mathematics

Article

Design of Polynomial Observer-Based Control of Fractional-Order Power Systems

Hamdi Gassara [1], Imen Iben Ammar [2], Abdellatif Ben Makhlouf [3], Lassaad Mchiri [4] and Mohamed Rhaima [5,*]

[1] Laboratory of Sciences and Techniques of Automatic Control and Computer Engineering, National School of Engineering of Sfax, University of Sfax, P.O. Box 1173, Sfax 3038, Tunisia
[2] GREAH Laboratory, University of Le Havre Normandy, 75 Rue Bellot, 76600 Le Havre, France
[3] Department of Mathematics, Faculty of Sciences, University of Sfax, P.O. Box 1171, Sfax 3038, Tunisia; abdellatif.benmakhlouf@fss.usf.tn
[4] ENSIIE, University of Evry-Val-d'Essonne, 1 Square de la Résistance, 91025 Évry-Courcouronnes cedex, France; lassaad.mchiri@u-paris2.fr
[5] Department of Statistics and Operations Research, College of Sciences, King Saud University, Riyadh 11451, Saudi Arabia
* Correspondence: mrhaima.c@ksu.edu.sa

Abstract: This research addresses the problem of globally stabilizing a distinct category of fractional-order power systems (F-OP) by employing an observer-based methodology. To address the inherent nonlinearity in these systems, we leverage a Takagi–Sugeno (TS) fuzzy model. The practical stability of the proposed system is systematically established through the application of a sum-of-squares (SOS) approach. To demonstrate the robustness and effectiveness of our approach, we conduct simulations of the power system using SOSTOOLS v3.00. Our study contributes to advancing the understanding of F-OP and provides a practical framework for their global stabilization.

Keywords: power system; fractional derivative; Lyapunov function

MSC: 26A33; 34A08

Citation: Gassara, H.; Iben Ammar, I.; Ben Makhlouf, A.; Mchiri, L.; Rhaima, M. Design of Polynomial Observer-Based Control of Fractional-Order Power Systems. *Mathematics* **2023**, *11*, 4450. https://doi.org/10.3390/math11214450

Academic Editors: Vishnu Suresh and Dominika Kaczorowska

Received: 11 September 2023
Revised: 16 October 2023
Accepted: 19 October 2023
Published: 27 October 2023

1. Introduction

The applicationm of stability theory is one of the most important techniques for the qualitative study of power systems. The application of linearized models in power systems [1] has frequently been utilized due to their straightforward design and usefulness in implementation. For example, in [2], a novel sliding mode controller was used for a linearized power system, incorporating both a superconducting magnetic energy storage and wind model. However, this linear model limitation depends on its ability to ensure stability within the vicinity of an operational point. Starting from 1990, a significant number of researchers have focused on the TS fuzzy model due to its effective representation of the nonlinear dynamics in numerous engineering systems, such as that of an inverted pendulum on a cart [3], electronic circuits [4], vehicles [5], and mechanical systems [6]. Thanks to its robust capacity for approximating nonlinear dynamics, the TS fuzzy model can be applied in the modeling and control of power systems. In [7], the TS model was employed to formulate a controller for a single machine, using an infinite bus system that encounters perturbations due to faults. In this control design approach, the expectation is that the state variables can be accessed by means of output measurements. However, this assumption is not universally applicable. In reality, it is known that in the majority of real-world scenarios, to obtain state variables directly from output measurements is often not applicable. Consequently, to design a controller that can operate effectively without needing complete access to the entire state vector is more practical. For instance, as demonstrated in reference [8], when designing a feedback controller for a power system, it is necessary to possess information about the damper currents. However, these damper currents are not attainable through direct measurement. As a result, an approach is suggested for creating an observer for the damper currents.

Developing observers to estimate unmeasured states continues to be a vibrant field of research, not only within the realm of power systems but also across a wide array of real-world applications and diverse categories of complex models, e.g., the regular TSF model [9], the singular model [10,11], and networked TSF systems [12]. It is important to note that in all the results cited previously, the stabilization observer-based controller (O-BC) is formulated by considering linear matrix inequalities (LMIs). These LMIs can be effectively dealt with by using the LMI tool v1.00. Later, a polynomial convex optimization tool, called SOSTOOLS [13], emerged as a substitute for the conventional LMI toolbox. The primary benefit of this tool, by comparison to the LMI tool, is its capability to address polynomial LMI problems. These problems are simpler than the standard LMI problem when all the polynomials are constrained to constants. Following the introduction of the aforementioned tool, there has been expansion in the realm of polynomial models, controllers, and observers. In this extended framework, matrices and gains are no longer fixed constants; instead, they vary according to the polynomial functions. In such instances, the derived conditions are expressed using sum-of-squares (SOS) formulations, which can be addressed through partially symbolical methods using the newly devised SOSTOOLS. Many variations of O-BC designs utilizing polynomial matrices have been formulated. Many variations of observer-based control designs utilizing polynomials have been used. For example, in [14], the authors proposed a polynomial O-BC scheme for a permanent magnet linear synchronous motor.

Fractional calculus (FC) is a mathematical concept that generalizes traditional integer-order calculus (differentiation and integration) by including non-integer orders. Instead of working with integer derivatives and integrals (e.g., first-order, second-order), fractional calculus deals with fractional derivatives and integrals (e.g., 0.5-order, 1.5-order) [15]. FC has found applications in various scientific and engineering fields, including physics [16], engineering [17], signal processing [18], and control theory [19,20]. It offers a more flexible way to model systems with memory effects and complex dynamics that cannot be accurately captured using classical integer-order calculus.

Fractional-order systems (F-OS) are described by fractional-order differential equations. This fractional order introduces a memory property, meaning that the system's behavior is influenced not only by its current state but also by its past states [21]. The rapid development of FC has attracted many researchers to investigate the stability analysis of F-OS, leading to the development of specialized techniques and methodologies. Researchers have explored stability criteria, Lyapunov functions [22–25], and stability regions in the fractional-order space [26]. As the understanding of stability in F-OS continues to evolve, it has enriched both the theoretical foundations of mathematics and practical applications across disciplines, like control theory, physics, engineering, biology, and more [27]. This intersection of mathematics and other fields demonstrates the interdisciplinary nature of FC and its significant impact on modern research and technology.

Later, the authors of reference [28] highlighted the presence of fractional-order characteristics within power systems. To date, several outcomes have been reported concerning the modeling and stabilization of power systems with fractional-order characteristics, e.g., using an LMI method in [29], and using an SOS method in [30]. In this sense, the main highlights of this paper are as follow:

(i) presenting an initial attempt to develop an O-BC for a fractional-order power (F-OP) system.
(ii) using an SOS method for a fractional-order power (F-OP) system.

This paper is organized into three main sections. Section 2 considers the foundational results, Section 3 introduces a practical observer-based controller for an F-OP system, and Section 4 illustrates the theoretical findings through examples, enhancing real-world understanding.

Notations: $[\mathcal{Z}]_{sy}$ denotes $\mathcal{Z} + \mathcal{Z}^T$.

2. Preliminaries

In this section, we provide an introductory overview of the π-fractional integral and the SOS approach.

Definition 1 ([22])**.** *Let $\varpi > 0$ and $\pi \in C^1[b_0, b_1]$ with $\pi'(\eta) > 0$ for every $\eta \in [b_0, b_1]$. Then, the π–fractional integral of order ϖ of a locally integrable function W is given by*

$$I_{b_0^+}^{\varpi,\pi} W(\eta) = \frac{1}{\Gamma(\varpi)} \int_{b_0}^{\eta} (\pi(\eta) - \pi(s))^{\varpi-1} \pi'(s) W(s) ds,$$

where Γ is the gamma function.

Definition 2 ([22])**.** *Let $0 < \varpi < 1$, $\pi \in C^1[b_0, b_1]$ with $\pi'(\eta) > 0$ for every $\eta \in [b_0, b_1]$ and $W \in AC[b_0, b_1]$. The π–Caputo fractional derivative of order ϖ of W is given by*

$$^{C}D_{b_0^+}^{\varpi,\pi} W(\eta) = \frac{1}{\Gamma(1-\varpi)} \int_{b_0}^{\eta} (\pi(\eta) - \pi(s))^{-\varpi} \frac{d}{ds} W(s) ds. \tag{1}$$

Remark 1. *In the case when $\pi(\eta) = \eta$, we have $^{C}D_{b_0^+}^{\varpi,\pi} W(\eta) = {}^{C}D_{b_0^+}^{\varpi} W(\eta)$ (see [22]).*

Definition 3 ([23])**.** *Let $\alpha > 0$, $\mu > 0$. The Mittag–Leffler function is defined as*

$$E_{\alpha,\mu}(\eta) = \sum_{k=0}^{+\infty} \frac{\eta^k}{\Gamma(k\alpha + \mu)}, \ \eta \in \mathbb{C}$$

If $\mu = 1$, we write $E_\alpha(\eta) := E_{\alpha,1}(\eta)$.

Lemma 1 ([23])**.** *For $a > 0$ and $0 < \alpha < 1$, the function*

$$t \longmapsto \int_{b_0}^{t} (t-s)^{\alpha-1} E_{\alpha,\alpha}\big(-a(t-s)^{\alpha}\big) ds$$

is bounded.

Lemma 2 ([22])**.** *Let $0 < \varpi < 1$ and $\pi \in C^1[b_0, b_1]$ with $\pi'(\eta) > 0$ for every $\eta \in [b_0, b_1]$. The solution of the following system*

$$^{C}D_{b_0{}^+}^{\varpi,\pi} W(\eta) = lW(\eta) + m(\eta), \tag{2}$$

where $W \in \mathbb{R}^n$ is given by

$$\begin{aligned} W(\eta) &= E_\varpi\big(l(\pi(\eta) - \pi(b_0))^{\varpi}\big) W(b_0) \\ &+ \int_{b_0}^{\eta} (\pi(\eta) - \pi(s))^{\varpi-1} E_{\varpi,\varpi}\big(l(\pi(\eta) - \pi(s))^{\varpi}\big) \pi'(s) m(s) ds. \end{aligned} \tag{3}$$

Lemma 3 ([22])**.** *Let $0 < \varpi < 1$, $\pi \in C^1[b_0, b_1]$ with $\pi'(\eta) > 0$ for every $\eta \in [b_0, b_1]$, S be a symmetric and definite positive function and $W(\eta) \in \mathbb{R}$ be an absolutely continuous function, then*

$$^{C}D_{b_0^+}^{\varpi,\pi} W^T S\, W(\eta) \le 2W^T(\eta) S\, {}^{C}D_{b_0^+}^{\varpi,\pi} W(\eta). \tag{4}$$

If we consider:

$$\begin{aligned} ^{C}D_{b_0}^{\varpi,\pi} X(\eta) &= G(\eta, X), \ \eta \ge b_0, \\ X(b_0) = X_0, \end{aligned} \tag{5}$$

where $b_0 \in \mathbb{R}_+$ and $G \in C(\mathbb{R}_+ \times \mathbb{R}^n, \mathbb{R}^n)$.

Definition 4. *The above system is practical π–Mittag–Leffler stable (π–PMLS), if there is $l_k > 0$, $k = 1, \ldots, 3$ and $\delta \geq 0$, such that:*

$$\|X(\eta; b_0, X_0)\| \leq l_1 \|X_0\| \Big(E_{\varpi}\big(- l_2(\pi(\eta) - \pi(b_0))^{\varpi}\big) \Big)^{l_3} + \delta, \ \forall \eta \geq b_0 \geq 0. \tag{6}$$

Remark 2. *When $\delta = 0$, the above system is said to be π–Mittag–Leffler stable (see [22]). When $\pi(t) = t$, we obtain the practical Mittag–Leffler stability (see [31]).*

Definition 5 ([32])**.** *Consider $\omega(g) = \omega(g_1, g_2, \ldots, g_r)$, (in which $g \in \mathbb{R}^q$) is a polynomial. and $\omega(g)$ is an SOS if there exist polynomials $e_1(g), e_2(g), \ldots,$ and $e_g(g)$, such that*

$$\omega(g) = \sum_{j=1}^{g} e_j^2(g). \tag{7}$$

In the rest, S_{SOS} denotes the set of SOS.

It is clear that $\omega(g) \in S_{SOS}$ implies that $\omega(g) \geq 0, \ \forall y \in \mathbb{R}^q$.

Lemma 4 ([32])**.** *Consider $\mathcal{T}(g)$ a $t \times t$ symmetric polynomial matrix and a vector $y \in \mathbb{R}^q$ which do not depend on g and a known positive polynomial $\gamma(g)$, then*

$$-y^T \Big(\mathcal{T}(g) + \gamma(g)\Big) y \in S_{SOS} \tag{8}$$

implies that

$$\mathcal{T}(g) < 0. \tag{9}$$

Lemma 5 ([33])**.** *The inequality mentioned below holds for any scalar $d > 0$ and matrices $\mathcal{J}_1$ and $\mathcal{J}_2$ with suitable dimensions*

$$\mathcal{J}_1^T \mathcal{J}_2 + \mathcal{J}_2^T \mathcal{J}_1 \leq d \mathcal{J}_1^T \mathcal{J}_1 + d^{-1} \mathcal{J}_2^T \mathcal{J}_2. \tag{10}$$

3. Practical Observer-Based Controller for an F-Op System

The F-OP system can be defined as follows [29]:

$$\begin{cases} {}^{C}D_{b_0}^{\varpi,\pi} \theta(t) = v(t) \\ {}^{C}D_{b_0}^{\varpi,\pi} v(t) = -\rho \sin(\theta(t)) - \beta v(t) + \xi + \sigma \cos(\phi t) \end{cases} \tag{11}$$

where $\theta(t)$ is the rotor angle, $v(t)$ is the rotating speed, and ϕ denotes the disturbance power frequency, $\rho = \frac{S_e}{G}, \beta = \frac{T}{G}, \xi = \frac{S_m}{G}, \sigma = \frac{S_e}{G}$, in which S_e refers to the electrical power, T refers to the damping coefficient, G refers to the inertia time constant, S_m refers to the mechanical power, and S_e refers to the disturbance power amplitude.

In the subsequent content, for the sake of brevity, we exclude the time variable t. The PS with a stabilizing controller $z \in \mathbb{R}^2$ is given by:

$${}^{C}D_{b_0}^{\varpi,\pi} \vartheta = \mathcal{A}(\theta)\vartheta + \mathcal{C} + z \tag{12}$$

where

$$\vartheta = \begin{bmatrix} \vartheta_1, & \vartheta_2 \end{bmatrix}^T = \begin{bmatrix} \theta, & v \end{bmatrix}^T, \mathcal{A}(\theta) = \begin{bmatrix} 0 & 1 \\ -\rho \frac{\sin(\theta)}{\theta} & -\beta \end{bmatrix}, \mathcal{C} = \begin{bmatrix} 0 \\ \xi + \sigma \cos(\phi t) \end{bmatrix}$$

Assumption 1. *θ is detected through a sensor.*

By considering $x = \frac{\sin(\theta)}{\theta}$ as a premise variable, and using the sector nonlinearity concept [34], we attain the subsequent Takagi–Sugeno fuzzy model, which can precisely depict the dynamics of the PS:

$$\begin{cases} {}^{C}D_{b_0}^{\varpi,\pi}\vartheta = \sum_{i=1}^{2}\kappa_i(\theta)\mathcal{A}_i\vartheta + \mathcal{C} + z \\ \mathcal{O} = \mathcal{B}\vartheta \end{cases} \tag{13}$$

where

$$\kappa_1(\theta) = \frac{\sin(\theta) + 0.2172\theta}{1.2172\theta}, \kappa_2(\theta) = \frac{\theta - \sin(\theta)}{1.2172\theta},$$
$$\mathcal{A}_1 = \begin{bmatrix} 0 & 1 \\ -\rho & -\beta \end{bmatrix}, \mathcal{A}_2 = \begin{bmatrix} 0 & 1 \\ -0.2172\rho & -\beta \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

In the rest, κ_i is used to denote $\kappa_i(\theta)$.

Remark 3. *It is noted that the model of PS in [28,29] is represented by the following form:*

$$\begin{cases} {}^{C}D_{b_0}^{\varpi,\pi}\vartheta = \mathcal{A}\vartheta + g(\theta) + \mathcal{C} + z \\ \mathcal{O} = \mathcal{B}\vartheta \end{cases} \tag{14}$$

where

$$\mathcal{A} = \begin{bmatrix} 0 & 1 \\ 0 & -\beta \end{bmatrix}, g(\theta) = \begin{bmatrix} 0 \\ -\rho\sin(\theta) \end{bmatrix}$$

in which $g(\theta)$ satisfies $\|g(\theta)\| \le \zeta\|\vartheta\|$, $\zeta \in \mathbb{R}$.

The main advantage of the TS model (13) compared with (14) is that it allows us to deal with the PS without imposing any bounds on $g(\theta)$.

The objective is to design a polynomial observer-based controller z, described as follows:

$$\begin{cases} {}^{C}D_{b_0}^{\varpi,\pi}\hat{\vartheta} = \sum_{i=1}^{2}\kappa_i\Big(\mathcal{A}_i\hat{\vartheta} + \mathcal{L}_i(\hat{\vartheta},\mathcal{O})(\mathcal{O} - \hat{\mathcal{O}})\Big) + \mathcal{C} + z \\ \hat{\mathcal{O}} = \mathcal{B}\hat{\vartheta} \\ z = \sum_{i=1}^{2}\kappa_i\mathcal{N}_i(\hat{\vartheta},\mathcal{O})\hat{\vartheta} \end{cases} \tag{15}$$

where $\hat{\vartheta}$ is the estimated value of ϑ, $\mathcal{N}_i(\hat{\vartheta},\mathcal{O})$ are the polynomial controller gains, and $\mathcal{L}_i(\hat{\vartheta},\mathcal{O})$ are the polynomial observer gains.

We define $\vartheta_e = \vartheta - \hat{\vartheta}$. The augmented system can be described as:

$$ {}^{C}D_{b_0}^{\varpi,\pi}\tilde{\vartheta} = \sum_{i=1}^{2}\kappa_i\widetilde{\mathcal{A}}_i(\hat{\vartheta},\mathcal{O})\tilde{\vartheta} + \widetilde{\mathcal{C}} \tag{16}$$

where:

$$\tilde{\vartheta} = \begin{bmatrix} \hat{\vartheta} \\ \vartheta_e \end{bmatrix}, \widetilde{\mathcal{A}}_i(\hat{\vartheta},\mathcal{O}) = \begin{bmatrix} \mathcal{A}_i + \mathcal{N}_i(\hat{\vartheta},\mathcal{O}) & \mathcal{L}_i(\hat{\vartheta},\mathcal{O})\mathcal{B} \\ 0 & \mathcal{A}_i - \mathcal{L}_i(\hat{\vartheta},\mathcal{O})\mathcal{B} \end{bmatrix}, \widetilde{\mathcal{C}} = \begin{bmatrix} \mathcal{C} \\ 0 \end{bmatrix}$$

in which $\widetilde{\mathcal{C}}$ satisfies $\|\widetilde{\mathcal{C}}\|^2 = (\xi + \sigma\cos(\phi t))^2 \le 2(\xi^2 + \sigma^2)$.

Theorem 1. *For the given positive scalars γ_1, γ_2, γ_3 and the positive polynomial $\gamma_4(\hat{\vartheta},\mathcal{O})$, the closed-loop system (16) is π–PMLS if there exist symmetric positive definite matrices $\mathcal{Q}_1 \in \mathbb{R}^{2\times 2}$,*

$\mathcal{Q}_2 = diag(q_{11}, q_{22})$ in which q_{11} and q_{22} are two scalars, $\mathcal{P} \in \mathbb{R}^{4\times 4}$, and the polynomial matrices are $\mathcal{H}_i(\hat{\vartheta}, \mathcal{O})$, $\mathcal{M}_i(\hat{\vartheta}, \mathcal{O})$, such that the following SOS-based conditions are satisfied:

Minimize ϵ, satisfying the following conditions:

$$\varphi_1^T \Big(\mathcal{Q}_1 - \gamma_1 I\Big)\varphi_1 \ \in \mathcal{S}_{SOS}, \tag{17}$$

$$\varphi_1^T \Big(\mathcal{Q}_2 - \gamma_2 I\Big)\varphi_1 \ \in \mathcal{S}_{SOS}, \tag{18}$$

$$\varphi_2^T \Big(\mathcal{P} - \gamma_3 I\Big)\varphi_2 \ \in \mathcal{S}_{SOS}, \tag{19}$$

$$-\varphi_3^T \Big(\Theta_i(\hat{\vartheta}, \mathcal{O}) + \mathcal{P} + \gamma_4(\hat{\vartheta}, \mathcal{O}) I\Big)\varphi_3 \ \in \mathcal{S}_{SOS}, \tag{20}$$

$$\Theta_i(\hat{\vartheta}, \mathcal{O}) \quad = \quad \begin{bmatrix} \Big[\mathcal{A}_i \mathcal{Q}_1 + \mathcal{M}_i(\hat{\vartheta}, \mathcal{O})\Big]_{sy} + \frac{1}{\epsilon} I & \mathcal{H}_i(\hat{\vartheta}, \mathcal{O})\mathcal{B} \\ * & \Big[\mathcal{A}_i \mathcal{Q}_2 - \mathcal{H}_i(\hat{\vartheta}, \mathcal{O})\mathcal{B}\Big]_{sy} + \frac{1}{\epsilon} I \end{bmatrix} \tag{21}$$

where $\varphi_1 \in \mathbb{R}^{2\times 1}$, φ_2 and $\varphi_3 \in \mathbb{R}^{4\times 1}$ are symbolic vectors in which φ_3 is independent of $\hat{\vartheta}$ and $\mathcal{O}$. In this case, $\mathcal{N}_i(\hat{\vartheta}, \mathcal{O}) = \mathcal{M}_i(\hat{\vartheta}, \mathcal{O})\mathcal{Q}_1^{-1}$, $\mathcal{L}_i(\hat{\vartheta}, \mathcal{O}) = \frac{1}{q_{11}}\mathcal{H}_i(\hat{\vartheta}, \mathcal{O})$

Proof. Take into account the subsequent Lyapunov function:

$$\mathcal{V}(\tilde{\vartheta}) = \quad \tilde{\vartheta}^T \mathcal{R} \tilde{\vartheta}, \tag{22}$$

where

$$\mathcal{R} \quad = \quad \begin{bmatrix} \mathcal{R}_1 & 0 \\ 0 & \mathcal{R}_2 \end{bmatrix}$$

in which $\mathcal{R}_1 = \mathcal{Q}_1^{-1}$ and $\mathcal{R}_2 = \mathcal{Q}_2^{-1}$.

Based on Lemma 2, we obtain

$${}^{C}D_{b_0}^{\varpi,\pi}\mathcal{V}(\widetilde{\vartheta}) \leq \quad 2\,{}^{C}D_{b_0}^{\varpi,\pi}\widetilde{\vartheta}^T\mathcal{R}\widetilde{\vartheta} = \sum_{i=1}^{2}\kappa_i\widetilde{\vartheta}^T[\mathcal{R}\widetilde{\mathcal{A}}_i(\hat{\vartheta})]_{sy}\widetilde{\vartheta} + \widetilde{\mathcal{C}}^T\mathcal{R}\widetilde{\vartheta} + \widetilde{\vartheta}^T\mathcal{R}\widetilde{\mathcal{C}} \tag{23}$$

where

$$[\mathcal{R}\widetilde{\mathcal{A}}(\hat{\vartheta})]_{sy} = \begin{bmatrix} \Big[\mathcal{R}_1\Big(\mathcal{A}_i(\hat{\vartheta}) + \mathcal{N}_i(\hat{\vartheta}, \mathcal{O})\Big)\Big]_{sy} & \mathcal{R}_1\mathcal{L}(\hat{\vartheta}, \mathcal{O})\mathcal{B} \\ 0 & \Big[\mathcal{R}_2\Big(\mathcal{A}_i(\vartheta) - \mathcal{L}_i(\hat{\vartheta}, \mathcal{O})\mathcal{B}\Big)\Big]_{sy} \end{bmatrix}$$

Let $c = 2(\xi^2 + \sigma^2)$. By applying lemma 5, we obtain

$$\widetilde{\mathcal{C}}^T\mathcal{R}\widetilde{\vartheta} + \widetilde{\vartheta}^T\mathcal{R}\widetilde{\mathcal{C}} \quad \leq \quad \widetilde{\vartheta}^T\Big(\frac{1}{\epsilon}\mathcal{R}\mathcal{R}\Big)\widetilde{\vartheta} + \epsilon c^2 \tag{24}$$

Let $\mathcal{Q} = \begin{bmatrix} \mathcal{Q}_1 & 0 \\ 0 & \mathcal{Q}_2 \end{bmatrix}$

$${}^{C}D_{b_0}^{\varpi,\pi}\mathcal{V}(\tilde{\vartheta}) \quad \leq \quad \sum_{i=1}^{2}\kappa_i\nu^T\Xi_i(\hat{\vartheta}, \mathcal{O})\nu + \epsilon c^2 \tag{25}$$

where

$$\nu = \mathcal{Q}^{-1}\tilde{\vartheta}, \quad \Xi_i(\hat{\vartheta},\mathcal{O}) = \begin{bmatrix} \left[\mathcal{A}_i\mathcal{Q}_1 + \mathcal{M}_i(\hat{\vartheta},\mathcal{O})\right]_{sy} + \frac{1}{\epsilon}I & \mathcal{L}_i(\hat{\vartheta},\mathcal{O})\mathcal{B}\mathcal{Q}_2 \\ * & \left[\mathcal{A}_i\mathcal{Q}_2 - \mathcal{L}_i(\hat{\vartheta},\mathcal{O})\mathcal{B}\mathcal{Q}_2\right]_{sy} + \frac{1}{\epsilon}I \end{bmatrix}$$

$\mathcal{L}_i(\hat{\vartheta},\mathcal{O})\mathcal{B}\mathcal{Q}_2$ can be expressed as:

$$\begin{aligned} \mathcal{L}_i(\hat{\vartheta},\mathcal{O})\mathcal{B}\mathcal{Q}_2 &= \mathcal{L}_i(\hat{\vartheta},\mathcal{O})\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \\ &= \mathcal{L}_i(\hat{\vartheta},\mathcal{O})\begin{bmatrix} q_{11} & 0 \end{bmatrix} \\ &= \mathcal{L}_i(\hat{\vartheta},\mathcal{O})q_{11}\mathcal{B} \\ &= \mathcal{H}_i(\hat{\vartheta},\mathcal{O})\mathcal{B} \end{aligned}$$

The SOS condition (20) implies that $\Xi_i(\hat{\vartheta},\mathcal{O}) < -\mathcal{P}$. Then, we obtain

$$^{C}D_{b_0}^{\varpi,\pi}\mathcal{V}(\tilde{\vartheta}) \leq -\lambda_{min}(\mathcal{P})\|\tilde{\vartheta}\|^2 + \epsilon c^2 \tag{26}$$

Therefore,

$$^{C}D_{b_0}^{\varpi,\pi}\mathcal{V}(\tilde{\vartheta}) \leq -\frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}\mathcal{V}(\tilde{\vartheta}) + \epsilon c^2$$

Let $M(t) =^{C} D_{b_0}^{\varpi,\pi}\mathcal{V}(\tilde{\vartheta}) + \frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}\mathcal{V}(\tilde{\vartheta})$; we obtain from Lemma 2

$$\begin{aligned} \mathcal{V}(\tilde{\vartheta}) &= E_{\varpi}\Big(-\frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}(\pi(t)-\pi(b_0))^{\varpi}\Big)\mathcal{V}(\tilde{\vartheta}(b_0)) \\ &\quad + \int_{b_0}^{t}(\pi(t)-\pi(s))^{\varpi-1}E_{\varpi,\varpi}\Big(-\frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}(\pi(t)-\pi(s))^{\varpi}\Big)\pi'(s)M(s)ds \\ &\leq E_{\varpi}\Big(-\frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}(\pi(t)-\pi(b_0))^{\varpi}\Big)\mathcal{V}(\tilde{\vartheta}(b_0)) \\ &\quad + \epsilon c^2\int_{b_0}^{t}(\pi(t)-\pi(s))^{\varpi-1}E_{\varpi,\varpi}\Big(-\frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}(\pi(t)-\pi(s))^{\varpi}\Big)\pi'(s)ds \end{aligned} \tag{27}$$

Using the change of variable $u = \pi(s)$ and Lemma 1, we obtain

$$\begin{aligned} \mathcal{V}(\tilde{\vartheta}) &\leq E_{\varpi}\Big(-\frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}(\pi(t)-\pi(b_0))^{\varpi}\Big)\mathcal{V}(\tilde{\vartheta}(b_0)) \\ &\quad + \epsilon c^2\int_{\pi(b_0)}^{\pi(t)}(\pi(t)-u)^{\varpi-1}E_{\varpi,\varpi}\Big(-\frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}(\pi(t)-u)^{\varpi}\Big)du \\ &\leq E_{\varpi}\Big(-\frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}(\pi(t)-\pi(b_0))^{\varpi}\Big)\mathcal{V}(\tilde{\vartheta}(b_0)) + M \end{aligned} \tag{28}$$

where M is a positive constant. Therefore,

$$\|\tilde{\vartheta}\| \leq \sqrt{\frac{\lambda_{max}(\mathcal{R})}{\lambda_{min}(\mathcal{R})}}\Big(E_{\varpi}\Big(-\frac{\lambda_{min}(\mathcal{P})}{\lambda_{max}(\mathcal{R})}(\pi(t)-\pi(b_0))^{\varpi}\Big)\Big)^{\frac{1}{2}}\|\tilde{\vartheta}(b_0)\| + \sqrt{\frac{M}{\lambda_{min}(\mathcal{R})}},$$

for every $t \geq t_0$; hence, the system (5) is $\pi-$PMLS. □

In Figure 1, the block diagram illustrates the proposed polynomial observer-based controller strategy. It represents the interaction structure diagram of the controller, the observer (15), and the Takagi–Sugeno fuzzy model (13) used to describe the dynamics of the PS (12).

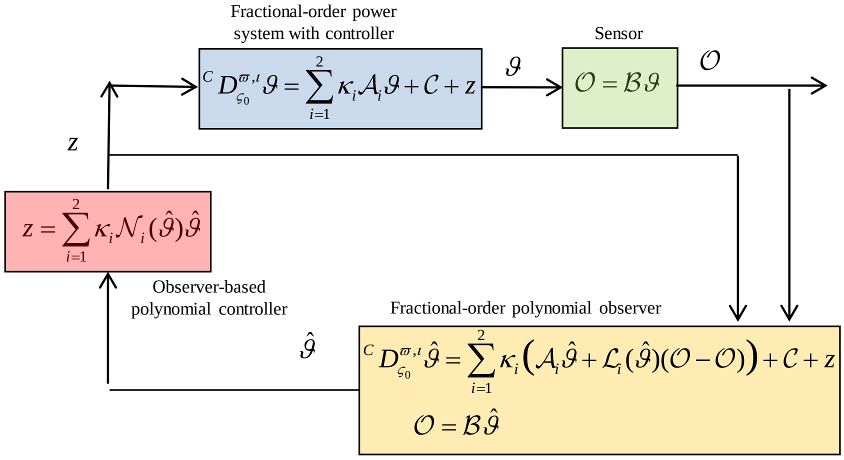

Figure 1. Polynomial observer-based controller scheme.

4. Illustrative Example

Regarding paper [29], the power system parameters were chosen as follows:

$$\rho = 1, \quad \beta = 0.0052, \quad \xi = 0.03, \quad \phi = 0.026, \quad \sigma = 0.5.$$

Now, we apply Theorem 1 to the TS model (13). By choosing $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4(\hat{\vartheta}, \mathcal{O}) = 10^{-6}$ and the degrees of $\mathcal{H}_i(\hat{\vartheta}, \mathcal{O})$ and $\mathcal{M}_i(\hat{\vartheta}, \mathcal{O})$ are 2 in $\mathcal{O}$, we obtain the following gains:

$$\mathcal{N}_1(\hat{\vartheta}, \mathcal{O}) = \begin{bmatrix} -0.676 \times \mathcal{O}^2 - 1.083 & -0.0504 \times \mathcal{O}^2 - 0.109 \\ -0.041\mathcal{O}^2 - 0.105 & -0.645 \times \mathcal{O}^2 - 1.248 \end{bmatrix}$$

$$\mathcal{N}_2(\hat{\vartheta}, \mathcal{O}) = \begin{bmatrix} -0.676 \times \mathcal{O}^2 - 1.202 & -0.0504 \times \mathcal{O}^2 - 0.791 \\ -0.042 \times \mathcal{O}^2 - 0.717 & -0.645 \times \mathcal{O}^2 - 1.681 \end{bmatrix}$$

$$\mathcal{L}_1(\hat{\vartheta}, \mathcal{O}) = \begin{bmatrix} 0.324 \times \mathcal{O}^2 + 0.602 \\ 0.240 \end{bmatrix}, \mathcal{L}_2(\hat{\vartheta}, \mathcal{O}) = \begin{bmatrix} 0.324 \times \mathcal{O}^2 + 0.822 \\ 1 \end{bmatrix}$$

To initiate the trajectory simulation of the following system (11), (13), and (15), we set the initial conditions as follows: $b_0 = 0.2$, $(\theta(b_0), v(b_0)) = (\vartheta_1(b_0), \vartheta_2(b_0)) = (0.1, -0.1)$, $(\hat{\vartheta}_1(b_0), \hat{\vartheta}_2(b_0)) = (0.11, -0.09)$, $\varpi = 0.89$, and $\pi(t) = t^2 + t$. In Figure 2, we present the trajectory simulation of system (11). Furthermore, we maintain the same parameter values for Figures 3 and 4, which depict the trajectory simulations of system (13) and (15). These simulations effectively demonstrate the practical stability of systems (13) and (15).

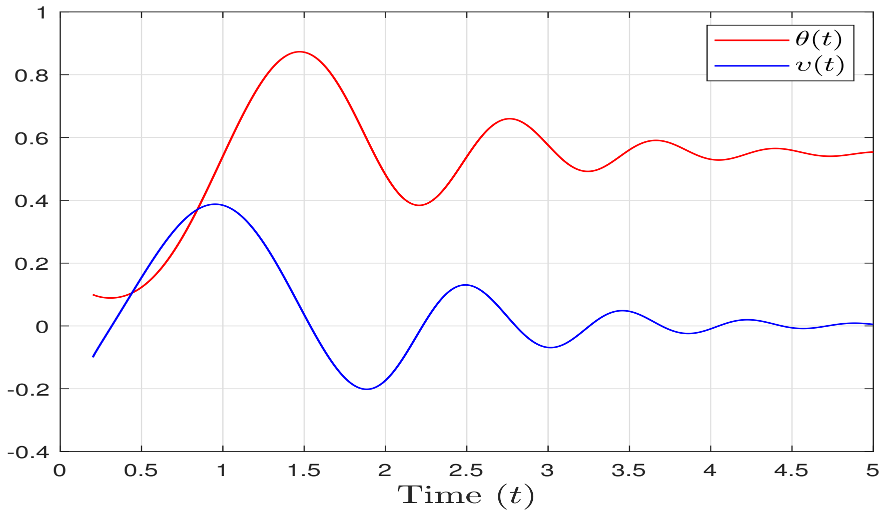

Figure 2. Time evolution of the states $\theta(t)$ and $v(t)$ of system (11) for $t \in [0.2, 5]$.

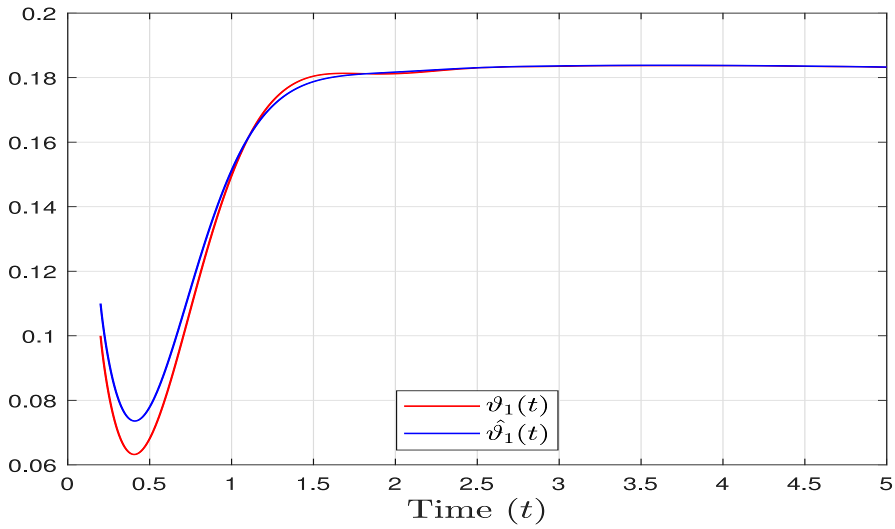

Figure 3. Time evolution of the states $\vartheta_1(t)$ (respectively, $\hat{\vartheta}_1(t)$) of system (13) (respectively, (15)) for $t \in [0.2, 5]$.

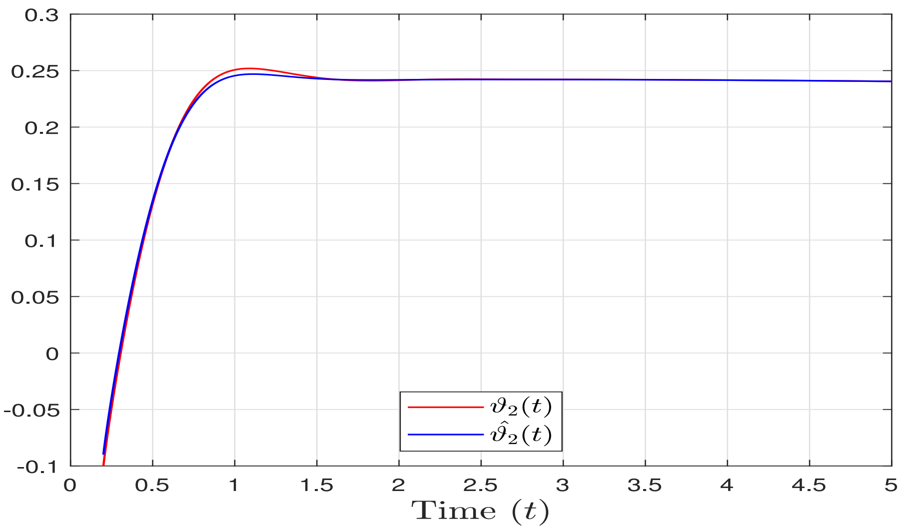

Figure 4. Time evolution of the states $\vartheta_2(t)$ (respectively, $\hat{\vartheta}_2(t)$) of system (13) (respectively, (15)) for $t \in [0.2, 5]$.

5. Conclusions

In summary, this paper has extensively explored the problem of achieving the globally practical stabilization for a particular category of F-OP systems. Our approach, grounded in observer-based techniques and sum-of-SOS methods, has been demonstrated to be effective in addressing the complexities inherent in these systems. The practicality and efficacy of our theoretical contributions were substantiated through thorough simulations using SOSTOOLS. This research contributes significantly to the field of power system stability and control, providing a robust framework with tangible applications in real-world scenarios. Looking forward, future endeavors may extend this work to address more complex scenarios and could explore the integration of advanced methodologies, such as adaptive control strategies, for further enhancement.

Author Contributions: Methodology, A.B.M.; Software, M.R.; Formal analysis, I.I.A.; Investigation, H.G. and M.R.; Writing—original draft, H.G. Writing—review & editing, L.M. All authors have read and agreed to the published version of the manuscript.

Funding: The Deputyship for Research and Innovation, "Ministry of Education" in Saudi Arabia for funding this research (IFKSUOR3-312-3).

Data Availability Statement: No data were used to support this study.

Acknowledgments: The authors extend their appreciation to the Deputyship for Research and Innovation, "Ministry of Education" in Saudi Arabia for funding this research (IFKSUOR3-312-3).

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Kundur, P. *Power System Stability and Control*; McGraw-Hill: New York, NY, USA, 1994.
2. Yang, F.; Shao, X.; Muyeen, S.M.; Li, D.; Lin, S.; Fang, C. Disturbance Observer based Fractional-order Integral Sliding Mode Frequency Control Strategy for Interconnected Power System. *IEEE Trans. Power Syst.* **2021**, *36*, 5922–5932.
3. Wang, H.O.; Tanaka, K.; Griffin, M.F. An approach to fuzzy control of nonlinear systems: Stability and design issues. *IEEE Trans. Fuzzy Syst.* **1996**, *4*, 14–23. [CrossRef]
4. Zhao, D.; Lam, H.K.; Li, Y.; Ding, S.X.; Liu, S. A novel approach to state and unknown input estimation for Takagi–Sugeno fuzzy models with applications to fault detection. *IEEE Trans. Circuits Syst.* **2020**, *67*, 2053–2063. [CrossRef]
5. El Youssfi, N.; El Bachtiri, T.; Zoulagh, T.; El Aiss, H. Unknown input observer design for vehicle lateral dynamics described by Takagi–Sugeno fuzzy systems. *Optim. Control. Appl. Methods* **2022**, *43*, 354–368.
6. Zhang, H.; Mu, Y.; Gao, Z.; Wang, W. Observer-based fault reconstruction and fault-tolerant control for nonlinear systems subject to simultaneous actuator and sensor faults. *IEEE Trans. Fuzzy Syst.* **2022**, *30*, 2971–2980.
7. Salah, R.B.; Kahouli, O.; Hadjabdallah, H. A nonlinear Takagi-Sugeno fuzzy logic control for single machine power system. *Int. J. Adv. Manuf. Technol.* **2017**, *90*, 575–590.
8. Ouassaid, M.; Maaroufi, M.; Cherkaoui, M. Observer-based nonlinear control of power system using sliding mode control strategy. *Electr. Power Syst. Res.* **2012**, *84*, 135–143.
9. Gassara, H.; El Hajjaji, A.; Chaabane, M. Observer-based robust H_∞ reliable control for uncertain T-S fuzzy systems with state time delay. *IEEE Trans. Fuzzy Syst.* **2010**, *18*, 1027–1040. [CrossRef]
10. Kchaou, M.; Gassara, H.; El Hajjaji, A. Robust observer-based control design for uncertain singular systems with time-delay. *Int. J. Adapt. Control. Signal Process.* **2014**, *28*, 169–183. [CrossRef]
11. Gassara, H.; El Hajjaji, A.; Kchaou, M.; Chaabane, M. Observer based $(Q, V, R) - \alpha$ dissipative control for TS fuzzy descriptor systems with time delay. *J. Frankl. Inst.* **2014**, *351*, 187–206.
12. Sun, H.; Han, H.-G.; Qiao, J.-F. Observer-based control for networked Takagi-Sugeno fuzzy systems with stochastic packet losses. *Inf. Sci.* **2023**, *644*, 119275. [CrossRef]
13. Prajna, S.; Papachristodoulou, A.; Seiler, P.; Parrilo, P.A. SOSTOOLS and its control applications. In *Positive Polynomials in Control (Part of the Lecture Notes in Control and Information Science)*; Henrion, D., Garulli, A., Eds.; Springer: Cham, Switzerland, 2015; Volume 312, pp. 273–292.
14. Ting, C.-S.; Chang, Y.-N.; Chiu, T.-Y. An SOS Observer-Based Sensorless Control for PMLSM Drive System. *J. Control. Autom. Electr. Syst.* **2020**, *31*, 760–776. [CrossRef]
15. Kilbas, A.A.; Srivastava, H.M.; Trujillo, J.J. Theory and applications of fractional differential equations. In *North-Holland Mathematics Studies*; Elsevier Science B.V.: Amsterdam, The Netherlands, 2006; Volume 204.
16. Ahmad, I.; Ahmad, H.; Thounthong, P.; Chu, Y.M.; Cesarano, C. Solution of Multi-Term Time-Fractional PDE Models Arising in Mathematical Biology and Physics by Local Meshless Method. *Symmetry* **2022**, *12*, 1195. [CrossRef]
17. Mohamed, E.A.; Aly, M.; Watanabe, M. New Tilt Fractional-Order Integral Derivative with Fractional Filter (TFOIDFF) Controller with Artificial Hummingbird Optimizer for LFC in Renewable Energy Power Grids. *Mathematics* **2022**, *10*, 3006. [CrossRef]
18. Liu, K.; Chen, Y.Q.; Domański, P.D.; Zhang, X. A Novel Method for Control Performance Assessment with Fractional Order Signal Processing and Its Application to Semiconductor Manufacturing. *Algorithms* **2018**, *11*, 90. [CrossRef]
19. Williams, W.K.; Vijayakumar, V.; Udhayakumar, R.; Panda, S.K.; Nisar, K.S. Existence and controllability of nonlocal mixed Volterra-Fredholm type fractional delay integro-differential equations of order $1 < r < 2$. *Numer. Methods Partial. Differ. Equ.* **2020**, *2020*, 1–18.
20. Chen, L.; Chai, Y.; Wu, R.; Yang, J. Stability and Stabilization of a Class of Nonlinear Fractional-Order Systems with Caputo Derivative. *IEEE Trans. Circuits Syst. II: Express Briefs* **2012**, *59*, 602–606. [CrossRef]
21. Baleanu, D.; Machado, J.A.T.; Luo, A.C.J. *Fractional Dynamics and Control*; Springer: New York, NY, USA, 2011.
22. Abdeljawad, T.; Madjidi, F.; Jarad, F.; Sene, N. On Dynamic Systems in the Frame of Singular Function Dependent Kernel Fractional Derivatives. *Mathematics* **2019**, *7*, 946. [CrossRef]
23. Ben Makhlouf, A. Partial practical stability for fractional-order nonlinear systems. *Math. Methods Appl. Sci.* **2022**, *45*, 5135–5148. [CrossRef]
24. Chen, L.; Huang, T.; Machado, J.T.; Lopes, A.M.; Chai, Y.; Wu, R. Delay-dependent criterion for asymptotic stability of a class of fractional-order memristive neural networks with time-varying delays. *Neural Netw.* **2019**, *118*, 289–299. [CrossRef]

25. Ben Makhlouf, A.; Hammami, M.A.; Sioud, K. Stability of fractional-order nonlinear systems depending on a parameter. *Bull. Korean Math. Soc.* **2017**, *54*, 1309–1321.
26. Matignon, D. Stability results on fractional differential equations to control processing. In Proceedings of the Computational Engineering in Syatems and Application Multiconference, Lille, France, 9–12 July 1996; IMACS, IEEE-SMC: Lille, France, 1996; Volume 2, pp. 963–968.
27. Zhou, Y.; Ionescu, C.; Tenreiro Machado, J.A. Fractional dynamics and its applications. *Nonlinear Dyn.* **2015**, *80*, 1661–1664. [CrossRef]
28. Yu, Z.; Sun, Y.; Dai, X.; Ye, Z. Stability analysis of interconnected nonlinear fractional-order systems via a single-state variable control. *Int. J. Robust Nonlinear Control* **2019**, *29*, 6374–6397. [CrossRef]
29. Yu, Z.; Sun, Y.; Dai, X. Stability and Stabilization of the Fractional-Order Power System with Time Delay. *IEEE Trans. Circuits Syst. II Express Briefs* **2021**, *68*, 3446–3450. [CrossRef]
30. Gassara, H.; Kharrat, D.; Makhlouf, A.B.; Mchiri, L.; Rhaima, M. SOS Approach for Practical Stabilization of Tempered Fractional-Order Power System. *Mathematics* **2023**, *11*, 3024. [CrossRef]
31. Ahmed, H.; Jmal, A.; Ben Makhlouf, A. A practical observer for state and sensor fault reconstruction of a class of fractional-order nonlinear systems. *Eur. Phys. J. Spec. Top.* **2023**. [CrossRef]
32. Tanaka, K.; Yosihida, H.; Ohtake, H.; Wang, H. A sum of squares approach to modeling and control of nonlinear dynamical systems with polynomial fuzzy systems. *IEEE Trans. Fuzzy Syst.* **2009**, *17*, 911–922. [CrossRef]
33. Petersen, I.R. A stabilization algorithm for a class of uncertain linear systems. *Syst. Control Lett.* **1987**, *8*, 351–357. [CrossRef]
34. Tanaka, K.; Wang, H.O. *Fuzzy Control Systems Design and Analysis. A Linear Matrix Inequality Approach*; John Wiley: New York, NY, USA, 2001.