Editorial

Preface to the Special Issue "Abstract Fractional Integro-Differential Equations and Fixed Point Theory with Applications"

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Fractional calculus has played a significant role in modeling complex systems in disciplines such as mathematics, physics, biology, and engineering. Abstract fractional integro-differential equations arise from approximation theory and operator theory, numerical computational methods, the modeling of nonlinear phenomena, the optimal control of complex systems, and other scientific research areas. For more details about the subject, we refer readers to research monographs [1–7]. Fixed point theory and its application have contributed to different fields for more than eight decades, including nonlinear functional analysis, differential equations, economics, game theory, optimization, dynamic system theory, and signal and image processing, which has enhanced our understanding of the world around us. For more details, we refer readers to [8–14] and the references cited therein.

This Special Issue focuses on the originality of recent results concerning abstract (degenerate) fractional integro-differential equations in Banach spaces and locally convex spaces, the corresponding semilinear Cauchy problems, and applications of fixed point theory. We are particularly interested in the qualitative analysis of solutions for various classes of abstract fractional integro-differential equations and new results concerning the existence and uniqueness of almost periodic solutions (almost automorphic solutions, hypercyclic and topologically mixing solutions) of abstract fractional integro-differential equations.

Following a comprehensive review process as per the journal's policy and guidelines, this Special Issue reports 12 research papers of a total of 36 submissions received (around a 33% acceptance rate). The list of published contributions is as follows:

i. Kostić, M.; Du, W.-S.; Fedorov, V.E. Doss ρ-Almost Periodic Type Functions in $\mathbb{R}^n$. *Mathematics* **2021**, *9*, 2825. https://doi.org/10.3390/math9212825;

ii. Fedorov, V.E.; Du, W.-S.; Kostić, M.; Abdrakhmanova, A.A. Analytic Resolving Families for Equations with Distributed Riemann-Liouville Derivatives. *Mathematics* **2022**, *10*, 681. https://doi.org/10.3390/math10050681;

iii. Huang, H.; Todorčević, V.; Radenović, S. Remarks on Recent Results for Generalized *F*-Contractions. *Mathematics* **2022**, *10*, 768. https://doi.org/10.3390/math10050768;

iv. Elshenhab, A.M.; Wang, X.; Cesarano, C.; Almarri, B.; Moaaz, O. Finite-Time Stability Analysis of Fractional Delay Systems. *Mathematics* **2022**, *10*, 1883. https://doi.org/10.3390/math10111883;

v. Karapinar, E.; Fulga, A. Contraction in Rational Forms in the Framework of Super Metric Spaces. *Mathematics* **2022**, *10*, 3077. https://doi.org/10.3390/math10173077;

vi. Janngam, K.; Suantai, S. An Inertial Modified S-Algorithm for Convex Minimization Problems with Directed Graphs and Its Applications in Classification Problems. *Mathematics* **2022**, *10*, 4442. https://doi.org/10.3390/math10234442;

vii.   Zuo, Z.; Huang, Y.; Huang, H.; Wang, J. The Gao-Type Constant of Absolute Normalized Norms on $\mathbb{R}^2$. *Mathematics* **2022**, *10*, 4591. https://doi.org/10.3390/math10234591;

viii.   Jitpeera, T.; Padcharoen, A.; Kumam, W. On New Generalized Viscosity Implicit Double Midpoint Rule for Hierarchical Problem. *Mathematics* **2022**, *10*, 4755. https://doi.org/10.3390/math10244755;

ix.   Liu, X.; Chen, L.; Zhao, Y. Existence Theoremsfor Solutions of a Nonlinear Fractional-Order Coupled Delayed System via Fixed Point Theory. *Mathematics* **2023**, *11*, 1634. https://doi.org/10.3390/math11071634;

x.   Noorwali, M.; Shagari, M.S. On Two-Point Boundary Value Problems and Fractional Differential Equations via New Quasi-Contractions. *Mathematics* **2023**, *11*, 2477. https://doi.org/10.3390/math11112477;

xi.   Dai, Q.; Zhang, Y. Stability of Nonlinear Implicit Differential Equations with Caputo-Katugampola Fractional Derivative. *Mathematics* **2023**, *11*, 3082. https://doi.org/10.3390/math11143082;

xii.   Zaslavski, A.J. Global Convergence of Algorithms Based on Unions of Non-Expansive Maps. *Mathematics* **2023**, *11*, 3213. https://doi.org/10.3390/math11143213.

In this Editorial, we express our heartfelt appreciation to all authors and reviewers who contributed to this Special Issue. It was with the enthusiasm and spirit of the authors and reviewers that we could make the Special Issue an extraordinary success. The 30 authors of these 12 papers are as follows:

Aliya A. Abdrakhmanova (see ii)	Barakah Almarri (see iv)
Clemente Cesarano (see iv)	Lili Chen (see ix)
Qun Dai (see xi)	Wei-Shih Du (see i, ii)
Ahmed M. Elshenhab (see iv)	Vladimir E. Fedorov (see i, ii)
Andreea Fulga (see v)	Huaping Huang (see iii, vii)
Yimin Huang (see vii)	Kobkoon Janngam (see vi)
Thanyarat Jitpeera (see viii)	Erdal Karapinar (see v)
Marko Kostić (see i, ii)	Wiyada Kumam (see viii)
Xin Liu (see ix)	Osama Moaaz (see iv)
Maha Noorwali (see x)	Anantachai Padcharoen (see viii)
Stojan Radenović (see iii)	Mohammed Shehu Shagari (see x)
Suthep Suantai (see vi)	Vesna Todorčević (see iii)
Jing Wang (see vii)	Xingtao Wang (see iv)
Alexander J. Zaslavski (see xii)	Yunying Zhang (see xi)
Yanfeng Zhao (see ix)	Zhanfei Zuo (see vii)

The published contributions to this Special Issue can be divided according to the following scheme considering their main purposes:

- Abstract fractional integro-differential equations and their applications (see i, ii, iv, ix, x, xi);
- Fixed point theory and its applications (see iii, v, vii, ix, x, xi, xii);
- Convex analysis and optimization (see vi, viii);
- Functional analysis (see i, ii, vii, xii).

We hope that researchers and practitioners find these papers interesting and inspirational for future research work in these exciting areas.

As Guest Editors, we believe that the contributions to this Special Issue provide new insights on several important issues while, at the same time, providing new research problems or avenues that undoubtedly exceed our original aim. Finally, we would like to express our sincere thanks to the Editorial team and the reviewers of *Mathematics*, particularly the Editor-in-Chief Prof. Dr. Francisco Chiclana, for their great support throughout the editing process of our Special Issue.

References

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7. Vejvoda, O. *Partial Differential Equations: Time-Periodic Solutions*; Herrmann, L., Lovicar, V., Eds.; Martinus Nijhoff Publishers: Hague, The Netherlands, 1981.
8. Du, W.-S. A note on cone metric fixed point theory and its equivalence. *Nonlinear Anal.* **2010**, *72*, 2259–2261. [CrossRef]
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13. Takahashi, W. *Nonlinear Functional Analysis*; Yokohama Publishers: Yokohama, Japan, 2000.
14. Zaslavski, A.J. *Approximate Solutions of Common Fixed Point Problems*; Springer Optimization and Its Applications; Springer: Cham, Switzerland, 2016.