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Negentropy as a Measure to Evaluate the Resilience in Industrial Plants

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Abstract: Resilience is an essential quality of systems. This characteristic is based on the ability of a system to cope with disruptive events. To prevent decreases in system functionality and performance and to respond promptly to unexpected situations or shocks, systems must possess this capacity. One challenge lies in identifying and measuring resilience. Recently, various metrics have been proposed in the literature to represent the resilience of systems. Despite this, there is still no global resilience measure that can be used in any type of industrial system. This work investigated a series of moment statistics and explored the field of entropy in the search for a general resilience indicator. A set of 27 hypothetical cases were proposed to calculate the indices under evaluation. Then, a series of comparisons were made between these indices and two resilience indicators found in the literature. The main results of this work lead to the overall conclusion that it is possible to use some of these indicators as potential resilience indicators for engineering systems and production lines. Specifically, negentropy appears to be a good option for this purpose.

Keywords: resilience; entropy; negentropy; engineered systems; moments statistics

MSC: 62P30

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1. Introduction

Resilience refers to the capacity of a system to tolerate and cope with the challenges and shocks that are presented by its contexts. Resilience emerged as a concept within the domain of ecology [1]. Later, this term was expanded to other domains such as social sciences, civil and urban infrastructure, and industry [2]. In the industrial context, the notion of resilience emerges as a means of assessing how effectively a physical asset or a system of physical assets can swiftly respond to disruptive events and restore its operational condition. According to the American Society of Mechanical Engineers (ASME) [3], resilience refers to the capacity to endure internal and external disruptions, without interrupting operation. In addition, if interruption occurs, resilience also indicates the system's ability to recover fully and quickly.

Despite its relevance, there is no unanimous or universal indicator for such a concept, nor a single way to measure it. Several authors have proposed resilience metrics in industrial sectors such as gas supply [4], electric distribution networks [5], hydraulic distribution systems [6], and wind farms [7]. According to Sterbenz et al. [8], the definition of the concept of resilience and its measurement combines quantitative and qualitative aspects [9].

Some of the terms used in association with resilience are: the speed at which the system recovers from a failure [10] and vulnerability, which is used by Ji et al. as the characteristic of "non-resilience" [11]. Meanwhile, Cai et al. [7,12] combine the following terms to explain resilience: adaptability, robustness, redundancy, flexibility, survivability, recoverability, swiftness, ingenuity, and reparability.

Resilience captures the recovery process through the rate between the recovered functionality and the lost functionality [13]. Cholda [14] proposes the concept of "resilience quality" as the combination of frequency and magnitude of interruptions. Tierney and Bruneau [15] propose four basic properties that explain the concept of resilience: robustness, redundancy, swiftness, and ingenuity.

In summary, most resilience metrics are derived from functions of one or several parameters, particularly the feature and consequence of the failure, the recovery time, and the performance measured over a period of time or at a given instant, as well as some probabilistic indicators [16].

One author highlights the need to implement a learning process during the system's operability restoration process [17]. In addition, according to the same work, the system's capacity to enhance its management by leveraging the insights acquired from each disruptive event encountered is needed. Essentially, this is connected to the organization's available resources and their associated costs. However, it is known that one of the main limitations to increasing the resilience of systems is the limitation of their resources. Therefore, it is not feasible to allocate resources to every element of a system. Consequently, it becomes essential to identify critical components that, when enhanced, will significantly enhance the system's resilience or, conversely, whose deterioration will have the greatest overall impact on the system [18]. In order to unequivocally identify the effect that a system's characteristics may have on its resilience, it is essential to quantify resilience and its behavior over time. However, it is important to mention that no general metric capable of quantifying the resilience of an engineering system has yet been developed. This makes it very complex to compare resilience between different systems or equipment.

This paper presents a comparative study of some moment statistics and two quantitative measures of resilience found in the literature. In addition, the potential use of the negentropy, based on the concept of Shannon's entropy as a resilience measure, is explored.

The following chapters are structured as follows: Section 2 discusses the most widespread resilience metrics. Then, in Section 3, the study of negentropy as a resilience measure is proposed. Section 4 details the proposed methodology. Section 5 presents a set of case studies as a way of validating the proposed resilience indicator based on the concept of negentropy. Finally, in the last part of this work, the results and aspects related to the decision-making process based on the proposed indicator are discussed, along with the conclusions drawn from this study.

2. Resilience Metrics

There are a number of metrics that exist to assess and measure the resilience of an engineered system. Selecting the appropriate resilience metric presents a challenge as it must consider crucial factors such as performance, temporality, and ease of implementation. Tierney and Bruneau [15] suggested measuring resilience based on the observation that resilient systems reduce the probability of failure and improve recovery. Therefore, resilience can be measured by the behavior of a system's functionality after experiencing a shock, including the time it takes the system to return to its initial level of performance.

In 2016, Yodo and Wang [19] presented a comprehensive review of the literature on resilience of engineered systems. They suggest that there is a need to develop a general-purpose engineering resilience analysis framework. Cholda [14] highlights that resilience ought to be viewed as a comprehensive evaluation derived from the structure of the system, encompassing all its components, interconnections, and elemental operations, as well as how they affect the whole. According to Bishop et al. [20], it is necessary to consider, in a holistic way, the functionality of a system. Thus, different levels of abstraction are possible and necessary. Bishop et al. [20], highlight their concerns about the relationship between the systems' structure and the possible cascade effects that will occur in the presence of either external or internal shocks.

Albasrawi et al. [13] propose a resilience metric for comparing recovery strategies for cyber infrastructure systems failures. Through simulation, based on a stochastic reliability model, they obtained information on the potential effects that cascading failures may cause and how this impacts the resilience.

Most of the cited papers do not specify the variable that represents the level of functionality or performance of the system. They merely refer to it as a measurable indicator (F(t)).

On the other hand, various authors relate the concept of resilience to a specific indicator or a performance parameter of the system under analysis. Ibrahim and Alkhraibat [21] suggested a set of factors through which it is possible to quantify the resilience in microgrid systems. Such indicators include: voltage percentage, percentage of load reduction, recovery time, and time to reach power equilibrium state. Attoh-Okine et al. [22] propose another metric based on the concepts of robustness, redundancy, speed, and resources in infrastructure systems. Hu and Mahadevan [23] propose a metric to measure the resilience of mechanical systems by tracking reliability over time. Other authors express the behavior of resilience through system performance over time, using indicators such as productivity, overall equipment effectiveness (OEE), overall throughput effectiveness (OTE), and system availability. According to [16], the system's performance or functionality variation behaves mainly in one of four different ways, which can be seen in Figure 1, which shows (a) deterioration caused by a failure and its subsequent recovery, (b) deterioration caused by an irreparable failure, (c) sudden deterioration followed by a recovery process, and (d) deterioration caused by a failure and delayed recovery. Further details about the behavior of each type of curve can be found in [16].

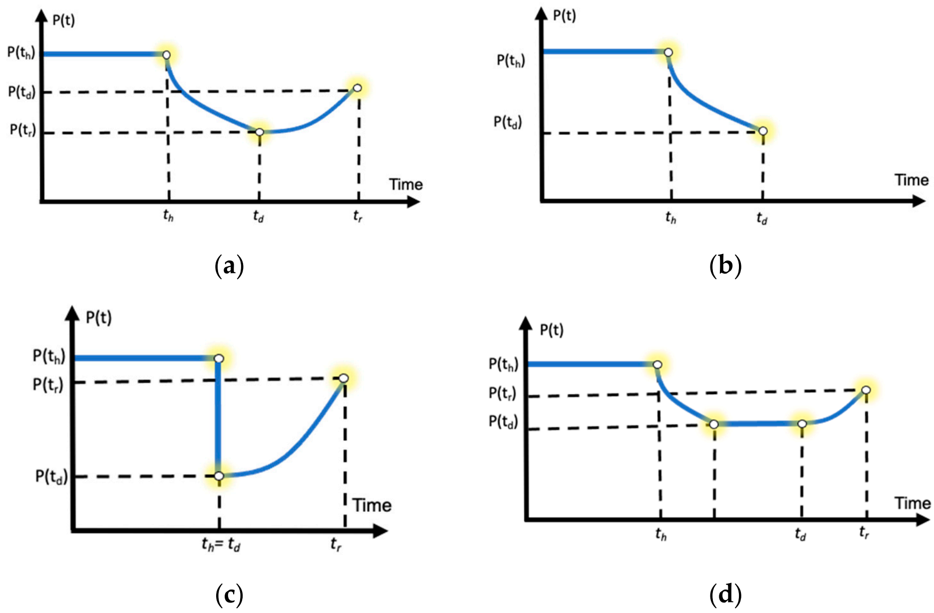

Figure 1. Performance behavior for normal systems under performance loss and delayed recovery [16].

The variables in the figure are:

- t: arbitrary time instant;
- t_h: time at which a failure or threat occurs;
- t_d: time at which performance degrades to an unacceptable level, at which point recovery (if possible) begins;
- t_r: arbitrary time instant during the recovery period;
- *P(t)*: system performance level at time instant t.

The model proposed by Cai et al. [12] is the only analytical model that relates system resilience (ρ_A) to system availability in the presence of a given set of n disruptive events. Considering Figure 2, it is possible derive Equation (1):

$$\rho_A = \frac{A_1}{n\,(\ln(t_1))} \sum_{i=1}^{n} \frac{A_2^i A_3^i}{\ln\left(t_3^i - t_2^i\right)} \tag{1}$$

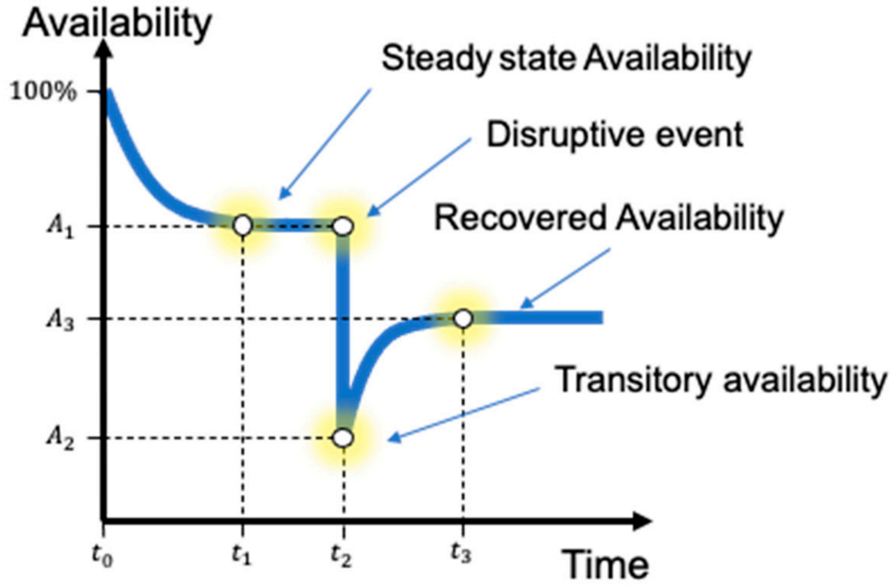

Figure 2. System availability subject to degradation and shock (adapted from [12]).

Regarding Equation (1) and Figure 2, A_1 corresponds to the system availability in steady state. This occurs from the initial moment until time t_1. At time t_2, there is a sudden decrease in system availability. From that moment on, availability presents a transient state or value A_2. Next, the system recovers its availability to a new equilibrium state A_3. This occurs at t_3. The number of availability drops is denoted by "n". Consequently, ${A_2}^i$ and ${A_3}^i$ represent the corresponding values associated with each of the shocks i ($i \leq n$).

Another model that stands out in the literature is the resilience triangle model, proposed by Davis et al. [24], see Figure 3.

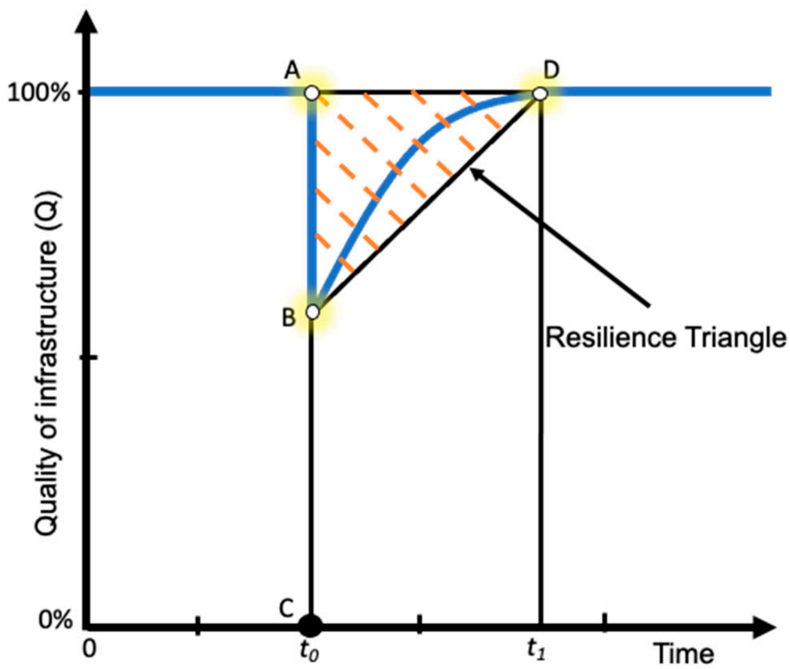

Figure 3. Resilience properties and triangle [24].

For the fundamental case of linear recovery, the resilience metric considering a specific shock consists, basically, of the ratio of two areas: the rectangular area tQ_{100}, divided by tQ_{100} minus the area of the triangle, representing the loss of system functionality. The resilience for the case presented in Figure 3 is then expressed as follows [24]:

$$\rho_B = \frac{(t_r - t_i)(Q_{100} - Q_r)}{2Q_{100}t} \tag{2}$$

We will use these two models as a basis for comparison with the indicators proposed in this article. These models will be identified as ρ_A in the case of the model proposed by [12], and ρ_B for the model proposed by [24].

3. Concept of Entropy and Negentropy

Identifying patterns in a time series can be a challenging task, as there can be multiple ways in which complexity can appear in a dataset. However, one of the techniques commonly used to identify patterns of complexity in a time series is entropy [25]. The entropy of a time series can be used as an approximate measure of the complexity and variability of the data, aspects closely related to the concept of resilience. The higher the entropy, the greater the complexity existing in the time series.

Entropy can be calculated from the probability distribution of the values taken by a time series. In information theory, Shannon entropy [25] is proposed as one of the most widely used methods for determining the amount of information contained in a time series. Shannon entropy is defined as:

$$H = -\sum_{i=1}^{k} (p_i \, log_2 p_i) \tag{3}$$

where p is the probability of occurrence of each value in the time series.

On the other hand, in 1944, Erwin Schrödinger [26] coined the concept of negentropy, which refers to the ability of some systems to increase their organization and complexity over time, against the natural tendency towards increasing entropy or disorder. In other words, negentropy describes how some systems are capable of generating order, complexity, and life from chaos, disorganization, and entropy. Thus, negentropy refers to the ability of some systems to store information and energy and use them to maintain their organization by performing useful work. That is, generating structures and functions of greater complexity is only possible when there is negentropy.

It could be said that negentropy represents a tendency towards the creation of order and organized structures, and is a force that opposes the natural tendency of matter to become disordered and disorganized. This is how we can draw a parallel between negentropy and resilience since both terms refer to a certain ability to maintain order and organization within a system. Therefore, in this work, we propose the following hypothesis:

> *The higher the negentropy of a system, measured from the analysis of the behavior over time of some performance variable, the higher its resilience.*

For these purposes, we will make use of one of the accepted forms for the concept of negentropy (N), that is, the reciprocal value of Shannon's entropy:

$$N = (1/H) \tag{4}$$

By dividing 1 by the Shannon's entropy (H), an estimate of the number of equally probable possible outcomes that could occur for the given random variable is obtained. However, it is important to note that this estimate may not always be accurate or significant in practical applications, as the actual number of possible outcomes may be influenced by other factors beyond the scope of Shannon's entropy. In order to prove the aforementioned hypothesis, in the next chapter, we will describe the proposed methodology.

4. Proposed Methodology

The procedures followed to carry out this work are summarized in the flowchart shown in Figure 4.

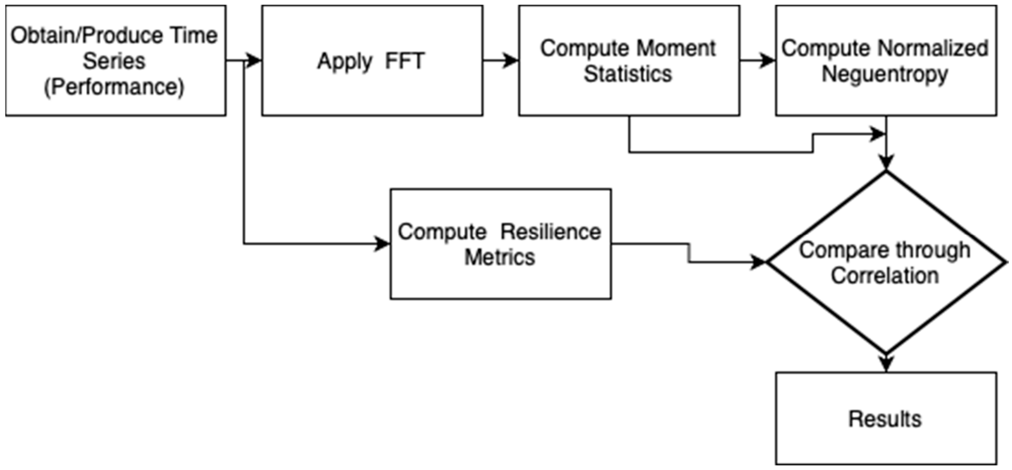

Figure 4. The proposed methodology.

The objective of this work was to propose a resilience indicator using as a basis a time series that contained the performance behavior of an equipment or a system. To expand this research, the performance behavior in the frequency domain using the fast Fourier transform (FFT) was also analyzed, from which a second dataset was obtained. Both data sets were structured in the form of vectors.

It was of interest for this research to analyze the behavior of the indicators in two domains: time series and their respective frequency histograms. In addition, we considered two types of scenarios under analysis: time series with a single shock and time series that have more than one shock. To evaluate the potential of being used as resilience indicators, various statistics were applied to both data sets. These statistics were: standard deviation, skewness, and kurtosis. Subsequently, the calculated statistics were compared with the resilience values obtained by using the models of [12,24]. Such comparisons were made using the Pearson linear correlation coefficient.

In the second stage, the use of the concept of negentropy was explored. Thus, negentropy values were computed considering the same time series and frequency histograms. These indicators were also compared with the indicators ρ_A and ρ_B. Before comparing the values of resilience from both models with the values of entropy and negentropy, the latter were normalized so that they were expressed in the same range as the values obtained by both models in the literature. Normalized negentropy is obtained using Equation (5):

$$N_{norm} = \frac{N}{N_{max}} \tag{5}$$

where N_{max} corresponds to the highest value of N from the set of cases.

To verify the applicability and potential of the resilience metrics for an equipment or a system, the next chapter demonstrates the application of this procedure on a set of synthetic cases.

4.1. Case Study

This procedure was based on a set of case studies aimed at applying the methodology described above. The experiment was based on a set of 27 cases, each one, represented by a correspondent time series. Each one of them was synthetically created and represented the behavior of values corresponding to any typical performance measure (productivity, OEE, availability, etc.) over time. Table 1 shows the vectors with the values of each of the

27 cases. These cases were grouped according to the characteristics of the shock records they contained, based on the terms “prolongation” and “intensity”. Prolongation of the shock involves both the duration of the performance decline and its recovery time. Similarly, the term “intensity” refers to the magnitude of the performance decrease caused by each shock. In this way, cases could be grouped into two large categories: cases with a single shock and cases with more than one shock. Specifically, there were four categories according to the following logic:

- Only one shock with different prolongations and intensities (cases 1 to 9);
- Six brief shocks with different intensities (cases 10 to 12);
- Six brief shocks with diverse and interleaved intensities (cases 13 to 18);
- Several shocks with different prolongations and intensities (cases 19 to 27).

Table 1. Vectors with the performance values of each case.

Case #	Values
1	[0.95 0.95 0.95 0.95 0.95 0.95 0.75 0.95 0.95 0.95 0.95 0.95]
2	[0.95 0.95 0.95 0.95 0.95 0.95 0.55 0.95 0.95 0.95 0.95 0.95]
3	[0.95 0.95 0.95 0.95 0.95 0.95 0.35 0.95 0.95 0.95 0.95 0.95]
4	[0.95 0.95 0.95 0.95 0.95 0.85 0.75 0.85 0.95 0.95 0.95 0.95]
5	[0.95 0.95 0.95 0.95 0.95 0.75 0.55 0.75 0.95 0.95 0.95 0.95]
6	[0.95 0.95 0.95 0.95 0.95 0.65 0.35 0.65 0.95 0.95 0.95 0.95]
7	[0.95 0.95 0.95 0.95 0.883 0.816 0.75 0.816 0.883 0.95 0.95 0.95]
8	[0.95 0.95 0.95 0.95 0.816 0.683 0.55 0.683 0.816 0.95 0.95 0.95]
9	[0.95 0.95 0.95 0.95 0.75 0.55 0.35 0.55 0.75 0.95 0.95 0.95]
10	[0.95 0.75 0.95 0.75 0.95 0.75 0.95 0.75 0.95 0.75 0.95 0.75]
11	[0.95 0.55 0.95 0.55 0.95 0.55 0.95 0.55 0.95 0.55 0.95 0.55]
12	[0.95 0.35 0.95 0.35 0.95 0.35 0.95 0.35 0.95 0.35 0.95 0.35]
13	[0.95 0.75 0.95 0.75 0.95 0.75 0.95 0.35 0.95 0.35 0.95 0.35]
14	[0.95 0.35 0.95 0.35 0.95 0.35 0.95 0.75 0.95 0.75 0.95 0.75]
15	[0.95 0.75 0.95 0.75 0.95 0.35 0.95 0.35 0.95 0.75 0.95 0.75]
16	[0.95 0.75 0.95 0.35 0.95 0.75 0.95 0.35 0.95 0.75 0.95 0.35]
17	[0.95 0.35 0.95 0.75 0.95 0.35 0.95 0.75 0.95 0.35 0.95 0.75]
18	[0.95 0.35 0.95 0.35 0.95 0.75 0.95 0.75 0.95 0.35 0.95 0.35]
19	[0.95 0.85 0.75 0.85 0.95 0.85 0.75 0.85 0.95 0.85 0.75 0.85]
20	[0.95 0.883 0.816 0.75 0.816 0.883 0.95 0.883 0.816 0.75 0.816 0.883]
21	[0.95 0.90 0.85 0.80 0.75 0.80 0.85 0.90 0.95 0.90 0.85 0.80]
22	[0.95 0.75 0.55 0.75 0.95 0.75 0.55 0.75 0.95 0.75 0.55 0.75]
23	[0.95 0.816 0.683 0.55 0.683 0.816 0.95 0.816 0.683 0.55 0.683 0.816]
24	[0.95 0.85 0.75 0.65 0.55 0.65 0.75 0.85 0.95 0.85 0.75 0.65]
25	[0.95 0.65 0.35 0.65 0.95 0.65 0.35 0.65 0.95 0.65 0.35 0.65]
26	[0.95 0.75 0.55 0.35 0.55 0.75 0.95 0.75 0.55 0.35 0.55 0.75]
27	[0.95 0.80 0.65 0.50 0.35 0.50 0.65 0.80 0.95 0.80 0.65 0.50]

Subsequently, all cases were transformed into the frequency domain using the fast Fourier transform. The respective time series representation and frequency histograms are shown in Figures 5 and 6.

The next step was to calculate resilience indices according to [12,24] and Equations (2) and (3), respectively. Table 2 shows these results.

To compare the resilience values obtained from both models for the 27 cases, a scatter plot was created, which is shown in Figure 7. In this graph, four zones or categories have been defined, identified by colors, and according to the criteria: not resilient, low resilience, resilient, and highly resilient.

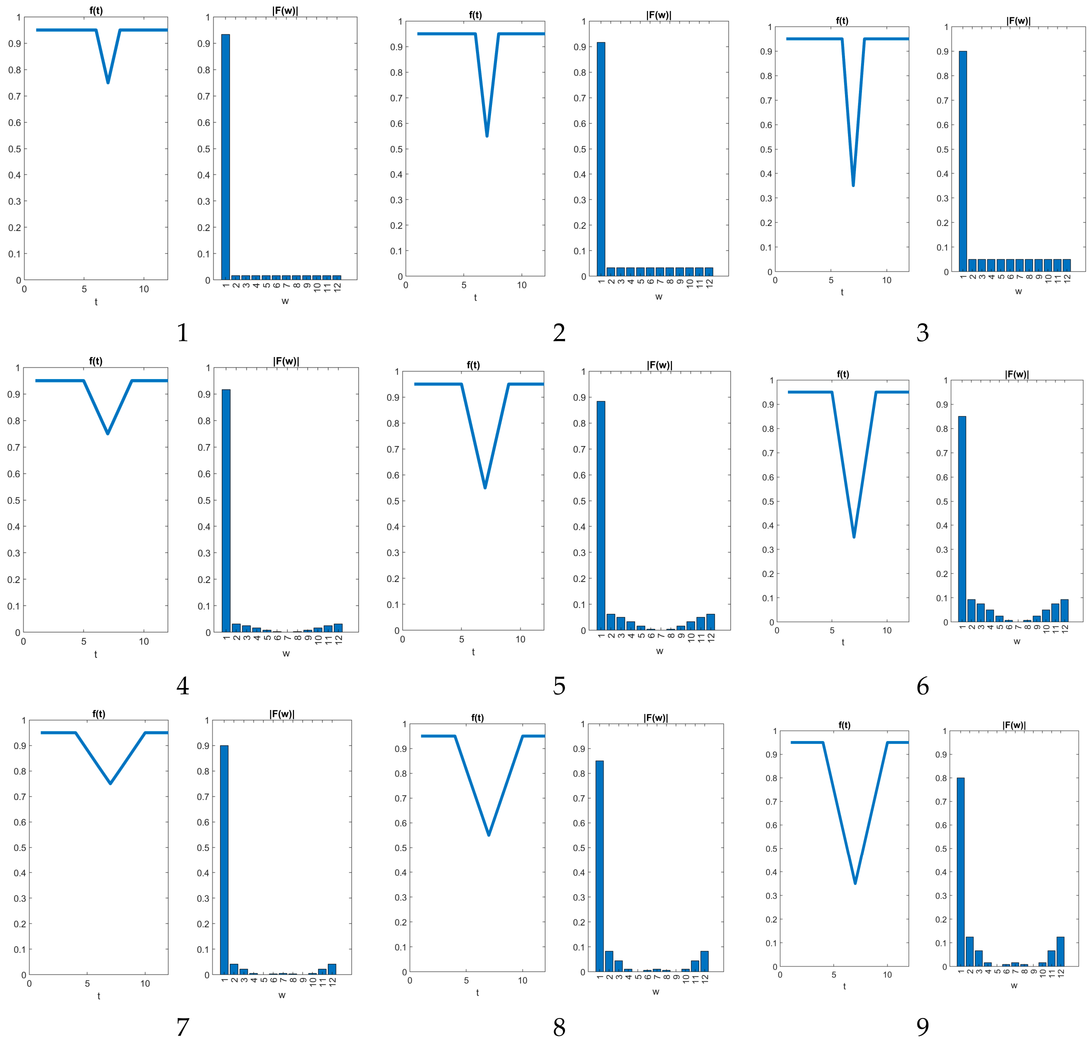

Figure 5. Time series and frequency histograms of cases 1 to 9.

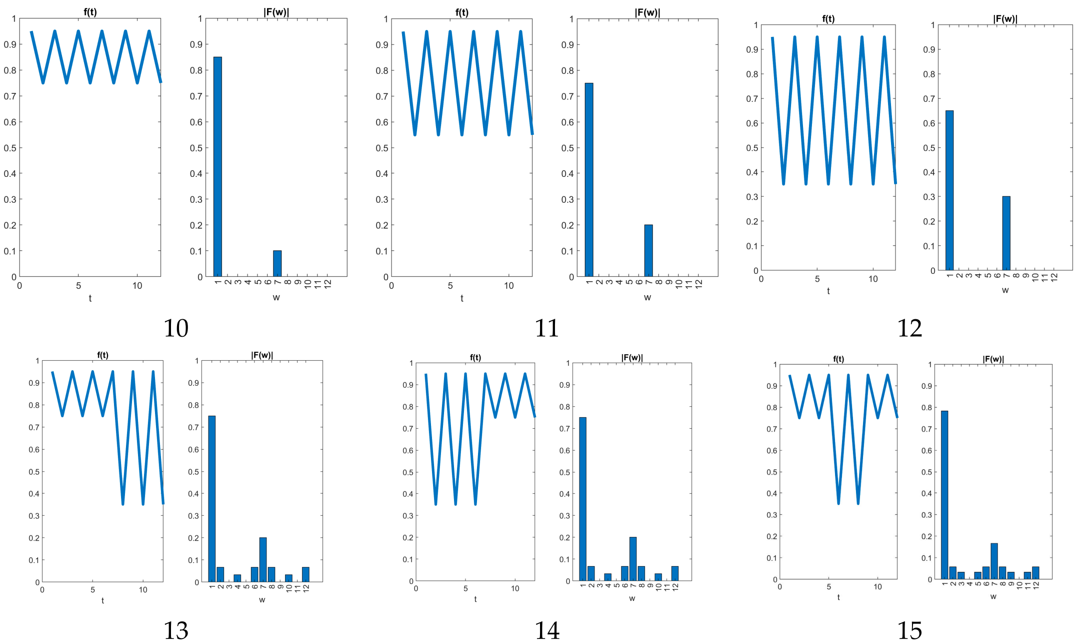

Figure 6. *Cont.*

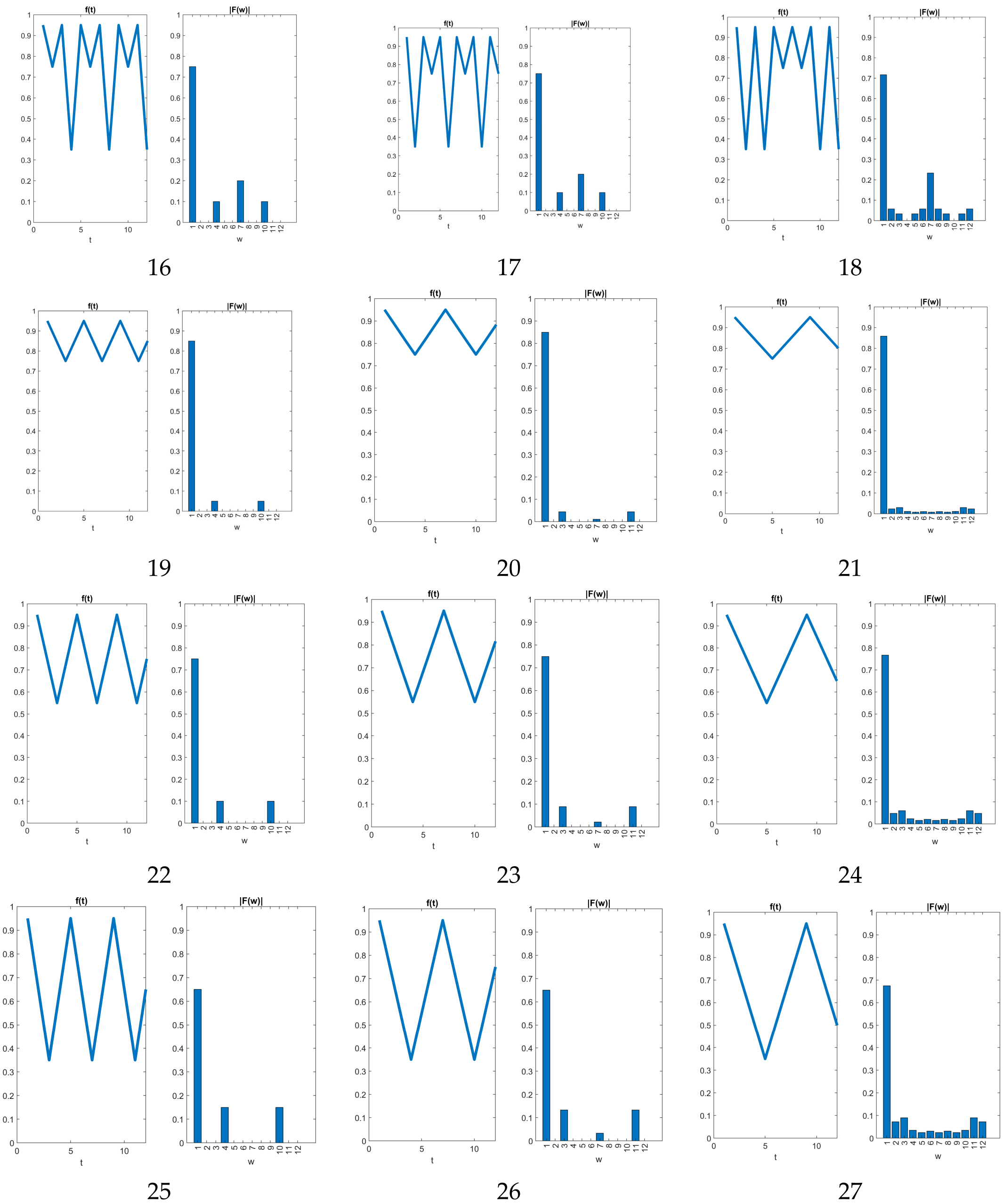

Figure 6. Time series and frequency histograms of cases 10 to 27.

Table 2. Resilience values obtained from the models developed by Cai et al. [12] and Davis et al. [24].

Case #	Ro Cai	Ro Bilal
1	0.59	0.99
2	0.44	0.98
3	0.28	0.97
4	0.54	0.98
5	0.39	0.96
6	0.25	0.95
7	0.51	0.97
8	0.37	0.95
9	0.24	0.92
10	0.59	0.99
11	0.44	0.98

Table 2. *Cont.*

Case #	Ro Cai	Ro Bilal
12	0.28	0.97
13	0.47	0.98
14	0.40	0.98
15	0.47	0.98
16	0.47	0.98
17	0.40	0.98
18	0.40	0.98
19	0.54	0.99
20	0.50	0.98
21	0.49	0.96
22	0.38	0.97
23	0.36	0.96
24	0.36	0.93
25	0.23	0.96
26	0.22	0.94
27	0.23	0.89

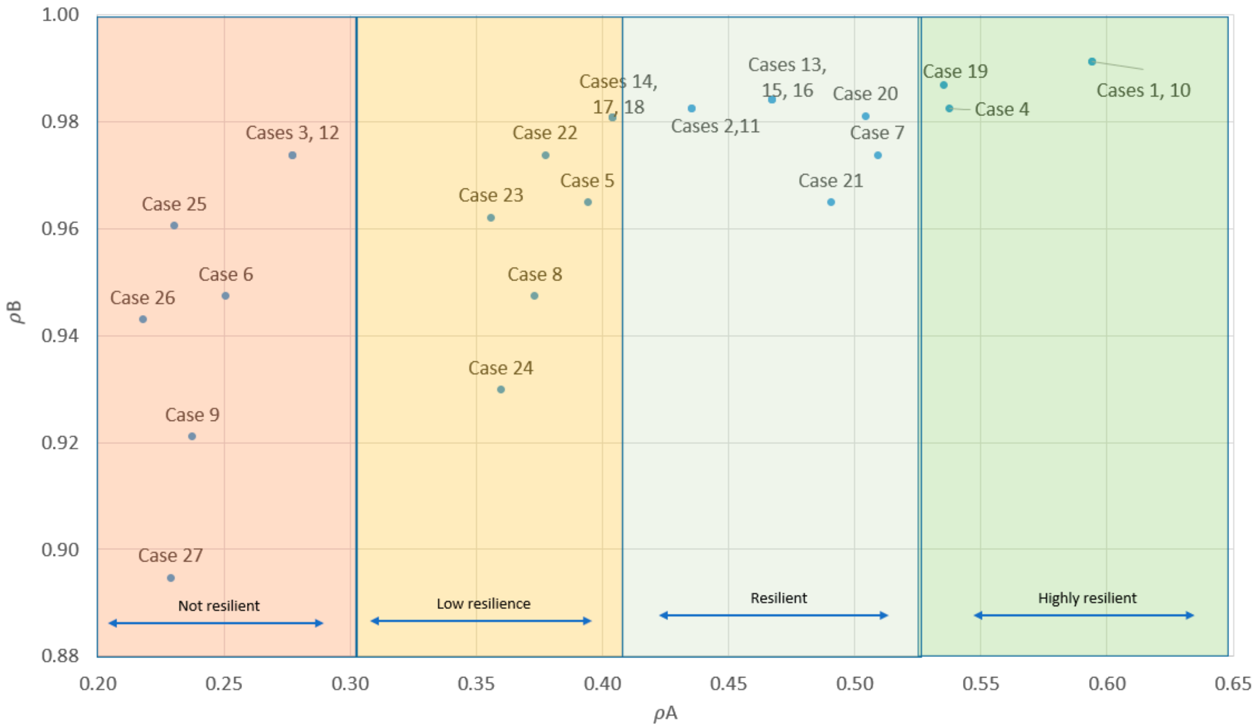

Figure 7. Resilience values scatter plot (ρ_A vs. ρ_B).

Table 3. Statistical moments obtained in the time domain.

Case #	StDev	Skewness	Kurtosis
1	0.06	−3.02	10.09
2	0.12	−3.02	10.09
3	0.17	−3.02	10.09
4	0.07	−1.68	4.53
5	0.13	−1.68	4.53
6	0.2	−1.68	4.53
7	0.07	−0.99	2.6
8	0.14	−1.00	2.61
9	0.21	−1.00	2.62
10	0.1	0.00	1.00

Table 3. *Cont.*

Case #	StDev	Skewness	Kurtosis
11	0.21	0.00	1.00
12	0.31	0.00	1.00
13	0.26	−0.82	2.00
14	0.26	−0.82	2.00
15	0.22	−1.16	3.02
16	0.26	−0.82	2.00
17	0.26	−0.82	2.00
18	0.28	−0.52	1.44
19	0.07	0.00	2.00
20	0.07	0.01	2.05
21	0.06	−0.04	1.99
22	0.15	0.00	2.00
23	0.13	0.01	2.06
24	0.13	−0.04	1.99
25	0.22	0.00	2.00
26	0.20	0.00	2.06
27	0.19	−0.04	1.99

To obtain the moment statistics (standard deviation, skewness, and kurtosis) in the frequency domain, the fast Fourier transform was initially applied to the 27 cases. In this way, the frequency histograms of each of these cases were obtained. These histograms are shown alongside the time series of each case in Figures 5 and 6. Following this, moment statistics were calculated on the vectors that represent the frequency histograms. Tables 3 and 4 show the values of standard deviation, skewness, and kurtosis, obtained from both domains.

Table 4. Statistical moments obtained in the frequency domain.

Case #	StDev	Skewness	Kurtosis
1	0.26	3.02	10.09
2	0.26	3.02	10.09
3	0.25	3.02	10.09
4	0.26	3.01	10.06
5	0.25	2.97	9.93
6	0.23	2.91	9.70
7	0.26	3.00	10.02
8	0.24	2.93	9.78
9	0.22	2.80	9.30
10	0.24	2.95	9.82
11	0.22	2.70	8.77
12	0.20	2.30	6.84
13	0.21	2.67	8.75
14	0.21	2.67	8.75
15	0.22	2.82	9.34
16	0.22	2.59	8.41
17	0.22	2.59	8.41
18	0.20	2.60	8.39
19	0.24	2.98	9.97
20	0.24	2.99	10.00
21	0.24	3.01	10.07
22	0.21	2.85	9.46
23	0.21	2.89	9.60
24	0.21	2.98	9.96
25	0.19	2.57	8.31
26	0.19	2.65	8.67
27	0.18	2.91	9.70

In a second stage, potential uses of the negentropy computed using the time series and frequency histograms were explored. Normalized values of negentropy (N_{norm}) were calculated in both domains. Finally, Pearson correlation values were calculated between the resilience values obtained by the [12,24] models and the aforementioned moment statistics, as well as the normalized negentropy in both time and frequency domains (Tables 5 and 6). At the end of each table, the average values of each correlation coefficient are reported.

Table 5. Pearson coefficients calculated among resilience indicators (time domain).

Time Domain		Standard Deviation	Skewness	Kurtosis	N_{norm}
Cases 1–9	ρ_B	−81.70%	−66.70%	65.30%	93.60%
Case 1–9	ρ_A	−99.00%	−21.10%	21.10%	67.30%
Cases 10–27	ρ_A	−41.70%	−24.40%	−4.90%	92.30%
Cases 10–27	ρ_B	9.60%	−37.30%	−15.60%	57.60%
	Corr.Avg	−53.20%	−37.38%	16.48%	77.70%

Table 6. Pearson coefficients calculated among resilience indicators (frequency domain).

Freq. Domain		Standard Deviation	Skewness	Kurtosis	N_{norm}
Case 1–9	ρ_B	96.40%	96.50%	96.40%	43.10%
Case 1–9	ρ_A	92.00%	68.00%	67.60%	80.50%
Cases 10–27	ρ_A	90.30%	44.40%	43.80%	47.90%
Cases 10–27	ρ_B	56.80%	−20.60%	−20.50%	41.40%
	Corr.Avg	83.88%	47.08%	46.83%	53.23%

It was observed that the highest correlations values were found between the normalized negentropy values and ρ_B for the time series in cases where only one shock was recorded (cases 1 to 9). On the other hand, for cases with multiple shocks (10 to 27), the highest correlation was also found between the normalized negentropy values and ρ_A applied in the time domain.

In the frequency domain, it was observed that the standard deviation showed higher correlations with the values of ρ_A and ρ_B for all cases.

The variable behavior of the signs of the Pearson correlation coefficients between different experiments should be noted. This erratic behavior reduces the utility of this statistic as an alternative resilience indicator to those obtained by the [12,24] models. For this reason, we suggest that the resilience indicator based on negentropy has a greater potential to be used as a resilience indicator in systems based on time series with performance values.

4.2. Discussion and Decisional Insights

The analysis of the resilience of a production system provides information for decision-making, helping organizations to optimize their production processes and minimize the risk of downtime.

The analysis of the resilience of productive systems enables the identification of critical points in the process. These are the points where a failure or interruption can have the greatest impact on the system's performance. Likewise, by performing a systemic resilience analysis, specifically based on the system's availability, companies can prioritize their maintenance efforts and allocate resources more effectively, ensuring that these areas are adequately protected.

Another decision-making perspective that can be derived from the analysis of the resilience of a production system is the identification of potential risks and vulnerabilities.

These may include risks related to equipment failure, supply chain interruptions, or other external factors that may affect production continuity. By identifying these risks and vulnerabilities, the company can take proactive measures to mitigate their impact and minimize the risk of downtime.

The analysis of the recovery capability of a production line can also provide insights for decision-making related to the optimization of maintenance and restoration activities. By analyzing data on equipment reliability and availability, as well as maintenance activities records, companies can identify patterns and trends that can help them optimize their maintenance programs and procedures. This can help minimize the risk of unexpected downtime and prolong the life of critical equipment.

Finally, the analysis of the recovery capability of performance levels can provide insights for decision-making related to the development of contingency plans. By analyzing potential scenarios and their impact on the production line, companies can develop response plans that allow them to respond quickly and effectively in the event of an interruption. This may include investments in backup equipment, alternative supply chains, or other measures to mitigate the impact of unexpected events.

In conclusion, resilience analysis in a production line can provide insights for decision-making that help companies optimize their production processes and minimize the risk of downtime. By identifying critical points, potential risks and vulnerabilities, optimization opportunities, and contingency plans, companies can develop a more robust and resilient production line that can withstand unexpected events and interruptions.

5. Conclusions

The analysis of the resilience models used by [12,24] suggests that, despite exhibiting some level of similarity in their behavior, the resilience values obtained do not show a high correlation between both models when obtained simultaneously (Pearson coefficient equal to 71.9%). Although no clear trend was observed, as resilience approaches higher values, the trend becomes more similar.

Another important point that can be observed from the results obtained is that the resilience values of systems subjected to one or more shocks were of equal value, as long as the magnitude and duration of the shocks was the same for both systems. That is, the number of shocks a system experiences was not relevant, whereas the magnitude and duration was. This is because both models work with an average resilience value.

On the other hand, when looking at the correlation values for different statistical indicators, it was found that the best results, in terms of correlation with the resilience values obtained by the Cai et al. [12] and Davis et al. [24] models, were obtained when compared with the values of standard deviation, in both the frequency and time domains. However, it is worth noting that the variability of the signs of the correlation values between the standard deviation and the values of ρ_A and ρ_B shows an erratic behavior reducing the utility of this statistic as an alternative resilience indicator. Negentropy is shown as a good and reliable resilience indicator when compared to the resilience values obtained by Cai's [12] and Davis's [24] models, in both time and frequency domains.

The only potential bias we observed is the time scale used to record the system performance values. Depending on the chosen time scale, the resilience values may vary. However, this issue is not specific to our proposed methodology but rather a general concern across other existing resilience indices in the literature.

The selection of the 27 hypothetical cases used to calculate the indices under evaluation was based on the aim of capturing a diverse range of situations and resilience values. These cases were intended to represent a variety of industries and scenarios to ensure a comprehensive assessment of resilience.

It is important to note that the resilience values obtained by the models developed by Davis et al. [24] and Cai et al. [12], for these 27 cases exhibited a wide distribution, as shown in Figure 7. This distribution indicates the variability in resilience levels across different systems' resilience capabilities. The inclusion of this diverse set of cases allowed for a more robust evaluation and comparison of the resilience indices, providing insights into their effectiveness across various contexts.

In conclusion, a new resilience indicator is presented that, when compared with indicators reported in the literature, shows high Pearson correlation values, which is promising. The proposed indicator derives from a well-known and accepted parameter, the Shannon's entropy. Furthermore, this indicator is independent of the scale on which the system performance level is measured, and it is easy to automate from field information and does not require complex calculations.

Explanatory analysis was explored when we established causal relationships with the resilience values obtained from two existing models in the literature, and this implies a degree of validation of the hypothesis put forward in our work. This does not prevent us from continuing to conduct explanatory tests in the future. Therefore, and in order to perform more statistical analysis, further developments can be expected based on the presented new resilience indicator. Specifically, the possibility to apply real-world validation studies. The proposed indicator has the potential to become a reliable and widely accepted measure of resilience in different types of systems. Since the proposed indicator is derived from Shannon's entropy, future studies may explore additional parameters or modifications to refine the measurement and capture more aspects of system resilience.

Negentropy is a well-suited indicator for evaluating resilience in engineering systems due to its advantageous characteristics: (1) it has the ability to maintain functionality and adaptability in the face of disruptive events; (2) it offers a holistic perspective by considering the system as a whole, taking into account the interdependencies and interactions between its components; and (3) it is sensitive to changes in the system's behavior and performance over time, enabling it to capture variations and fluctuations in its response to disruptions, thus reflecting its adaptive capacity. Finally, the quantifiability and analytical feasibility of negentropy make it a practical choice for evaluating resilience, as it can be calculated using a simple equation, derived from the Shannon's entropy equation, and facilitates the development of quantitative metrics and indices. Thus, its calculations are often straightforward and can be automated, enabling its implementation in engineering systems. In summary, negentropy offers a robust and comprehensive approach to assessing and benchmarking the resilience of engineering systems, thanks to its ability to capture variability, its universality, its system-level perspective, and its sensitivity to change.

Moreover, the independence of the proposed indicator from the scale of system performance measurement is a valuable characteristic. This implies that the indicator can be applied to diverse systems without the need for scaling adjustments, facilitating its implementation in different contexts. Additionally, its simplicity and ease of automation using field information make it a practical tool for assessing resilience in real-world scenarios. As future developments unfold, researchers may explore ways to further streamline the calculation process or incorporate additional relevant factors to enhance the indicator's accuracy and applicability in a wide range of domains and using big datasets and field information.

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