mathematics

Review

Research Trend, Logical Structure and Outlook on Complex Economic Game

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Abstract: Diseases, natural disasters, and other emergencies force the economy and management system to confront nonlinear and random changes. In recent years, complexity science has attracted much attention. Complex economics believes that economic models are dynamic, stochastic, and unpredictable, and that equilibrium and stability are temporary. It is changing traditional economic theory. Based on complexity theory, bibliometric theory, nonlinear theory, and game theory, combined with knowledge graph methods, the article analyzed 200 papers from the Web of Science, covering the period 1998–2022. This research presents the research structure and theoretical evolution of complex economic games through visualization techniques. The clusters of keywords and the logical relationships between them are discussed. Then, the analysis of hot keywords and co-occurrence keywords is carried out. Finally, future research directions for complex economic games are given: (1) the market complexity that comes with intelligent expectations, (2) complex characteristics of the data trading market, and (3) complex risk control for emergencies. The innovation lies in the use of data analysis software combined with manual knowledge to overcome the shortcomings of inflexible software analysis, as well as weak manual storage and computation. This research builds a theoretical foundation for grasping the research direction and selecting advanced topics.

Keywords: complexity; stability; period; bifurcation; chaos; big data; artificial intelligence; risk warning

MSC: 91A11; 91A25; 93C10; 34A34

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1. Introduction

Complexity theory is profoundly changing economic theory. Neoclassical economics believes that statics and equilibrium are constant states. Players makes decisions in a world of equilibrium. Conversely, complex economics accepts randomness, uncertainty, nonequilibrium, and dynamic change. They also consider that economic systems can self-renew, self-assess, and are uncertain.

Complexity theory originated in the 19th century and did not originally emerge from economics. In chemistry, biology, and physics, scientists found a class of unbalanced, changing, periodic, and random systems and called them complex systems. The complex systems were composed of many individuals that could interact with each other, share information, and adjust their behaviors. So, they were also called complex adaptive systems [1]. Social systems were complex adaptive systems in which individuals communicated with each other and adapted their strategies [2]. In recent years, under the impact of sudden events, such as diseases, technological revolution, natural disasters, and climate change, the economic system has shown clearly complex characteristics. In 1987, the Santa Fe Institute (SFI) located in New Mexico, USA, formally proposed that "The economy can be viewed as an evolving complex system." In 1999, Arthur first proposed the concept "Complex Economics" [3]. Complex economics is a theory that goes beyond equilibrium; it is uncertain, unpredictable, process-dependent, and evolutionary.

This study provides a review of complex economic games. Based on complexity theory, bibliometric theory, nonlinear theory, and game theory, combined with knowledge graph methods, this paper analyzed 200 articles from 1998 to 2022. This research presents the research structure and theoretical evolution of complex economic games through visualization techniques and scholars' subjective experience. The clusters of keywords and the logical relationships between them are discussed. Additionally, the analysis of hot keywords and co-occurrence keywords is carried out. Then, future research directions are proposed. The structure of the article is as follows: Section 1 is the introduction; Section 2 is the research framework; Section 3 is cluster analysis and hotspots analysis of existing studies; Section 4 is the overview of related research; Section 5 is the research outlook; and Section 6 is the conclusion.

2. Research Framework

This research includes four steps: data collection, data analysis, review of research hotspots, and future research outlook, as shown in Figure 1. Literature data came from the Web of Science, covering the period 1998–2022. The cluster analysis and hotspots analysis are carried out by CiteSpace, based on the results of the software analysis. The logical framework of the study is given manually.

Figure 1. Research Framework.

The retrieval formula is: "complex" or "complexity" or "chaos" or "stability" or "bifurcation" or "market" or "game" or "competition". After manual selection one by one, 200 papers are chosen from 1998 to 2022. The data analysis process is as Table 1 shows:

Table 1. Content of bibliometric analysis.

Step	Details
① Data Format	Data from Web of Science is exported as "text file".
② Data pre-processing	Delete duplicate literature.
③ Data conversion	Import data to "CiteSpace 6.1.R6" software, and execute "conversion" command.
④ Data analysis, data visualization, manual summarization	Clustering, hot keywords and their co-occurrence keywords, research logic analysis.

3. Cluster Analysis, Hotspots Analysis, and Research Logic Analysis

3.1. Cluster Analysis and Research Logic Analysis

(1) Cluster Analysis

The research is clustered as shown in Figure 2, and the studies are clustered into 12 categories.

Each study cluster is typical and representative. Table 2 explains each cluster.

(2) Research Logic Analysis

The results of the analysis of the software are not sufficient to express the logical relationships between the studied clusters. By consulting experts, the logical relationships between clusters are given in the research discussions. Figure 3 shows the logical relationship between clusters. The relationship between each cluster is depicted in Figure 3.

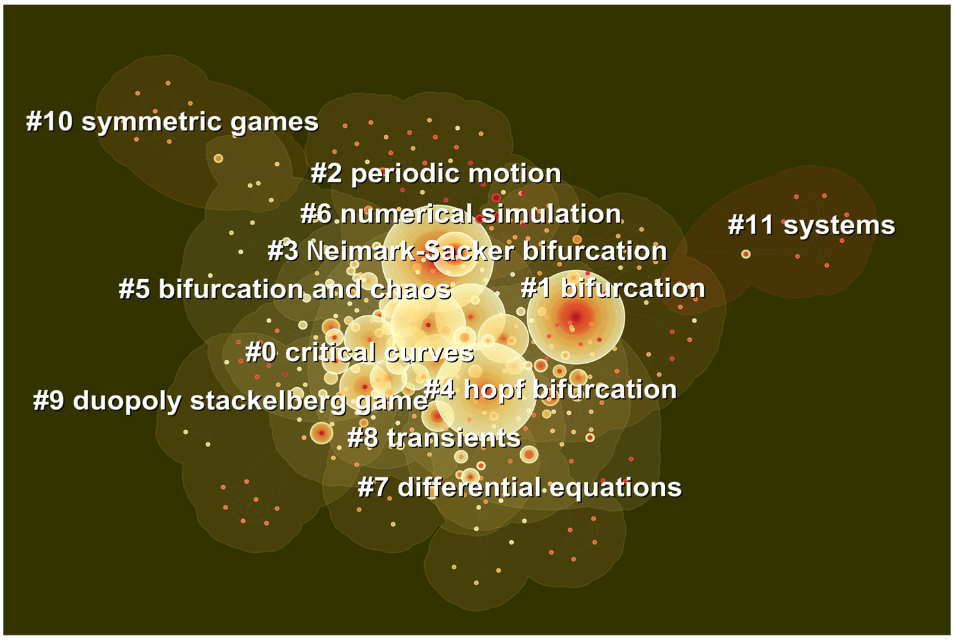

Figure 2. The cluster of keywords.

Table 2. Cluster explanation.

Cluster Label	Cluster Explanation
#0 Critical curves	Calculate the boundaries of equilibrium, period, and chaos.
#1 Bifurcation	#3 Neimark–Sacker bifurcation#5 bifurcation and chaos#4 Hopf bifurcation
#2 Periodic motion	Periodic characteristics of economic fluctuations.
#6 Numerical simulation	Dynamic systems, periodic fluctuations, bifurcation, and chaos were analyzed by numerical simulation.
#7 Differential equations	The economic iterative models were built through differential equations.
#8 Transients	Instantaneous dynamics of system through time series methods.
#10 Symmetric games	Calculating static Nash equilibrium through symmetry games.
#11 Systems	Dynamic Systems.

Stimulated by complex factors, economic systems evolve from static equilibrium (“Symmetric games”) to “periodic motion”, then to “bifurcation” and chaos. Period and bifurcation studies are part of “dynamic system” analysis. “Critical curves” analysis includes the boundary from static equilibrium to periodic fluctuations and the boundary from periodic fluctuations to bifurcation. It is also a hot research topic in academia. The “transients” of various states are analyzed by time series; the time series model can be constructed by “differential equations”, which represents the recurrence relation, as Equation (1) shows.

$$\begin{cases} x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + \ldots + a_n x_{t-n} \\ y_t = b_0 + b_1 y_{t-1} + b_2 y_{t-2} + \ldots + b_n y_{t-n} \\ z_t = c_0 + c_1 z_{t-1} + c_2 z_{t-2} + \ldots + c_n z_{t-n} \end{cases} \tag{1}$$

Equation (1) is a 3-dimension differential equation group. However, when dimensions increase, the equilibrium solutions, period, and bifurcation research cannot be analyzed by mathematical methods. Therefore, it is necessary to resort to “numerical simulations”.

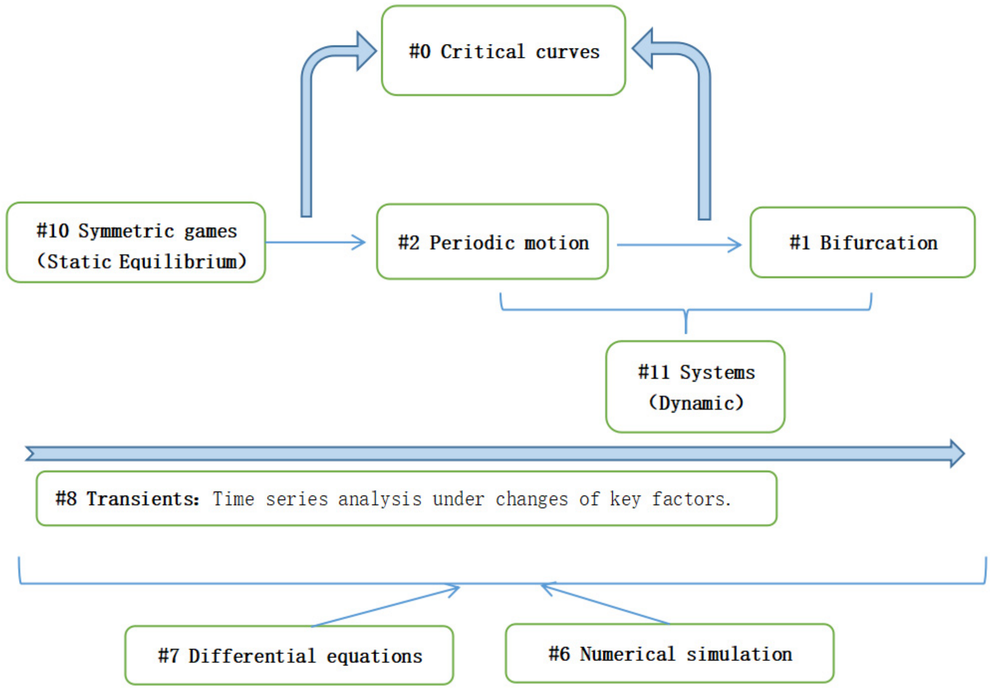

Figure 3. The logical relationships between clusters.

3.2. Hotspots Analysis and Research Logic Analysis

(1) Hotspots Analysis by CiteSpace

Figure 4 shows hot keywords of 200 papers, and larger font size indicates higher frequency of occurrence.

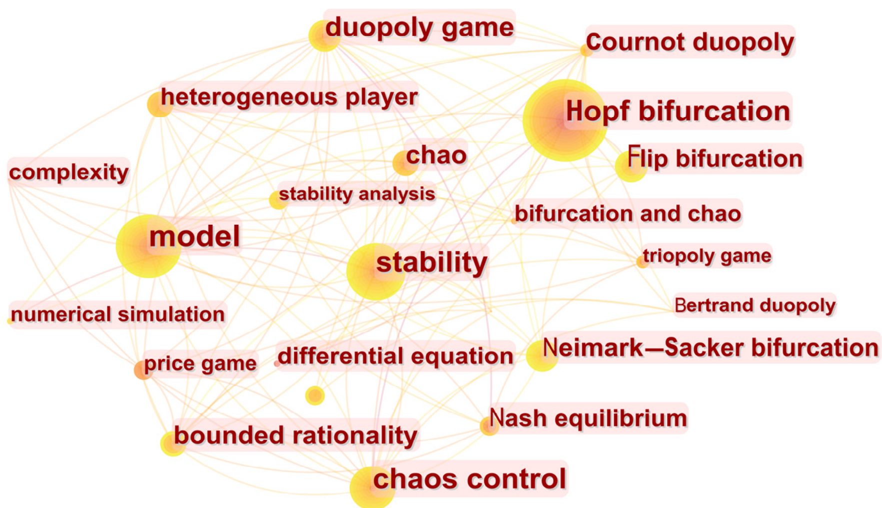

Figure 4. Hot keywords of these articles.

(2) Research Logic Analysis of Hot Keywords

The logical relationship of the research between the hot keywords is represented by Figure 5.

The relationship between hot keywords also reflects the research structure of complex economic games. Firstly, "Numerical simulation" is often used to study complex economics. For example, Lampart and Lampartov [4] studied the control method of heterogeneous Cournot oligopoly by numerical simulation. Ma and Zhang [5] investigated the stability, bifurcation, and chaos in the Chinese insurance market through numerical simulations. Secondly, "model" is the basis for studying complex economic models. The most common models in the literature are "Duopoly game", "Cournot game", "price (Bertnard) game", "Triopoly game", "Output game", etc. Thirdly, the "heterogenous players" in the model is important for reasons of complexity, and includes heterogenous expectations [6], variation in goals [7], number of players, etc. Fourthly, the "stability" studies include the "Nash equilibrium" and its "stability analysis"; the transition path from stable equilibrium to unstable state. Fifthly, when the system breaks stable equilibrium and enters complex fluctuation, the economic model will exhibit periods and bifurcations. The types of bifurcation include Neimark–Sarker bifurcation [8], Hopf bifurcation [9], Flip bifurcation [10], intermittent bifurcation [11], etc. Finally, bifurcation and chaos imply random and unpredictable fluctuations; therefore, chaos control is also a hot topic. In Section 4, the review of the hot keywords bolded in Figure 5 will be carried out.

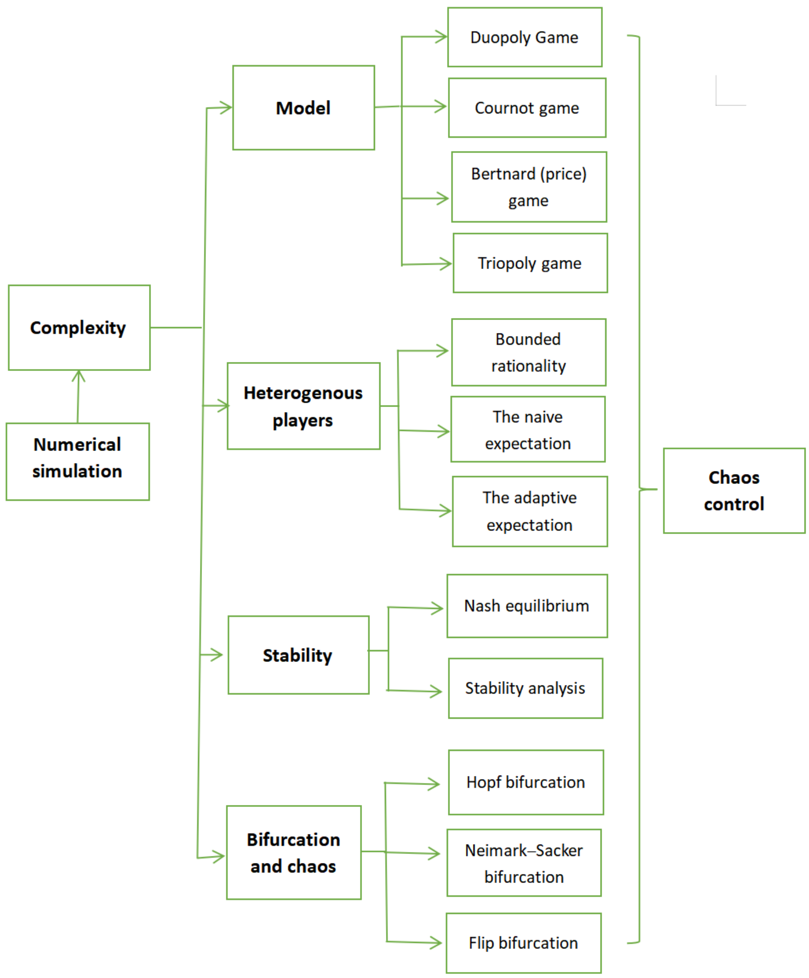

Figure 5. The logical relationship of hot keywords.

4. Related Research

This section provides a review of the hot keywords bolded in Figure 5.

4.1. The Complexity

The Santa Fe Institute (SFI, 1984–now) is a famous research center of complexity in physical, computer, biological, economic, and social systems. The research results of SFI are divided into five parts: system dynamics, adaptive research, chaos research, cross domain research, and structure research. The research of system dynamics uses differential equations to model the complex problems of self-learning organizations [12]. The adaptive research includes springing up, evolution, and chaos edge [13]. Cross-domain research is the intersection of multiple theories, such as the intersection between physics and economics [8]. The structure research [14,15] uses graphical and Boolean methods to describe complex systems, such as complex networks.

In the viewpoint of complex economic game, the focuses include modeling studies, differentiated-player games, stability, numerical simulation, bifurcation and chaos, and chaos control, as Figure 5 shows. Next, we will analyze these hotpots one by one.

4.2. The Model

In these 200 articles, the keyword "model" appears 52 times, as Figure 6 shows. In addition, the keywords that appear together with "model" are also shown. The larger font size indicates a higher frequency of co-occurrence.

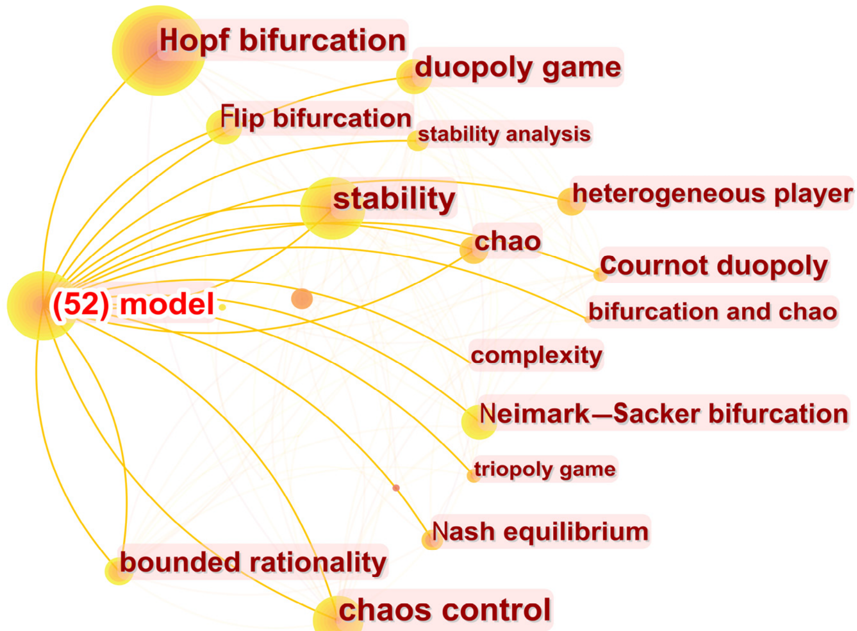

Figure 6. The co-occurrence key words of "model".

In competitive viewpoint, "model" includes duopoly game, triopoly game, Cournot game, Bertrand (price) game, etc. In model structure viewpoint, differential equation models based on game theory are usually used, including discrete and continuous models. For example, Xin et al. [16] constructed a discrete, differential Cournot model based on Caputo fractional-order difference calculus and discussed its long-memory effects. Ledvina [17] constructed a continuous Bertrand oligopoly model based on differential equations. The effects of firm size and product sustainability on equilibrium stability and dynamic characteristics were also investigated. Aska [18] constructed a 3-dimensional, discrete Stackelberg model based on the bounded rationality, obtained the equilibrium solution, and analyzed its periodic and chaotic behavior. Xin [19] constructed a Bertnard model of a master–slave relationship and analyzed its complexity. Cavalli [20] constructed an oligopoly model under resource constraints and studied its multi-attractors. Elsadany [21] studied the complexity and chaos control of the heterogeneous expectation duopoly game. Si [22] constructed a triopoly game with multiple delays, and analyzed the impact of consumers' willingness to purchase green products on complexity. The above models were constructed based on differential equations.

From an evolutionary viewpoint, some scholars have studied the dynamic evolutionary path of economic models, such as stability, period, and complex fluctuation. For the economic period, Ma [23] used fractional order theory to study complex periods in macroeconomic models; he [24] also constructed a money supply model with time delays and investigated the effect of delayed changes on its stability, periodic, and chaotic fluctuations. For the electricity market, Yang [25] constructed a dynamic model to analyze the stability and fluctuations under grid constraints. Wu [26] studied the relationship between the diffusion of agricultural IoT technologies and the complex dynamics of agricultural markets. In conclusion, the evolutionary paths of different economic models differ in complexity dimension, attractor, initial value sensitivity, and bifurcation.

4.3. Heterogenous Player

In dynamic models, heterogeneity of players is an important reason for complexity, and its occurrence frequency is 8 in 200 articles. As can be seen from Figure 7, the heterogeneous player model has typical complex features, such as stability evolution, bifurcation, chaos, chaos control, etc. Furthermore, the "heterogeneous player" has several scenarios. Some researchers focus on the differentiated market expectations of players; the most famous expectation is bounded rationality, with which players determine their next decision based on the most recent historical information. The bounded rationality expectation exhibits rich dynamic complexity features [27–29]. In addition, there are naive expectations, adaptive expectations, and mixed expectations. For examples, Elsadany [21] investigated the complex characteristics of a game model consisting of adaptive expectation firms and natural expectation firms. Wu [6] built a triopoly multiproduction model with heterogeneous expectation: naive expectation, adaptive expectation, and the bounded rationality expectation, and analyzed its complexity. From the viewpoint of player decision order, Xiao [30] investigated the dynamic behavior of the Stackelberg game, in which players take strategies sequentially. From a participant status viewpoint, Wu and Ma [8] studied the complex dynamics of multiproduct supply chains, in which firms had epiphytic relationships; multiple paths of evolution from stable to chaotic were found.

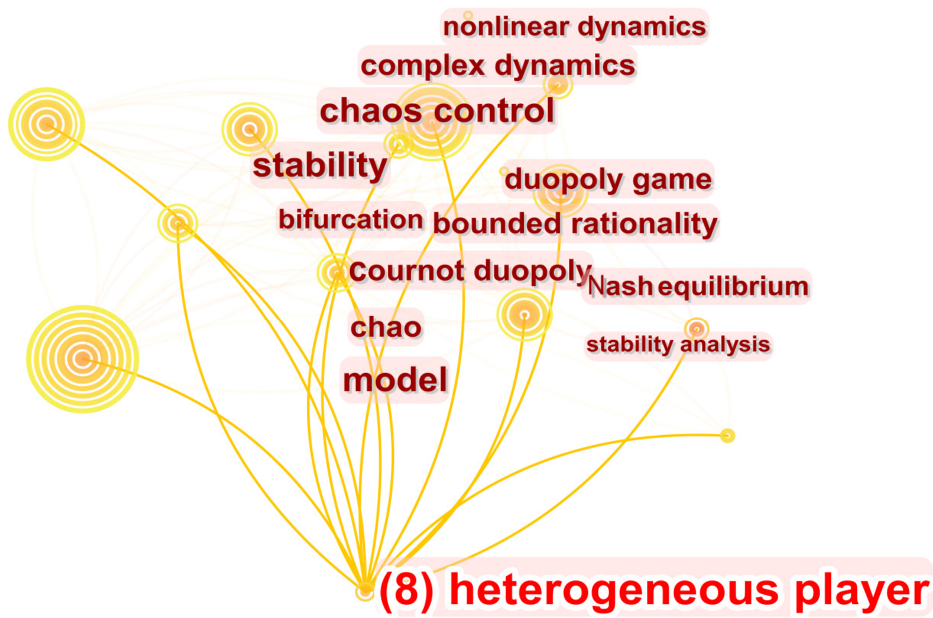

Figure 7. "Heterogeneous player" and its co-occurrence key words.

4.4. Stability

The occurrence frequency of "Stability" is 8 in these 200 articles. As shown in Figure 8, related research has been conducted in the following viewpoint.

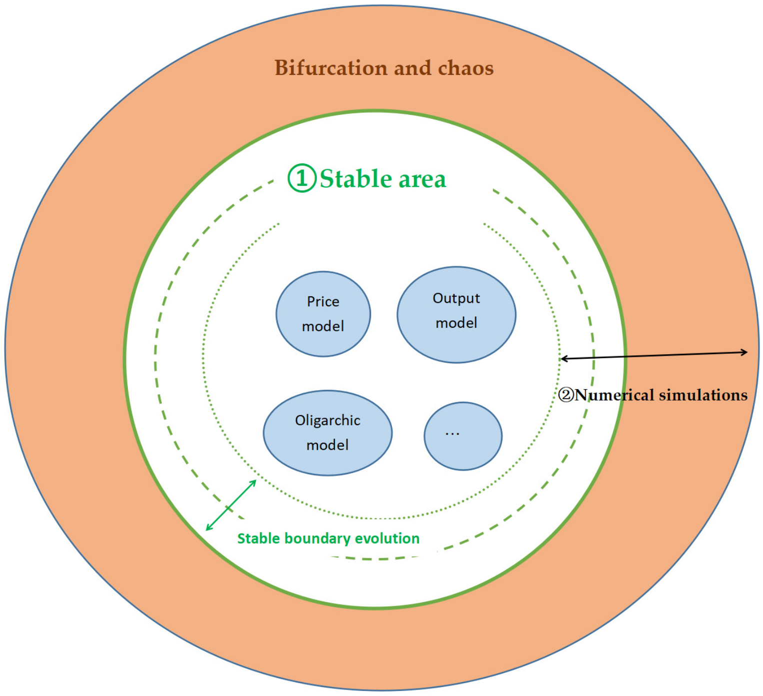

Figure 8. Stability and its evolutionary analysis.

① Stable area. Traditional economics pursues stable equilibrium and optimal solutions under static decisions. Optimal stability studies include: price competition, output competition, production operation model [31], oligarchic models, technology diffusion models [26], queue models [32], Bitcoin trading [33], etc. Nowadays, scholars have gradually focused on stability boundary evolution under nonlinearity and uncertainty. The stability boundary evolution can be analyzed by mathematical theorems [34–40], such as Jury condition. These researchers used mathematics as a tool to analyze equilibrium stability.

② Numerical simulation method. However, mathematical analytical methods cannot meet the increased complexity. Therefore, scholars started to use numerical simulations. Through simulation, higher dimensional and more octants' stability and evolutionary analysis can be developed [8–11].

4.5. Numerical Simulation

Recent years, numerical simulation has replaced analytical methods for studying complex model. In Figure 9, "numerical simulation" is used to analyze the complex "dynamics" of the game model (such as "duopoly game", "Cournot duopoly", etc.). "Dynamics" means dynamic changes of the stability with multiple influencing factors in high-dimensional models. For example, Ma and Wu [6] built a complex dynamics of a 6-dimension, discrete dynamical system, to investigate the changes of "stability" boundaries, "periods", the path to chaos ("dynamics"), and "bifurcation control" by numerical simulation. Additionally, Kabir [41] constructed a complex network model for epidemic transmission, investigating the impact of information spreading in vaccination game on epidemic dynamics.

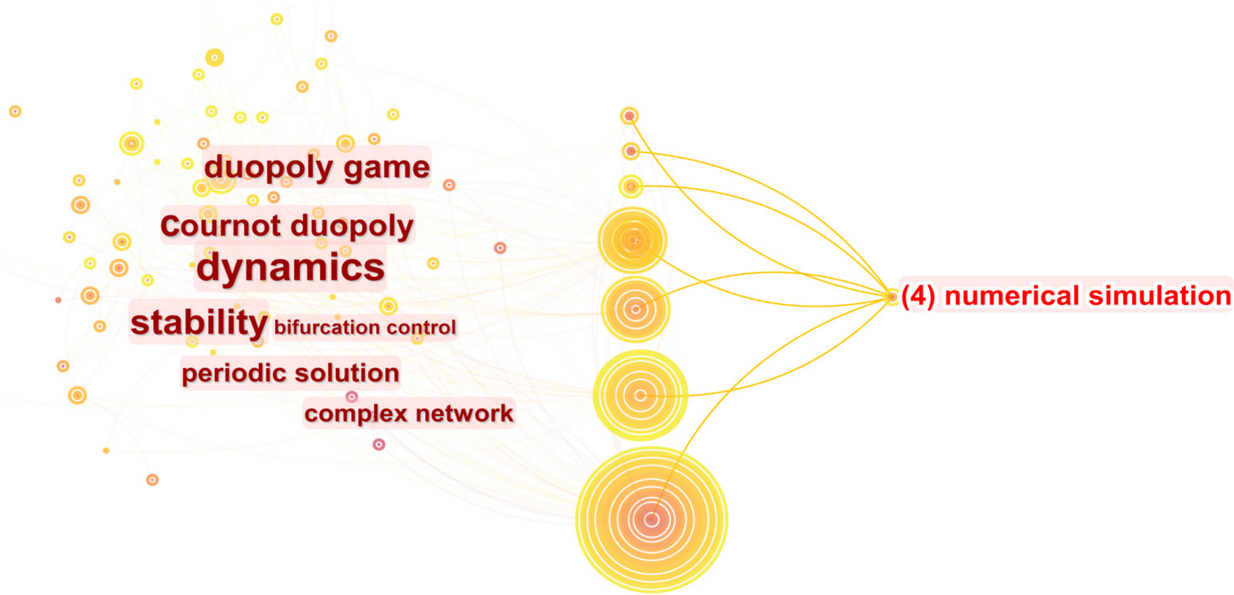

Figure 9. "Numerical simulation" and its co-occurrence key words.

4.6. Bifurcation, Chaos, and Control

When a nonlinear economic system breaks the stable boundary the whole system enters bifurcation and chaos. This is shown in the yellow area of Figure 10, which is a part of Figure 3. In this section, existing research will be organized as: bifurcation, chaos, and chaos control.

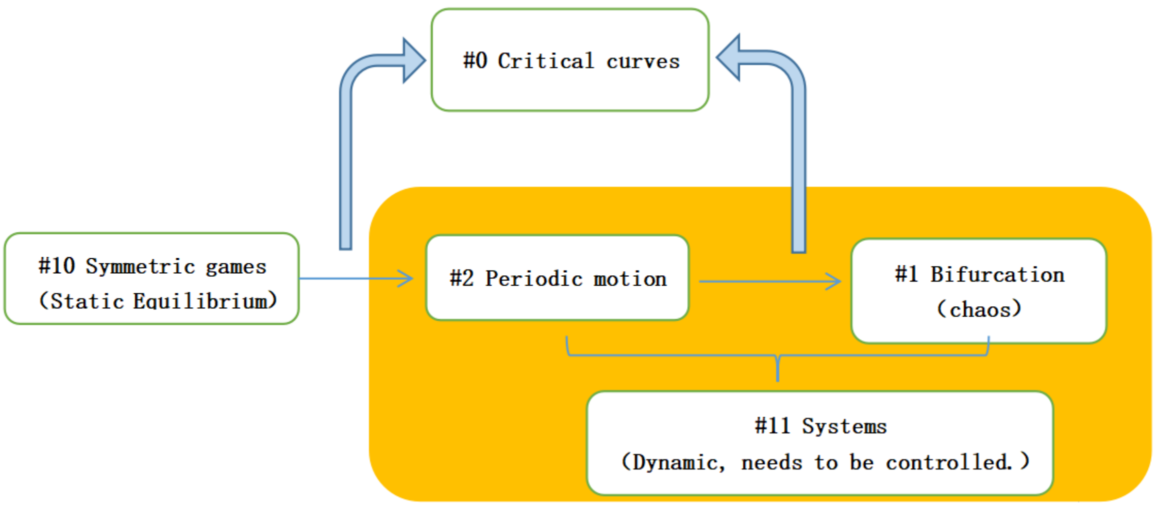

Figure 10. Evolutionary paths of bifurcation and chaos.

4.6.1. Bifurcation and Chaos

The most common one is the period-doubling bifurcation, also named as Flip bifurcation, that is a typical path to chaos. For example, in the energy market, electricity price fluctuations are characterized by Flip bifurcation [8]. Figure 11 shows the change in prices of three competitors in the market. The horizontal axis is the complexity influencing factor, the speed of price adjustment, and the vertical axis is the price. At the beginning, the price of each competitor is stable and the market is in Nash equilibrium. As the price adjustment speed increases, the price enters a state of 2-period oscillation, which means that the price fluctuates at two points. Then, the price enters 4-period oscillation, where the price changes in four states. Finally, the price enters chaotic state, in which the price of the market is unpredictable and the production plan of the company faces great uncertainty.

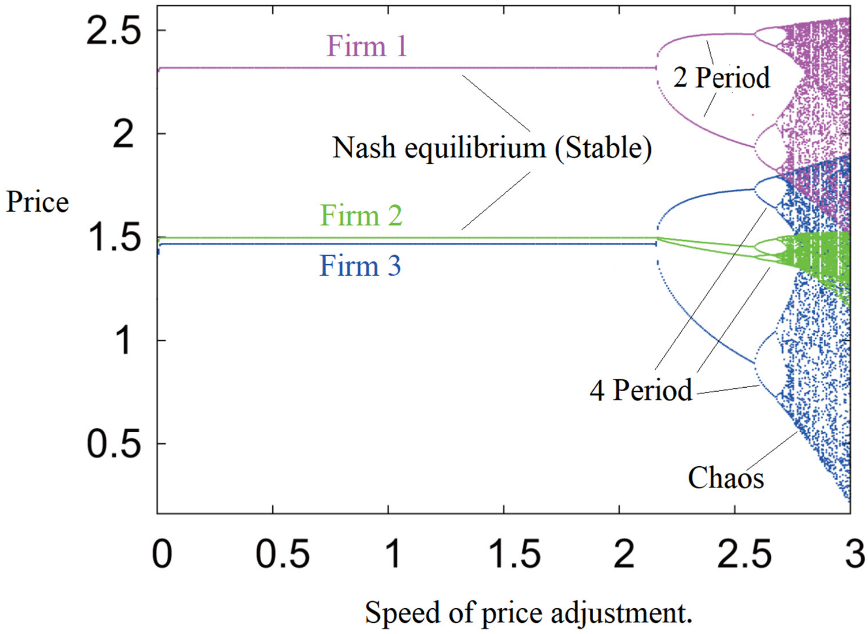

Figure 11. The Flip bifurcation [11].

Another path to chaos is Hopf bifurcation, also known as Neimark–Sacker bifurcation. It does not pass through periods and goes to the chaotic state directly, as Figure 12 shows. However, bifurcations do not always follow Flip or Hopf rules; abrupt changes occur in some nonlinear systems, which lead to tangential bifurcation, as shown in Figure 13 (a three-period bifurcation change to a Hopf bifurcation).

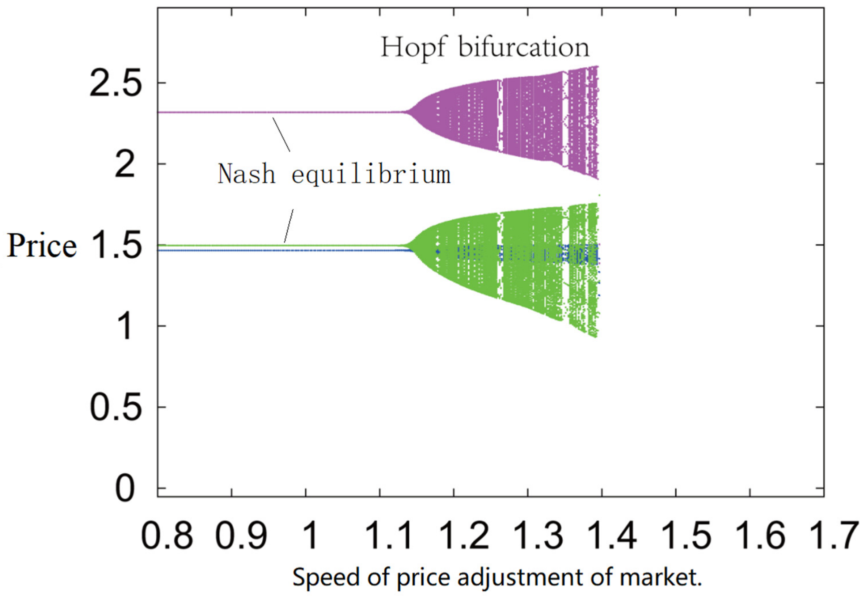

Figure 12. The Hopf bifurcation [11].

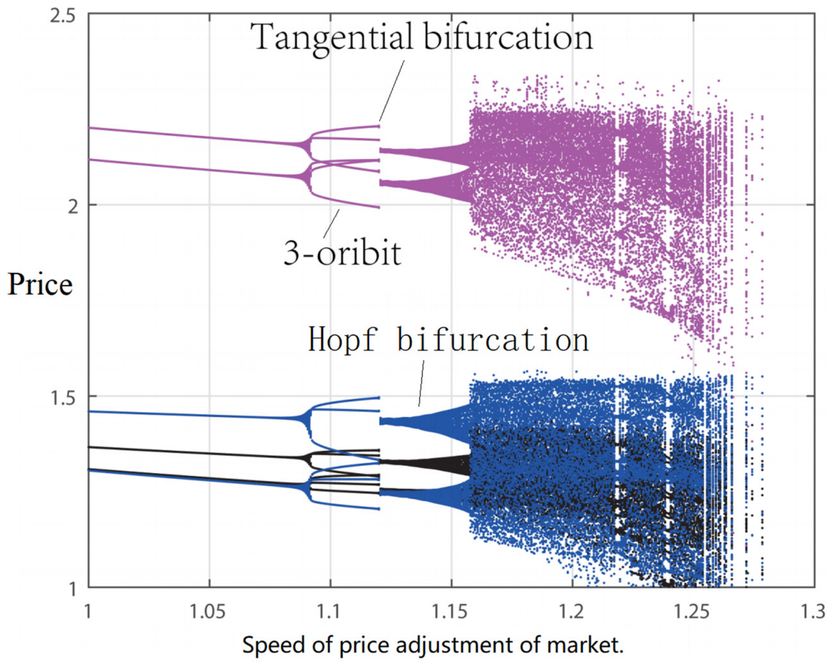

Figure 13. Three-period bifurcation and tangential bifurcation [11].

Different economic structures can cause different types of bifurcation, which are summarized in Table 3.

Table 3. Types of bifurcation.

Type	Application Fields
Hopf bifurcation	Finance system [42]; Macroeconomics ISLM model [23]; Money supply [24]; Delay, Price game [9]; Financial Risks [43]; Insurance industry [44].
Flip bifurcation	Oligopoly game of semi-collusion in production [36]; The oligopoly game considering consumer surplus [10]; Stackelberg game [45].
Tangential bifurcation	Sticky price [11].

4.6.2. Chaos Control

In the market, sudden events, such as natural disasters, supply chain disruptions, and climate change, can lead to dramatic changes in supply and demand, and these external shocks have a high potential to trigger chaos. If chaos occurs, the price and output of goods in the market will change randomly and unpredictably; therefore, business decisions face huge uncertainty challenges. Additionally, a small disturbance can trigger huge and unpredictable fluctuations, such as the butterfly effect. Disequilibrium, period, various types of bifurcations, chaotic fluctuations, and initial value sensitivity can be suppressed by chaos control. Among these 200 articles, 35 articles studied chaos control from 2002 to 2022, distributed by year as shown in Figure 14.

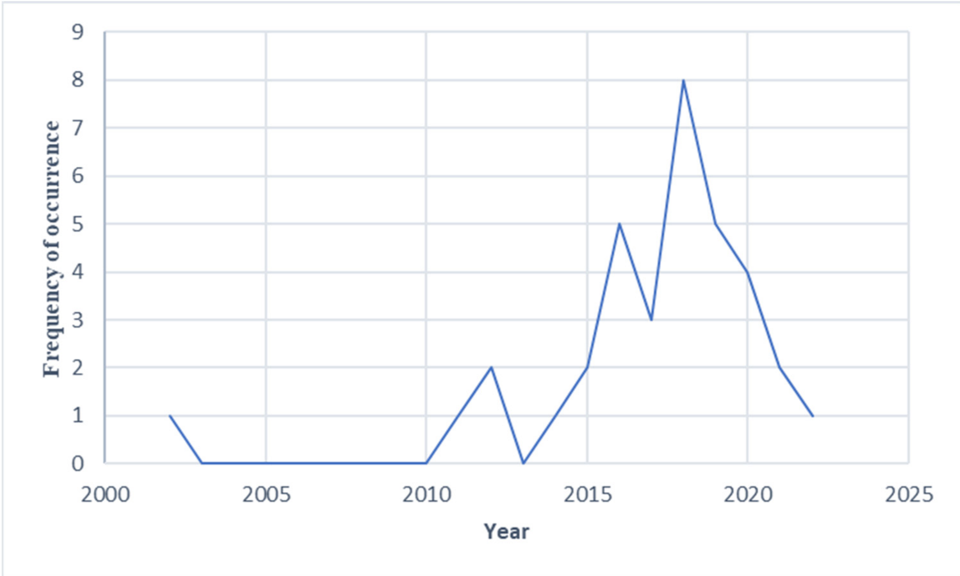

Figure 14. Study distribution of "chaos control" from 2002 to 2022.

5. Research Outlook

5.1. Market Expectations Based on Big Data and Artificial Intelligence

Market expectations are main reasons for complexity, especially the bounded rational expectation; decisions of current production are based on the previous period. By 2023, more than half of GDP will be related to AI-transformed products or services in the world [46]. Big data and AI have changed the structure of market expectations. In this paper, intelligent expectations are defined, based on historical data, as machine learning algorithms that perform case derivation, analogy, statistics to get knowledge, and can estimate the future. Policy makers anticipate the future based on their own experience and the results of AI calculations. The intelligent expectation also reflects the complex characteristics of adaptability, dynamism, and evolution. The basic structure of new expectation rule driven by big data and AI is: "data" → "algorithm" → "human–machine collaboration" → "intelligent expectation", as Figure 15 shows. Firstly, the right result can be drawn only on the basis of high-quality data. Then, intelligent algorithms are run on the correct data, whereby the effectiveness of the algorithms also directly affects the correctness of decision. Subsequently, when the results are generated, the player makes a decision based on the results given by the AI, which incorporates the player's preferences and emotions. Finally, a new market expectation of human–machine mixed emerges.

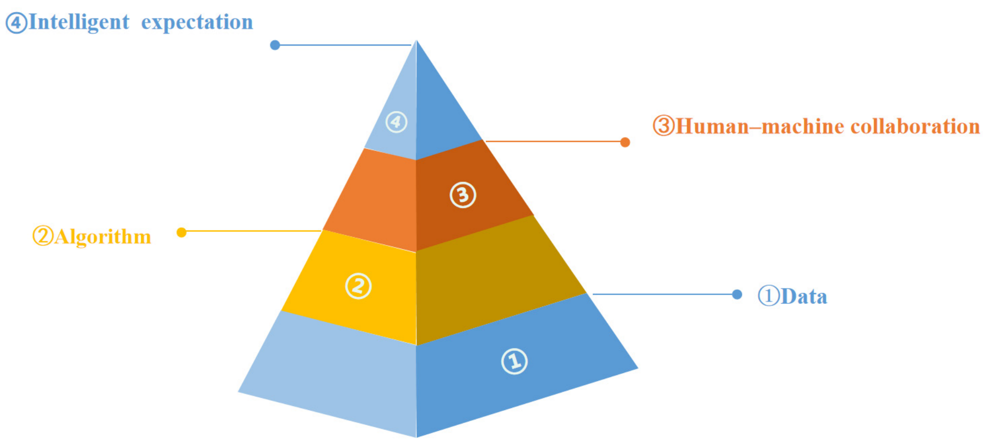

Figure 15. The structure of Intelligent Expectations.

For example, on the GitHub website (an open-source code base), scholars already provide intelligent programs by Python to make predictions for stock markets. For Apple Inc. stock forecasts, the process is as Figure 16 follows: Firstly, the past sequence data of NASDAQ Apple Inc. are collected through Python. Then, artificial intelligence programs, such as LSTM (Long Short-Term Memory) models, are built by Python to analyze the data and give future stock trends. The user considers the AI predictions and personal preferences to make the final decision.

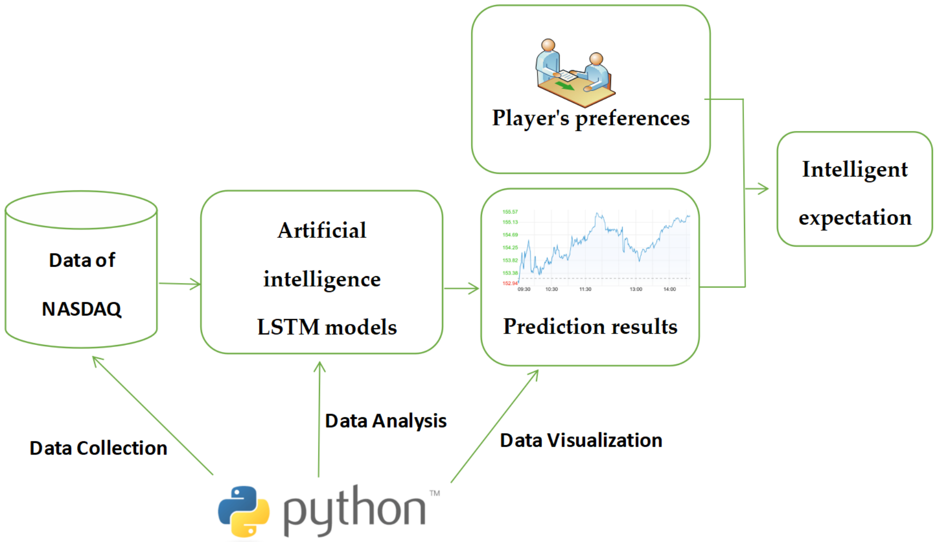

Figure 16. The Intelligent Expectations by Python.

So, what will the new market expectations bring to the economy? It is a subject that needs to be studied urgently.

5.2. Complex Risk of Data Trading

The complex risk of the data trading market is a new issue that needs to be studied. The Internet of Things and cloud computing technologies lead to huge amounts of data that are mined for training artificial intelligence algorithms and supporting decisions. Data are purchased through the data market. Because of the particularity of data product, the data market is different from the ordinary product market: ① the data are reusable and shared. When the data are mined to generate new knowledge and decisions, the original data cannot be consumed and can be reused. ② Data have quality attributes. The collected data need to be preprocessed before it can be used for decisions. Data quality is the basis of decision quality. Better data quality leads to better decisions, which leads to better profits. ③ The benefit distribution in data creation is also different from traditional products. For the final benefit of data, the original data producer and the last data processor should all enjoy the distribution. In addition, data privatization makes the data exchange market lag. The characteristics of data market are shown in Figure 17. The complex dynamics and evolution of the data trading markets is a new research topic.

5.3. Emergency Warning Based on Complexity Theory

A crisis event is uncertain, nonlinear, and difficult to identify. Its occurrence could trigger a new series of crises, via the butterfly effect. Artificial intelligence and the internet have greatly expanded information dissemination and risk derivation channels of emergencies. Typical emergencies were the terrorist attacks on the U.S. World Trade Center and the Pentagon in Washington, D.C. on 11 September 2001. The 911 emergency brought a series of derivative risks to the U.S. and world economies, with the Dow Jones index involving travel, insurance, and aviation plunging and gasoline prices plummeting. The butterfly effect, caused by unexpected events, has imposed a negative impact on the global economy. It is a new research topic using complexity theory to forewarn the derived risks of emergencies. Research can be carried out from the following aspects: ① influencing factors that may trigger derivative risk events; ② the evolutionary path of complex risk and the butterfly effect; ③ chaos risk control, including wave control and system robustness study.

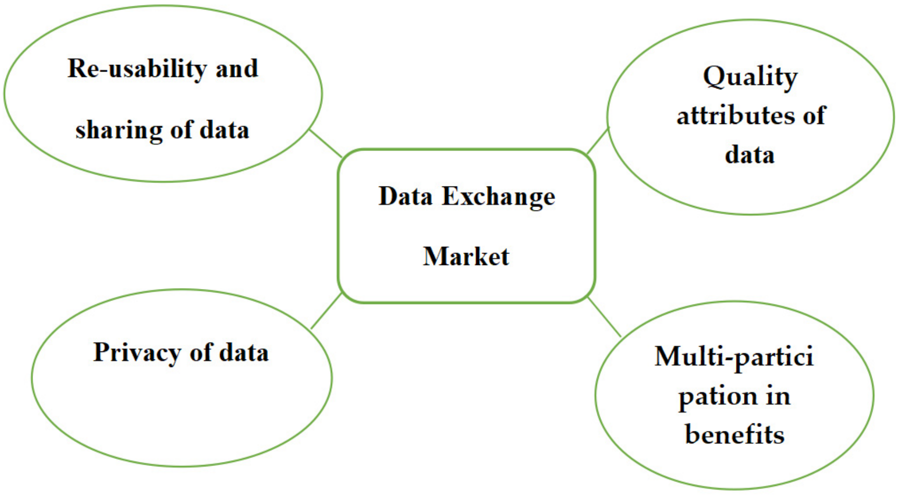

Figure 17. Data market characteristics.

6. The Conclusions

Based on complexity theory, game theory, bibliometrics, and visualization technology, this study provides a review of complex economic games. For 200 articles from 1998 to 2022 in the Web of Science, the study investigates the keyword clusters, hot keywords, and co-occurrence keywords. Subsequently, the logical associations between clusters and the research associations between keywords are given by manual analysis. Finally, future research directions for complex economic games are given: (1) based on historical data, machine learning algorithms perform case derivation, analogy, statistics to get knowledge, and estimate the future. Policy makers anticipate the future based on their own experiences and the results of AI calculations. This intelligent expectation also reflects the complex characteristics of adaptability, dynamism, and evolution. It is a subject that needs to be studied urgently. (2) Data trading markets have special characteristics: data can be shared and reused; data have quality that affects the effectiveness of decisions; and everyone involved in data production should participate in the benefits distribution. Therefore, the complex risks of data trading markets are a new research topic. (3) Uncertainty about emergencies often triggers a series of derivative risks. The influencing factors, evolutionary paths, and control of derivative risks are also issues worthy of study. The innovation of this article lies in the use of big data analysis combined with manual knowledge to overcome the shortcomings of inflexible software analysis and weak manual storage and computation. This research provides a theoretical reference for grasping the research direction and selecting advanced topics.

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