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Article

Adaptive Nonsingular Terminal Sliding Mode Control for Performance Improvement of Perturbed Nonlinear Systems

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Abstract: In this study, an adaptive nonsingular terminal sliding mode control technique according to the barrier function is designed for the performance improvement and robust stability of nonlinear systems with outdoor disturbances. For this reason, a novel nonlinear sliding surface is presented based on the states of the system. The nonlinear sliding surface forces the states of the system to converge from initial conditions to zero. Subsequently, a non-singular terminal sliding control scheme is advised for the purpose of finite-time stability of the nonlinear switching surface. Finite-time stabilization of the non-singular terminal sliding surface is verified by the Lyapunov theory. For improvement of the system performance against exterior perturbation, the barrier function adaptive technique is employed to estimate the unknown upper bounds of the exterior disturbance. Finally, the advantage and productivity of the recommended control method is investigated based on the simulation results. In the simulation part, the plasma torch jerk chaotic system is considered as a case study, such that the obtained results are given in different scenarios.

Keywords: chaotic systems; performance improvement; non-singular control; sliding mode control; external disturbance

MSC: 34H10; 34C28; 62F35; 93C40; 93C10; 93D09

1. Introduction

Chaotic behavior as the actual phenomenon originating from the nonlinear dynamical systems has been distinguished in several types of devices and mechanisms [1–3]. Under-actuation is a technical condition that can happen when the number of the actuators of a system are less than its number of degrees-of-freedom (DoF) [1]. Due to this definition, some mechanical and robotic systems, including mass-spring system [4], rotary inverted pendulum (RIP) system [5], quad-rotor unmanned aerial vehicles (UAVs) [6], robot manipulators [7], and chaotic systems [8], are under-actuated systems. These systems have various advantages compared to fully-actuated systems, as they are light weight, use less energy, and need simple communication apparatuses and fault-tolerant methods [9]. It is impossible to directly control each portion of under-actuated systems; hence, the expansion of control techniques, particularly model-free approaches, is still an important challenge. Therefore, the stabilization/tracking control of these systems have fascinated many researchers around the world during recent years [10]. Many control techniques have been applied for stabilization and tracking control of the under-actuated systems. One of the most significant nonlinear methods is the sliding mode control (SMC) procedure, which has a high robustness and rapid performance under external disturbances [11–13]. Thus, the main advantage of the SMC technique is to force state trajectories to converge to the switching surface and remain on it [14–17]. However, in practice, it is important for a controller to force trajectories of the system to be converged in the least possible time; so, the terminal sliding mode control (TSMC) tactic is planned, which can improve the convergence performance of the system [18–21]. The TSMC method suffers from singularity, where it is combined with a non-singular control scheme to remove the singularity problem [22–24]. All of the controllers that are designed for mechanical and robotic under-actuated systems have to deal with external disturbances that can occur at any moment. Therefore, some control techniques like adaptive procedures are applied to estimate the unknown bounded external disturbances [25–29]. However, the common adaptive control mechanisms suffer from two issues: one is the variation in the adaptive gain, because of changes of disturbances and the second issue is the asymptotic stability of the trajectories of the system. For this reason, the adaptive barrier function is presented to enhance the performance of the system [30]. The first benefit of the barrier function is to remove the variation due to the increase in disturbance amplitude. Another advantage of the adaptive barrier function is finite-time stability of the system states [31–33].

In [34], a robust fuzzy SMC technique is designed for stability of uncertain nonlinear systems. Hence, the whole system is converted to the second-order nonlinear dynamics, such that SMC is applied for stability control of closed-loop system. Moreover, the fuzzy control technique is employed for rejection of the uncertainty; however, the subjects of the exterior perturbation and finite-time convergence have not been considered. In [35], the integral TSMC method combined with model predictive control (MPC) is applied for performance improvement of discrete-time nonlinear systems in the existence of perturbation and uncertainty. Moreover, the chattering phenomenon is reduced using the MPC method, although no control technique such as adaptive control is adopted for the rejection of disturbance and uncertainty. In [36], a new TSMC surface is planned for stability control of uncertain systems. Then, based on this new surface, a fractional-order TSMC surface is designed for fast convergence of fractional-order chaotic systems and for removal of the singularity problem; although, the impact of the outdoor perturbation is declared. In [37], an integral fractional-order TSMC scheme is proposed for the stability control of nonlinear second-order system in the existence of uncertainty and disturbance. Moreover, for performance improvement of a system under uncertainties and disturbances, the radial basis function neural-network (RBFNN) has been adopted. Nevertheless, the performance of the system is changed by variations in the disturbance and uncertainty. In [38], the adaptive SMC technique is recommended for nth-order nonlinear system in the existence of perturbation. For fast convergence, the TSMC method combined differential term is suggested and the adaptive control technique is also applied for the estimation of the upper bounds of uncertainty and perturbation. However, a significant change in disturbance and uncertainty can damage the performance of the system. In [39], a super-twisting SMC method combined with interval type-2 fuzzy fractional scheme is planned for under-actuated nonlinear systems in the appearance of parametric uncertainties. Nonetheless, no control technique is applied for the removal of uncertainty. In [40], for the development of the robustness of an under-actuated system against bounded disturbances, an adaptive self-tuning technique is offered according to the linear-quadratic-regulator (LQR) technique. Nevertheless, the fast stability control of the system is overlooked in this article. In [41], a non-singular TSMC method is recommended for the fourth-order under-actuated uncertain system with unknown bounded external disturbances. Furthermore, for the rejection of disturbances, a finite-time disturbance observer is proposed. However, the problem of the chattering phenomenon is denied in this work. In [42], an input−output feedback linearization control technique using online optimal control based on a multi-crossover genetic algorithm is suggested for an underactuated system under parametric uncertainty. However, in this work, the fast convergence and effects of the external disturbances are ignored. In [43], an adaptive fuzzy state-feedback control approach is combined with barrier function for tracking the control of nonlinear system with constraints on states and uncertainties. Therefore, outputs of the system are converged to the setpoint in finite time. Moreover, for the rejection of the constraints on states, the integral barrier function is used. Furthermore, the fuzzy-control scheme is applied for estimation of the nonlinear uncertainties. However, in this work, the impression of the exterior perturbation is not examined in the control strategy. In [44], an adaptive barrier function method is used for a nonlinear system with unmodeled dynamics. Hence, the barrier function theory is applied to compensate the unmodeled dynamics, and the backstepping procedure is proposed for the stability of the system. However, the disturbance rejection is not investigated in [44]. In [45], the adaptive fuzzy controller combined with the barrier function scheme is recommended for nonlinear systems in the existence of constraints on the states. Therefore, the backstepping technique is applied for tracking control purposes and the barrier function is adopted for compensation of the constraints on states. Nevertheless, the impacts of exterior disturbances are denied in the above-mentioned works. In [46], a nonlinear differential equation in the presence or absence of the perturbation and existence of the multiple constant delay is considered. Moreover, the boundedness and stability analysis of the system are demonstrated using two new Lyapunov−Krasovskii functionals. However, the impression of the exterior perturbation is not examined in the control strategy. In [47], the boundedness and stability analysis based on the Lyapunov−Krasovskii functionals are presented for linear/nonlinear differential forms of first-order systems with time-varying delays. A barrier function adaptive higher-order SMC design approach is proposed in [48] for the fast finite time stability control of a chain of integrators with bounded perturbations, where the parametric uncertainties are unknown. In [49], a barrier function variable-gain adaptive super-twisting control technique is recommended for the first-order nonlinear systems with external disturbance and model uncertainty, where the time-derivative of disturbance term is unknown and bounded. A barrier function adaptive SMC methodology is suggested in [50] for the robust tracking of linear motor positioners in the existence of payload uncertainty and time-varying disturbance. In [51], a barrier function distributed backstepping adaptive control technique is advised for the three-order nonlinear connected and automated vehicles in the existence of full-state constraints and parameter uncertainties. In [52], the improved adaptive continuous barrier-function TSMC method is employed for the robotic manipulator in the existence of exterior disturbances, where the designed absolute function according to the fractional power of sliding surface causes smooth continuous control input. However, the mentioned method in [52] is not a non-singular finite time approach and the performance improvement of the closed-loop control system has not been considered in the mentioned research. Table 1 is provided to present the advantages and disadvantages of the proposed method in comparison with the above-mentioned research works.

According to the investigation and analysis of the above-studied articles, it can be stated that little consideration has been given to the stability control of perturbed nonlinear under-actuated systems based on the barrier function adaptive non-singular TSMC approach. Moreover, no work has been reported for the performance improvement and robust stability of the nonlinear under-actuated systems based on the barrier-function adaptive control technique. Therefore, the substantial contributions of this study are as follows:

(i) Design of a nonlinear sliding surface for stabilization of under-actuated nonlinear systems in the appearance of exterior perturbation with unknown bounds;
(ii) Proposition of a non-singular terminal sliding surface for the convergence of a nonlinear sliding surface in the finite time;
(iii) Employment of a nonlinear function φ in the sliding function for performance improvement of the closed-loop control system;

(iv) Design of a barrier function adaptive scheme to satisfy the system's robust performance against perturbation.

Table 1. Comparison of the proposed method with other research works.

Article	Advantages	Disadvantages
Method in [34]	Rejection of uncertainty using fuzzy control technique.	No consideration of exterior perturbation and finite-time convergence.
Method in [35]	Reduction of chattering phenomenon via MPC method.	No control technique such as adaptive control for rejection of disturbance and uncertainty.
Method in [36]	Removal of singularity problem.	Declaration of impact of the outdoor perturbation.
Method in [37]	Radial basis function neural-network (RBFNN) for performance improvement under uncertainties and disturbances.	Change of the system's performance by variations of disturbance and uncertainty.
Method in [38]	Suggestion of TSMC for fast convergence and adaptive controller for the estimation of upper bounds of perturbations.	Damage of the system's performance by a significant change in disturbance and uncertainty.
Method in [39]	Fast convergence of perturbed and uncertain nonlinear system.	No control technique for the removal of uncertainty.
Method in [40]	Adaptive self-tuning technique according to the linear-quadratic-regulator (LQR).	No consideration of the fast stability control of system.
Method in [41]	Proposition of a finite-time disturbance observer for disturbance rejection.	The chattering problem is denied.
Method in [42]	Suggestion of input-output feedback linearization via online optimal control based on multi-crossover genetic algorithm.	Fast convergence and effects of the external disturbances are ignored.
Method in [43]	Using the integral barrier function-based fuzzy control for rejection of state constraints and estimation of the nonlinear uncertainties.	No examination of the impression of exterior perturbation in the control strategy.
Method in [44]	Compensation of the unmodeled dynamics by barrier function theory and offering a backstepping procedure for stability of the system.	No investigation of disturbance rejection.
Method in [45]	Backstepping technique for tracking control and barrier function for compensation of the states' constraints.	The impacts of exterior disturbances are denied.
Method in [46]	Two new Lyapunov–Krasovskii functionals for the boundedness and stability analysis of system.	No examination of the impression of the exterior perturbation in the control strategy.
Method in [47]	Non-singular finite time control approach.	No consideration of the performance improvement of the closed-loop control system.

This article is prepared and prearranged as follows: the explanation of the considered system and preliminaries are given in Section 2. The proposition of a nonlinear sliding surface, design of non-singular terminal sliding surface, and usage of the barrier function adaptive approach are given in Section 3. Simulation outcomes are provided in Section 4, which shows the applicability and success of the planned scheme. Lastly, conclusions are reported in Section 5.

2. Problem Definition and Preliminaries

In this part, at first, a nonlinear under-actuated system under exterior perturbation is presented. Then, for the stability control of the considered system, a nonlinear sliding surface is defined. Afterward, for fast convergence of states of systems, a non-singular TSMC surface is proposed. Finally, for the disturbance rejection, the barrier function adaptive technique is combined with non-singular terminal sliding mode control.

An under-actuated nonlinear system in the appearance of bounded external disturbance is considered

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) \tag{1}$$

$$\dot{x}_2(t) = A_{21}x_1(t) + A_{22}x_2(t) + f(x) + Bu(t) + w(t). \tag{2}$$

where $x_1(t) \in R^{n-m}$ and $x_2(t) \in R^m$ are the states of the system. The expression $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ is the matrix of system with components of $A_{11} \in R^{n-m\times n-m}$, $A_{12} \in R^{n-m\times m}$, $A_{21} \in R^{m\times n-m}$ and $A_{22} \in R^{m\times m}$. The function $f(x)$ is the nonlinear function. The matric $B \in R^{m\times m}$ is non-singular and the term $w(t) \in R^m$ is the external disturbance with an unknown bound $v \in R$, i.e., $\|w(t)\| \le v$.

The main aim of this study is the stability control of the above-considered under-actuated system in the presence of bounded exterior perturbation.

Assumption 1. *The pair (A_{11}, A_{12}) is assumed completely controllable; hence, for any specified symmetric Positive-Definite (PD) matrix Q, there exists a unique symmetric PD matrix P with*

$$(A_{11} - A_{12}F)^T P + P(A_{11} - A_{12}F) = -Q \tag{3}$$

3. Main Results

3.1. Nonlinear SMC Surface

For convergence of a nonlinear under-actuated system (1)–(2), a nonlinear sliding function is defined by

$$s(t) = C_1x_1(t) + x_2(t), \tag{4}$$

with $C_1 = \left(F - \varphi(x_1(t))A_{12}^T P\right) \in R^{m\times(n-m)}$, where P denotes an $(n-m)\times(n-m)$ positive-definite matrix, F specifies an $m\times(n-m)$ constant matrix, and $\varphi(x_1(t))$ represents an $m\times m$ diagonal matrix with non-positive nonlinear functions of $x_1(t)$. The function $\varphi(x_1(t))$ is given as a diagonal matrix with exponential terms as

$$\varphi(x_1(t)) = \mathrm{diag}(\varphi_1(x_1(t)), \ldots, \varphi_m(x_1(t))) \tag{5}$$

$$\varphi_i(x_1(t)) = -\gamma_i e^{-\left(\frac{x_1(t)}{x_{1_0}}\right)^2} \tag{6}$$

where x_{1_0} is the initial state of x_1 and γ_i denotes a positive coefficient. The value of $\varphi_i(x_1(t))$ is changed from zero to the negative value $-\gamma_i$ as x_1 goes from the initial value to the origin. When the nonlinear sliding function $s(t) = 0$ is attained, Equation (4) yields

$$x_2(t) = \left(\varphi(x_1(t))A_{12}^T P - F\right)x_1(t). \tag{7}$$

From (1) and (7), the sliding dynamics is obtained as

$$\dot{x}_1(t) = \left\{A_{11} + A_{12}\left(\varphi(x_1(t))A_{12}^T P - F\right)\right\}x_1(t). \tag{8}$$

Theorem 1. *Consider the under-actuated nonlinear system (1)–(2), the Lyapunov equality (3), and nonlinear sliding function (4). The sliding dynamics (8) converges to the origin exponentially.*

Proof. Construct the positive-definite Lyapunov function $V_1(t) = {x_1}^T(t)Px_1(t)$, where differentiating it along the sliding dynamics (8), we have

$$\begin{aligned}\dot{V}_1(t) &= \dot{x}_1{}^T(t)Px_1(t) + {x_1}^T(t)P\dot{x}_1(t) \\ &= {x_1}^T(t)\left(\left\{{A_{11}}^T + \left(\varphi(x_1(t))PA_{12} - F^T\right){A_{12}}^T\right\} + P\{A_{11} \right. \\ &\quad \left. + A_{12}\left(\varphi(x_1(t))A_{12}^T P - F\right)\}\right)x_1(t)\end{aligned} \tag{9}$$

From (3) and (9), we obtain

$$\dot{V}_1(t) = x_1{}^T(t)\Big(2PA_{12}\varphi(x_1(t))A_{12}^T P - Q\Big)x_1(t) \tag{10}$$

where, because $Q > 0$ and $\varphi(x_1) < 0$, Equation (10) gives

$$\dot{V}_1(t) \le -x_1{}^T(t)Qx_1(t) < -\rho V_1(t) < 0 \tag{11}$$

with $\rho = \frac{\lambda_{min}(Q)}{\lambda_{max}(P)} > 0$, where $\lambda_{min}(Q)$ and $\lambda_{max}(P)$ are the lowest eigenvalue of Q and largest eigenvalue of P, respectively. □

3.2. Non-Singular TSMC

In this part, for convergence of the sliding function $s(t)$ to zero in the finite time, the nonsingular TSMC manifold is designed as

$$\sigma(t) = s(t) + \varpi \int_0^t s(t)^{a/b} d\tau, \tag{12}$$

where a and b are the odd integers with $1 < a/b < 2$ and ϖ is a positive coefficient.

Theorem 2. *Consider the nonlinear system with external disturbance as (1)–(2) and the nonsingular TSMC manifold (12). If the control input is designed as*

$$u = -B^{-1}\Big(\Big(C_1A_{11} + A_{21} - \dot{\varphi}(x_1(t))A_{12}^T P\Big)x_1(t) + (C_1A_{12} + A_{22})x_2(t) + f(x) + \varpi s^{\frac{a}{b}} + (v+\mu)sgn(\sigma(t))\Big), \tag{13}$$

with $\|w(t)\| \le v$, where v and μ are the positive constants, then the state trajectories of (1)–(2) are moved from any initial condition to the nonsingular manifold (12) and remain on it later.

Proof. Differentiating the nonsingular TSMC manifold (12) along the trajectories of (1)–(2), we have

$$\dot{\sigma}(t) = \dot{s}(t) + \varpi s(t)^{a/b} = \Big(\dot{C}_1 + C_1A_{11} + A_{21}\Big)x_1(t) + (C_1A_{12} + A_{22})x_2(t) + Bu + f(x) + \varpi s(t)^{a/b} + w(t). \tag{14}$$

Construct the positive-definite Lyapunov function $V_2(t) = 0.5\sigma^T(t)\sigma(t)$, where its time-derivative is obtained as

$$\begin{aligned}\dot{V}_2(t) &= \sigma^T(t)\dot{\sigma}(t) \\ &= \sigma^T(t)\big\{\big(-\dot{\varphi}(x_1(t))A_{12}^T P + C_1A_{11} + A_{21}\big)x_1(t) + (C_1A_{12} \\ &\quad + A_{22})\, x_2(t) + Bu + f(x) + \varpi s(t)^{\frac{p}{q}} + w(t)\Big\}.\end{aligned} \tag{15}$$

Replacing the controller signal (13) into (15), one has

$$\begin{aligned}&\dot{V}_2(t) = -\sigma^T(t)((v+\mu)sgn(\sigma(t)) - w(t)) \\ &\le -(v - \|w(t)\|)\|\sigma(t)\| - \mu\|\sigma(t)\| \le -\mu\|\sigma(t)\| \le -\sqrt{2}\mu V_2(t)^{0.5}.\end{aligned} \tag{16}$$

Finally, the trajectories of states of dynamics (1)–(2) converge to the nonsingular manifold (12) from any initial condition and remain on it afterward. □

Using Theorem 2, the nonsingular TSMC manifold (12) converges to origin in the finite time. Hence, from $\sigma(t) = 0$, one can obtain

$$s(t) = -\varpi \int_0^t s(\tau)^{a/b} d\tau, \tag{17}$$

where, assuming $h = \int_0^t s(\tau)d\tau$ and $\dot{h} = s$, one can find from (17):

$$\frac{\dot{h}}{h^{a/b}} = -\varpi. \tag{18}$$

By integration of both sides of Equation (18) from t_0 to t_s, it yields

$$\int_{h(t_0)}^{h(t_s)} h^{-a/b}\dot{h}dh = -\int_{t_0}^{t_s} \varpi dt = -\varpi(t_s - t_0), \tag{19}$$

where

$$t_s = t_0 + \frac{h(t_0)^{1-a/b}}{\varpi(1-a/b)} = t_0 + \frac{s(t_0)^{1-a/b}}{\varpi(1-a/b)}. \tag{20}$$

3.3. Barrier-Function Adaptive Non-Singular TSMC

In practice, the adaptive controller is presented to overcome the unknown external perturbations, where the adaptive-tuning gains are varied with the changes of the exterior disturbances. Now, a barrier function adaptive-tuning control input is designed. By employing the barrier-function adaptive TSMC approach, the external perturbation is estimated more successfully, the control performance of the system is improved, and the closed-loop system becomes more stable. The new adaptive control input is updated as

$$u = -B^{-1}\Big(\Big(C_1A_{11} + A_{21} - \dot{\varphi}(x_1(t))A_{12}^T P\Big)x_1(t) + (C_1A_{12} + A_{22})x_2(t) + f(x) + \varpi s^{\frac{a}{b}} + (\hat{v} + \mu)sgn(\sigma(t))\Big) \tag{21}$$

with

$$\hat{v} = \begin{cases} v_a, & 0 < t \leq \bar{t} \\ v_{psb}, & t > \bar{t} \end{cases} \tag{22}$$

where $\bar{t}$ denotes the time that the states are moved from initial conditions to the neighborhood ε of a non-singular TSMC manifold σ. The adaptive gain and positive semi-definite (PSD) barrier function are specified as

$$\dot{v}_a = \Gamma\|\sigma(t)\| \tag{23}$$

$$v_{psb} = \frac{\|\sigma(t)\|}{\varepsilon - \|\sigma(t)\|}, \tag{24}$$

where ε and Γ are two positive coefficients. Using (23), the adaptive controller coefficient is adjusted to be enlarged until the states reach the neighborhood ε of the non-singular TSMC manifold at the time $\bar{t}$. Then, for times bigger than $\bar{t}$, the adaptive coefficient switches to barrier function (24), which can decrease the convergence region and keep the states in the region ε. The system's stability is divided into the following two parts:

Condition (I): $0 < t \leq \bar{t}$

Theorem 3. *Consider the nonlinear disturbed system (1)–(2), nonlinear sliding function (4), and nonsingular TSMC manifold (12). Using the adaptive controller (21) and considering $\hat{v} = v_a$, then the states reach the neighborhood ε of the nonsingular TSMC manifold.*

Proof. Consider the Lyapunov function $V_3(t) = 0.5(\sigma(t)^T\sigma(t) + \kappa^{-1}(v_a - v)^2)$, where κ and v are the positive constants. The time-derivative of this Lyapunov function is obtained as

$$\dot{V}_3(t) = \sigma^T(t)\dot{\sigma}(t) + \kappa^{-1}(v_a - v)\dot{v}_a, \tag{25}$$

where, when replacing (14) and (23) into the above equation, one attains

$$\dot{V}_3(t) = \sigma^T(t)\{(C_1A_{11} + A_{21} - \dot{\varphi}(x_1(t))A_{12}^T P)x_1(t) + (C_1A_{12} + A_{22})x_2(t) + Bu + f(x) + \varpi s(t)^{\frac{a}{b}} + w(t)\} + \kappa^{-1}\Gamma(v_a - v)\|\sigma(t)\| \quad (26)$$

Substituting the adaptive controller signal (21) into (26) gives

$$\begin{aligned} \dot{V}_3(t) &= \sigma^T(t)\{w(t) - (\hat{v} + \mu)sgn(\sigma(t))\} + \kappa^{-1}\Gamma(v_a - v)\|\sigma(t)\| \\ &\leq -\mu\|\sigma(t)\| + \|\sigma(t)\|\|w(t)\| - \sigma(t)^T v_a sgn(\sigma(t)) + \kappa^{-1}\Gamma(v_a - v)\|\sigma(t)\| \\ &\leq \|w(t)\|\|\sigma(t)\| - v_a\|\sigma(t)\| + \kappa^{-1}\Gamma(v_a - v)\|\sigma(t)\| + v\|\sigma(t)\| - v\|\sigma(t)\| \\ &\leq -(v - \|w(t)\|)\|\sigma(t)\| - (1 - \kappa^{-1}\Gamma)(v_a - v)\|\sigma(t)\| \end{aligned} \quad (27)$$

where, since $v - \|w(t)\| > 0$ and $\kappa^{-1}\Gamma < 1$, the above equation can be expressed as

$$\begin{aligned} \dot{V}_3(t) &\leq -\sqrt{2}(v - \|w(t)\|)\tfrac{\|\sigma(t)\|}{\sqrt{2}} - \sqrt{2\kappa}(1 - \kappa^{-1}\Gamma)\|\sigma(t)\|\tfrac{v_a - v}{\sqrt{2\kappa}} \\ &\leq -\min\left\{\sqrt{2}(v - \|w(t)\|), \sqrt{2\kappa}(1 - \kappa^{-1}\Gamma)\|\sigma(t)\|\right\}\left(\tfrac{\|\sigma(t)\|}{\sqrt{2}} + \tfrac{\|v_a - v\|}{\sqrt{2\kappa}}\right) \\ &\leq -\Omega V_3(t)^{0.5} \end{aligned} \quad (28)$$

where $\Omega = \min\left\{\sqrt{2}(v - \|w(t)\|), \sqrt{2\kappa}(1 - \kappa^{-1}\Gamma)\|\sigma(t)\|\right\}$. □

Condition (II): $t > \bar{t}$

Theorem 4. *Consider the nonlinear system (1)–(2) with external disturbance, the nonlinear sliding function (4), and the nonsingular TSMC manifold (12). If the adaptive control signal is designed as (21) with $\hat{v} = v_{psb}$ (Equation (24)), that is,*

$$u = -B^{-1}((C_1A_{11} + A_{21} - \dot{\varphi}(x_1(t))A_{12}^T P)x_1(t) + (C_1A_{12} + A_{22})x_2(t) + f(x) + \varpi s^{\frac{a}{b}} + \left(\tfrac{\|\sigma(t)\|}{\varepsilon - \|\sigma(t)\|} + \mu\right)sgn(\sigma(t))) \quad (29)$$

then the system states reach the region $\|\sigma(t)\| \leq \varepsilon$.

Proof. Consider the positive-definite Lyapunov function $V_4(t) = 0.5(\sigma^T(t)\sigma(t) + (v_{psb} - v_{psb}(0))^2)$. The time-derivative of the above Lyapunov function is obtained as

$$\dot{V}_4(t) = \sigma^T(t)\dot{\sigma}(t) + \left(v_{psb} - v_{psb}(0)\right)\dot{v}_{psb}, \quad (30)$$

where substituting $\dot{\sigma}$ and $v_{psb}(0) = 0$ into (30) yields

$$\dot{V}_4(t) = \sigma^T(t)\{(C_1A_{11} + A_{21} - \dot{\varphi}(x_1(t))A_{12}^T P)x_1(t) + (C_1A_{12} + A_{22})x_2(t) + Bu + f(x) + \varpi s^{\frac{a}{b}} + w(t)\} + v_{psb}\dot{v}_{psb}. \quad (31)$$

Substituting the control signal (31) into the above equation results in the following

$$\begin{aligned}\dot{V}_4(t) &= \sigma^T(t)\Big\{w(t) - \Big(v_{psb} + \mu\Big)sgn(\sigma(t))\Big\} \\ &\quad + v_{psb}\varepsilon(\varepsilon - \|\sigma(t)\|)^{-2} sgn(\sigma)\dot{\sigma}(t) \\ &\le -\mu\|\sigma\| + \|w(t)\|\|\sigma\| - v_{psb}\|\sigma\| \\ &\quad + v_{psb}\varepsilon(\varepsilon - \|\sigma(t)\|)^{-2} sgn(\sigma(t))\Big\{w(t) - \Big(v_{psb} + \mu\Big)sgn(\sigma)\Big\} \\ &\le -\mu\Big(\|\sigma\| + v_{psb}\varepsilon(\varepsilon - \|\sigma\|)^{-2}\Big) - \Big(v_{psb} - \|w(t)\|\Big)\|\sigma(t)\| \\ &\quad - \varepsilon(\varepsilon - \|\sigma(t)\|)^{-2}\Big\{v_{psb} - \|w(t)\|\Big\}v_{psb}\end{aligned} \tag{32}$$

where, since $v_{psb} > \|w\|$ and $\varepsilon(\varepsilon - \|\sigma(t)\|)^{-2} > 0$, we have

$$\begin{aligned}&\dot{V}_4(t) \le -\sqrt{2}\Big(v_{psb} - \|w(t)\|\Big)\tfrac{\|\sigma(t)\|}{\sqrt{2}} - \sqrt{2}\varepsilon(\varepsilon - \|\sigma(t)\|)^{-2}\Big\{v_{psb} - \|w(t)\|\Big\}\tfrac{v_{psb}}{\sqrt{2}} \\ &\le -\sqrt{2}\Big(v_{psb} - \|w(t)\|\Big)\min\Big\{1, \varepsilon(\varepsilon - \|\sigma(t)\|)^{-2}\Big\}\Big(\tfrac{\|\sigma(t)\|}{\sqrt{2}} + \tfrac{v_{psb}}{\sqrt{2}}\Big) \le -\Xi V_4(t)^{0.5}\end{aligned} \tag{33}$$

where $\Xi = \sqrt{2}\Big(v_{psb} - \|w(t)\|\Big)\min\Big\{1, \varepsilon(\varepsilon - \|\sigma(t)\|)^{-2}\Big\}$. □

The flow-chart of the proposed control strategy according to the barrier function adaptive non-singular TSMC scheme is shown in Figure 1.

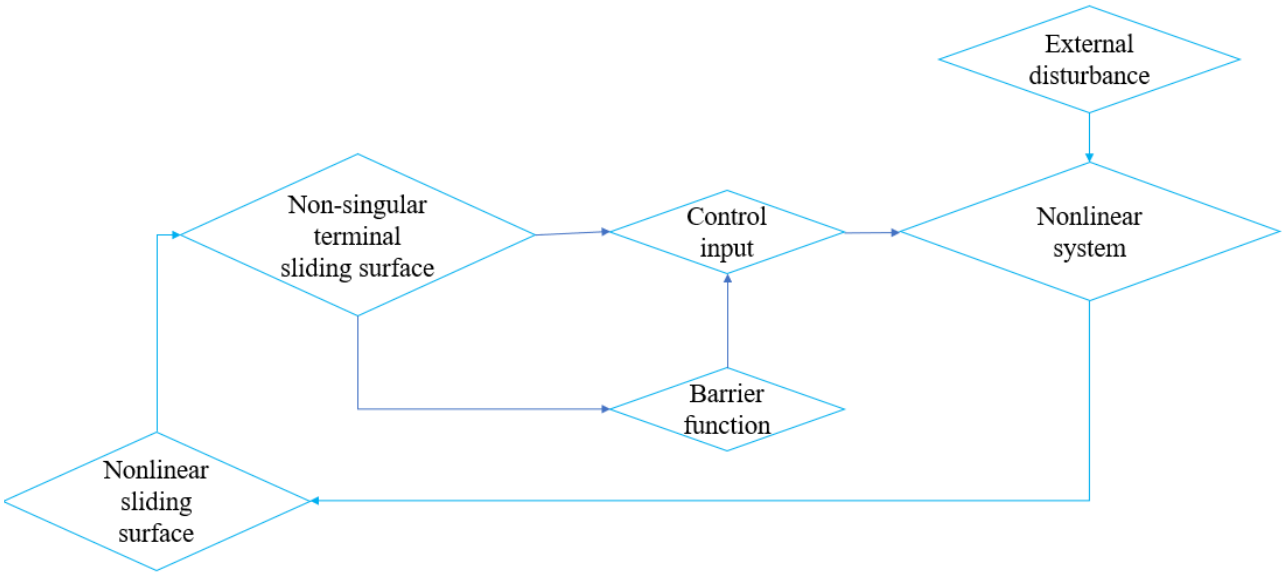

Figure 1. Block diagram of the above-designed technique.

Remark 1. *The following steps are presented to clarify the control strategy:*

(a) Firstly, the nonlinear system under external disturbance is defined;
(b) Afterward, the nonlinear sliding surface based on the system states is defined for convergence of the system states to the origin;
(c) Then, the nonsingular terminal sliding surface based on the nonlinear sliding surface is defined for fast convergence of the nonlinear sliding surface;
(d) For rejection of the external disturbances, a barrier function is defined;
(e) At last, the control input is achieved to enter to the nonlinear system for stability control of the system states;
(f) This closed-loop control procedure is repeated at any moment.

4. Simulation Results

This section is presented in three parts to prove the success and advantages of the suggested method for the stability control of a nonlinear under-actuated system in the existence of unknown bounded disturbances. In the first part, a third-order nonlinear underactuated chaotic system is introduced as a case study. Simulation results based on the barrier function adaptive non-singular TSMC are prepared in comparison with the results of the method of [1] in the second part. In the third part, simulation is repeated in the appearance of the abrupt change in external disturbance to prove the robustness of the proposed method.

4.1. Introduction of Chaotic System

The third-order nonlinear under-actuated plasma torch jerk chaotic system with two nonlinearities is considered as [53]

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = ax - by - z - cxy - x^3 + u + w(t) \end{cases} \tag{34}$$

where x, y, and z signify the system states, and a, b, and c are constant parameters of the system. In order to use of the control strategy, the above system equation is rewritten as

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 \tag{35}$$

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + f(x) + u + w(t). \tag{36}$$

where $x_1 = [x, y]^T$, $x_2 = z$ are the states of the system; $A_{11} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $A_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $A_{21} = [a - b]$, $A_{22} = -1$; $f(x) = \begin{bmatrix} -cxy \\ -x^3 \end{bmatrix}$ is the nonlinear vector function. The initial states are assumed as $x_1(0) = [0.4, 0.2]^T$, $x_2(0) = 0.2$, $v_a(0) = 0.2$, and the external disturbance is considered as $w(t) = 0.2\sin(1.5\pi t)$.

4.2. Simulation Results without Abrupt Change

The control strategy parameters, which are obtained based on the trial and error method, are given as $a = 5$, $b = 3$, $\gamma = 0.1$, $\varpi = 0.1$, $\varepsilon = 0.3$, $\mu = 150$, and $\Im = 0.3$. If the gain F is considered as $F = [5\ 5]$, so, from Equation (3), the matrix $P = \begin{bmatrix} 0.6 & -0.5 \\ -0.5 & 0.6 \end{bmatrix}$ is obtained.

In this part, at first, simulation results are obtained and compared with the existing method [1]. In Figure 2, the stability of the states of the considered chaotic system is shown. One can observe that the system states are converged to zero and the proposed method offers a fast and accurate response compared with the method of [1]. Time histories of the nonlinear sliding and non-singular terminal sliding surfaces are displayed in Figures 3 and 4, correspondingly. As one can apperceive in these plots, the nonlinear sliding surface is converged to zero in the finite time and the proposed non-singular terminal sliding surface has a fast convergence rate with respect to the non-singular terminal sliding surface of [1]. The adaptive law obtained from [1] and the time trajectory of the above-designed barrier function are depicted in Figure 5. From this figure, it can be inferred that the barrier function is not sensitive to the external disturbance, and has more and accurate performance compared with the adaptive control law of [1]. The time trajectories of the control input obtained by the recommended method and the method of [1] are illustrated in Figure 6. It can be proven that the above-designed controller based on the barrier function adaptive non-singular TSMC technique has a better transient and steady-state performance compared to the method of [1]. Moreover, there is no chattering in the proposed control input.

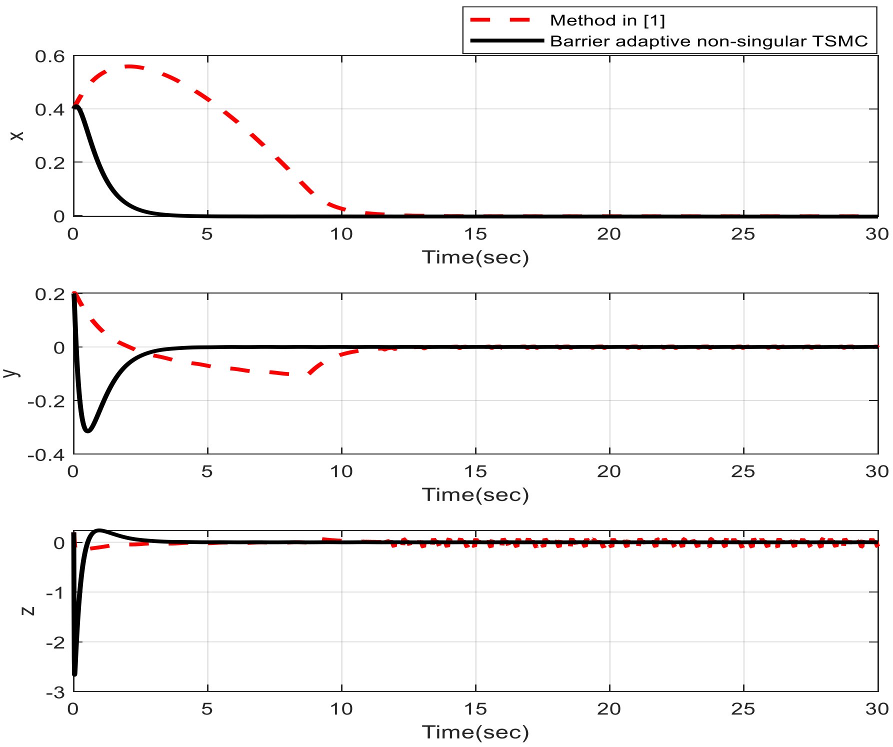

Figure 2. Trajectory stability of the state of system.

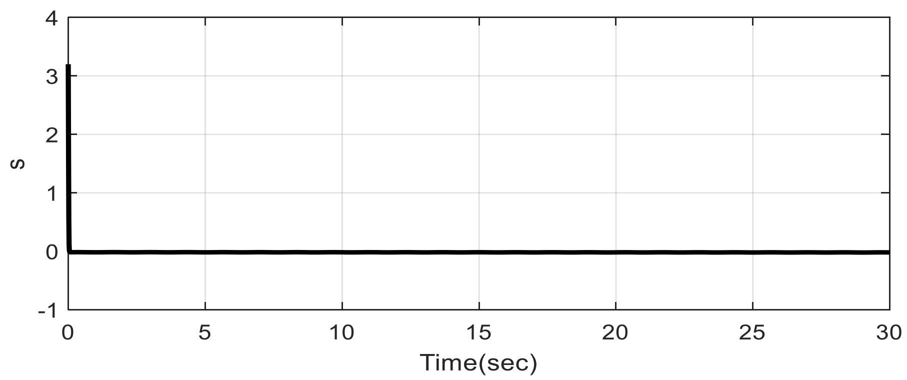

Figure 3. Time histories of the nonlinear sliding surface.

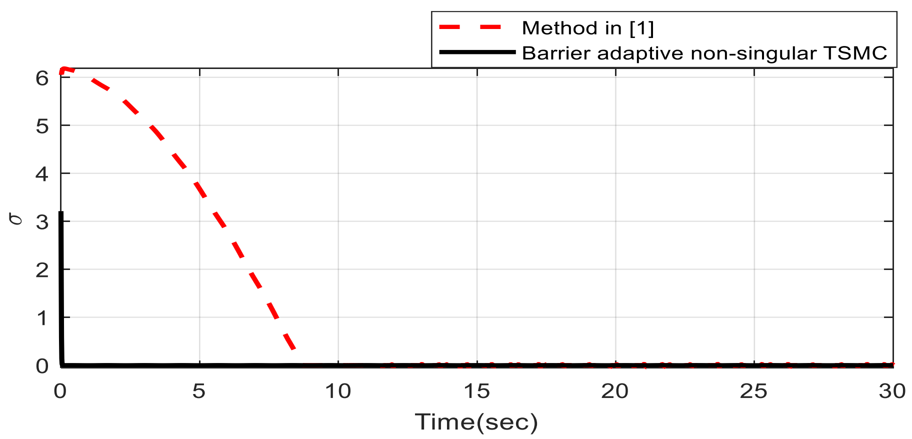

Figure 4. Time responses of the non-singular terminal sliding surface.

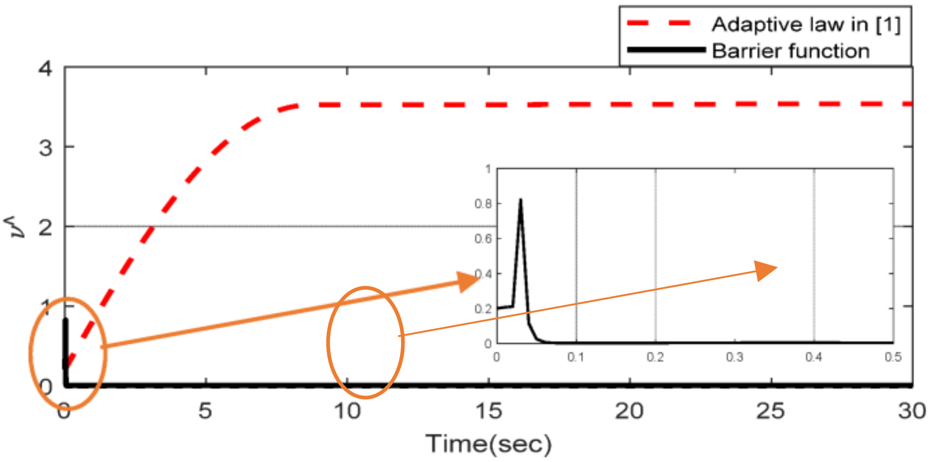

Figure 5. Time responses of the barrier function and adaptive law.

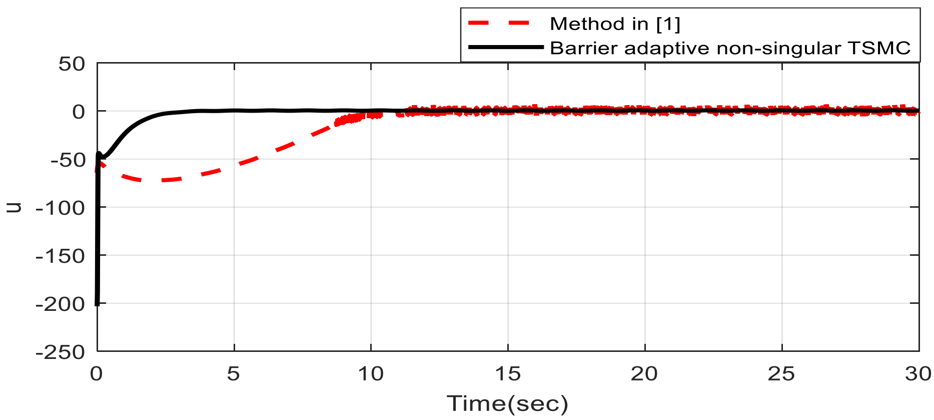

Figure 6. Trajectory of the control inputs.

4.3. Simulation Result in the Existence of Abrupt Change

In the second part, the simulation results according to the barrier function adaptive nonsingular TSMC technique and the method of [1] are obtained and compared in the existence of measurement noise and abrupt changes of disturbances. For this reason, measurement noise is considered as a bound-limited noise (power 0.0001, sample time 0.01). Moreover, the external disturbance is presumed as $w(t) = 7.5\sin(1.5\pi t)$ in the time interval $(5, 10)$, which confirms the abrupt change of disturbance. The time trajectories of the system states are exhibited in Figure 7 under measurement noise and abrupt change of disturbances. It is evidenced that the states are stabilized faster and better using the recommended scheme with respect to the technique of [1]. From Figure 8, the nonlinear sliding surface is converged to origin in the finite time in the appearance of measurement noise and the abrupt change of disturbance. According to Figure 9, the proposed non-singular terminal sliding surface compared with the non-singular terminal sliding surface of [1] can converge to origin in the finite time in the existence of measurement noise and the abrupt change of disturbance. Figure 10 shows the control inputs achieved by the planned methodology and the technique of [1] under measurement noise and abrupt change of disturbance. Thus, the validity and proficiency of the suggested controller with respect to another technique are proven.

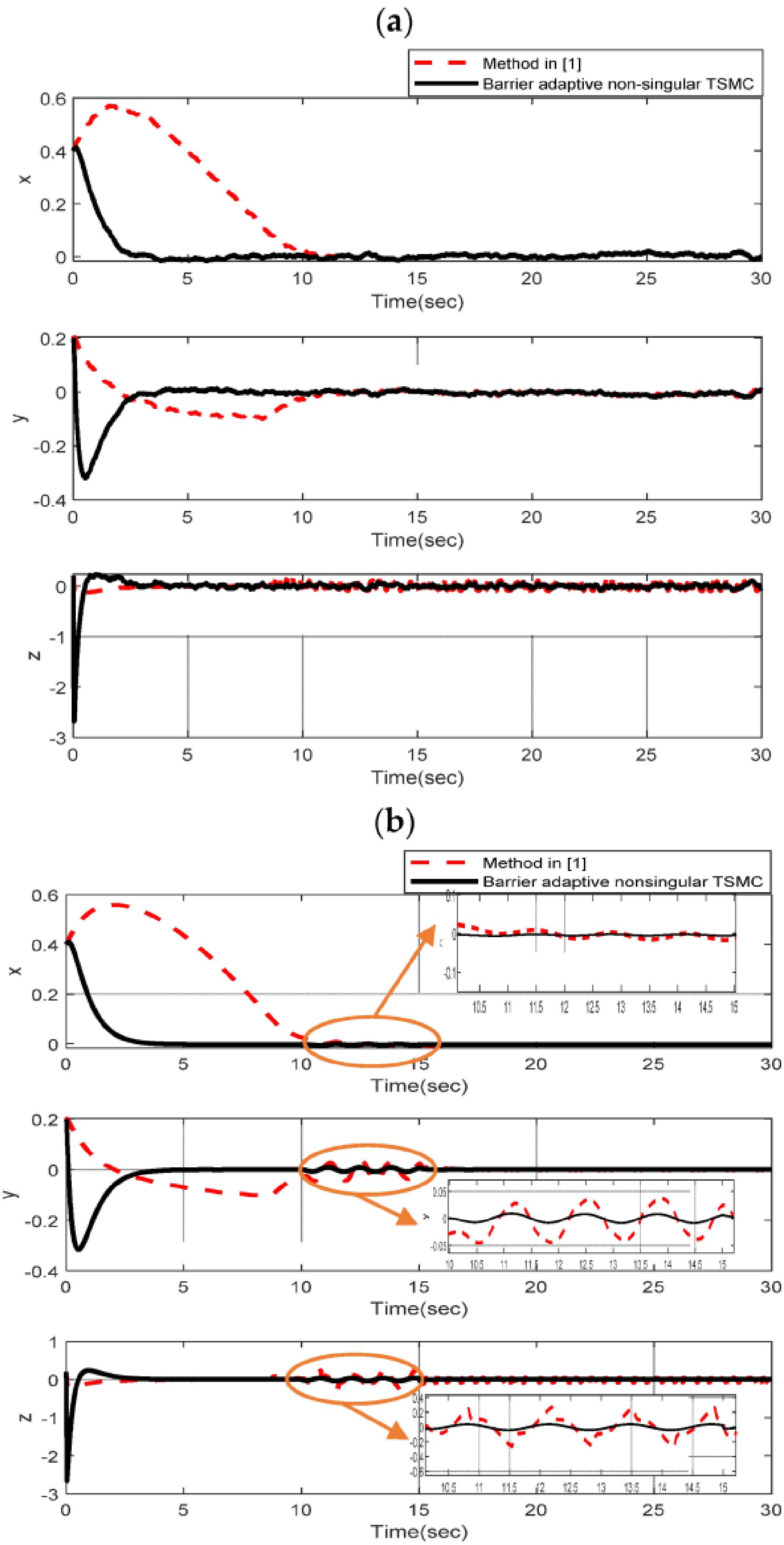

Figure 7. State trajectories of a system (**a**) with noise (**b**) in the presence of an abrupt change in disturbance.

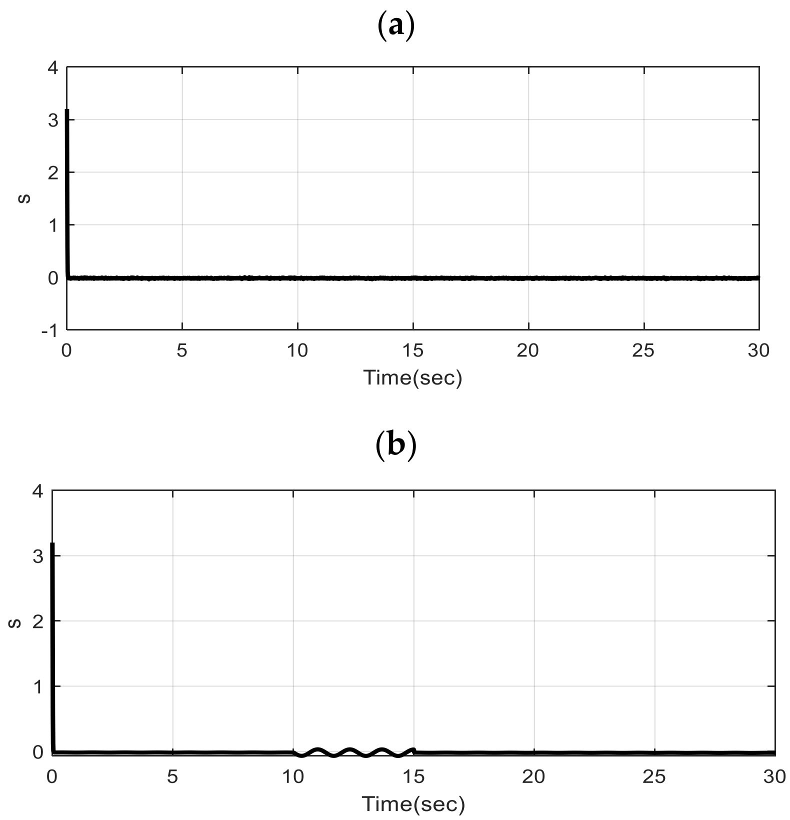

Figure 8. Time histories of a nonlinear sliding surface (**a**) with noise (**b**) in the presence of an abrupt change in disturbance.

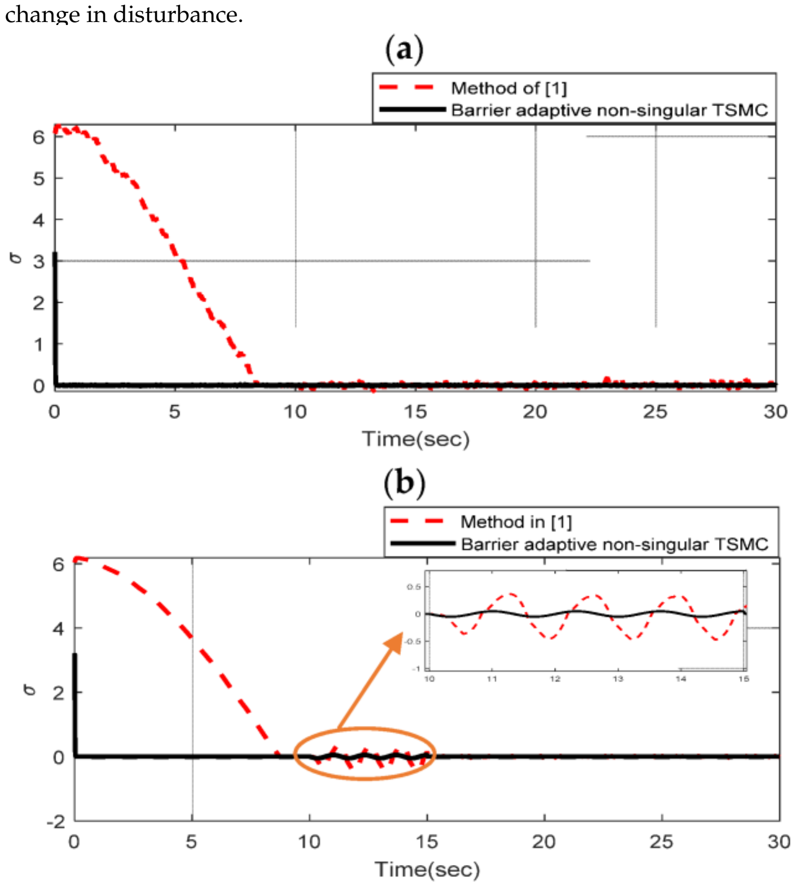

Figure 9. Time responses of a non-singular terminal sliding surface (**a**) with noise (**b**) in the presence of abrupt change in disturbance.

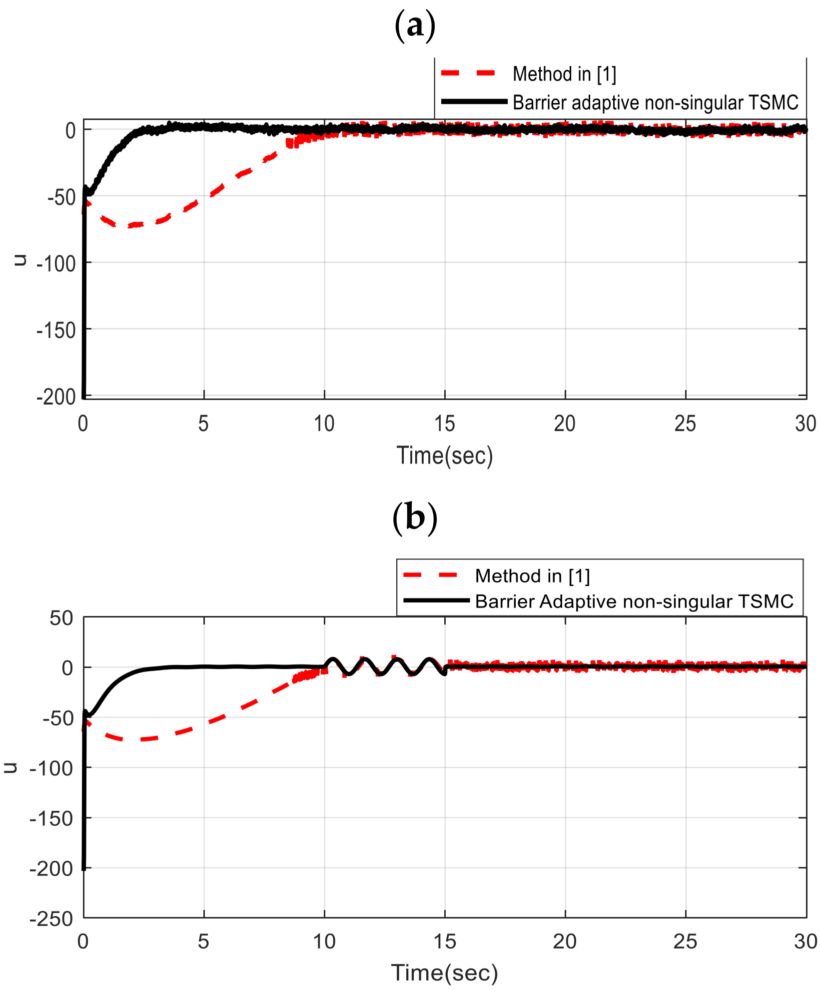

Figure 10. Trajectory of control inputs (**a**) with noise (**b**) in the presence of an abrupt change in disturbance.

5. Conclusions

In this paper, the robust performance improvement of a nonlinear under-actuated system under external perturbation has been investigated using the barrier function adaptive non-singular terminal sliding mode control method. A nonlinear under-actuated system in the presence of unknown bounded exterior perturbation has been considered. In the target of the stabilization control of system states, a novel nonlinear sliding surface with respect to the states of the system has been defined. Thus, the states of the system can reach origin using this nonlinear sliding surface. Moreover, a non-singular terminal sliding surface has been proposed to force the nonlinear sliding surface to be converged to origin in the finite time and to remain on it. The finite-time convergence of the non-singular terminal siding surface has been proven using the Lyapunov theory. For the robustness enhancement of the system against the exterior disturbances, the barrier function adaptive procedure has been adopted to estimate the unknown upper bounds of perturbations. Thus, the controlled system is robust against variations of external disturbances. At last, validity verification of the suggested method has been done using the simulation results on the plasma torch jerk chaotic system.

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