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A Combinatorial Optimization Approach for Air Cargo Palletization and Aircraft Loading

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Abstract: The current air cargo loading plan handles the Air Cargo Palletization Problem (ACPP) and the Aircraft Weight and Balance Problem (WBP) separately, which has an impact on the optimization of the payload and the aircraft's center of gravity (CG). Thanks to improvements in computer processing power, the joint combinatorial optimization of ACPP and WBP is now feasible. Three integer linear programming models are proposed: a Bi-objective Optimization Model (BOM), a Combinatorial Optimization Model (COM), and an Improved Combinatorial Optimization Model (IOM). The objectives of the models are the maximum loading capacity and the lowest CG deviation from a specified target CG. The models also consider a wide range of restrictions in the actual packing and stowage procedures, such as volume, weight, loading position, aircraft balance, and other aspects of aircraft and unit load devices. Four scenarios with various conditional metrics for three models are solved for the B777F aircraft using Gurobi. The results of the computations demonstrate that the BOM has the fastest solution speed, but the CG deviation is the largest, and in several cases the CG deviation results are unacceptable. The COM has the longest solution time, which is difficult to tolerate in practice. Despite taking a little longer to solve computationally than the BOM, the IOM offers the best optimization solution.

Keywords: constrained optimization; transportation; air cargo; loading problems; packing problems; load balance; aircraft weight and balance; stowage

MSC: 90B80; 90C10

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1. Introduction

One of the most significant sources of income for airlines is air cargo. In 2019, USD 101 billion was produced, followed by USD 129 billion in 2020 and 155 billion in 2021. In its 2022 annual review report, the International Air Transport Association noted that in 2021 air cargo revenue made up more than one-third of airline revenue.

A significant challenge in air transportation planning for air cargo is the cargo loading problem (ACLP) [1]. This is the problem of developing a plan for allocating unit load devices (ULDs) and bulk onto a cargo aircraft in a reasonable manner in order to ensure flight safety and maximize revenues. Loadmasters for airlines must select, assign, and load all goods on board. A rational loading plan should maximize the volume of transportation while maintaining the aircraft's balance and safety during flight. If the plan is irrational, it will interfere with the aircraft's normal operation and may potentially have serious repercussions. In the past, many aviation accidents involved issues with loading planning.

The two primary physical processes in the ACLP are the Air Cargo Palletization Problem (ACPP) and the Aircraft Weight and Balance Problem (WBP) [2]. The assignment of cargo to ULDs will be determined by the ACPP. With a variety of constraints, its objective is to load as much cargo as possible. In consideration of loading and balancing constraints, the WBP must choose the loading positions for ULDs inside the aircraft's compartments.

Air cargo palletization can improve bulk cargo transportation plans. A thoughtful packing strategy can help maximize container space use, cut down on wasteful packaging materials, and save on transport costs [3]. Additionally, individual products can be joined into units of a standard size unit, which enables quick loading and unloading onto aircraft with compatible handling, increasing operational efficiency and minimizing damage. The operators packing goods rely mainly on their experience during the ULD build-up in practical work. More goods can be loaded with the same volume by a skilled loadmaster, who can also achieve high room utilization, an appropriate contour, and a reasonable height. However, a novice may pack a poor-quality ULD with low loading capacity, wasting the pallet's space. It is challenging to develop a suitable pallet loading standard when the procedure is fully dependent on expertise. It makes subsequent aircraft loading operations difficult.

Aircraft weight and balance planning involves assigning palleted ULDs to various positions in the aircraft cargo holds in order to maximize loading and minimize deviation from the center of gravity (CG). The arrangement of ULDs in holds has an impact on transit efficiency, economy, flying safety, and maneuverability [4]. A proper weight and balance plan can ensure airplane safety; reduce fuel consumption, airline costs, and carbon dioxide emissions; and promote "emission peak" and "carbon neutrality" realization. An improper plan can have disastrous impacts.

The development of air cargo transportation will inevitably integrate WBP and ACPP optimizations. The results of the aircraft weight and balance tests are directly related to the palletization of air cargo. Because a ULD weighs more than a piece of cargo, switching the positions of two ULDs during the loading process causes a significant shift in the weight and balance moment. As a result, the CG adjustment is drastic as well. Minimizing the CG offset from a target CG is one of the main objectives of the WBP to maintain aircraft balance and cut fuel usage. We lose precise control of the CG due to the ULD's heavy weight. However, if the WBP is taken into account for the bulk cargo items and we can precisely place each piece of cargo, it will improve the accuracy of CG control and adjustment. Maximizing the entire payload to maximize profit is another crucial objective. The appropriate cargo items of weight and volume can be chosen to completely utilize the flight transport capacity by guiding the air cargo palletization by weight and balance.

Dahmani & Krichen (2013) and (2016) developed a multi-objective particle swarm optimization approach for the problem of packing a set of items into a minimum number of containers, which are subsequently stowed into a predefined number of compartments in an aircraft [5,6]. It was known as the Bi-objective Aircraft Cargo Loading Problem (BOACLP). Maximum total weight and loaded cargo priority were the two objectives. To the best of our knowledge, they are the first to suggest the joint combinatorial optimization of the ACPP and WBP, despite being far from flawless. In the field of air cargo transportation, this approach is valuable commercially. The model, however, is overly simplistic when compared with real-world work. The primary limitations are weight and container capacity constraints as well as load balance constraints, which are far from what the actual work demands of the air cargo transportation companies. Their test plane was a Hercules C-130, which can carry up to six pallets at a time. However, it is too small for commercial planes.

The purpose of this paper is to apply the joint combinatorial optimization of the ACPP and WBP to heavy commercial aircraft with the airline's practical constraints. We hope it can facilitate the work of cargo palletization and making weight and balance plans for air cargo.

The novelty of our study is its extension to include commercial aircraft for large-scale bulk cargo and offering three integer linear programming (ILP) models for the joint combinatorial optimization of ACPP and WBP. The objectives are to maximize the overall payload and minimize CG deviation from a specified target value. The restrictions on

ULDs' weight and capacity, as well as the aircraft's positions, weight limits, and balance, all stem from the specifications of the balance chart and aircraft load sheet used by air cargo carriers.

Consequently, the contributions addressed in this study focus on WBP and ACPP combinatorial optimization as follows:

(1) For the joint combinatorial optimizations of the WBP with the ACPP, we suggest three integer linear programming models: a Bi-objective Optimization Model (BOM), a Combinatorial Optimization Model (COM), and an Improved Combinatorial Optimization Model (IOM).
(2) To lower the decision variable dimension from 3D to 2D, we provide a dimension reduction technique in the IOM.

We created an experimental data set for the B777F heavy jet. The results demonstrate that the IOM can assign thousands of items in a reasonable solution time, achieve a highly precise CG, and obtain the maximum payload.

The rest of this study is divided into the following sections. The essential literature on the WBP and ACPP is reviewed in Section 2. The problem description and a summary of the constraints we took into consideration are presented in Section 3. Three distinct mathematical models are presented in Section 4. Section 5 discusses the implementation and results. Finally, Section 6 concludes.

2. Literature Review

The first overview of the WBP for airplanes was presented by Martin-Vega (1985) [7]. He noted that, up to the time of writing, the majority of research had been on manual and computer aid rather than computer development of load plans. Instead of producing an optimal plan, planners' primary goal was to develop a feasible one. The CG was generally addressed via pyramid loading, which assigns the heaviest items to central positions and alternately loads towards the front and back of the aircraft. A comprehensive literature assessment of commercial air cargo operations was presented by Feng et al. (2015) [8]. They stated that the airplane loading problem is one of the fundamental issues in combinational optimization and described it as an NP-hard problem. Brandt & Nickel (2019) divided the ACLP into four subproblems: the aircraft configuration problem, build-up scheduling problem, ACPP, and WBP. They emphasized that the WBP is not yet sufficiently addressed to the scale of real-world problems or that some practically significant constraints are missing from the models [4].

On the bin packing problem, there is a wealth of literature; the key publications related to the ACPP are listed below. Padberg (2000) and Fasano (2004) provide various ILPs for non-uniform rectangular box packing into 3D containers [9,10]. A nonlinear mixed-ILP (MILP) model was presented by Yan et al. (2008) for the cargo container loading plan problem. With the constraints of origin and destination demand, container capacity, number of containers, container handling capability for each gateway, and aircraft capacity, its objective is to minimize container handling costs [11]. Liu et al. (2008) investigated multi-objective two-dimensional bin packing problems using a multi-objective evolutionary particle swarm technique. The objective is to minimize both the number of bins employed and the average deviation from an ideal CG; the challenge combines bin packing optimization with bin load-balancing objectives [12]. Li et al. (2009) presented a compromise large-scale search neighborhood to help freight forwarders reduce overall freight expenses given a finite number of rental containers [3]. Tang (2011) developed a strategy for resolving an air express cargo loading problem with stochastic demands by integrating scenario decomposition and a genetic algorithm [13]. The problem of maximizing the loading of boxes into airplane containers to reduce the unused volume was covered by Paquay et al. [14,15].

The literature has presented many WBPs for air cargo methods. Heuristic approaches and MILP modeling methods are two categories into which they can be classified.

The early literature provided various heuristic techniques. An interactive computerized procedure for developing load plans to load standardized containers and pallets onto a Boeing 747 combi was described by Larsen & Mikkelsen (1980) [16]. The load plans were handled by two heuristic approaches, and as a result, ground stability, combined load restrictions, position and compartment capacity constraints, and balancing were dealt with. The load plans are nearly optimal in that the number of items changing position in the airplane at intermediate airports is close to the minimum. However, no information is available about the CG. Brosh (1981) looked at a case study regarding arranging cargo allocations and determining the best load layout, which was formulated as a fractional programming problem and solved as a series of linear programming problems [17]. Given the volume, weight, and structure of the cargo as well as the CG limitations, the objective is loading maximization with the assumption that the cargo is homogeneous. According to Martin-Vega (1985), this is a unique method of mathematical programming that was applicable in real life before 1985 [7]. However, the author did not concentrate on putting the ULDs in an airplane in precise locations.

Furthermore, the nonlinear constraints in the formulation are produced by the CG limits. Amiouny et al. (1992) developed several heuristics for the unique situation in which all items of a given length and weight must be loaded and arranged in a row, i.e., a one-dimensional (1D) hold, so that the CG is within an error bound on the deviation from a target value [18]. They provided a reduction, demonstrating that even the 1D aircraft load balancing problem is NP-hard. Their heuristics algorithm only considers a 1D balance problem in which items cannot be placed side by side. The concepts of Amiouny et al. (1992) were expanded to higher dimensions by Wodziak & Fadel (1994) using genetic algorithms [19]. Nevertheless, this entails a considerable computational burden.

Numerous studies have looked at the WBP in terms of a "bin packing problem" or "knapsack problem", where airplane holds were viewed as bins or knapsacks and ULDs as objects. Mathur (1998) proposed a greedy heuristic for the 1D aircraft load balancing problem, in which the objective is to pack homogeneous blocks of specified lengths and weights in a container such that the CG of the packed blocks is as near as practicable to a target point [20]. The proposed algorithm, which is based on the approximate solution of this problem as a knapsack problem, has the same computational complexity but performs better in the worst-case scenario than the algorithm proposed by Amiouny et al. (1992).

Heidelberg et al. (1998) viewed cargo packing aboard an aircraft as a two-dimensional (2D) bin packing problem [21]. The constraints included weight limits for axles and contact points resting on the cargo floor, weight limits within regions on the cargo floor, and requirements for longitudinal and lateral balance. However, they disregarded the aircraft center of balance, the multi-holds, and the spatial limitations of the goods. These heuristic methods, such as the differential evolution algorithm, were unable to achieve optimal solutions.

In recent studies, a number of ILP and MILP models have been developed. For loading cargo containers into the light aircraft that Federal Express uses, Thomas et al. (1998) presented an ILP formulation [22]. They considered a specific variation of the shear constraint, which is imposed on the load in this category of light aircraft but not in heavier commercial aircraft. In their formulation, they presumptively assigned a predetermined set of homogeneous items with varying weights to predetermined locations in the aircraft.

An ILP model was developed by Mongeau & Bès (2003) that maximizes the total amount of cargo carried by aircraft while guaranteeing that the CG is within a predetermined range of a target value [23]. They mainly took into account constraints on the arrangement of uniform containers in defined positions, aircraft volume capacity, and a single container for each compartment. They used Airbus aircraft to show the algorithm's effectiveness. Although they utilized ILP software (the subroutine H02BBF in NAG, The Numerical Algorithms Group (NAG), Oxford, UK) that is available commercially, the system required more than 6 min to solve the problem. Without utilizing the Airbus aircraft's CG envelope, they took the CG constraint within a specific deviation from a target value into consideration.

Kaluzny & Shaw (2009) developed an MILP model to solve the load balance optimization problem specific to a military application where the 2D layout is of key significance when arranging a set of items in a cargo hold. The objective was to load all items to achieve an optimal balance or to maximize a function of the number of items taken. Realistic problem characteristics, such as rectangles of various sizes and CG offsets from the geographical center, were modeled in items [24]. However, apart from the limits on item spacing and CG envelope, the model formulation does not take into account structural and safety constraints, such as those relating to floor strength and cumulative load constraints, which are crucial for commercial air cargo transportation.

An MILP model was produced by Limbourg et al. (2012) to place a set of items in certain positions aboard an aircraft [25]. Its objective is to minimize the moment of inertia of the cargo to increase stability, alleviate strain on the aircraft's structure, and save fuel. They also considered actual limitations such as payload weight, CG, lateral balance, combined load limits, and cumulative load limits. The presumption that all containers can be loaded is restrictive when there are a lot of containers available. The CG constraint does not take the aircraft's CG envelope into account; it is within a specific deviation from a target value. In their computing experiments, the minimal CG solve time was under 441.9 s.

Vancroonenburg et al. (2014) proposed MILP models for the aircraft WBP [26]. The objective was to either determine the most profitable payload selection from a set of cargo to be loaded onto an aircraft or minimize deviation from the target aircraft's CG. The model is subject to constraints to guarantee the aircraft's structural stability and integrity, the crew's and cargo's safety, and the safe and efficient loading and unloading of cargo. However, the CG constraint, as opposed to the CG envelope, is instead within a specific deviation from a target value. The effect of the deviation parameter from the target CG was not addressed in their computational experiments. The maximum number of ULDs was 50, and the solve time was less than 7.5 s for a B747 airplane.

The work of Limbourg et al. (2012) was expanded by Lurkin & Schyns (2015) into two legs with pickups and deliveries at an intermediate airport [27]. To reduce CG offset and handling operation costs at the intermediate airport, they developed an MILP model. The model took pickup and delivery constraints into account and can provide the best ULD unloading and loading operations.

Brandt (2017) provided the first ILP formulation of the WBP for any number of flight legs [4]. Reducing the total number of ULDs that are loaded and unloaded at the intermediate airport was one of the objectives. However, several types of weight limitations were generalized to cumulative load constraints.

Dahmani & Krichen (2013) and (2016) were the first and only studies to propose the combinatorial optimization of the ACPP and WBP [5,6]. However, its model is far from the actual work carried out in air cargo transportation companies. We extend the work in this study.

The combinatorial optimization of the ACPP and WBP is a multi-objective optimization problem. Over the last two decades, multi-objective optimization problems have become popular among researchers and engineers [28,29]. Kumar et al. (2023) address two conflicting objectives for a complex bridge network [30]. We employed multi-objective optimization in our models too, one for maximizing aircraft payload and another for minimizing the CG deviation. These two objectives, however, do not completely conflict with each other. The first objective takes precedence over the second. As a result, we take a hierarchical approach to problem solving.

3. Problem Description and Model Formulation

3.1. Problem Description

Figure 1 describes the problem in detail. Under a variety of constraints, a set of cargo items must be assigned to a number of ULDs, and then these loaded ULDs must be assigned to the positions of the aircraft's cargo holds that are specifically designated for ULDs.

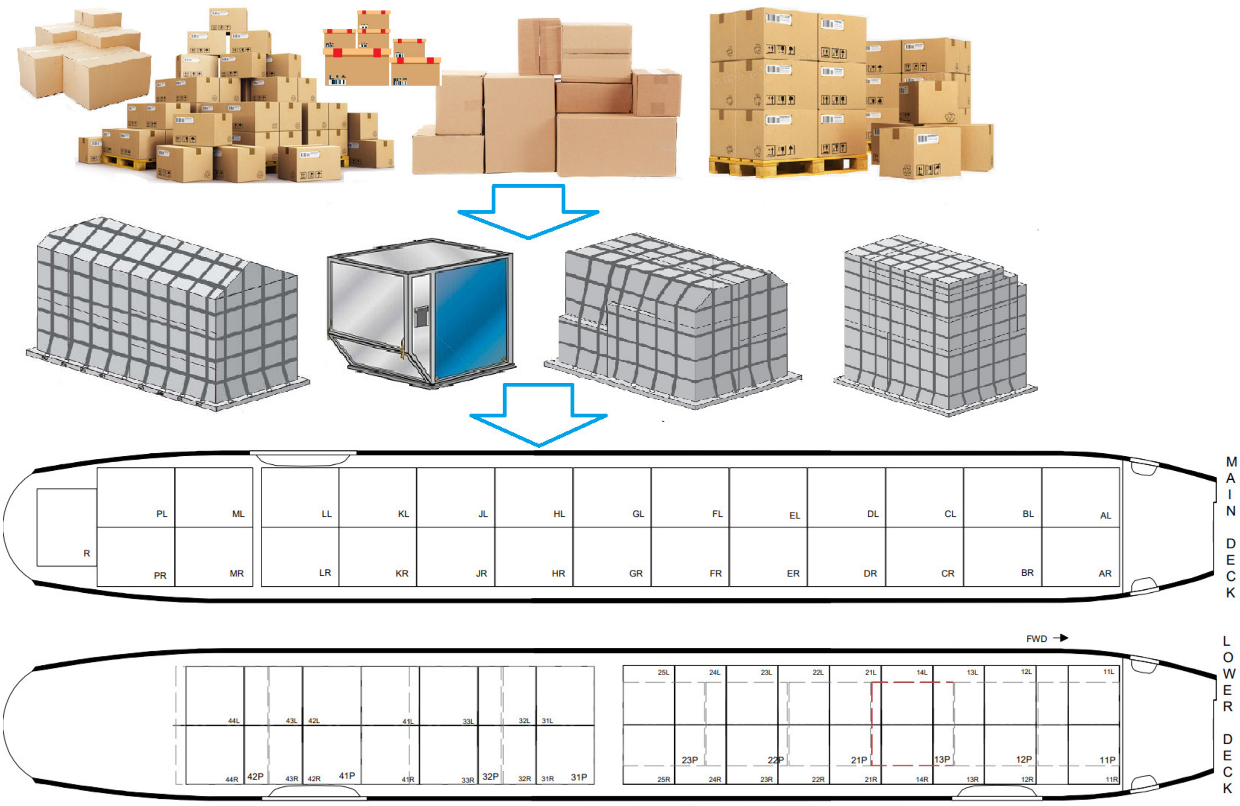

Figure 1. Problem description.

This article aims to provide mathematical methods for Table 1.

Table 1. Problem description of the joint combinatorial optimizations of the WBP with the ACPP.

Maximize		Total payload.
Minimize		The CG deviation from target one.
Subject to	ULD constraints	Weight limit. Capacity limit. ULD and loading position assignment.
	Position constraints	Adapting position constraints. Overlapping position constraints. Loading dependencies. ULD separation.
	Weight constraints	Position weight limit. The maximum combined load limit. Main deck unsymmetrical load limit. Maximum allowable payload constraints.
	Balance constraints	Lateral imbalance constraints. The CG envelope constraints.

3.2. Notation

The notations used in the mathematical models are shown in Tables 2 and 3.

Table 2. Aircraft parameters and definitions.

Para.	Explanation	Para.	Explanation
j	Index of predefined positions.	(M)TOW	(Maximum) takeoff weight
$\mathbf{P}$	The set of all available positions.	(M)LW	(Maximum) landing weight
w_j	The loading weight in position j.	(M)ZFW	(Maximum) zero-fuel weight
W_j	The maximum weight of position j.	MPL	Maximum payload of aircraft
T_j	The type of position j	OEW	Operation empty weight
BA_j	The balance arm of position j.	CG_{TOW}	The CG of TOW
%MAC	The value of CG.	$\mathrm{CG}_{\mathrm{target}}$	The given target CG
$\mathbf{O}_j$	The overlapping position set of j.	TOF	Takeoff fuel
Sp	A set of pairs of positions side by side in the main deck.	TF	Trip fuel
Lp	The set of left positions.	FI	Index of TOF
Rp	The set of right positions.	IND_ω	The *INDEX* at weight ω
Cp	A set of pairs of combined positions that have approximate values of BAs between main deck and low decks.	$\mathrm{IND}_\omega^{\mathrm{FWD}}$	The forward index limit at weight ω
		$\mathrm{IND}_\omega^{\mathrm{AFT}}$	The aft index limit at weight ω

Table 3. Items of bulk cargo and ULD parameter definitions.

Para.	Explanation	Para.	Explanation
i	Index of items of bulk cargo	u	Index of ULD
$\mathbf{I}$	The set of items available	U	The set of ULDs available
N_{I}	The total number of items available	N_{U}	The total number of ULDs available
w_i	The weight of item i	w_u	The weight of ULD u
v_i	The volume of item i	T_u	Type of ULD u
W_u	Maximum weight of ULD u	V_u	Maximum volume of ULD u

3.3. Bi-Objective Optimization Model (BOM)

Instead of the bin packing issue, we will refer to the ACPP as the air cargo assignation problem. It is considered that the assignation results of ULDs can be packed if the constraints are guaranteed, excluding the geographic overlapping limits of the items.

We assigned a great number of items of bulk cargo to *Nu* ULDs and maximized the total weight within the constraints of the ULDs. We consider variables x_{iu} as taking a value of 1 if the item i is in ULD u; the formulation for ACPP is given by:

$$\text{Maximize} \quad \sum_{u\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i x_{iu} \tag{1}$$

$$\text{Subject to} \quad \sum_{u\in\mathbf{U}} x_{iu} \le 1 \ \forall i \in \mathbf{I} \tag{2}$$

$$w_u = \sum_{i\in\mathbf{I}} w_i x_{iu} \le W_u \ \forall u \in \mathbf{U} \tag{3}$$

$$\sum_{i\in\mathbf{I}} v_i x_{iu} \le V_u \ \forall u \in \mathbf{U} \tag{4}$$

$$x_{iu} \in \{0,1\} \ \forall i \in \mathbf{I}, u \in \mathbf{U} \tag{5}$$

Objective (1) is to maximize the total cargo weight. Constraint (2) ensures that each item is allocated to exactly one ULD. Constraints (3) and (4) ensure that items do not exceed their ULD maximum weight and capacity.

After ULD build-up, ULDs should be allocated to predefined positions of the aircraft holds. We consider variables y_{uj} as taking a value of 1 if ULD u is assigned to position j. The WBP is:

$$\text{Maximize} \quad \sum_{j\in\mathbf{P}}\sum_{u\in\mathbf{U}} w_u \cdot y_{uj} \tag{6}$$

$$\text{Minimize} \quad \left|CG_{TOW} - \mathrm{CG}_{\mathrm{target}}\right| \tag{7}$$

$$\text{Subject to} \quad \sum_{j\in\mathbf{P}} y_{uj} \leq 1 \ \forall u \in \mathbf{U} \tag{8}$$

$$\sum_{u\in\mathbf{U}} y_{uj} \leq 1 \ \forall j \in \mathbf{P} \tag{9}$$

$$\mathrm{M}\cdot(1-y_{uj}) \geq \left|T_u - T_j\right| \ \forall u \in \mathbf{U}, j \in \mathbf{P} \tag{10}$$

$$y_{u_1,j_1} + y_{u_2,j_2} \leq 1 \ \forall u_1, u_2 \in \mathbf{U},\ \ u_1 \neq u_2; \forall j_1 \in \mathbf{P}; \forall j_2 \in \mathbf{O}_{j_1} \tag{11}$$

$$\sum_{u\in\mathbf{U}} w_u y_{uj} \leq W_j \ \forall j \in \mathbf{P} \tag{12}$$

$$\alpha_{j_\mathrm{m}} \cdot \sum_{u_1\in\mathbf{U}} w_{u_1} \cdot y_{u_1,j_\mathrm{m}} + \sum_{u_2\in\mathbf{U}} w_{u_2} \cdot y_{u_2,j_1} \leq \mathrm{MaxW}_{(j_\mathrm{m},\,j_1)} \ \forall (j_\mathrm{m}, j_1) \in \mathbf{C}_\mathrm{P} \tag{13}$$

$$\sum_{u_1\in\mathbf{U}} w_{u_1} \cdot y_{u_1,j_\mathrm{left}} \leq a_\mathrm{unsym} \sum_{u_2\in\mathbf{U}} w_{u_2} \cdot y_{u_2,j_\mathrm{right}} + b_\mathrm{unsym} \quad \forall (j_\mathrm{left}, j_\mathrm{right}) \in \mathbf{S}_\mathbf{P} \tag{14}$$

$$\sum_{u_1\in\mathbf{U}} w_{u_1} \cdot y_{u_1,j_\mathrm{right}} \leq a_\mathrm{unsym} \sum_{u_2\in\mathbf{U}} w_{u_2} \cdot y_{u_2,j_\mathrm{left}} + b_\mathrm{unsym} \quad \forall (j_\mathrm{left}, j_\mathrm{right}) \in \mathbf{S}_\mathbf{P} \tag{15}$$

$$\sum_{\forall j\in\mathbf{P}}\sum_{\forall i\in\mathbf{U}} w_u \cdot y_{uj} \leq \mathrm{MPL} \tag{16}$$

$$\left|\sum_{j_\mathrm{left}\in\mathbf{P_L}}\sum_{\forall u_1\in\mathbf{U}} w_{u_1} \cdot y_{u_1,j_\mathrm{left}} - \sum_{j_\mathrm{right}\in\mathbf{P_R}}\sum_{\forall u_2\in\mathbf{U}} w_{u_2} \cdot y_{u_2,j_\mathrm{right}}\right| \leq a_\mathrm{LatTOW} \cdot TOW + b_\mathrm{LatTOW} \tag{17}$$

$$\left|\sum_{j_\mathrm{left}\in\mathbf{P_L}}\sum_{\forall u_1\in\mathbf{U}} w_{u_1} \cdot y_{u_1,j_\mathrm{left}} - \sum_{j_\mathrm{right}\in\mathbf{P_R}}\sum_{\forall u_2\in\mathbf{U}} w_{u_2} \cdot y_{u_2,j_\mathrm{right}}\right| \leq a_\mathrm{LatLW} \cdot LW + b_\mathrm{LatLW} \tag{18}$$

$$\mathrm{IND}_\omega^{FWD} \leq IND_\omega \leq \mathrm{IND}_\omega^{AFT} \ \omega \in \{TOW, LW, ZFW\} \tag{19}$$

$$y_{uj} \in \{0,1\} \ \forall u \in \mathbf{U}, j \in \mathbf{P} \tag{20}$$

Objective (6) consists of maximizing the payload, and Objective (7) is to minimize the aircraft takeoff CG deviation from a given target CG. The airplane's CG can be expressed as Balance Arms (BA) or the percentage of the Mean of Aerodynamic Chord (%MAC). We use %MAC here. The relation between BA and %MAC is $CG_\mathrm{TOW} = 100\cdot(BA_\mathrm{TOW} - l_\mathrm{emac})/l_\mathrm{mac}$, where l_mac is the length of the Mean of Aerodynamic Chord, l_emac is the length from the datum to the leading edge of the Mean of Aerodynamic Chord, and BA_TOW is

$$BA_\mathrm{TOW} = \frac{\sum_{j\in\mathbf{P}}\sum_{u\in\mathbf{U}} BA_j \cdot w_u \cdot y_{uj} + \mathrm{OEW}\cdot\mathrm{BA}_\mathrm{OEW} + \mathrm{TOF}\cdot\mathrm{BA}_\mathrm{TOF}}{\sum_{j\in\mathbf{P}}\sum_{u\in\mathbf{U}} w_u \cdot y_{u,j} + \mathrm{OEW} + \mathrm{TOF}} \tag{21}$$

Constraints (8)–(11) are position constraints. Constraints (8) and (9) ensure that each ULD is allocated to exactly one position and each position loads exactly one ULD. Constraint (10) ensures that the type of ULD and a predefined position are the same if the ULD is allocated to this position. M is a big scalar, and we can define $\mathrm{M} := \max\{T_u - T_j | u \in \mathbf{U}, j \in \mathbf{P}\} + 1$. In cases where various types of ULDs' predefined positions share the same area in the holds, one position may overlap with others. Constraint (11) describes that if one of these overlapping positions is occupied, the others cannot be allocated anymore.

Constraints (12)–(16) are weight constraints. Constraint (12) ensures that ULDs are allocated to positions without exceeding their weight limit. Constraint (13) ensures that the total loading weight of the main deck and lower deck cargo must not exceed the maximum combined linear load limits. $(j_m, j_l) \in \mathbf{C}_\mathrm{p}$ is a pair of combined positions, where a_{j_m} is a constant, j_m is one of the positions {A, B, ..., R} in the main deck, and j_l is a combined low deck position slot as shown in Figure 2. The weight of ULDs located side by side on the front or rear of the main deck must satisfy the unsymmetrical linear load limits. This leads to Constraints (14) and (15), where $(j_\mathrm{left}, j_\mathrm{right}) \in \mathbf{S}_\mathrm{p}$ is a pair of positions side by side, and a_unsym and b_unsym are constants. Constraint (16) guarantees that the total weight of the ULDs is loaded onto the aircraft without exceeding the maximum payload.

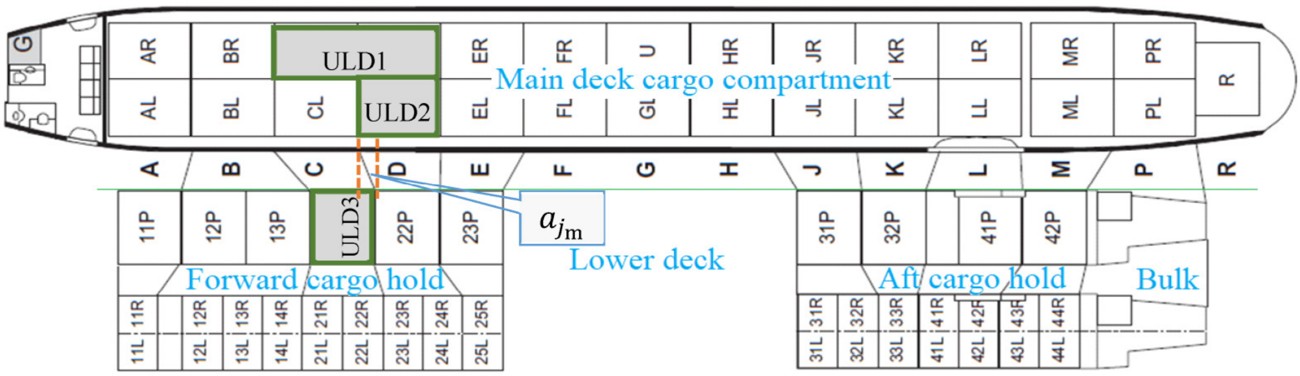

Figure 2. Combined positions.

Constraints (17)–(19) are balance constraints that limit the CG in a certain area of the aircraft. Constraints (17) and (18) are lateral balance constraints that limit the maximum weight difference between the left side and the right side, where

$$TOW := \textstyle\sum_{j\in\mathbf{P}} \sum_{i\in\mathbf{U}} w_u \cdot y_{u,j} + \mathrm{OEW} + \mathrm{TOF} \tag{22}$$

$$LW := \textstyle\sum_{j\in\mathbf{P}} \sum_{i\in\mathbf{U}} w_u \cdot y_{u,j} + \mathrm{OEW} + \mathrm{TOF} - \mathrm{TripF} \tag{23}$$

$$ZLW := \textstyle\sum_{j\in\mathbf{P}} \sum_{i\in\mathbf{U}} w_u \cdot y_{u,j} + \mathrm{OEW} \tag{24}$$

To keep the aircraft in balance in flight, the CG of the aircraft must be in the CG envelope. The CG envelope is described with INDEX, an expression of the balance moment. This leads to Constraint (18). IND_ω denotes the INDEX of an aircraft at weight ω:

$$\begin{aligned} &IND_\omega = \mathrm{DOI} + \mathrm{FI}_\omega + \sum_{i\in\mathbf{U}} \sum_{j\in\mathbf{P}} \frac{w_i \cdot (BA_j - \mathrm{CG}_\mathrm{DATUM})}{\mathrm{CI}} \cdot y_{uj} \\ &\forall\omega \in \{TOW, LW, ZFW\} \end{aligned} \tag{25}$$

where DOI, CI, and $\mathrm{CG}_\mathrm{DATUM}$ are constant for the aircraft, and FI_ω is constant at a given ω.

3.4. Combinatorial Optimization Model (COM)

We consider variables x_{iuj} taking a value of 1 if item i is in ULD u and u is allocated to position j; the formulation is given by:

$$\text{Maximize} \quad \sum_{j\in\mathbf{P}}\sum_{u\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i x_{iuj} \tag{26}$$

$$\text{Minimize} \quad \left|CG_{TOW} - \mathrm{CG}_{\text{target}}\right| \tag{27}$$

$$\text{Subject to:} \quad \sum_{j\in\mathbf{P}}\sum_{u\in\mathbf{U}} x_{iuj} \le 1 \;\forall i \in \mathbf{I} \tag{28}$$

$$\sum_{j\in\mathbf{P}} x_{iuj} \le 1 \;\forall i \in \mathbf{I}, u \in \mathbf{U} \tag{29}$$

$$\sum_{u\in\mathbf{U}} x_{iuj} \le 1 \;\forall i \in \mathbf{I}, j \in \mathbf{P} \tag{30}$$

$$\sum_{i\in\mathbf{I}} v_i x_{iuj} \le V_u \; u \in \mathbf{U}, j \in \mathbf{P} \tag{31}$$

$$\mathrm{M}\cdot(1 - x_{iuj}) \ge \left|T_u - T_j\right| \;\forall i \in \mathbf{I}, u \in \mathbf{U}, j \in \mathbf{P} \tag{32}$$

$$x_{i_1,u_1,j_1} + x_{i_2,u_2,j_2} \le 1 \;\forall i_1, i_2 \in \mathbf{I}, u_1, u_2 \in \mathbf{U},\; j_1 \in \mathbf{P}, j_2 \in \mathbf{O}_{j_1} \tag{33}$$

$$\sum_{i\in\mathbf{I}} w_i x_{iuj} \le \min\left\{W_u, W_j\right\} \;\forall u \in \mathbf{U}, j \in \mathbf{P} \tag{34}$$

$$\begin{aligned}&\alpha_{j_\mathrm{m}} \cdot \sum_{u_1\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_1,j_\mathrm{m}} + \sum_{u_2\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_2,j_\mathrm{l}} \le \mathrm{MaxW}_{(j_\mathrm{m},\, j_\mathrm{l})} \\ &\forall (j_\mathrm{m}, j_\mathrm{l}) \in \mathbf{C_P}\end{aligned} \tag{35}$$

$$\begin{aligned}&\sum_{u_1\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_1,j_{\text{left}}} \le a_{\text{unsym}} \sum_{u_2\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_2,j_{\text{right}}} + b_{\text{unsym}} \\ &\forall (j_{\text{left}}, j_{\text{right}}) \in \mathbf{S_P}\end{aligned} \tag{36}$$

$$\begin{aligned}&\sum_{u_1\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_1,j_{\text{right}}} \le a_{\text{unsym}} \sum_{u_2\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_2,j_{\text{left}}} + b_{\text{unsym}} \\ &\forall (j_{\text{left}}, j_{\text{right}}) \in \mathbf{S_P}\end{aligned} \tag{37}$$

$$\sum_{\forall j\in\mathbf{P}}\sum_{\forall i\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{iuj} \le \mathrm{MPL} \tag{38}$$

$$\begin{aligned}&\left|\sum_{j_{\text{left}}\in\mathbf{P_L}}\sum_{\forall u_1\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_1,j_{\text{left}}} - \sum_{j_{\text{right}}\in\mathbf{P_R}}\sum_{\forall u_2\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_2,j_{\text{right}}}\right| \\ &\le a_{\text{LatTOW}} \cdot TOW + b_{\text{LatTOW}}\end{aligned} \tag{39}$$

$$\begin{aligned}&\left|\sum_{j_{\text{left}}\in\mathbf{P_L}}\sum_{\forall u_1\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_1,j_{\text{left}}} - \sum_{j_{\text{right}}\in\mathbf{P_R}}\sum_{\forall u_2\in\mathbf{U}}\sum_{i\in\mathbf{I}} w_i \cdot x_{i,u_2,j_{\text{right}}}\right| \\ &\le a_{\text{LatLW}} \cdot LW + b_{\text{LatLW}}\end{aligned} \tag{40}$$

$$\mathrm{IND}_\omega^{FWD} \le IND_\omega \le \mathrm{IND}_\omega^{AFT} \;\forall l \in \mathbf{L}, \omega \in \{TOW, LW, ZFW\} \tag{41}$$

$$x_{iuj} \in \{0, 1\} \;\forall i \in \mathbf{I}, u \in \mathbf{U}, j \in \mathbf{P} \tag{42}$$

Objective (26) consists of maximizing the payload, and Objective (27) consists of minimizing the aircraft takeoff CG deviation from a given target CG. Constraints (28)–(30) ensure that each item is allocated to exactly one ULD and one position, each ULD is allocated to exactly one position, and each position loads exactly one ULD. Constraint (31) ensures that items do not exceed their ULD's capacity. Constraint (32) ensures that the type of ULD and a predefined position are the same if the ULD is allocated to this position. Constraint (33) prevents the ULDs from overlapping. Constraint (34) ensures that items do not exceed the weight limits of both their ULD and position. Constraint (35) is the maximum combined linear load limits of the main deck and lower deck. Constraints (36) and (37) are the unsymmetrical linear load limits. Constraint (38) is the maximum payload limits. Constraints (39) and (40) are lateral balance constraints. Constraint (41) is the CG envelope limits.

3.5. Improved Combinatorial Optimization Model (IOM)

Because COM uses 3D variables, it takes too long to solve. Therefore, we reduced the dimension of decision variables. We consider variables x_{ij} taking a value of 1 if item i is allocated to position j in the aircraft holds; the formulation is given by:

$$\textit{Maximize} \quad \sum_{j \in \mathbf{P}} \sum_{i \in \mathbf{I}} w_i \cdot x_{ij} \tag{43}$$

$$\textit{Minimize} \quad \left| CG_{TOW} - \mathrm{CG}_{\mathrm{target}} \right| \tag{44}$$

$$\textit{Subject to:} \quad \sum_{i \in \mathbf{I}} x_{ij} \le 1 \; j \in \mathbf{P} \tag{45}$$

$$\sum_{i \in \mathbf{I}} w_i \cdot x_{ij} \le W_j \; \forall j \in \mathbf{P} \tag{46}$$

$$\sum_{i \in \mathbf{I}} v_i \cdot x_{ik} \le V_j \; j \in \mathbf{P} \tag{47}$$

$$x_{i_1, j_1} + x_{i_2, j_2} \le 1 \; \forall i_1, i_2 \in \mathbf{I}, \; i_1 \ne i_2; \forall j_1 \in \mathbf{P}; \forall j_2 \in \mathbf{O}_{j_1} \tag{48}$$

$$\alpha_{j_{\mathrm{m}}} \cdot \sum_{i \in \mathbf{I}} w_i \cdot x_{ij_{\mathrm{m}}} + \sum_{i \in \mathbf{I}} w_i \cdot x_{ij_{\mathrm{l}}} \le \mathrm{MaxW}_{(j_{\mathrm{m}},\, j_{\mathrm{l}})} \; \forall (j_{\mathrm{m}}, j_{\mathrm{l}}) \in \mathbf{C_P} \tag{49}$$

$$\sum_{i \in \mathbf{I}} w_i \cdot x_{i, j_{\mathrm{left}}} \le a_{\mathrm{unsym}} \sum_{i \in \mathbf{I}} w_i \cdot x_{i, j_{\mathrm{right}}} + b_{\mathrm{unsym}} \; \forall (j_{\mathrm{left}}, j_{\mathrm{right}}) \in \mathbf{S_P} \tag{50}$$

$$\sum_{i \in \mathbf{I}} w_i \cdot x_{i, j_{\mathrm{right}}} \le a_{\mathrm{unsym}} \sum_{i \in \mathbf{I}} w_i \cdot x_{i, j_{\mathrm{left}}} + b_{\mathrm{unsym}} \; \forall (j_{\mathrm{left}}, j_{\mathrm{right}}) \in \mathbf{S_P} \tag{51}$$

$$\sum_{\forall j \in \mathbf{P}} \sum_{i \in \mathbf{I}} w_i \cdot x_{ij} \le \mathrm{MPL} \tag{52}$$

$$\left| \sum_{j_{\mathrm{left}} \in \mathbf{P_L}} \sum_{i \in \mathbf{I}} w_i \cdot x_{i, j_{\mathrm{left}}} - \sum_{j_{\mathrm{right}} \in \mathbf{P_R}} \sum_{i \in \mathbf{I}} w_i \cdot x_{i, j_{\mathrm{right}}} \right| \le a_{\mathrm{LatTOW}} \cdot TOW + b_{\mathrm{LatTOW}} \tag{53}$$

$$\left| \sum_{j_{\mathrm{left}} \in \mathbf{P_L}} \sum_{i \in \mathbf{I}} w_i \cdot x_{i, j_{\mathrm{left}}} - \sum_{j_{\mathrm{right}} \in \mathbf{P_R}} \sum_{i \in \mathbf{I}} w_i \cdot x_{i, j_{\mathrm{right}}} \right| \le a_{\mathrm{LatLW}} \cdot LW + b_{\mathrm{LatLW}} \tag{54}$$

$$\mathrm{IND}_{\omega}^{FWD} \le IND_{\omega} \le \mathrm{IND}_{\omega}^{AFT} \; \forall l \in \mathbf{L}, \omega \in \{TOW, LW, ZFW\} \tag{55}$$

$$x_{ij} \in \{0, 1\} \; \forall i \in \mathbf{I}, j \in \mathbf{P} \tag{56}$$

Objective (43) consists of maximizing the payload, and Objective (44) consists of minimizing the aircraft takeoff CG deviation from a given target CG. Constraint (45) ensures that each item is allocated to exactly one position. Constraint (46) ensures that the total weight of items assigned in a position does not exceed the weight limit of the position and the ULD that has the same type as the position. Here, W_j denotes the minimum weight limit of a position and its compatible ULD. Constraint (47) ensures that the volume of items assigned in a position does not exceed its compatible ULD's capacity. Constraint (48) prevents ULDs from overlapping. Constraint (49) is the maximum combined linear load limits of the main deck and lower deck. Constraints (50) and (51) are the unsymmetrical linear load limits. Constraint (52) is the maximum payload limits. Constraints (53) and (54) are lateral balance constraints. Constraint (55) is the CG envelope limits.

4. Computational Experiments

The experiments were run on a portable computer with an Intel(R) Core (TM) i7- 4720HQ processor (CPU@3.20 GHz, 3.00 RAM) and Windows 11 operating system.

The model was implemented with Python and solved with Gurobi 9.5.0. Because each of the models developed in Section 3 has multiple objectives, we employed Gurobi's hierarchical approach. The *setObjectiveN* method was used. For the BOM, the maximization of total cargo weight (1) had the highest priority, then the maximation of payload (6), and the minimization of CG deviation (7) had the lowest priority. For the COM and IOM, the priority of maximizing payload ((26) and (43)) was higher than minimizing CG deviation ((27) and (44)). All other solver parameters were maintained at their default values.

In the following tables, weight is in kilograms, time is in seconds, and CG is in %MAC.

4.1. Test Case Generation and Parameter Setting

The test was conducted using a Boeing 777F. It had four holds, as shown in Figure 2. They were the main deck cargo compartment, the forward cargo hold, the aft cargo hold, and the bulk hold. There were 27 and 10 positions for ULDs of type "PMC" in the main deck and lower holds, respectively. PMC was its most used type of ULD. We used 37 PMC ULDs for the Boeing 777F here. The maximum weight of each ULD (W_u) was 6804 kg. Their length, width, and height were 318 cm, 244 cm, and 163 cm, respectively. The aircraft data were from the Boeing 777F Weight and Balance Control and Loading Manual. The aircraft's OEW was 141,750 kg, MTOW was 34,745 kg, MLW was 260,815 kg, MZFW was 248,115 kg, MPL was 102,300 kg, and target CG was 28% MAC.

Each item's weight and volume were determined at random (see Table 4). A total of 4 scenarios and 25 cases were produced. The number of items to be packed is indicated in column #. w_i and v_i were randomly generated in different ranges.

Table 4. Test data.

Scenarios	Tests	#	w_i	v_i	$PLA=\sum w_i$	$VA=\sum v_i$
I: $PLA \leq$ MPL $VA \leq$ MV	1-1	340	[290, 305]	[1.0, 1.2]	101,299	372.49
	1-2	720	[132, 152]	[0.35, 0.45]	102,136	288.19
	1-3	760	[127, 140]	[0.35, 0.45]	101,389	304.03
	1-4	800	[120, 135]	[0.35, 0.45]	101,944	319.82
	1-5	780	[115, 135]	[0.4, 0.55]	97,483	371.01
II: $PLA \geq$ MPL $VA \leq$ MV	2-1	1080	[86, 106]	[0.3, 0.4]	103,772	378.25
	2-2	760	[50, 260]	[0.01, 1]	118,344	391.43
	2-3	1020	[95, 110]	[0.3, 0.4]	104,649	357.75
	2-4	1040	[95, 110]	[0.3, 0.4]	106,730	364.68
	2-5	1100	[84, 104]	[0.3, 0.45]	103,459	410.15
	2-6	340	[51, 600]	[0.01, 2.59]	109,934	452.58
	2-7	360	[52, 599]	[0.01, 2.71]	117,501	460.52
	2-8	380	[52, 598]	[1, 1.2]	122,523	416.63
	2-9	400	[50, 549]	[1, 1.2]	122,160	438.49
	2-10	1060	[93, 108]	[0.3, 0.4]	106,592	371.54
III: $PLA \leq$ MPL $VA \geq$ MV	3-1	400	[51, 448]	[0.03, 2.39]	100,377	469.6
	3-2	740	[123, 138]	[[0.01, 1.5]	96,386	536.6
	3-3	780	[115, 135]	[0.01, 1.4]	97,483	563.34
	3-4	760	[125, 140]	[0.02, 1.6]	100,824	621.6
	3-5	800	[120, 135]	[0.01, 1.4]	101,944	569.44
IV: $PLA \geq$ MPL $VA \geq$ MV	4-1	360	[51, 598]	[0.01, 2.69]	111,772	490.57
	4-2	380	[50, 500]	[0.03, 2.41]	107,475	480.7
	4-3	800	[50, 230]	[0.01, 1.4]	114,198	569.44
	4-4	820	[115, 140]	[0.01, 1.3]	104,584	563.02
	4-5	840	[115, 130]	[0.01, 1.3]	102,933	566.94

PLA and *VA* are the weight and volume of the total items. The number of ULDs was 37, according to the number of positions in the aircraft. MPL is the maximum payload weight of the aircraft. MV is the sum of a total of 37 ULDs' volume, which is the maximum volume that can be used. In scenario I, as $PLA \leq$ MPL and $VA \leq$ MV, all items can be loaded, whereas in scenarios II, III, and IV, only partial objects can be loaded. When the optimal solution is reached for each instance, or when the solver's running duration reaches 1 hour (3600 s) and it finds a feasible solution, the solver will immediately terminate.

4.2. Test Results Analysis

The main results of the three approaches are given in Table 5. Column *PL* denotes the weight of the total payload. $|\Delta CG| := |CG_{\text{TOW}} - \text{CG}_{\text{target}}|$ is the CG deviation from the target one. *Time* corresponds to the total computational time in seconds. The volume loading ratio in column *VLR* indicates the loading volume capacity of the aircraft given a set of items to be transported.

$$VLR = \begin{cases} \sum\limits_{j \in \mathbf{P}} \sum\limits_{u \in \mathbf{U}} \sum\limits_{i \in \mathbf{I}} \frac{v_i}{V_u} \cdot x_{iu} \cdot y_{uj} \cdot 100 \text{ for BOM} \\ \sum\limits_{j \in \mathbf{P}} \sum\limits_{u \in \mathbf{U}} \sum\limits_{i \in \mathbf{I}} \frac{v_i}{V_u} \cdot x_{iuj} \cdot 100 \quad \text{for COM} \\ \sum\limits_{j \in \mathbf{P}} \sum\limits_{i \in \mathbf{I}} \frac{v_i}{V_j} \cdot x_{ij} \cdot 100 \quad \text{for IOM} \end{cases} \tag{57}$$

Table 5. Solution of tests.

Tests	BOM				COM				IOM			
	PL	$\|\Delta CG\|$	Time	VLR	PL	$\|\Delta CG\|$	Time	VLR	PL	$\|\Delta CG\|$	Time	VLR
1-1	101,299	0.87	0.3	79.6	101,299	1.21	115.2	79.6	101,299	0.70	1.7	79.6
1-2	102,136	1.62	0.2	61.6	102,136	0.81	337.1	61.6	102,136	0.71	3.1	61.6
1-3	101,389	1.73	0.2	65.0	101,389	0.70	493.7	65.0	101,389	0.79	2.3	65.0
1-4	101,944	2.00	0.2	68.3	101,944	0.75	469.5	68.3	101,944	0.69	2.3	68.3
1-5	97,483	1.43	0.1	79.3	97,483	0.82	430.7	79.3	97,483	0.68	4.1	79.3
Mean	100,850.2	1.53	0.2	70.7	100,850.2	0.86	369.2	70.7	100,850.2	0.71	2.7	70.7
2-1	100,446	1.48	0.4	78.1	102,297	0.7	700.0	79.6	102,296	0.83	5.6	79.6
2-2	102,294	0.85	0.4	72.0	102,291	0.70	679.3	71.4	102,294	0.70	7.5	65.7
2-3	101,084	0.69	1.9	73.8	102,293	0.80	308.0	74.6	102,300	0.76	9.0	74.6
2-4	102,292	0.84	0.4	74.6	102,292	0.69	1027.1	74.5	102,300	0.74	5.2	74.7
2-5	101,780	0.69	0.4	86.1	102,295	0.73	1064.9	86.5	102,296	0.91	8.4	86.5
2-6	102,299	1.06	0.3	88.1	102,299	0.98	678.6	88.9	102,291	0.81	1.1	82.3
2-7	102,295	2.04	0.3	83.2	102,297	0.71	397.3	81.5	102,299	0.94	1.5	79.9
2-8	102,300	0.87	0.2	73.1	102,296	0.74	142.0	68.0	102,294	0.94	1.1	79.7
2-9	102,299	1.49	0.3	77.4	102,294	1.08	290.9	75.0	102,294	1.15	0.9	71.0
2-10	102,294	1.50	0.7	76.2	102,291	0.79	939.6	76.1	102,292	0.70	6.5	74.9
Mean	101,938	1.15	0.5	78.3	102,295	0.72	622.8	77.6	102,296	0.85	4.7	76.9
3-1	100,323	0.71	41.7	99.9	100,263	0.68	3600.0	99.5	100,274	0.68	3600.0	99.6
3-2	90,215	0.69	3.9	100.0	89,892	0.64	3600.0	99.4	90,214	0.65	750.7	100.0
3-3	88,612	1.49	92.9	100.0	88,283	0.64	3600.0	99.6	88,605	1.08	3600.0	100.0
3-4	88,612	3.66	232.9	100.0	86,936	0.63	3600.0	99.6	87,107	0.70	3600.0	99.9
3-5	87,167	1.91	952.5	100.0	92,006	0.65	3600.0	99.6	92,230	0.86	3600.0	100.0
Mean	90,985.8	1.69	264.8	100.0	91,476	0.65	3600.0	99.5	91,686	0.79	3030.1	99.9
4-1	102,297	0.77	192.9	91.8	102,291	0.99	491.8	85.8	102,292	1.05	1.9	85.4
4-2	102,291	0.85	807.6	94.5	102,292	0.87	474.1	91.9	102,298	0.76	2.0	90.2
4-3	102,281	0.69	3600.0	94.6	102,299	0.74	1365.5	98.0	102,297	0.87	4.4	94.6
4-4	95,029	1.10	2.1	100.0	94,844	0.67	3600.0	99.7	95,019	0.66	3600.0	100.0
4-5	93,184	0.77	1.0	100.0	92,719	0.77	3600.0	99.2	93,112	0.66	3600.0	99.9
Mean	99,016	0.84	920.7	96.2	98,889	0.81	1906.3	94.9	99,004	0.80	1441.7	94.0

The three approaches in scenario I yield the same weight and volume because the same solver is used and all items are loaded. It is clear that the COM takes the longest to compute, with a mean time of 369.2 s. The IOM reduces the mean duration to 2.7 s and is more effective than the COM. The BOM takes the shortest amount of time, with a mean of 0.17 s. However, the BOM has the highest $|\Delta CG|$, with a mean of 1.53% MAC and a peak of 2.00% MAC for tests 1–4. It is unacceptable if the $|\Delta CG|$ exceeds 2% MAC since, in practice, most airlines can only operate with a 2% CG deviation. In terms of $|\Delta CG|$, the COM and IOM have respective means of 0.86% MAC and 0.71% MAC.

In scenario II, all three models produced results that were extremely similar for PL and *VLR*. Compared with the BOM's mean, the COM and IOM can get a little more PL. However, the COM takes too much time, with a mean consumption of 622.8 s and a peak of 1027.1 s. The average computation time for the BOM and IOM is 4.7 and 0.5 s, respectively. The BOM has the highest |ΔCG| as well, with a mean of 1.15% MAC.

In scenario III, the BOM achieves the best outcomes. All COM instances and 4 out of the 5 IOM computational times are up to the time limit (3600 s), and they do not produce the best results due to the limits of the total volume. However, in both PL and |ΔCG|, the BOM is the worst.

Due to the volume and total weight restrictions in scenario IV, all models take excessive amounts of time and cannot be solved within the allowed computing time. The outcomes of PL and VLR are remarkably similar.

We reject the COM since it takes the longest of the three methods, but also because its *PL* and $|\Delta CG|$ are inferior to the IOM's. Figure 3 compares the BOM with IOM in terms of $|\Delta CG|$ and computational time. It demonstrates that the BOM's $|\Delta CG|$ is worse, though it has the fastest computational time. Some $|\Delta CG|$ solutions are unworkable in real-world situations. Despite being slightly slower than the BOM, the IOM's $|\Delta CG|$ and *PL* are the best. In actual work, the BOM can be solved first, and the load master can decide whether or not the $|\Delta CG|$ is acceptable. The IOM model can be solved once again if it is unable to satisfy the CG deviation requirement.

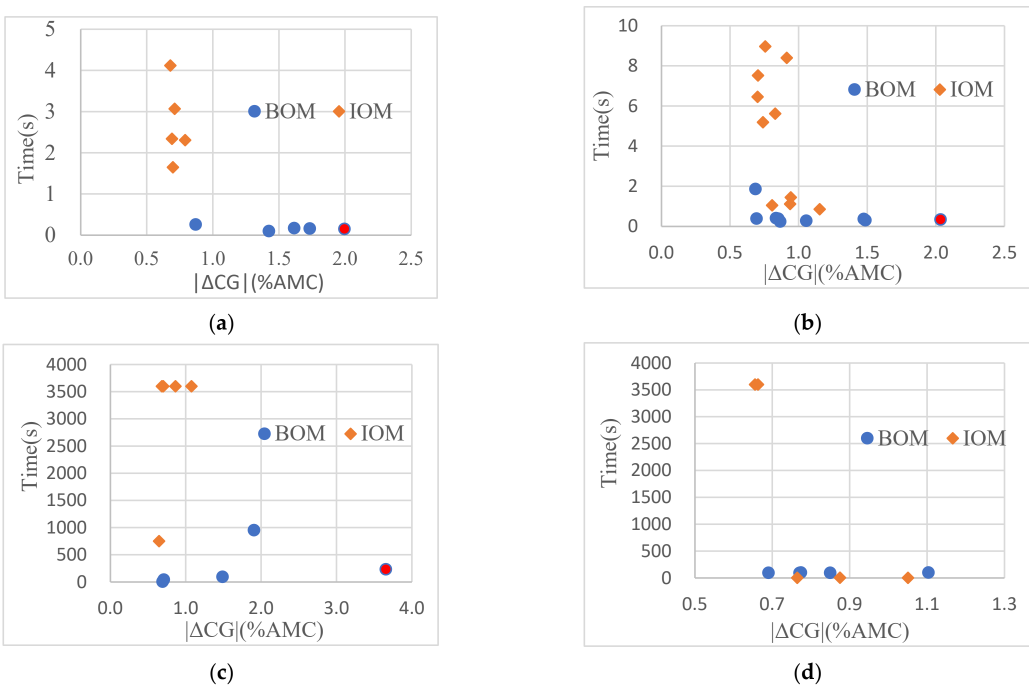

Figure 3. $|\Delta CG|$ and computational time comparison of BOM with IOM. (**a**) Scenario I; (**b**) scenario II; (**c**) scenario III; (**d**) scenario IV.

5. Discussion

The purpose of this study was to develop a preliminary loading solution combining the ACPP and WBP to subsequently be used to guide packing optimization.

It can be used by cargo airlines as a preloading plan to carry out overall cargo palletization planning prior to transporting cargo. The cargo to be transported arrives at the origin airport terminal around 3–6 h before departure, at which point the bulk cargo must be packaged into ULDs. Our models can be used to determine which cargo should be assigned to which container. On the one hand, it can be utilized to direct bulk cargo ULD allocation. On the other hand, the weight of the ULD loading cargo according to the assignment plan can be estimated, ensuring the accuracy of the subsequent development of an aircraft weight and balance plan [6]. As a result, the aircraft capacity can be fully utilized, increasing the transporting payload and optimizing the aircraft CG.

It can also be used to determine the weight of each preloaded ULD, which can be used as a reference for airlines selling pallet weights to freight forwarders.

The main limitations of our approach are that the three models investigated in this study only consider the overall volume constraint for the cargo; they do not take into account how the cargos will be packed in respect to one another. Therefore, the solution for a bin packing problem of air cargo palletization is required in practical work, which can be developed in further research.

6. Conclusions

The combinatorial optimization of the ACPP and WBP was studied in this paper. Three ILP models, the BOM, COM, and IOM, are proposed based on the procedure of cargo loading into ULDs and their subsequent loading into holds in airplanes. In order to reduce the computational time of the COM in solving, a dimension reduction strategy was employed to reduce the dimension of the decision variable from 3D to 2D. As a result, we developed the IOM, which significantly speeds up the solution.

Using a Boeing 777F, 4 scenarios and 25 cases were tested. After comparing and contrasting the test results, the following conclusions were obtained. Because the BOM's variable feasible domain is limited, the optimization result is insufficient, and the CG deviation is occasionally too large, which is unsatisfactory in actual operation. Despite obtaining a good solution, the COM takes too long to solve, making it difficult to meet the needs of the actual scene. Even though the IOM accelerates the solving process as compared with the COM, some scenarios will still necessitate a significant amount of computation time when the volume exceeds the aircraft volume limit. In practice, the BOM can be used to generate a set of solutions and then evaluate their acceptability. If this is not acceptable, we may alter the IOM.

In order to use the IOM to directly guide real applications rather than attempting the BOM first and then the IOM, in future work the solving method for the IOM should be further investigated to speed up the calculation. Additionally, further study is needed in the area of stowage solutions as a guide for packing optimization.

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