_L^0 = 1 - L\left(k_0 + Z_{1?\alpha}\sqrt{g(k_0)}\right)$.

Consider C_o = 1, a = 2, and b = 1. For testing $H_0 : C_L \le 0.8$ with β = 0.15, α = 0.05, p = 0.01, c_1 = 0.95, L = 0.1, and T = 0.8, the curve of total cost with $m = 1{:}m_0$ is displayed in Figure 2a. It can be seen that the minimum total cost occurred at m = 2, with a total cost of 13.8. For a different set up of parameters β = 0.25, α = 0.05, p = 0.05, c_1 = 0.90, another curve of total cost with $m = 1{:}m_0$ is displayed in Figure 2b. You can see that the minimum total cost occurred at m = 2, with a total cost of 25.8.



(**a**) (**b**)

Figure 2. (**a**): Total cost curve at $m = 1{:}m_0$. (**b**) Total cost curve at $m = 1{:}m_0$.

For testing $H_0 : C_L \le 0.80$, the required minimal inspection intervals m^* and the related sample size n^* to yield the minimum total cost $TC(m^*)$ with $m < 50$ are tabulated in Tables A1 and A2 at the conditions of α = 0.01, 0.05, 0.1, β = 0.25, 0.20, 0.15, p = 0.01, 0.025, 0.05, 0.15, 0.25 for c_1 = 0.825, 0.85 and c_1 = 0.875, 0.90 respectively. Tables A1 and A2 also contain the relevant critical values.

Looking at Table A2, suppose that the experimenter wants to conduct the level 0.01 hypothesis test with c0 = 0.80 and $1 - \beta = 0.85$ at $c_1 = 0.90$, $p = 0.05$. We find that the required minimal number of inspection intervals is 2 and the sample size is determined as 14 with the minimum total cost $TC^* = 18.8$ and the critical value of 0.877235.

From Tables A1 and A2, the optimal number of inspection intervals m is nonincreasing when c_1 is increasing and the range of m is 2~6. In Figure 3, the plot of the minimum total cost TC^* vs. c_1 for $\alpha = 0.01, 0.05, 0.1$ at $\beta = 0.25$ and $p = 0.05$ is displayed. In Figure 4, the plot of the minimum total cost TC^* vs. $1 - \beta = 0.75, 0.80, 0.85$ at $\alpha = 0.1$ and $p = 0.05$ is displayed. In Figure 5, the plot of the minimum total cost TC^* vs. $p = 0.05, 0.075, 0.1$ is displayed at $\alpha = 0.1$ and $\beta = 0.25$. From Figure 3, it can be observed that, as the level of significance increases, there is a decrease in the minimum total cost TC^*. From Figure 4, it can be observed that, as the test power increases, there is an increase in the minimum total cost TC^*. From Figure 5, it can be observed that, as the removal rate p increases, there is an increase in the minimum total cost TC^*. Furthermore, these three figures show that the minimum total cost TC^* is a deceasing function of c_1. According to Tables A1 and A2, the required minimal number of inspection intervals is inversely proportional to c_1.

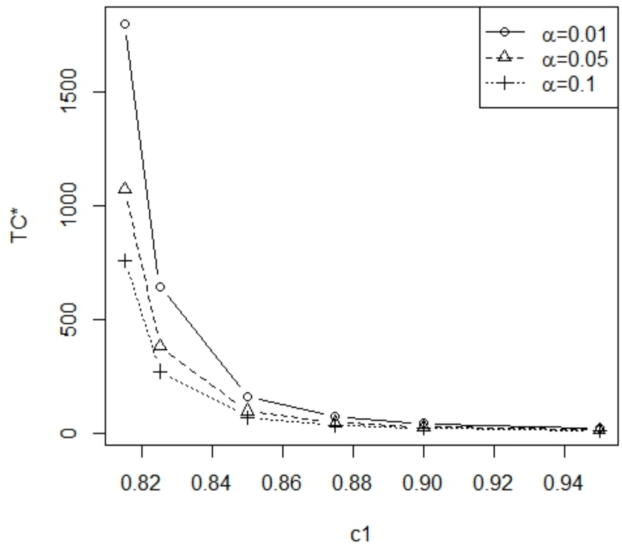


Figure 3. The minimum total cost curve for $\alpha = 0.01, 0.05, 0.1$.

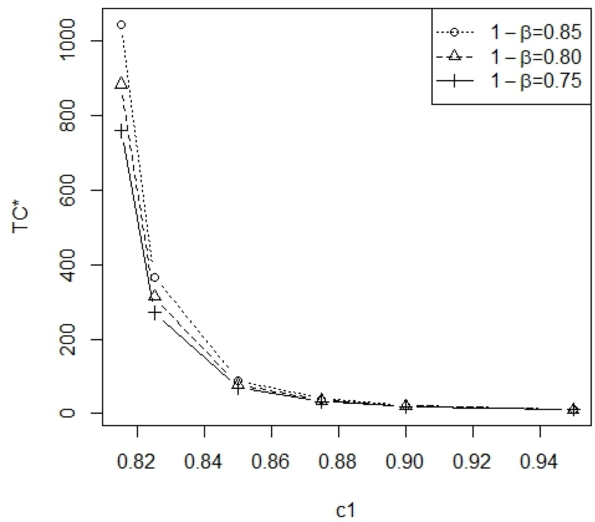


Figure 4. The minimum total cost for $1 - \beta = 0.75, 0.80, 0.85$.

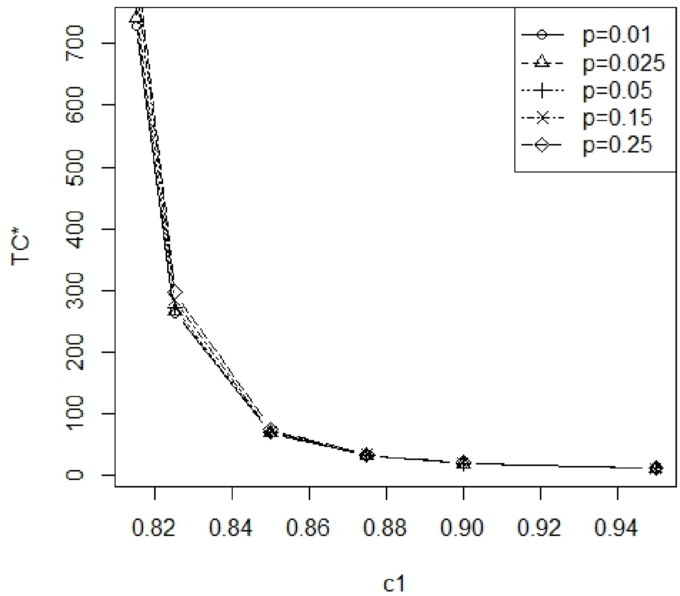


Figure 5. The minimum total cost for p = 0.01, 0.025, 0.05.

3.2. The minimal Required *m*, *t*, and *n* When the Interval Time of the Experiment Is Unfixed

When the equal interval time t is not fixed, we would like to determine the optimal (m,t) to yield the minimum total cost incurred for the progressive type I interval censored sampling. The total cost becomes

$$TC(m,t) = \ mC_I + ceiling\left(\frac{Z_\beta\sqrt{w^{-1}(k_1)} + Z_\alpha\sqrt{w^{-1}(k_0)}}{k_1 - k_0}\right)^2 C_s \ + \ mtC_o. \tag{12}$$

We use the numeration method to search for the optimal (m,t), and the steps of the algorithm are as follows:

Step 1: Specify the pre-assigned values of m = 1, c_0, c_1, α, β and p, L and m_0 (the default value is 30), and the costs C_I = aC_o, C_s = bC_o, C_o.

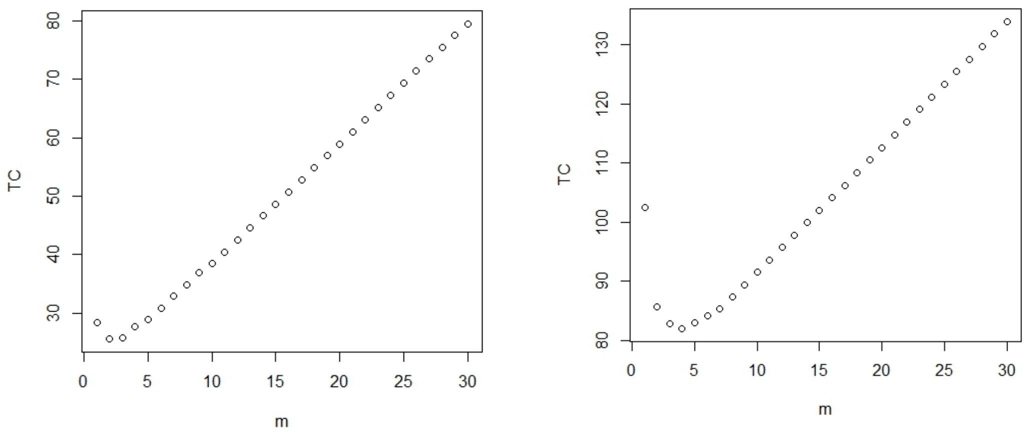
Step 2: Determine the optimal solution, denoted as t*, to minimize the total cost $TC(m,t)$, as described in Equation (12). Put m = m and t = t* in Equation (11) so that the sample size n can be computed. Subsequently, the corresponding total cost $TC(m,t^*)$ in Equation (12) is computed.

Step 3: If $m < m_0$, then let $m = m + 1$ and go to Step 2; otherwise go to Step 4.

Step 4: For an array of total costs $TC(1,t^*)$, . . . , $TC(m_0,t^*)$, the optimal solution of m* is the minimum value of m such that $TC(m^*,t^*) = TC^{**} = \min\limits_{m \le m_0} TC(m,t^*)$ is achieved. Put $m = m^*$ and $t = t^*$ in Equation (11), and then the related sample size n* in Equation (11) can be computed.

Step 5: Calculate the value of $k_0 = \frac{1-c_0}{L}$ followed by determining the critical value of $C_L^0 = 1 - L\left(k_0 + Z_{1-\alpha}\sqrt{g(k_0)}\right)$.

Consider C_o = 1, a = 2, and b = 1. For testing $H_0 : C_L \le 0.8$ when β = 0.25, α = 0.05, p = 0.025, c_1 = 0.9, m_0 = 50, L = 0.05, T = 0.8, we plot m = 1:m_0 against its corresponding total cost in Figure 6a. We find that the curve is a concave upward curve and the minimum total cost occurred at m = 2 with a total cost of 25.563. For another set up of parameters β = 0.15, α = 0.01, p = 0.05, ^{c1} = 0.875, another curve of total cost with m = 1:m_0 is given in Figure 6b. It can be seen that it is a concave upward curve and the minimum total cost occurred at m = 4 with a total cost of 81.962. For other combinations of setups, we can also find similar patterns.



(**a**) (**b**)

Figure 6. (**a**) The minimum total cost at m = 1:m_0. (**b**) The minimum total cost at m = 1:m_0.

For testing $H_0 : C_L \leq 0.8$, the required minimum inspection intervals m^*, the inspection interval time length t^*, and sample size n^* to yield the minimum total cost $TC(m^*,t^*)$ are tabulated in Tables A3 and A4 at α = 0.01, 0.05, 0.1, β = 0.25, 0.20, 0.15, p = 0.01, 0.025, 0.05, 0.15, 0.25 for c_1 = 0.825, 0.85, and c_1 = 0.875, 0.90, respectively, under the constraint of $m < m_0$, with m_0 = 50. Tables A3 and A4 also contain the relevant critical values.

Looking at Table A4, if the experimenter wants to conduct a level 0.05 hypothesis test under a power of 0.75 at c_1 = 0.90 and p = 0.05, the minimum required sample size is obtained as 21, the minimum number of inspection intervals is obtained as 2, and the optimal inspection interval time length is 0.28. For this case, the minimum total cost is TC^{**} = 25.55 and the relevant critical value is 0.881071.

From Tables A3 and A4, the optimal required minimal number of inspection intervals is inversely proportional to c_1 and the range of m is 2~10. The optimal length of inspection interval t^* is within 0.15 and 0.22 unit of times for c_1 = 0.825. The values of t^* are within 0.18 and 0.28 units of time for c_1 = 0.875. The values of t^* are within 0.21 and 0.37 units of time for c_1 = 0.875. The values of t^* are within 0.23 and 0.37 units of time for c_1 = 0.90. Figure 7 displays a graph showing the relationship between the minimum total cost TC^{**} and c_1 for α = 0.01, 0.05, 0.1 at β = 0.25, and p = 0.05. Figure 8 displays a graph showing the relationship between the minimum total cost TC^{**} and c_1 for $1 - \beta$ = 0.75, 0.80, 0.85 at α = 0.1, and p = 0.05. Figure 9 displays a graph showing the relationship between the minimum total cost TC^{**} and c_1 for p = 0.01, 0.025, 0.05, 0.15, 0.25 at α = 0.1 and β = 0.25. From Figures 7 and 8, it can be seen that the minimum total cost TC^{**} is a decreasing function of α or an increasing function of $1 - \beta$. From Figure 9, it can be seen that, as the removal rate p increases, there is an increase in the minimum total cost TC^{**}. Furthermore, the minimum total cost TC^{**} deceases as c_1 increases.

3.3. Example

The dataset utilized in this study, obtained from Xie and Lai [17], comprises the failure times (measured in number of cycles in 100,000 times) of n = 18 electronic devices, which are provided as 0.05, 0.11, 0.21, 0.31, 0.46, 0.75, 0.98, 1.22, 1.45, 1.65, 1.95, 2.24, 2.45, 2.93, 3.21, 3.30, 3.50, and 4.20.

To test the goodness of fit of the Chen distribution, we employ the Gini statistic suggested by Gill and Gastwirth [18]. The p-value of this test is a function of β and the p-value versus the β value from 0 to 1.0 is given in Figure 10. From Figure 10, the value of β = 0.64 is determined with the largest p-value of 0.9788521. The large p-value indicated that the data fitted the Chen distribution very well. We also conducted the Kolmogorov-Smirnov test (ks. test in R) with a p-value of 0.781, which fitted the Chen distribution as well.

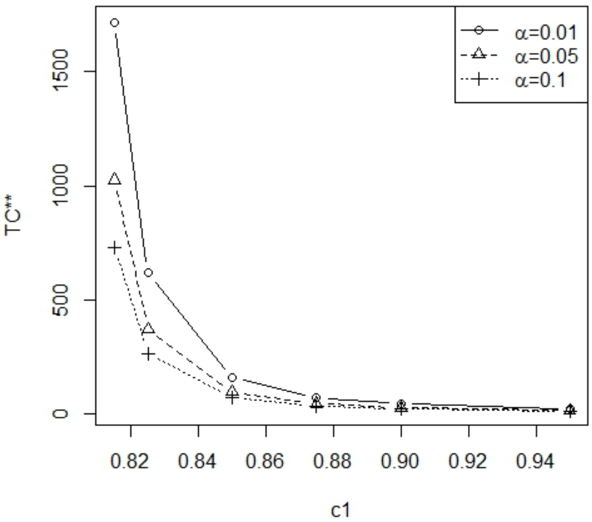


Figure 7. The minimum total cost curve or α = 0.01, 0.05, 0.1.

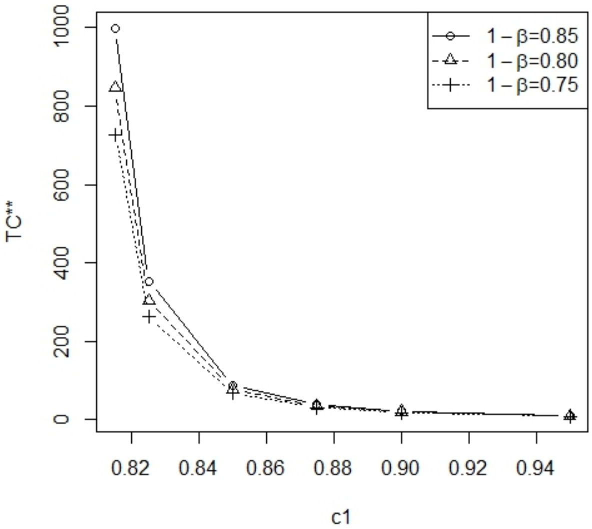


Figure 8. The minimum total cost curve or $1 - \beta$ = 0.75, 0.80, 0.85.

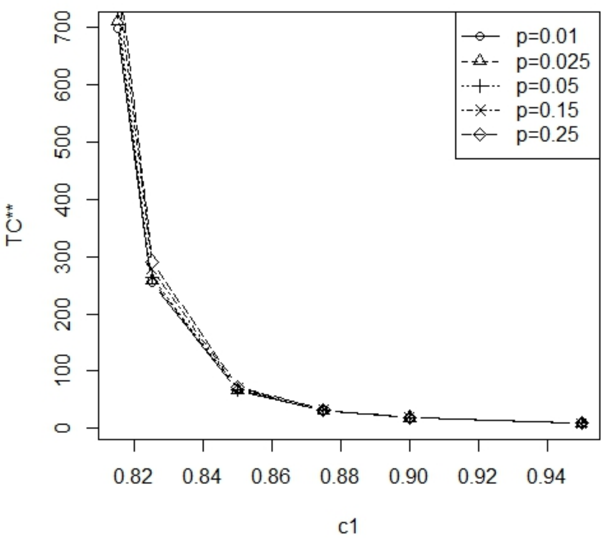


Figure 9. The minimum total cost curve for p = 0.01, 0.025, 0.05.

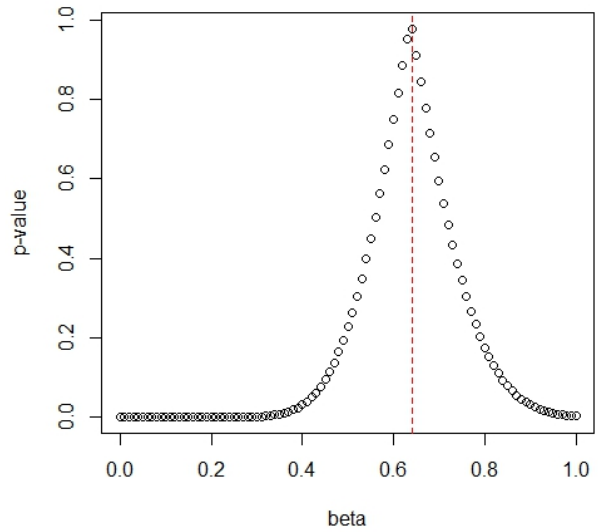


Figure 10. The p-value vs. the β values.

Using this example, the implementation of Sections 3.1 and 3.2 is given as follows: Suppose we want to test $H_0 : C_L \leq 0.8$. Refer to Section 3.1, the case of α = 0.1, the power $1 - \beta$ = 0.75 at c_1 = 0.90, p = 0.05 and T = 0.8 is considered, where the termination time of experiment T is fixed. From Table A2, we can find that the optimal sampling design is m^* = 2, n^* = 17 with critical value C_L^0 = 0.870090 and a minimum total cost of 21.8 units under the cost setup of C_o = 1, a = 2, and b = 1.

The procedure for testing is executed in the following manner:

Step 1 Take a random sample of size n = 17 from the data set. Observe the progressive type I interval censored sample (X_1,X_2) = (4,1) at the pre-set observation time points (t_1,t_2) = (0.4,0.8) with censoring schemes of (R_1,R_2) = (1,11).

Step 2 Obtain the MLE of k as $\hat{k} = 0.3155534$, and then we can obtain the test statistic $\hat{C}_L = 1 - \hat{k}L$ = 0.9684447.

Step 3 Compare the test statistic with the critical value. We have $\hat{C}_L$ = 0.9684447 > $C_L^0 =$ 0.870090. It can be inferred that the lifetime performance index of product surpasses the required level of 0.80.

Refer to Section 3.2, the case of α = 0.10, the power $1 - \beta$ = 0.85 at c_1 = 0.95 is considered. We can find that the optimal sampling design is m^* = 2, n^* = 16, and t^* = 0.34 with critical value C_L^0 =0.9266319 and a minimum total cost of 20.682 units under the cost setup of C_o = 1, a = 2, and b = 1 from our software.

The procedure for testing is executed in the following manner:

Step 1 Take a random sample of size n = 17 from the data set. Observe the progressive type I interval censored sample (X_1,X_2) = (4,1) at the pre-set observation time points (t_1,t_2) = (0.34,0.68) with censoring schemes of (R_1,R_2) = (0,12).

Step 2 Obtain the MLE of k as $\hat{k} = 0.3052468$, and then we can obtain the test statistic $\hat{C}_L = 1 - \hat{k}L$ = 0.9694753.

Step 3 Comparing the test statistic with the critical value, we have $\hat{C}_L$ = 0.9694753 > $C_L^0 = 0.9266319$. As a result, we arrived at the same conclusion of rejecting the null hypothesis.

4. Conclusions

The evaluation of the lifetime performance index for products is a crucial subject in various manufacturing industries, particularly when the product's lifetime follows a Chen distribution. To facilitate the collection of data, a sample was collected using the progressive type I interval censoring scheme. Our investigation aimed to determine the minimum number of inspection intervals required to achieve the given test power with a minimum total cost for a level α test when the total experimental time was fixed. When the total experimental time was not fixed, the required minimum sample size, number of inspection intervals, and the equal inspection interval time length were determined to achieve the given test power with a minimum total cost for a level α test under progressive type I interval censoring. The influences of various structures of level α, the power, and p on the minimum total cost were analyzed for the given c_1 value. Nine figures for the total cost vs. c_1 value in the alternative hypothesis were displayed and analyzed. We also observed that, in all cases, the minimum total cost decreased as c_1 increased.

Author Contributions: Conceptualization, S.-F.W.; methodology, S.-F.W.; software, S.-F.W. and M.-Z.S.; validation, M.-Z.S.; formal analysis, S.-F.W.; investigation, S.-F.W. and M.-Z.S.; resources, S.-F.W.; data curation, S.-F.W. and M.-Z.S.; writing—original draft preparation, S.-F.W. and M.-Z.S.; writing—review and editing, S.-F.W.; visualization, M.-Z.S.; supervision, S.-F.W.; project administration, S.-F.W.; funding acquisition, S.-F.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research and the APC are funded by [National Science and Technology Council, Taiwan] NSTC 111-2118-M-032-003-MY2.

Data Availability Statement: Data available in a publicly accessible repository The data presented in this study are openly available in Xie and Lai [17].

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. The optimal m^*, n^*, related total cost TC^* and the critical value for c_1 = 0.825, 0.85 and p = 0.01, 0.025, 0.05, 0.15, 0.25 under m_0 = 30, L = 0.1 and c_0 = 0.80.

		c_1			0.825				0.85	
α	β	p	m^*	n^*	TC^*	C_L^0	m^*	n^*	TC^*	C_L^0
0.01	0.25	0.010	6	745	757.8	0.817927	4	177	185.8	0.837169
		0.025	5	760	770.8	0.817926	4	179	187.8	0.837135
		0.050	5	777	787.8	0.817918	4	181	189.8	0.837218
		0.150	3	828	834.8	0.817948	3	193	199.8	0.837176
		0.250	3	863	869.8	0.817928	3	201	207.8	0.837149
	0.20	0.010	6	668	680.8	0.818932	4	160	168.8	0.839094
		0.025	5	681	691.8	0.818937	4	162	170.8	0.839035
		0.050	5	696	706.8	0.818932	4	165	173.8	0.838981
		0.150	3	743	749.8	0.818947	3	175	181.8	0.839041
		0.250	3	774	780.8	0.818931	3	182	188.8	0.839040
	0.15	0.010	6	605	617.8	0.819893	4	147	155.8	0.840786
		0.025	5	617	627.8	0.819895	4	148	156.8	0.840840
		0.050	5	631	641.8	0.819883	4	151	159.8	0.840748
		0.150	3	673	679.8	0.819908	3	160	166.8	0.840830
		0.250	3	701	707.8	0.819892	3	167	173.8	0.840755
0.05	0.25	0.010	6	465	477.8	0.816044	4	108	116.8	0.833644
		0.025	5	474	484.8	0.816049	4	109	117.8	0.833648
		0.050	4	487	495.8	0.816043	3	113	119.8	0.833679
		0.150	3	517	523.8	0.816060	3	118	124.8	0.833617
		0.250	3	538	544.8	0.816055	3	123	129.8	0.833577
	0.20	0.010	5	407	417.8	0.817209	4	95	103.8	0.835873
		0.025	5	413	423.8	0.817194	4	96	104.8	0.835853
		0.050	4	423	431.8	0.817214	3	100	106.8	0.835801
		0.150	3	450	456.8	0.817214	3	104	110.8	0.835808
		0.250	3	468	474.8	0.817214	3	108	114.8	0.835833

Table A1. *Cont.*

		c_1			0.825				0.85	
α	β	p	m^*	n^*	TC^*	C_L^0	m^*	n^*	TC^*	C_L^0
	0.15	0.010	6	356	368.8	0.818336	3	87	93.8	0.838078
		0.025	5	363	373.8	0.818340	3	88	94.8	0.837975
		0.050	4	373	381.8	0.818331	3	89	95.8	0.837949
		0.150	3	396	402.8	0.818350	3	93	99.8	0.837866
		0.250	3	412	418.8	0.818346	2	99	103.8	0.838114
0.10	0.25	0.010	5	345	355.8	0.814563	3	81	87.8	0.830747
		0.025	5	350	360.8	0.814552	3	81	87.8	0.830839
		0.050	4	358	366.8	0.814578	3	82	88.8	0.830804
		0.150	3	381	387.8	0.814576	3	86	92.8	0.830680
		0.250	3	397	403.8	0.814562	2	91	95.8	0.830973
	0.20	0.010	5	293	303.8	0.815803	3	69	75.8	0.833314
		0.025	5	297	307.8	0.815797	3	70	76.8	0.833174
		0.050	4	305	313.8	0.815794	3	71	77.8	0.833104
		0.150	3	323	329.8	0.815831	3	74	80.8	0.833074
		0.250	3	337	343.8	0.815805	2	78	82.8	0.833455
	0.15	0.010	5	252	262.8	0.817040	3	61	67.8	0.835431
		0.025	5	255	265.8	0.817049	3	61	67.8	0.835537
		0.050	4	262	270.8	0.817041	3	62	68.8	0.835425
		0.150	3	278	284.8	0.817064	3	64	70.8	0.835564
		0.250	3	290	296.8	0.817037	2	69	73.8	0.835570

Table A2. The optimal m^*, n^*, related total cost TC^* and the critical value for c_1 = 0.875, 0.90 and p = 0.01, 0.025, 0.05, 0.15, 0.25 under m_0 = 30, L = 0.1 and c_0 = 0.85.

		c_1			0.875				0.90	
α	β	p	m^*	n^*	TC^*	C_L^0	m^*	n^*	TC^*	C_L^0
0.01	0.25	0.010	3	75	81.8	0.858004	3	39	45.8	0.880437
		0.025	3	76	82.8	0.857793	3	40	46.8	0.879663
		0.050	3	76	82.8	0.858082	3	40	46.8	0.880060
		0.150	3	80	86.8	0.857743	2	44	48.8	0.880105
		0.250	2	85	89.8	0.858175	2	45	49.8	0.879954
	0.20	0.010	3	69	75.8	0.860473	2	39	43.8	0.884001
		0.025	3	69	75.8	0.860654	3	37	43.8	0.882829
		0.050	3	70	76.8	0.860520	3	37	43.8	0.883242
		0.150	3	73	79.8	0.860448	2	41	45.8	0.882984
		0.250	2	78	82.8	0.860730	2	41	45.8	0.883764
	0.15	0.010	3	64	70.8	0.862791	3	34	40.8	0.886148
		0.025	3	64	70.8	0.862979	2	37	41.8	0.886359
		0.050	3	65	71.8	0.862804	2	37	41.8	0.886555
		0.150	3	68	74.8	0.862631	2	38	42.8	0.886197
		0.250	2	72	76.8	0.863210	2	39	43.8	0.885885
0.05	0.25	0.010	3	45	51.8	0.852946	2	25	29.8	0.874182
		0.025	3	45	51.8	0.853104	2	25	29.8	0.874283
		0.050	3	46	52.8	0.852786	2	25	29.8	0.874452
		0.150	2	50	54.8	0.853131	2	26	30.8	0.873680
		0.250	2	51	55.8	0.853103	2	26	30.8	0.874373
	0.20	0.010	3	40	46.8	0.856158	2	23	27.8	0.877341
		0.025	2	43	47.8	0.856640	2	23	27.8	0.877446
		0.050	3	41	47.8	0.855912	2	23	27.8	0.877622
		0.150	2	45	49.8	0.856005	2	23	27.8	0.878338
		0.250	2	46	50.8	0.855914	2	24	28.8	0.877410

Table A2. *Cont.*

		c_1			0.875				0.90	
α	β	p	m^*	n^*	TC^*	C_L^0	m^*	n^*	TC^*	C_L^0
	0.15	0.010	3	36	42.8	0.859195	2	21	25.8	0.880940
		0.025	2	39	43.8	0.859474	2	21	25.8	0.881050
		0.050	3	37	43.8	0.858857	2	21	25.8	0.881234
		0.150	2	40	44.8	0.859403	2	21	25.8	0.881984
		0.250	2	41	45.8	0.859226	2	22	26.8	0.880852
0.10	0.25	0.010	2	34	38.8	0.849561	2	17	21.8	0.870090
		0.025	3	32	38.8	0.849065	2	17	21.8	0.870185
		0.050	2	35	39.8	0.849025	2	18	22.8	0.868363
		0.150	2	35	39.8	0.849478	2	18	22.8	0.868994
		0.250	2	36	40.8	0.849245	2	18	22.8	0.869642
	0.20	0.010	2	30	34.8	0.852762	2	16	20.8	0.872247
		0.025	2	30	34.8	0.852833	2	16	20.8	0.872345
		0.050	2	31	35.8	0.852092	2	16	20.8	0.872510
		0.150	2	31	35.8	0.852573	2	16	20.8	0.873179
		0.250	2	32	36.8	0.852232	2	16	20.8	0.873867
	0.15	0.010	2	27	31.8	0.855616	2	14	18.8	0.877235
		0.025	2	27	31.8	0.855691	2	14	18.8	0.877340
		0.050	2	27	31.8	0.855818	2	14	18.8	0.877516
		0.150	2	28	32.8	0.855318	2	15	19.8	0.875579
		0.250	2	28	32.8	0.855838	2	15	19.8	0.876289

Table A3. The optimal (m^*,t^*), n^*, total cost TC^{**} and the critical value for c_1 = 0.825,0.85 and p = 0.01, 0.025, 0.05, 0.15, 0.25 under m_0 = 30, L = 0.1 and c_0 = 0.80.

		c_1				0.825					0.85	
α	β	p	m^*	t^*	n^*	TC^{**}	C_L^0	m^*	t^*	n^*	TC^{**}	C_L^0
0.01	0.15	0.010	10	0.14	702	723.36	0.817978	6	0.18	167	180.08	0.837372
		0.025	8	0.17	719	736.38	0.817985	5	0.21	171	182.07	0.837391
		0.050	7	0.20	739	754.37	0.817982	5	0.22	174	185.09	0.837367
		0.150	5	0.27	796	807.34	0.817984	4	0.27	186	195.09	0.837368
		0.250	5	0.29	836	847.44	0.817973	4	0.31	194	203.24	0.837387
	0.20	0.010	9	0.15	632	651.34	0.818984	6	0.20	151	164.18	0.839282
		0.025	9	0.16	643	662.43	0.818983	5	0.22	155	166.10	0.839262
		0.050	7	0.20	663	678.38	0.818985	5	0.21	158	169.07	0.839238
		0.150	6	0.25	712	725.50	0.818980	4	0.27	169	178.06	0.839203
		0.250	5	0.29	750	761.44	0.818975	4	0.31	176	185.26	0.839252
	0.25	0.010	9	0.15	573	592.35	0.819937	5	0.20	141	152.01	0.840983
		0.025	8	0.17	585	602.38	0.819938	5	0.22	142	153.12	0.841020
		0.050	7	0.18	602	617.30	0.819938	5	0.21	145	156.04	0.840959
		0.150	5	0.26	648	659.28	0.819930	4	0.26	155	164.04	0.840953
		0.250	5	0.29	680	691.43	0.819928	3	0.31	164	170.92	0.840931
0.05	0.15	0.010	8	0.16	441	458.30	0.816111	5	0.22	103	114.09	0.833883
		0.025	8	0.17	448	465.36	0.816109	5	0.23	104	115.14	0.833897
		0.050	6	0.21	463	476.28	0.816111	4	0.25	108	117.00	0.833837
		0.150	5	0.26	496	507.32	0.816107	3	0.30	116	122.91	0.833820
		0.250	4	0.30	523	532.19	0.816098	3	0.33	120	126.98	0.833832
	0.20	0.010	8	0.16	384	401.30	0.817265	5	0.21	91	102.06	0.836050
		0.025	8	0.17	390	407.39	0.817266	4	0.24	94	102.98	0.836036
		0.050	7	0.19	401	416.35	0.817262	4	0.23	96	104.90	0.835931
		0.150	5	0.26	432	443.30	0.817259	4	0.28	100	109.12	0.836031
		0.250	4	0.31	455	464.23	0.817261	3	0.32	106	112.95	0.835992

Table A3. *Cont.*

		c_1			0.825					0.85		
α	β	p	m^*	t^*	n^*	TC^{**}	C_L^0	m^*	t^*	n^*	TC^{**}	C_L^0
	0.25	0.010	7	0.17	341	356.19	0.818390	4	0.21	84	92.85	0.838071
		0.025	7	0.17	346	361.22	0.818393	4	0.24	84	92.95	0.838121
		0.050	7	0.20	353	368.41	0.818396	4	0.25	85	94.00	0.838141
		0.150	5	0.25	381	392.24	0.818381	3	0.27	92	98.82	0.838054
		0.250	4	0.30	401	410.19	0.818384	3	0.30	95	101.89	0.838044
0.10	0.15	0.010	7	0.17	327	342.20	0.814632	4	0.22	77	85.89	0.830936
		0.025	7	0.17	332	347.21	0.814629	4	0.25	77	85.99	0.831018
		0.050	6	0.20	341	354.22	0.814630	4	0.26	78	87.02	0.831024
		0.150	5	0.26	365	376.30	0.814629	3	0.30	84	90.90	0.830965
		0.250	4	0.29	385	394.18	0.814621	3	0.32	87	93.95	0.830953
	0.20	0.010	7	0.17	278	293.20	0.815869	4	0.24	66	74.95	0.833370
		0.025	6	0.20	284	297.22	0.815874	3	0.29	69	75.87	0.833391
		0.050	6	0.20	290	303.21	0.815865	3	0.27	70	76.82	0.833330
		0.150	4	0.28	313	322.12	0.815868	3	0.28	73	79.83	0.833249
		0.250	4	0.30	327	336.20	0.815862	3	0.32	75	81.95	0.833338
	0.25	0.010	7	0.18	239	254.23	0.817112	3	0.28	60	66.83	0.835706
		0.025	6	0.19	245	258.11	0.817096	4	0.24	58	66.97	0.835744
		0.050	5	0.22	252	263.10	0.817105	3	0.28	61	67.83	0.835685
		0.150	5	0.26	267	278.28	0.817104	3	0.32	63	69.95	0.835767
		0.250	4	0.28	282	291.13	0.817093	3	0.29	66	72.86	0.835591

Table A4. The optimal (m^*,t^*), n^*, total cost TC^{**} and the critical value for c_1 = 0.875, 0.90 and p = 0.01, 0.025, 0.05, 0.15, 0.25 under m_0 = 30, L = 0.1 and c_0 = 0.80.

		c_1			0.875					0.90		
α	β	p	m^*	t^*	n^*	TC^{**}	C_L^0	m^*	t^*	n^*	TC^{**}	C_L^0
0.01	0.15	0.010	4	0.21	72	80.86	0.858158	3	0.26	39	45.78	0.880480
		0.025	4	0.24	72	80.95	0.858235	3	0.28	39	45.83	0.880627
		0.050	4	0.24	73	81.96	0.858230	3	0.25	40	46.74	0.880226
		0.150	3	0.28	79	85.83	0.858019	3	0.29	41	47.86	0.880481
		0.250	3	0.33	81	87.98	0.858239	2	0.31	45	49.63	0.880063
	0.20	0.010	4	0.22	66	74.86	0.860656	3	0.28	36	42.83	0.883677
		0.025	4	0.24	66	74.96	0.860825	3	0.23	37	43.70	0.883314
		0.050	4	0.24	67	75.96	0.860781	3	0.25	37	43.76	0.883415
		0.150	3	0.29	72	78.88	0.860732	3	0.29	38	44.87	0.883598
		0.250	3	0.29	75	81.87	0.860607	2	0.37	41	45.74	0.883592
	0.25	0.010	4	0.22	61	69.88	0.863093	3	0.25	34	40.75	0.886296
		0.025	3	0.26	64	70.77	0.863016	3	0.27	34	40.80	0.886387
		0.050	4	0.24	62	70.97	0.863185	2	0.32	37	41.65	0.886264
		0.150	3	0.28	67	73.83	0.863001	2	0.31	38	42.63	0.886147
		0.250	3	0.31	69	75.93	0.863103	2	0.31	39	43.62	0.886001
0.05	0.15	0.010	3	0.25	45	51.75	0.853037	2	0.30	25	29.61	0.874037
		0.025	3	0.26	45	51.78	0.853136	2	0.31	25	29.62	0.874068
		0.050	3	0.29	45	51.87	0.853318	2	0.32	25	29.64	0.874201
		0.150	3	0.29	47	53.86	0.853148	2	0.30	26	30.59	0.873748
		0.250	2	0.34	51	55.68	0.853011	2	0.34	26	30.69	0.874244
	0.20	0.010	3	0.26	40	46.77	0.856188	2	0.28	23	27.55	0.877466
		0.025	3	0.27	40	46.82	0.856313	2	0.28	23	27.56	0.877600
		0.050	3	0.25	41	47.74	0.856028	2	0.29	23	27.58	0.877662
		0.150	3	0.29	42	48.86	0.856223	2	0.34	23	27.68	0.878102
		0.250	2	0.31	46	50.63	0.855990	2	0.30	24	28.61	0.877649

Table A4. *Cont.*

		c_1				0.875					0.90	
α	β	p	m^*	t^*	n^*	TC^{**}	C_L^0	m^*	t^*	n^*	TC^{**}	C_L^0
	0.25	0.010	3	0.27	36	42.80	0.859184	2	0.28	21	25.55	0.881071
		0.025	2	0.34	39	43.68	0.859205	2	0.28	21	25.56	0.881211
		0.050	3	0.25	37	43.74	0.858979	2	0.29	21	25.58	0.881276
		0.150	2	0.35	40	44.71	0.859213	2	0.35	21	25.70	0.881722
		0.250	2	0.35	41	45.69	0.859100	2	0.30	22	26.59	0.881101
0.10	0.15	0.010	2	0.37	34	38.75	0.849386	2	0.37	17	21.74	0.869842
		0.025	3	0.26	32	38.79	0.849094	2	0.27	18	22.54	0.868511
		0.050	3	0.29	32	38.86	0.849262	2	0.28	18	22.56	0.868540
		0.150	2	0.39	35	39.78	0.849421	2	0.31	18	22.63	0.868954
		0.250	2	0.36	36	40.72	0.849135	2	0.37	18	22.74	0.869499
	0.20	0.010	2	0.34	30	34.67	0.852513	2	0.26	16	20.52	0.872750
		0.025	2	0.35	30	34.70	0.852598	2	0.27	16	20.53	0.872667
		0.050	3	0.29	28	34.87	0.852663	2	0.27	16	20.55	0.872881
		0.150	2	0.34	31	35.67	0.852415	2	0.31	16	20.62	0.873136
		0.250	2	0.32	32	36.64	0.852231	2	0.37	16	20.73	0.873715
	0.25	0.010	2	0.29	27	31.59	0.855595	2	0.29	14	18.58	0.877207
		0.025	2	0.30	27	31.60	0.855599	2	0.30	14	18.59	0.877212
		0.050	2	0.31	27	31.63	0.855682	2	0.31	14	18.62	0.877327
		0.150	2	0.29	28	32.58	0.855473	1	0.43	17	19.43	0.877249
		0.250	2	0.34	28	32.68	0.855741	1	0.43	17	19.43	0.877249

References

1. Montgomery, D.C. *Introduction to Statistical Quality Control*; John Wiley and Sons Inc.: New York, NY, USA, 1985.
2. Balakrishnan, N.; Aggarwala, R. *Progressive Censoring: Theory, Methods and Applications*; Birkhäuser: Boston, MA, USA, 2000.
3. Aggarwala, R. Progressive interval censoring: Some mathematical results with applications to inference. *Commun. Stat.-Theory Methods* **2001**, *30*, 1921–1935.
4. Balakrishnan, N. Progressive Censoring Methodology: An Appraisal (with Discussions). *Test* **2007**, *16*, 211–296. [CrossRef]
5. Balakrishnan, N.; Cramer, E. *The Art of Progressive Censoring*; Birkhäuser: Boston, MA, USA, 2014.
6. Laumen, B.; Cramer, E. Inference for the lifetime performance index with progressively Type-II censored samples from gamma distributions. *Econ. Qual. Control* **2015**, *30*, 59–73. [CrossRef]
7. Bdair, O.M.; Abu Awwad, R.R.; Abufoudeh, G.K.; Naser, M.F.M. Estimation and Prediction for Flexible Weibull Distribution Based on Progressive Type II Censored Data. *Commun. Math. Stat.* **2020**, *8*, 255–277. [CrossRef]
8. Panahi, H. Interval estimation of Kumaraswamy parameters based on progressively type II censored sample and record values. *Miskolc Math. Notes* **2020**, *21*, 319. [CrossRef]
9. EL-Sagheer, R.M. Estimation of parameters of Weibull–Gamma distribution based on progressively censored data. *Stat. Pap.* **2018**, *59*, 725–757. [CrossRef]
10. Lee, W.C.; Wu, J.W.; Hong, C.W. Assessing the lifetime performance index of products with the exponential distribution under progressively type II right censored samples. *J. Comput. Appl. Math.* **2009**, *231*, 648–656. [CrossRef]
11. Wu, J.W.; Lee, W.C.; Lin, L.S.; Hong, M.L. Bayesian test of lifetime performance index for exponential products based on the progressively type II right censored sample. *J. Quant. Manag.* **2011**, *8*, 57–77.
12. Wu, S.F.; Lin, Y.P. Computational testing algorithmic procedure of assessment for lifetime performance index of products with one-parameter exponential distribution under progressive type I interval censoring. *Math. Comput. Simul.* **2016**, *120*, 79–90. [CrossRef]
13. Wu, S.F.; Liu, T.H.; Lai, Y.H.; Chang, W.T. A study on the experimental design for the lifetime performance index of Rayleigh lifetime distribution under progressive type I interval censoring. *Mathematics* **2022**, *10*, 517. [CrossRef]
14. Wu, S.F. The performance assessment on the lifetime performance index of products following Chen lifetime distribution based on the progressive type I interval censored sample. *J. Comput. Appl. Math.* **2018**, *334*, 27–38. [CrossRef]
15. Chen, Z. A new two-parameter lifetime distribution with bathtub shape or increasing failure rate function. *Stat. Probab. Lett.* **2000**, *49*, 155–161. [CrossRef]
16. Huang, S.R.; Wu, S.J. Reliability sampling plans under progressive type-I interval censoring using cost functions. *IEEE Trans. Reliab.* **2008**, *57*, 445–451. [CrossRef]

17. Xie, M.; Lai, C.D. Reliability analysis using an additiveWeibull model with bathtub-shaped failure rate function. *Reliab. Eng. Syst. Saf.* **1995**, *52*, 87–93. [CrossRef]
18. Gail, M.H.; Gastwirth, J.L. A scale-free goodness of fit test for the exponential distribution based on the Gini Statistic. *J. R. Stat. Soc. B* **1978**, *40*, 350–357. [CrossRef]