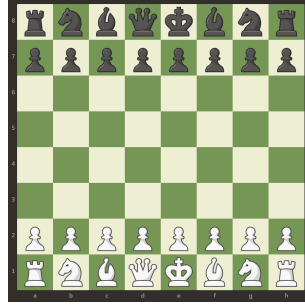


On Primitive Recursive Characteristics of Chess Supplementary Materials

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1 Piece and Position Encodings

Figures S1 and S2 from the main text of the article are repeated for ease of reference.



26	27	28	29	30	31	32	33
18	19	20	21	22	23	24	25
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17

Figure S1: Starting chess board (above) encoded as G-number b (below).

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Figure S2: Chess board positions.

2 Pairing

Every entry in the tables in Sections S3 and S4 below (Tables S1 – S6) is a natural number obtained with the pairing function defined in Equations (5) and (6) in the main text of the article. Thus, the top left and the bottom right entries in Table S1 are

$$\langle (2, 1), 37 \rangle = 153599;$$

$$\begin{aligned} \langle (9, 8), 47 \rangle = & 69466572136058514208415872544528802565625 \\ & 70577153014974960414575446172213049323421 \\ & 02633976062353111695519659537755468420726 \\ & 02072272114148033823871203883006537028435 \\ & 48168715321370271536199948398665525356790 \\ & 90261705244560394813252059630366957050144 \\ & 10463412405898749520163940968000465706684 \\ & 98118498927982883757846068893707040339362 \\ & 06470861534807436502575453800128128264386 \\ & 76244619399153688981412686851313061638911 \\ & 45075743494744079640280634406193672628203 \\ & 78808046341656366346236774252332189439939 \\ & 73373304143014671785270144021101847218762 \\ & 88098898779205152819124885590692251662709 \\ & 55250939685654230023836159370338086019892 \\ & 40129939162119665016145207868912916975034 \\ & 97068294925894449032737178165476863871011 \\ & 46351677825595545646806300185144004810056 \\ & 70111657067396948764954735870414538308931 \\ & 93117130546318562974619403690698766776372 \\ & 81334858932320563157238847812786172743537 \\ & 94281434855884904263681578864769874693777 \\ & 81231273752255191525112082780174864566554 \\ & 79779556775006171542210738196236808272552 \\ & 21313026218583404586674943753631620321677 \\ & 01944395345924255304597899976670261263011 \\ & 21671282486698599781163124551671406551657 \\ & 98564066827092774652044138306757406990702 \\ & 25700362813686891458837884959735709705005 \\ & 40156984130166665186004669493698528852586 \\ & 81157785047689191539898929562857069698697 \\ & 04076239461957908025429345046982978179860 \\ & 03354774877122791026040591848478834031462 \\ & 78417472993718462380055354511111481056031 \\ & 46071922549690949701724804011724007174893 \\ & 34586548883352072573192700048820920604790 \\ & 83698070082141618623680082291001461054065 \\ & 23256887534510146337816927404459033323675 \\ & 72612979924707958263476718204973727896612 \\ & 68557703961289293428972000272354784935054 \\ & 91751642929658633536334981878295753599489 \\ & 75577228186669448899543158297416031021592 \\ & 49331936418904670988976415661147200037056 \\ & 94937046848818190122268110573206040965158 \\ & 62900431444297259400215363733797772567211 \\ & 71394751872723350108388422323080499517435 \\ & 38074858241322387397909280704471350102965 \\ & 61965025316553899038040225489403922533887 \\ & 65273107772189121163401208604408781630349 \\ & 63109003295604283624242325630161180186762 \\ & 53291215452176241205193391787276793062787 \\ & 05399607454383208866201633973785816470860 \\ & 92085739446358511007789938911291432156514 \\ & 27945113986623386834494216901706788531343 \\ & 34010683310963364063178757197273574868047 \\ & 39364416467328259882514408339441222755144 \\ & 62680983356781453468967840371099195224619 \\ & 53502323468944166602561922196131616095526 \\ & 44224476179616485583337273447803438514877 \\ & 53624480440983321859255901790690988149105 \\ & 41614400710374833810323549534430446751427 \\ & 37487932925529428849185677874088753582933 \\ & 02531111492457892155480149811535213603358 \\ & 629244544553533090352415657693030645759. \end{aligned}$$

3 Black and White Metamorphic Pieces

Tables S1 and S2 give the numbers of the white and black metamorphic pieces packed into G-numbers in the main text of the article.

[illegible]

Table S1: Numbers of white metamorphic pieces packed into G-number ζ_μ^w in the main text of the article.

$\langle\langle 2, 2 \rangle, 43\rangle$	$\langle\langle 2, 4 \rangle, 43\rangle$	$\langle\langle 2, 6 \rangle, 43\rangle$	
$\langle\langle 3, 2 \rangle, 43\rangle$	$\langle\langle 3, 4 \rangle, 43\rangle$	$\langle\langle 3, 6 \rangle, 43\rangle$	$\langle\langle 3, 8 \rangle, 43\rangle$
$\langle\langle 4, 2 \rangle, 43\rangle$	$\langle\langle 4, 4 \rangle, 43\rangle$	$\langle\langle 4, 6 \rangle, 43\rangle$	$\langle\langle 4, 8 \rangle, 43\rangle$
$\langle\langle 5, 2 \rangle, 43\rangle$	$\langle\langle 5, 4 \rangle, 43\rangle$	$\langle\langle 5, 6 \rangle, 43\rangle$	$\langle\langle 5, 8 \rangle, 43\rangle$
$\langle\langle 6, 2 \rangle, 43\rangle$	$\langle\langle 6, 4 \rangle, 43\rangle$	$\langle\langle 6, 6 \rangle, 43\rangle$	$\langle\langle 6, 8 \rangle, 43\rangle$
$\langle\langle 7, 2 \rangle, 43\rangle$	$\langle\langle 7, 4 \rangle, 43\rangle$	$\langle\langle 7, 6 \rangle, 43\rangle$	$\langle\langle 7, 8 \rangle, 43\rangle$
$\langle\langle 8, 2 \rangle, 43\rangle$	$\langle\langle 8, 4 \rangle, 43\rangle$	$\langle\langle 8, 6 \rangle, 43\rangle$	$\langle\langle 8, 8 \rangle, 43\rangle$
$\langle\langle 9, 2 \rangle, 43\rangle$	$\langle\langle 9, 4 \rangle, 43\rangle$	$\langle\langle 9, 6 \rangle, 43\rangle$	$\langle\langle 9, 8 \rangle, 43\rangle$

Table S4: Numbers of metamorphic dark-color white bishops packed into G-number $\zeta_{\mu}^{dc,wb}$ in the main text of the article.

$\langle\langle 18, 58 \rangle, 61\rangle$	$\langle\langle 18, 60 \rangle, 61\rangle$	$\langle\langle 18, 62 \rangle, 61\rangle$	
$\langle\langle 19, 58 \rangle, 61\rangle$	$\langle\langle 19, 60 \rangle, 61\rangle$	$\langle\langle 19, 62 \rangle, 61\rangle$	$\langle\langle 19, 64 \rangle, 61\rangle$
$\langle\langle 20, 58 \rangle, 61\rangle$	$\langle\langle 20, 60 \rangle, 61\rangle$	$\langle\langle 20, 62 \rangle, 61\rangle$	$\langle\langle 20, 64 \rangle, 61\rangle$
$\langle\langle 21, 58 \rangle, 61\rangle$	$\langle\langle 21, 60 \rangle, 61\rangle$	$\langle\langle 21, 62 \rangle, 61\rangle$	$\langle\langle 21, 64 \rangle, 61\rangle$
$\langle\langle 22, 58 \rangle, 61\rangle$	$\langle\langle 22, 60 \rangle, 61\rangle$	$\langle\langle 22, 62 \rangle, 61\rangle$	$\langle\langle 22, 64 \rangle, 61\rangle$
$\langle\langle 23, 58 \rangle, 61\rangle$	$\langle\langle 23, 60 \rangle, 61\rangle$	$\langle\langle 23, 62 \rangle, 61\rangle$	$\langle\langle 23, 64 \rangle, 61\rangle$
$\langle\langle 24, 58 \rangle, 61\rangle$	$\langle\langle 24, 60 \rangle, 61\rangle$	$\langle\langle 24, 62 \rangle, 61\rangle$	$\langle\langle 24, 64 \rangle, 61\rangle$
$\langle\langle 25, 58 \rangle, 61\rangle$	$\langle\langle 25, 60 \rangle, 61\rangle$	$\langle\langle 25, 62 \rangle, 61\rangle$	$\langle\langle 25, 64 \rangle, 61\rangle$

Table S5: Numbers of metamorphic light-color black bishops packed into G-number $\zeta_{\mu}^{lc,bb}$ in the main text of the article.

$\langle\langle 18, 57 \rangle, 61\rangle$	$\langle\langle 18, 59 \rangle, 61\rangle$	$\langle\langle 18, 61 \rangle, 61\rangle$	$\langle\langle 18, 63 \rangle, 61\rangle$
$\langle\langle 19, 57 \rangle, 61\rangle$	$\langle\langle 19, 59 \rangle, 61\rangle$	$\langle\langle 19, 61 \rangle, 61\rangle$	$\langle\langle 19, 63 \rangle, 61\rangle$
$\langle\langle 20, 57 \rangle, 61\rangle$	$\langle\langle 20, 59 \rangle, 61\rangle$	$\langle\langle 20, 61 \rangle, 61\rangle$	$\langle\langle 20, 63 \rangle, 61\rangle$
$\langle\langle 21, 57 \rangle, 61\rangle$	$\langle\langle 21, 59 \rangle, 61\rangle$	$\langle\langle 21, 61 \rangle, 61\rangle$	$\langle\langle 21, 63 \rangle, 61\rangle$
$\langle\langle 22, 57 \rangle, 61\rangle$	$\langle\langle 22, 59 \rangle, 61\rangle$	$\langle\langle 22, 61 \rangle, 61\rangle$	$\langle\langle 22, 63 \rangle, 61\rangle$
$\langle\langle 23, 57 \rangle, 61\rangle$	$\langle\langle 23, 59 \rangle, 61\rangle$	$\langle\langle 23, 61 \rangle, 61\rangle$	$\langle\langle 23, 63 \rangle, 61\rangle$
$\langle\langle 24, 57 \rangle, 61\rangle$	$\langle\langle 24, 59 \rangle, 61\rangle$	$\langle\langle 24, 61 \rangle, 61\rangle$	$\langle\langle 24, 63 \rangle, 61\rangle$
	$\langle\langle 25, 59 \rangle, 61\rangle$	$\langle\langle 25, 61 \rangle, 61\rangle$	$\langle\langle 25, 63 \rangle, 61\rangle$

Table S6: Numbers of metamorphic dark-colored black bishops packed into G-number $\zeta_{\mu}^{dc,bb}$ in the main text of the article.