

Article

Deviance and Pearson Residuals-Based Control Charts with Different Link Functions for Monitoring Logistic Regression Profiles: An Application to COVID-19 Data

Maryam Cheema ¹, Muhammad Amin ¹ , Tahir Mahmood ^{2,3} , Muhammad Faisal ⁴ , Kamel Brahim ⁵ 
and Ahmed Elhassanein ^{5,*} 

¹ Department of Statistics, University of Sargodha, Sargodha 40100, Pakistan

² Industrial and Systems Engineering Department, College of Computing and Mathematics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

³ Interdisciplinary Research Centre for Smart Mobility and Logistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

⁴ Faculty of Health Studies, University of Bradford, Bradford BD7 1DP, UK

⁵ Department of Mathematics, College of Science, University of Bisha, P.O. Box 551, Bisha 61922, Saudi Arabia

* Correspondence: el_hassanein@yahoo.com

Abstract: In statistical process control, the control charts are an effective tool to monitor the process. When the process is examined based on an exponential family distributed response variable along with a single explanatory variable, the generalized linear model (GLM) provides better estimates and GLM-based charts are preferred. This study is designed to propose GLM-based control charts using different link functions (i.e., logit, probit, c-log-log, and cauchit) with the binary response variable. The Pearson residuals (PR)- and deviance residuals (DR)-based control charts for logistic regression are proposed under different link functions. For evaluation purposes, a simulation study is designed to evaluate the performance of the proposed control charts. The results are compared based on the average run length (ARL). Moreover, the proposed charts are implemented on a real application for COVID-19 death monitoring. The Monte Carlo simulation study and real applications show that the performance of the model-based control charts with the c-log-log link function gives a better performance as compared to model-based control charts with other link functions.

Keywords: ARL; control charts; COVID-19 data; deviance residuals; link functions; logistic profiling; Pearson residuals

MSC: 62P30; 62J02; 62J12; 62J20



Citation: Cheema, M.; Amin, M.; Mahmood, T.; Faisal, M.; Brahim, K.; Elhassanein, A. Deviance and Pearson Residuals-Based Control Charts with Different Link Functions for Monitoring Logistic Regression Profiles: An Application to COVID-19 Data. *Mathematics* **2023**, *11*, 1113. <https://doi.org/10.3390/math11051113>

Academic Editors: Wen Zhang, Xiaofeng Xu, Jun Wu and Kaijian He

Received: 16 January 2023

Revised: 11 February 2023

Accepted: 20 February 2023

Published: 23 February 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Quality is one of the main requirements of any organization's product and service goodwill. The control charts are the main tools of statistical process control (SPC) for monitoring the quality of products to improve the process capability [1]. In the last few decades, different control charts have been introduced and implemented in various industries to monitor the online process. These control charts include the Shewhart control chart [2], the cumulative sum (CUSUM) control chart [3] and the exponential weighted moving average (EWMA) control chart [4]. The Shewhart-type control chart is the memoryless control chart and is applied to detect the large shift in the mean and standard deviation of the process. However, EWMA and CUSUM are memory-type structures used to monitor small changes in the process parameters.

A single quality characteristic of a process is monitored by control charts, which may be quantitative or qualitative. Sometimes, the quality characteristic depends upon the explanatory variable(s). When the quality characteristic follows a normal distribution and

is linearly associated with the explanatory variable, it is termed a simple linear profile. In linear profiling, one may want to monitor the intercept, slope, and error variance [5].

In profiling, if the normality assumption is violated, we move towards the generalized linear model (GLM)-based profiling. The term GLM refers to a large class of models popularized by Nelder and Wedderburn [6]. In the literature, several monitoring studies were designed based on the GLM approach, and such charts are termed model-based control charts. Skinner et al. [7] proposed a model-based control chart using deviance residuals while the response variable follows a Poisson distribution, and they used the square root link function. Jearkpaporn et al. [8] proposed a model-based scheme based on the deviance residual for the system in which the response variable follows a Gamma distribution, and they used the log link function. In addition, Skinner et al. [9] studied the effectiveness of GLM-based control charts on the semiconductor process data based on the deviance residuals. Koosha and Amiri [10] considered the effect of autocorrelation presence between the observations in different levels of the independent variable in a logistic regression profile on the monitoring procedure (T^2 control chart) and proposed two remedies to account for the autocorrelation within logistic profiles. Shu et al. [11] reviewed the literature on regression control charts based on their importance and practical applications. Asgari et al. [12] proposed the GLM-based control chart to monitor a two-stage procedure under Poisson distribution. Amiri et al. [13] investigated the profiles with binomial and Poisson responses in phase I and monitored them using three methods, namely Hotelling T^2 statistic, the F method, and the likelihood ratio test (LRT). Amiri et al. [14] concentrated on Phase II monitoring and proposed procedures for monitoring multivariate linear and GLM regression profiles. Qi et al. [15] developed a control chart to monitor GLM profiles using the weighted likelihood ratio tests. Moheghi et al. [16] studied the robust estimation and monitoring of parameters in GLM profiles in the presence of outliers. Recently, Kinat et al. [17] proposed the Pearson and deviance residuals-based control charts for the inverse gaussian response. Moreover, recent studies on GLM-based control charts and their applications can be found in the literature [18–23].

One of the most important members of the GLM family is the logistic regression profile. When the quality characteristics follow the Bernoulli distribution, we use the logistic regression profile. Different link functions can be used for logistic regression profiling. These link functions include the logit, probit, log-log, complementary log-log (c-log-log), and cauchit link function. Koosha et al. [10] studied the effect of applying different link functions on the performance of the T^2 control chart in monitoring the parameters of logistic regression profiles, and they found that the logit link function is best for logistic regression. Yu et al. [24] analyzed the performance of the LRProb chart under the assumption that only a small number of predictable abnormal patterns are available. Hakimi et al. [19] proposed some robust approaches to estimate the logistic regression profile parameters to decrease the effects of outliers on the performance of the T^2 control chart. Khosravi et al. [25] proposed three self-starting control charts to monitor a logistic regression profile that models the relationship between a binomial response variable and explanatory variables. Alevizakos [26] proposed two indices, cp and Spmk, for logistic regression profiles using different link functions: the logit, probit, and the c-log-log. The value of each index is approximately the same regardless of the used link functions. Jahani et al. [27] developed two control charts based on Wald and Rao score test (RST) to monitor nominal logistic regression profiles in Phase II.

The available literature showed that most researchers focused on deviance residuals-based control charts with probit and logit link functions. In this study, we evaluate the performance of various link functions in logistic regression profiling based on Pearson and deviance residuals, and find out which one of the link functions shows better performance. The outline of the research work is described as: Section 2 involves the methodology. Section 3 presents the structure of the proposed control charts based on Pearson and deviance residuals. Section 4 consists of numerical evaluations, which include the simulation

study of the proposed control charts. Section 5 describes a real-life application, and finally, Section 6 consists of the conclusion and future recommendations.

2. The GLM-Based Control Charts

The GLM-based control chart is used to enhance the ability of linear profile when the variable of interest follows an exponential family distribution. In this study, GLM-based control charts are designed based on deviance residuals (DR) and Pearson residuals (PR) of the logistic regression. Figure 1 shows all the steps of our proposed approach.

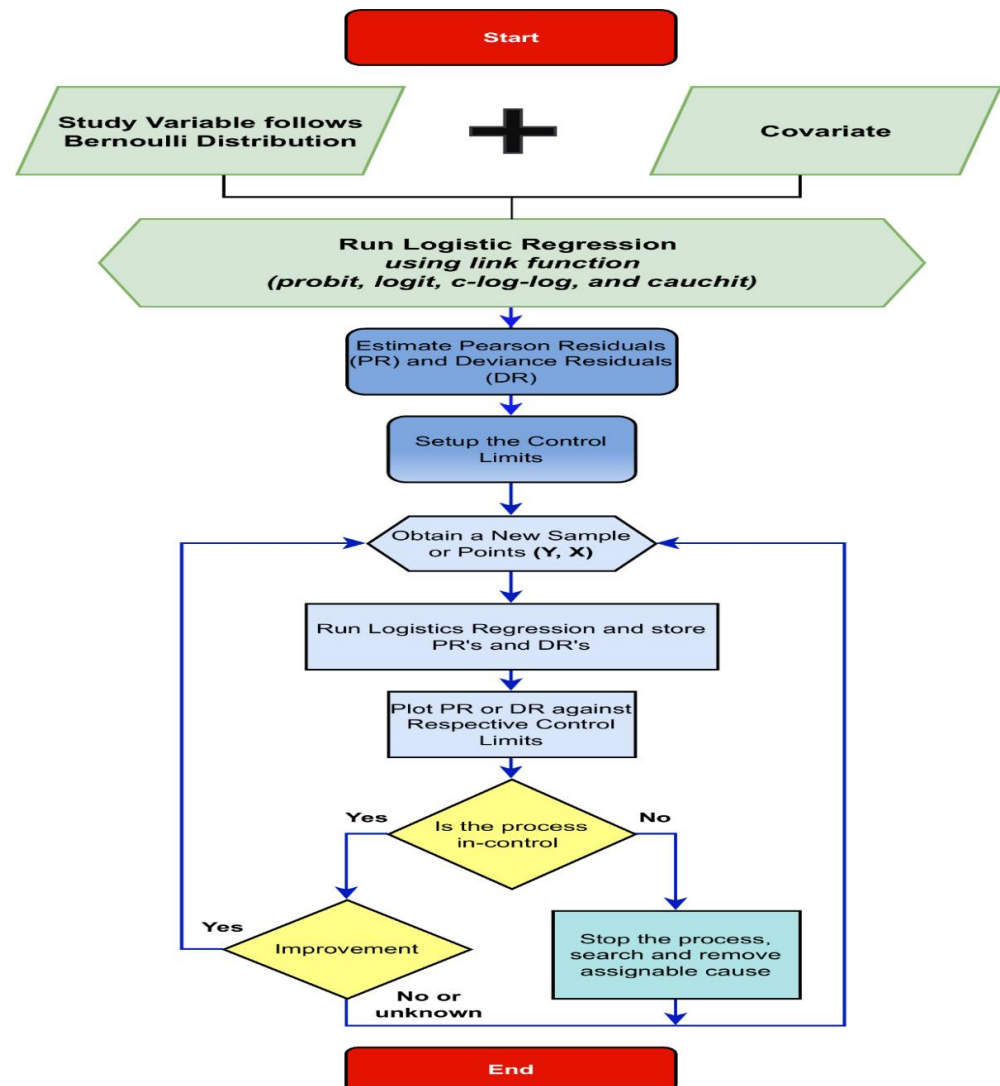


Figure 1. Follow chart of the process.

Suppose that our variable of interest (y) follows the Bernoulli distribution with probability mass function given by:

$$p(y_i|\pi_i) = \pi_i^{y_i}(1 - \pi_i)^{1-y_i}; y = 0, 1 \quad (1)$$

where π_i is the probability of success. The logistic regression model is a subset of the binomial regression model. When the response variable (y) of a regression model belongs to the Bernoulli distribution, the logistic regression model is the most commonly used

statistical model; i.e., $y_i \sim Be(\pi_i)$, where the probability π_i is a function of X_i and is defined by:

$$\pi_i = \frac{\exp(x'_i \beta)}{1 + \exp(x'_i \beta)}, \quad (2)$$

where x_i is the i th row of X , which is an $n \times (p + 1)$ data matrix with p explanatory variables, and β is a $(p + 1) \times 1$ vector of regression coefficients. The mean and variance of Bernoulli distribution are, respectively, given by $E(y_i) = \pi_i$ and $var(y_i) = \pi_i(1 - \pi_i)$.

To convert Equation (1) into an exponential format, we rewrite it as:

$$f(y|\pi_i) = \exp[\log(\pi_i^{y_i}(1 - \pi_i)^{1-y_i})] \quad (3)$$

$$= \exp(y_i \log \pi_i + (1 - y_i) \log(1 - \pi_i)) \quad (4)$$

$$= \exp(y_i \log \pi_i + \log(1 - \pi_i) - y_i \log(1 - \pi_i)) \quad (5)$$

$$= \exp(y_i \log \left(\frac{\pi_i}{1 - \pi_i} \right) - y_i \log(1 - \pi_i)) \quad (6)$$

The logit link function is suitable for linking π_i and X_i in the logistic regression profiles denoted by $g(\pi_i)$, which is defined by:

$$g(\pi_i) = \log \left(\frac{\pi_i}{1 - \pi_i} \right) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip}, \quad (7)$$

We also assume some other link functions such as the probit, c-log-log, and the cauchit to fit the logistic regression model. These link functions are given below:

$$\text{probit} \quad g(\pi_i) = \phi^{-1}(\pi_i)$$

$$\text{c-log-log} \quad g(\pi_i) = \ln(-\ln(1 - \pi_i))$$

$$\text{cauchit} \quad g(\pi_i) = \tan(\pi(\pi_i - 0.5))$$

The maximum likelihood estimator (MLE), based on the iterative reweighted least square technique (IRLS), is the most often used approach for estimating the parameter β , where the following log-likelihood of Equation (5) should be maximized.

$$l = \sum_{i=1}^n [y_i \log(\pi_i) + (1 - y_i) \log(1 - \pi_i)]$$

$$l = \sum_{i=1}^n [y_i \log(\pi_i) + \log(1 - \pi_i) - y_i \log(1 - \pi_i)]$$

$$l = \sum_{i=1}^n [\log(1 - \pi_i)] + \sum_{i=1}^n y_i [\log \pi_i - \log(1 - \pi_i)]$$

$$l = \sum_{i=1}^n [\log(1 - \pi_i)] + \sum_{i=1}^n y_i \left[\log \left(\frac{\pi_i}{1 - \pi_i} \right) \right]$$

Substituting Equations (1) and (7) in the above expression, we have:

$$l = \sum_{i=1}^n \left[\log \left(1 - \frac{\exp(x'_i \beta)}{1 + \exp(x'_i \beta)} \right) \right] + \sum_{i=1}^n y_i x'_i \beta$$

After some simplifications, the result is as follows:

$$l = - \sum_{i=1}^n [\log(1 + \exp(x_i \beta))] + \sum_{i=1}^n y_i x'_i \beta \quad (8)$$

Differentiating Equation (3) with respect to β and equating to zero yield:

$$\frac{\partial l}{\partial \beta} = \sum_{i=1}^n (y_i - \pi_i) x_i' = 0 \quad (9)$$

Equation (4) is solved using the IRLS algorithm and obtained:

$$\hat{\beta}_{MLE} = \left(X' \hat{W} X \right)^{-1} X' \hat{W} \hat{z}, \quad (10)$$

where

$$\hat{W} = \text{diag}(\hat{\pi}_i(1 - \hat{\pi}_i)), \text{ and } \hat{z}_i = \log(\hat{\pi}_i) + \frac{y_i - \hat{\pi}_i}{\hat{\pi}_i(1 - \hat{\pi}_i)}, \quad (11)$$

where $\hat{\pi}_i = \frac{\exp(x_i' \hat{\beta}_{MLE})}{1 + \exp(x_i' \hat{\beta}_{MLE})}$. The general form of the DR for the logistic regression model has the following form:

$$r_i^D = \text{sign}(y_i - \hat{\pi}_i) \sqrt{2\{y_i \log \hat{\pi}_i + (1 - y_i) \log(1 - \hat{\pi}_i)\}},$$

However, the PR for the logistic regression is defined as:

$$r_i^P = \frac{y_i - \hat{\pi}_i}{\sqrt{\hat{\pi}_i(1 - \hat{\pi}_i)}}$$

3. Structure of the Control Charts

The Shewhart control chart includes a baseline as well as an upper control limit (UCL) and lower control limit (LCL), which are represented as dashed lines that are symmetric around the baseline. Measurements are plotted against a timeline on the chart. Measurements that exceed the limits are considered out of control (OOC). The control chart's central line (CL) is the acceptable value, which is an average of the historical check standard values. If our interest is to plot or monitor the $\hat{\theta}$ statistic, then the UCL, CL, and LCL for $\hat{\theta}$ are, respectively, given by:

$$\text{UCL} = E(\hat{\theta}) + k\text{SD}(\hat{\theta})$$

$$\text{CL} = E(\hat{\theta})$$

$$\text{LCL} = E(\hat{\theta}) - k\text{SD}(\hat{\theta}),$$

where E is the expectation of $\hat{\theta}$, k is the charting constant, and SD is the standard deviation of $\hat{\theta}$.

3.1. Logistic Regression Model-Based Control Chart Based on PR

In the model-based control chart, control limits of the PR are obtained as:

$$\text{UCL} = E(r_i^P) + k_1 \sqrt{\text{Var}(r_i^P)}$$

$$\text{CL} = E(r_i^P)$$

$$\text{LCL} = E(r_i^P) - k_1 \sqrt{\text{Var}(r_i^P)},$$

where k_1 is a constant that defines the size of control limits and is selected based on the fixed in-control (IC) average run length (ARL_0). When any PR crosses the control limits, then the PR-logistic chart indicates the process is OOC otherwise, in IC condition.

3.2. Logistic Regression Model-Based Control Chart Based on DR

In the model-based control chart, control limits of DR are obtained by the following expressions:

$$UCL = E(r_i^D) + k_2 \sqrt{\text{Var}(r_i^D)}$$

$$CL = E(r_i^D)$$

$$LCL = E(r_i^D) - k_2 \sqrt{\text{Var}(r_i^D)},$$

where k_2 is a constant that defines the size of control limits and is obtained against the fixed ARL_0 . When any DR crosses the control limits, the DR-logistic chart indicates that the process is OOC otherwise, in IC condition.

4. Monte Carlo Simulation Study

In this section, we evaluate the performance of the proposed control charts under different link functions with the help of a simulation study.

4.1. Performance Measures

The performance of the proposed control charts is evaluated using average run length (ARL) (cf. [17,28]). The run length (RL) is described as the number of points till a signal is indicated, and ARL is the average of the RL. The ARL is divided into two categories: IC (ARL_0) and OOC (ARL_1). When the control chart is in a stable state, we use ARL_0 as the chart's performance measure, but when the process is in an unstable state, we use ARL_1 . A chart is considered to be the best among those under discussion if, on the fixed ARL_0 , it has the smallest ARL_1 . Hence, in this study, we focused on the ARL to evaluate the performance of the control charts under different link functions.

4.2. Data Generation

For the simulation purpose, firstly, we generate the response variable from the Bernoulli distribution with parameter π_i i.e., $y_i \sim Be(\pi_i)$, where $\pi = \frac{e^{(\beta_0 + \beta_1 x)}}{1 + e^{(\beta_0 + \beta_1 x)}}$, $x \sim \text{uniform}(0, 100)$. The true values of β_0 and β_1 are set to be 0.05 and 0.06, respectively. These variables are generated for a fixed sample size $n = 1000$.

4.3. Algorithm for Charting Constants

The following algorithm is used to find out charting constants.

- i Set the arbitrary value as a charting constant and set regression coefficients as $\beta_0 = 0.05$ and $\beta_1 = 0.06$;
- ii Generate 1000 observations of the input variable (x) from the uniform distribution;
- iii The mean of the logistic response variable (π_i) is determined for each observation using Equation (2);
- iv The response variable is generated from the Bernoulli distribution with parameter π_i defined in step ii;
- v Fit the logistic regression model and obtain the residuals DR and PR;
- vi For the control charts based on PR and DR, calculate the mean and standard error of DR and PR, respectively;
- vii Determine the UCL and LCL of each proposed control chart using steps (i) and (viii). Plot the PR and DR against their respective limits;
- vii To obtain specified ARL_0 , repeat steps i–vii, 10,000 times.

If specified ARL_0 does not achieve, then adjust the previous arbitrary value and repeat steps i–viii until specified ARL_0 is obtained. By using the above-stated algorithm, the control charting constants are reported in Table 1.

Table 1. Constants for the logistic regression profiling with $ARL_0 = 200$ under various link functions.

PR				DR			
logit	probit	c-log-log	cauchit	logit	probit	c-log-log	cauchit
1.045	1.0443	1.0462	1.0443	1.045	1.0443	1.0456	1.0457

4.4. Results and Discussion

This section discusses the simulated results of the logistic model-based control charts with different link functions. Furthermore, we evaluate the performance of control charts by inserting the additive shifts in parameters of the model, such as β_0 , β_1 , and μ_x .

Monitoring β_0, β_1 and μ_x in Terms of ARL

In this section, we evaluate the simulation results of the control charts based on PR and DR in terms of ARL. For the out-of-control scenario, we considered the shifts in the μ_i by changing the β_0 into $\beta_0 + \eta$, β_1 into $\beta_1 + \theta$, and by changing the μ_x into $\mu_x + \delta$. The OOC results in terms of ARL are reported in Tables 2–4.

Table 2. Performance of $ARL_0 = 200$ with shift in β_0 under considered link functions.

η	PR				DR			
	logit	probit	c-log-log	cauchit	logit	probit	c-log-log	cauchit
0	201.56	199.99	200.97	200.92	200.24	199.68	199.13	199.49
0.04	117.87	121.46	129.14	121.95	119.01	124.69	111.15	125.00
0.08	53.93	56.15	62.80	61.25	55.93	55.25	55.08	66.47
0.12	24.00	23.96	26.20	22.28	21.57	22.76	20.85	21.42
0.16	7.67	7.56	9.29	7.96	8.59	7.33	10.29	8.10
0.2	4.07	4.21	3.75	3.75	3.38	3.83	3.67	3.81
0.24	2.53	2.71	2.62	2.50	2.50	2.94	3.09	2.55
0.28	2.58	2.42	2.41	2.49	2.48	2.44	2.41	2.44

Table 3. Performance of $ARL_0 = 200$ with shift in β_1 under considered link functions.

θ	PR				DR			
	logit	probit	c-log-log	cauchit	logit	probit	c-log-log	cauchit
0	201.09	201.69	199.03	199.27	200.86	200.36	200.31	199.38
0.04	188.83	152.21	162.73	153.03	190.78	149.65	177.08	153.47
0.08	121.46	127.65	137.29	117.82	122.48	110.83	123.97	122.37
0.12	84.68	77.35	68.47	89.39	85.48	77.45	89.37	79.77
0.16	55.09	44.39	56.99	45.46	55.61	48.50	47.45	54.95
0.2	31.51	29.45	32.36	36.05	31.61	30.97	37.22	27.83
0.24	15.24	14.28	20.94	16.72	15.37	13.17	13.21	15.89
0.28	10.47	10.13	9.99	10.81	10.67	8.03	11.10	12.70

In Table 2, we evaluate the performance of the proposed control charts for monitoring β_0 in terms of ARL_1 under various link functions. The outcomes revealed that raising shifts minimizes the ARL_1 values for control charts based on DR and PR by using considered link functions. The estimated ARL_1 values at maximum shift ($\eta = 0.28$) for the logit, probit, c-log-log, and the cauchit link functions are 2.58, 2.42, 2.41, and 2.49, respectively. On comparing the performance of link functions in the PR-based control charts, we found that the c-log-log link function outperforms other link functions. The estimated ARL_1 of the DR-based control chart at the maximum shift ($\eta = 0.28$) for the logit, probit, c-log-log, and the cauchit link functions are 2.48, 2.44, 2.41, and 2.44, respectively. It is clear from the results that the c-log-log link function performs better than the other link functions. Finally, it is also observed that the DR-based control chart with the c-log-log link function outperformed the PR-based control chart with the c-log-log link function.

Table 4. Performance of $ARL_0 = 200$ with shift in μ_x under considered link functions.

δ	PR				DR			
	logit	probit	c-log-log	cauchit	logit	probit	c-log-log	cauchit
0	199.28	201.73	199.61	201.54	199.68	199.49	200.05	199.36
0.04	52.19	59.56	51.04	58.92	63.32	62.33	60.67	64.53
0.08	8.29	8.20	8.47	6.99	8.58	8.31	8.72	9.16
0.12	2.81	2.60	2.88	2.54	2.40	2.50	2.58	2.75
0.16	2.48	2.47	2.51	2.49	2.53	2.52	2.53	2.50
0.2	2.63	2.61	2.64	2.59	2.61	2.62	2.62	2.65
0.24	2.80	2.79	2.77	2.82	2.76	2.78	2.71	2.70
0.28	2.95	2.89	2.95	2.95	2.84	2.94	2.94	2.93

In Table 3, we evaluate the performance of the proposed control charts for monitoring β_1 in terms of ARL_1 under various link functions. The outcomes revealed that an increase in shift minimizes the ARL_1 values for control charts. The estimated ARL_1 values at maximum shift ($\theta = 0.28$) for the logit, probit, c-log-log, and the cauchit link functions are 10.47, 10.13, 9.99, and 10.81, respectively. On comparing the performance of link functions in PR-based control charts, we found that the c-log-log link function outperforms other link functions. The estimated ARL_1 of the DR-based control chart at the maximum shift ($\theta = 0.28$) for the logit, probit, c-log-log, and the cauchit link functions are 10.67, 8.03, 11.10, and 12.70, respectively, which reveals that the probit link function performs better than the other link functions. On comparing the performance of the DR- and the PR-based control charts in monitoring β_1 under different link functions, we observed that the DR-based control charts with probit link function perform better than the PR-based control charts with other link functions.

In Table 4, we evaluate the performance of the proposed control charts for the shifts in μ_x in terms of ARL'_1 s under various link functions. The outcomes revealed that raising shifts minimizes the ARL'_1 s values for control charts based on DR and PR by using considered link functions. Table 4 presents the estimated ARL'_1 values of the PR-based control charts under different link functions. The estimated ARL'_1 values report at maximum shift ($\delta = 0.28$) for the logit, probit, c-log-log, and the cauchit link functions are 2.95, 2.89, 2.95, and 2.95, respectively. On comparing the performance of link functions in PR-based control charts, we found that the probit link function outperforms other link functions. Further, the estimated ARL'_1 of the DR-based control chart at the maximum shift ($\delta = 0.28$) for the logit, probit, c-log-log, and the cauchit link functions are 2.84, 2.94, 2.94, and 2.93, respectively. It is clear from the results that the logit link function performs better than the other link functions. Finally, on comparing the performance of DR- and PR-based control charts in monitoring μ_x under different link functions, we observe that the DR-based control charts with the logit link function perform better than PR-based control charts as compared to the other link functions.

5. Application: COVID-19 Deaths Profile Monitoring

In this modern era, COVID-19 affects different people in different ways. Here, we monitor the mortality status of COVID-19 patients who were admitted to Benazir Bhutto Shaheed Hospital, Rawalpindi, Pakistan. This data had been already collected by Akhtar et al. [29], where they collected this data from three hospitals, and we are considering one of these hospital data sets. In this application, our response variable (Y) is binary ($y = 1$, if the COVID patient is discharged deceased; otherwise, $y = 0$). Therefore, we consider this application for the evaluation of our proposed control charts. The data about the demographic and vital signs of all adult COVID-19 patients who were discharged from Benazir Bhutto Shaheed Hospital, Rawalpindi, during the first wave of COVID-19 (February to August 2020) were retrospectively collected. The National Early Warning Score (NEWS) was calculated by following the work of the Royal College of Physicians (RCP) (2012, page 14), and considered an independent variable in this study. The NEWS is

based on a simple aggregate scoring system, which is computed based on vital signs (see Table 5 [30]).

Table 5. The strategy to estimate national early warning score (NEWS).

Physiological Parameters	3	2	1	0	1	2	3
Respiration Rate	≤ 8		9–11	12–20		21–24	≥ 25
Oxygen Saturations	≤ 91	92–93	94–95	≥ 96			
Any Supplemental Oxygen		Yes		No			
Temperature	≤ 35.0		35.1–36.0	36.1–38.0	38.1–39.0	≥ 39.1	
Systolic Blood Pressure	≤ 90	91–100	101–110	111–219			≥ 220
Heart Rate	≤ 40		41–50	51–90	91–110	111–130	≥ 131
Level of Consciousness				Alert			Voice, Pain, or Unconscious

We had data of a total of 916 COVID-19 patients, consisting of mortality status (y) and NEWS (x). Based on the available data, we run logistic regression by setting logit, probit, c-log-log, and cauchit link functions. The Pearson (PR) and deviance (DR) residuals were obtained for estimated logistic regression models under each link function, and their means and standard deviations (SD) are reported in Table 6. Further, we set $ARL_0 = 200$ and obtained the limits of PR and DR-based control charts, which are also given in Table 6. To make the OOC dataset, we have estimated a new response variable (Y_1) by using $\mu_Y - \text{uniform}(0, 0.2)$. Similarly, we estimated logistic regression models and obtained shifted PR's and DR's. The PR- and DR-based control charts under different link functions are presented in Figures 2–5. The points under the pink and white shaded areas belong to the IC and OOC situation, respectively. The blue color points indicate the IC PR's and DR's, while the red color shows the OOC points. The PR- and DR-based control charts under the logit function are presented in Figure 2, while proposed control charts based on the probit function are portrayed in Figure 3. Figures 4 and 5 consist of PR and DR-based control charts under c-log-log and cauchit link functions, respectively. The out-of-control points are counted for each chart and reported in Table 6. It is revealed that the PR- and DR-based control charts under the cauchit link function have captured a large number of OOC signals. The second largest OOC points were detected by the PR- and DR-based control charts for the c-log-log link function. Hence, PR- and DR-based control charts under cauchit link and c-log-log link functions outperform all their counterparts.

Table 6. For the COVID-19 data, the mean and standard deviation of logistic regression residuals, control limits and OOC signals under different link functions.

Link Functions	logit		probit		c-log-log		cauchit	
Residuals	PR	DR	PR	DR	PR	DR	PR	DR
Mean	−0.0019	−0.1476	−0.0011	−0.1468	−0.0015	−0.1473	−0.0078	−0.1529
SD	0.9971	1.0337	0.9979	1.034	0.9964	1.0345	0.9916	1.0352
LCL	−1.0234	−1.1939	−1.0152	−1.1873	−1.0375	−1.2051	−1.0591	−1.2222
UCL	2.256	1.899	2.277	1.9071	2.2192	1.8847	2.1154	1.8429
OOO Signals	83	97	83	83	97	97	144	144

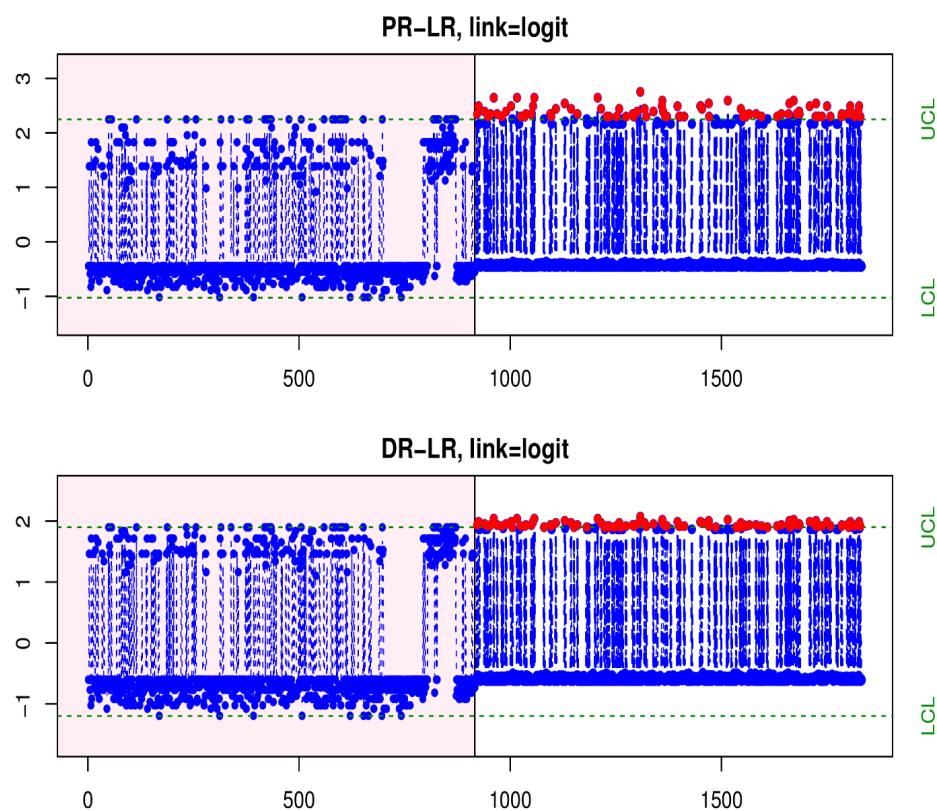


Figure 2. Control chart based on PR and DR with the logit link function.

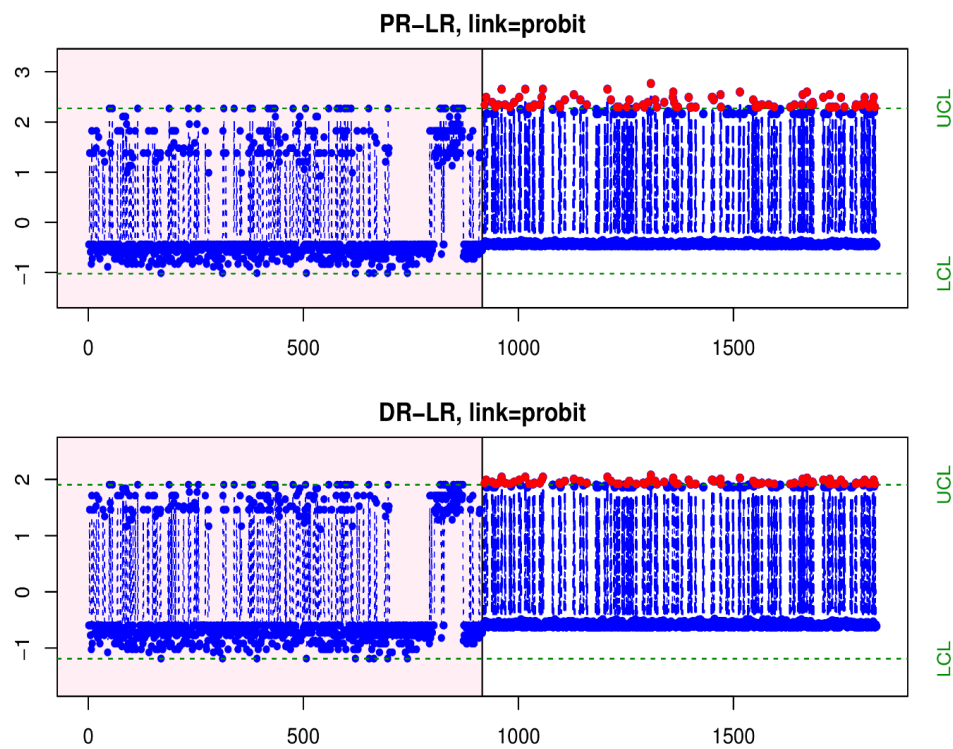


Figure 3. Control chart based on PR and DR with the probit link function.

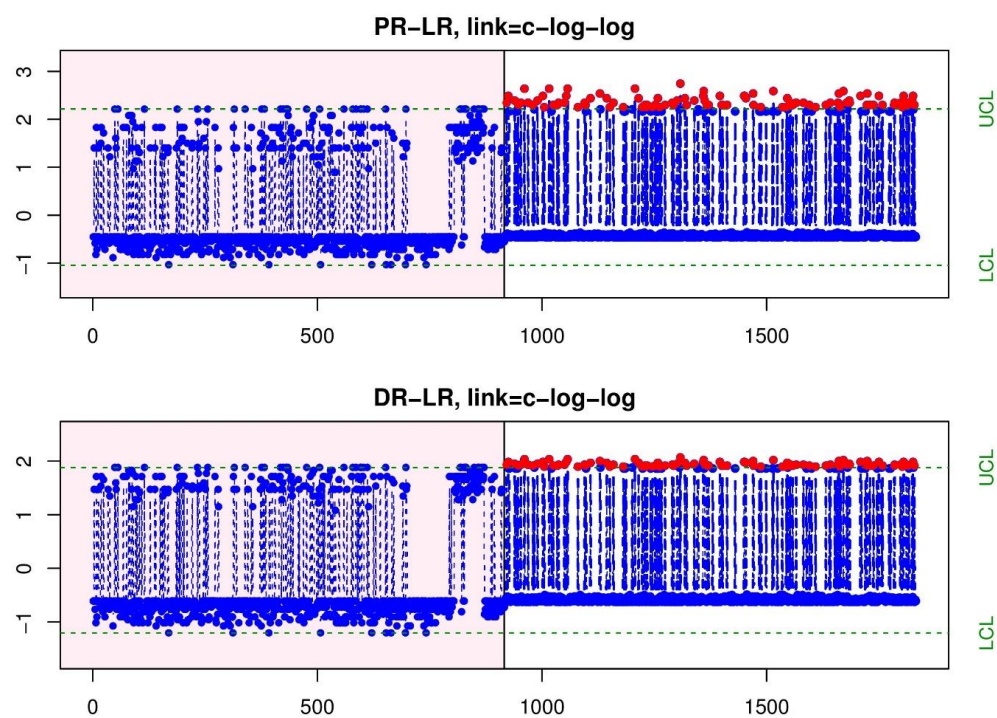


Figure 4. Control chart based on PR and DR with the c-log-log link function.

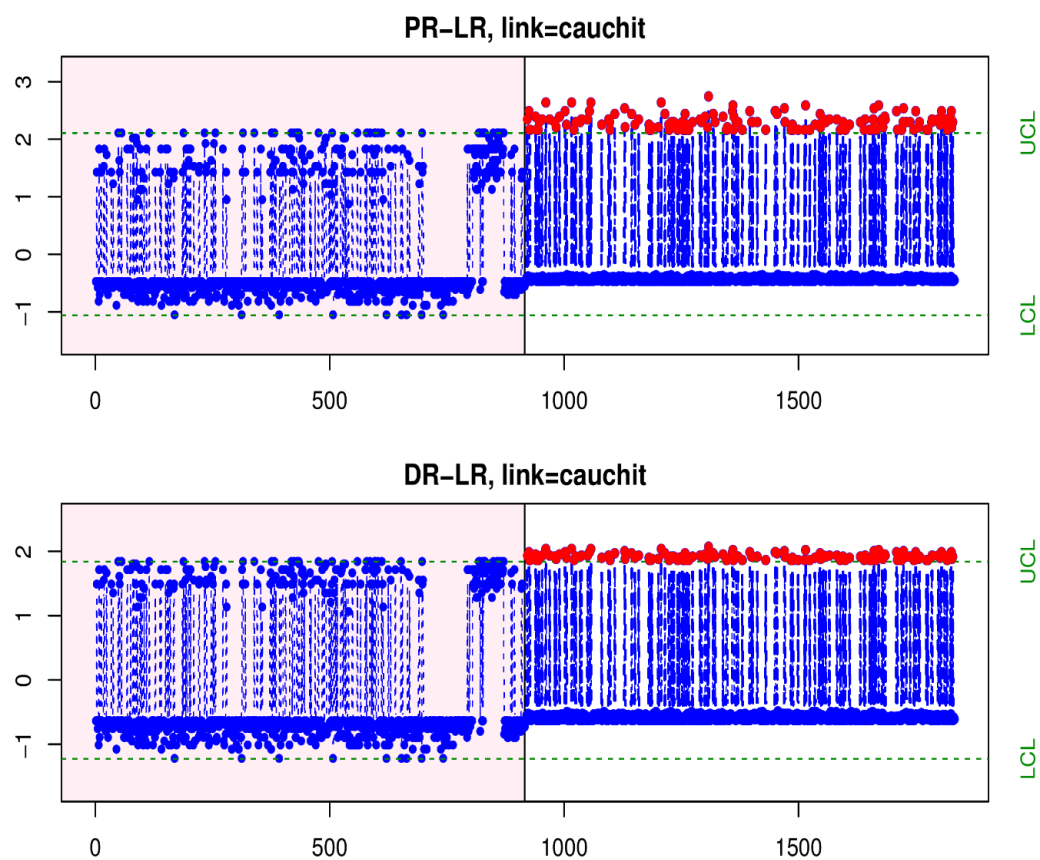


Figure 5. Control chart based on PR and Dr with the cauchit link function.

6. Conclusions

In this study, the control charts are constructed based on the LR residuals (Pearson residuals (PR) and deviance residuals (DR)) with different link functions (the logit, probit, c-log-log, and cauchit). We evaluate the performance of the control charts with the help of a simulation study, where ARL is considered as the evaluation criterion. The simulation study shows that in monitoring β_0 , the c-log-log link function showed good performance for both PR and DR control charts as compared to the other link functions. While monitoring β_1 , the DR-based control chart with the probit link function performs better as compared to the PR-based control charts and with other link functions due to its minimum ARL. We also conclude that in monitoring μ_x , the logit link function gives a better performance for the DR-based control chart as compared to the PR-based control charts with other link functions. For the COVID-19 death data, the PR- and DR-based control charts were implemented and showed that charts based on the cauchit link function indicated a large number of signals. The 2nd link function which gives the 2nd largest OOC is the c-log-log link function. Therefore, it is concluded from the results that mostly the DR-based control charts with the c-log-log link function give a better performance as compared to the other link functions.

Author Contributions: Methodology, M.C., M.A., T.M., M.F., K.B. and A.E.; Formal analysis, M.C., M.A., T.M., M.F., K.B. and A.E.; Writing—original draft, M.C.; Writing—review & editing, M.A., T.M., M.F., K.B. and A.E. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Deanship of Scientific Research at University of Bisha, grant number UB-GRP-42-1444.

Data Availability Statement: Not Applicable.

Acknowledgments: The authors extend their appreciation to the Deanship of Scientific Research at University of Bisha for funding this research through the general research project under grant number (UB-GRP-42-1444).

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Mahmood, T.; Xie, M. Models and monitoring of zero-inflated processes: The past and current trends. *Qual. Reliab. Eng. Int.* **2019**, *35*, 2540–2557. [CrossRef]
2. Shewhart, W.A. *Economic Control of Quality of Manufactured Product*; Macmillan And Co., Ltd.: London, UK, 1931.
3. Page, E.S. Continuous inspection schemes. *Biometrika* **1954**, *41*, 100–115. [CrossRef]
4. Roberts, S.W. Control chart tests based on geometric moving averages. *Technometrics* **2000**, *42*, 97–101. [CrossRef]
5. Zhu, J.; Lin, D.K. Monitoring the slopes of linear profiles. *Qual. Eng.* **2009**, *22*, 1–12. [CrossRef]
6. Nelder, J.A.; Wedderburn, R.W.M. Generalized linear models. *J. R. Stat. Soc. Ser. A* **1972**, *135*, 370–384. [CrossRef]
7. Skinner, K.R.; Montgomery, D.C.; Runger, G.C. Process monitoring for multiple count data using generalized linear model-based control charts. *Int. J. Prod. Res.* **2003**, *41*, 1167–1180. [CrossRef]
8. Jearkaporn, D.; Montgomery, D.C.; Runger, G.C.; Borror, C.M. Process monitoring for correlated gamma-distributed data using generalized-linear-model-based control charts. *Qual. Reliab. Eng. Int.* **2003**, *19*, 477–491. [CrossRef]
9. Skinner, K.R.; Montgomery, D.C.; Runger, G.C. Generalized linear model-based control charts for discrete semiconductor process data. *Qual. Reliab. Eng. Int.* **2004**, *20*, 777–786. [CrossRef]
10. Koosha, M.; Amiri, A. The effect of link function on the monitoring of logistic regression profiles. In Proceedings of the World Congress on Engineering 2011, London, UK, 6–8 July 2011.
11. Shu, L.; Tsui, K.L.; Tsung, F. Regression Control Charts. *Encyclopedia of Statistics in Quality and Reliability*. 2008. Available online: <https://onlinelibrary.wiley.com/doi/abs/10.1002/9780470061572.eqr260> (accessed on 15 January 2023).
12. Asgari, A.; Amiri, A.; Niaki, S.T.A. A new link function in GLM-based control charts to improve monitoring of two-stage processes with Poisson response. *Int. J. Adv. Manuf. Technol.* **2014**, *72*, 1243–1256. [CrossRef]
13. Amiri, A.; Koosha, M.; Azhdari, A.; Wang, G. Phase I monitoring of generalized linear model-based regression profiles. *J. Stat. Comput. Simul.* **2015**, *85*, 2839–2859. [CrossRef]
14. Amiri, A.; Yeh, A.B.; Asgari, A. Monitoring two-stage processes with binomial data using generalized linear model-based control charts. *Qual. Technol. Quant. Manag.* **2016**, *13*, 241–262. [CrossRef]
15. Qi, D.; Wang, Z.; Zi, X.; Li, Z. Phase II monitoring of generalized linear profiles using weighted likelihood ratio charts. *Comput. Ind. Eng.* **2016**, *94*, 178–187. [CrossRef]

16. Moheghi, H.R.; Noorossana, R.; Ahmadi, O. GLM profile monitoring using robust estimators. *Qual. Reliab. Eng. Int.* **2021**, *37*, 664–680. [[CrossRef](#)]
17. Kinat, S.; Amin, M.; Mahmood, T. GLM-based control charts for the inverse Gaussian distributed response variable. *Qual. Reliab. Eng. Int.* **2020**, *36*, 765–783. [[CrossRef](#)]
18. Garcia-Bustos, S.; Zambrano, G. Control charts for health surveillance based on residuals of negative binomial regression. *Qual. Reliab. Eng. Int.* **2022**, *38*, 2521–2532. [[CrossRef](#)]
19. Hakimi, A.; Amiri, A.; Kamranrad, R. Robust approaches for monitoring logistic regression profiles under outliers. *Int. J. Qual. Reliab. Manag.* **2017**, *34*, 494–507. [[CrossRef](#)]
20. Kim, J.-M.; Ha, I.D. Deep learning-based residual control chart for count data. *Qual. Eng.* **2022**, *34*, 370–381. [[CrossRef](#)]
21. Mahmood, T.; Iqbal, A.; Abbasi, S.A.; Amin, M. Efficient GLM-based control charts for Poisson processes. *Qual. Reliab. Eng. Int.* **2022**, *38*, 389–404. [[CrossRef](#)]
22. Soleimani, P.; Asadzadeh, S. Effect of non-normality on the monitoring of simple linear profiles in two-stage processes: A remedial measure for gamma-distributed responses. *J. Appl. Stat.* **2022**, *49*, 2870–2890. [[CrossRef](#)]
23. Yeganeh Chukhrova, N.; Johannssen, A.; Fotuhi, H. A network surveillance approach using machine learning based control charts. *Expert Syst. Appl.* **2023**, *219*, 119660. [[CrossRef](#)]
24. Yu, J.; Liu, J. LRProb control chart based on logistic regression for monitoring mean shifts of auto-correlated manufacturing processes. *Int. J. Prod. Res.* **2011**, *49*, 2301–2326. [[CrossRef](#)]
25. Khosravi, P.; Amiri, A. Self-Starting control charts for monitoring logistic regression profiles. *Commun. Stat.-Simul. Comput.* **2019**, *48*, 1860–1871. [[CrossRef](#)]
26. Alevizakos, V.; Koukouvinos, C.; Lappa, A. Comparative study of the Cp and Spmk indices for logistic regression profile using different link functions. *Qual. Eng.* **2019**, *31*, 453–462. [[CrossRef](#)]
27. Jahani, K.; Feili, H.; Ohadi, F. Phase II monitoring of the nominal logistic regression profiles based on Wald and Rao score test statistics (a case study in healthcare: Diabetic patients). *Int. J. Product. Qual. Manag.* **2019**, *27*, 161–176. [[CrossRef](#)]
28. Amin, M.; Mahmood, T.; Kinat, S. Memory type control charts with inverse-Gaussian response: An application to yarn manufacturing industry. *Trans. Inst. Meas. Control* **2021**, *43*, 656–678. [[CrossRef](#)]
29. Akhtar, H.; Akhtar, S.; Rahman, F.U.; Afridi, M.; Khalid, S.; Ali, S.; Akhtar, N.; Khader, Y.S.; Ahmad, H.; Khan, M.M. An overview of the treatment options used for the management of COVID-19 in Pakistan: Retrospective Observational Study. *JMIR Public Health Surve.* **2021**, *7*, e28594. [[CrossRef](#)]
30. Royal College of Physicians. *National Early Warning Score (NEWS): Standardising the Assessment of Acute Illness Severity in the NHS; Report of a Working Party*; RCP: London, UK, 2012.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.