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# Why Subsidize Independent Schools? Estimating the Effect of a Unique Canadian Schooling Model on Educational Attainment

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**Abstract:** Canada is recognized as one of the top 10 countries in secondary education according to PISA results. A particularly intriguing case in this country is the large system of highly subsidized independent schools in the province of Québec where students also perform extremely well in PISA testing. This paper uses the year 2000 PISA cohort of 15-year-olds in Québec to estimate the ATT effect of independent schooling on educational attainment. We find large, positive, robust, and statistically significant effects of independent schooling on attainment. The robustness of the results to omitted variable bias is addressed through a sensitivity analysis for matching estimators.

**Keywords:** YITS; high school graduation; independent schools; postsecondary education and professional programs enrollment and graduation; longitudinal data; treatment effect; entropy balancing



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#### 1. Introduction

There is undoubtedly a movement taking place around the western world in recent decades towards liberalizing schooling choices. Organizational changes in the education systems of OECD countries have taken place mostly in the 2000s, with governments intending to promote competition and quality among schools, and to improve students' results [1–3]. These decades saw the proliferation of charter schools in the United States, independent or subsidized religious schools in Sweden and in the Netherlands (see Böhlmark and Lindahl [4]), academies or foundations with public subsidies in England [5–7], with a variety of independent schools partially or completely subsidized by the state (e.g., France who subsidizes almost all schools, including those affiliated with religious denominations).

There are several studies across the world that consider the impact of independent schooling on achievement. Across all countries (see Table 2.8 of Hanushek and Woessman [8]), independent-school management tends to be positively associated with student achievement, corresponding to a difference, relative to publicly operated schools, of 16–20 percent of a standard deviation in the three subjects in PISA 2000 (math, reading, and science). Similar results are found with PISA 2003 [9].

This paper seeks to add to this literature by presenting estimates of the effect of independent schooling in high school on educational attainment in the province of Québec, Canada's second most populous province. More specifically, we estimate the average treatment on the treated effect (ATT) on educational attainment, with the treatment being attendance at an independent high school in Québec, and the treated being those attending an independent school in Québec.

In Québec, there has been a recognition of the right for two non-confessional education systems (public and independent, with very few non-subsidized independent schools) to coexist since the early 1980s. The public financing of the independent system is judicially guaranteed, making the province a social laboratory for the main topic of this paper. The educational system and high public subsidies paid to independent schools in Québec provide the opportunity for middle-class households to educate their children

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in independent institutions. In fact, in our sample of 15-year-old high school students, approximately 20% attend independent schools; clearly not all of these students are from very privileged households. Therefore, with the right data, good matches can be found in the public-school system for a large proportion of independent-school attendees so that matching or weighting methods can be efficiently used for estimation.

We use novel statistical techniques that we feel have been underused in previous studies seeking to estimate treatment effects in which the treatment is clearly endogenous. The principal method we use for estimation is called Entropy Balancing (EB). This method works very well to produce balancing weights that render the control group means of explanatory variables (matching variables) that cause changes in both outcome and treatment practically identical to the treatment group means, thus the term balancing. Matching methods can also be construed as a weighting method for estimating treatment effects. However, there is no guarantee that the weights produced will perfectly balance the means of treatment and control groups as in the case of EB. This is conditional, of course, on the EB algorithm, described in the methods section, finding a solution to the optimization problem to achieve the balancing weights. Once the weights are computed by the EB algorithm, the ATT is simply the mean outcome of the treated minus the mean outcome of the EB-weighted sample of the untreated.

This approach supposes that selection into each treatment is random and conditional on observable variables; hence, like matching methods, it necessitates a large number of key conditioning variables to produce credible estimates. For example, in Québec, parents must pay tuition fees (although these are relatively very low due to the very high public subsidies; see Tables A3 and A4) for their children to attend independent schools and therefore are on average from wealthier households. Hence, income data on families are crucial for our estimation of the effects of independent schooling. The education and social class of parents are also important factors in school choice. Finally, some independent schools choose students based on their results in admission tests. Therefore, children observed entering independent schools possess higher human capital and abilities than children in public schools because of these simple mechanisms.

Even in the case of data with a rich set of explanatory variables, estimation of treatment effects is subject to omitted variable bias, i.e., a variable missing in the set of observable variables used for conditioning the treatment. In this case, the conditional independence assumption (CIA) is violated. Therefore, it is crucial that some form of sensitivity analysis be performed to ascertain the size and direction of the bias (see Rosenbaum [10]).

To perform this exercise, we turn to the work of Nannicini [11]. He shows how the simulation of a confounding variable can be employed to assess the sensitivity of our estimates to omitted variable bias. The method consists in including a generated binary variable in the group of explanatory variables, which is both linked to treatment assignment and to the dependent variable of interest. The estimate of the ATT is then re-estimated for each simulation of the confounder. A comparison between the estimates without the confounder and the average ones obtained with the simulations provides a measure of the robustness of the matching estimator [11,12]. A more thorough explanation of the method appears in the Methods section.

We find that there are strong positive effects of independent schooling on postsecondary enrollment in the province of Québec. We also find that in general the results are not very sensitive to omitted variable bias. Other robustness exercises performed in the paper confirm the credibility of our results.

It is important to point out that more than 90 percent of students who transit from public to independent schools in Québec do so following the last year of elementary schooling (grade 6 in Québec). Furthermore, very few drop out of independent school to attend public schools (see [13]). Therefore, a large majority of students who attend independent high schools do not attend a public school at any time during their high school years. Lefebvre et al. [13] also show that very few students attend independent schools

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at the elementary level (also see Table A2). The treatment effect estimated in this paper is therefore very similar to the effect of attending an independent high school for five years.

It must be noted, in the latest PISA cycle in 2018, the Province of Québec ranked, as a region, fifth in the world in math, seventh in science, and eighth in reading (O'Grady et al. [14]; see Table A1). These rankings were higher than most of the other Canadian provinces (in math it is the top province, and in science it is ranked second in Canada). These results are remarkable, even more so given that Québec ranked fifth in 2000 for per capita GDP in Canada (52nd when considering all states and provinces in Canada and the United States). Furthermore, these numbers are not outliers as Québec's 15-year-olds have been doing well since the creation of the first PISA samples in 2000. Additionally, inequality measures are very favorable to Québec relative to other Canadian provinces, and this in a country that already does well along these metrics (see Lefebvre and Merrigan [15]). This excellent performance could be tied to the long-standing commitment in Québec to independent schools.

Other Studies on the Effects of Independent Schooling

In the United States, several studies have attempted to measure the effect of independent (essentially Catholic) schools, and more recently, that of charter schools with public funding, on achievement. As for elementary Catholic schools, recent studies show negative effects on reading and mathematics in the United States (for example, Elder and Jepsen [16]). The results in Morsya, Khavensonb, and Carnoy [17] indicate that attending a Catholic or independent school rather than a public school had no significant relationship with students' performance in either the PISA math or reading tests when students' and schools' socioeconomic status (SES) are controlled for in the regression analysis.

In the case of charter schools, there are a large number of studies considering their recent expansion. Those that use data from schools employing a lottery-based admission process find positive and significant effects on results in mathematics and reading, notably in urban settings where students hail from lower socioeconomic backgrounds [18–24].

Canada, where independent schools play an important part in the country's numerous education systems, has received significant attention worldwide because 15-year-old students achieve much higher average PISA scores than those of most OECD countries (regularly in the top 10 or 15 countries or regions in reading, math, and sciences; see Table A1) with an apparently equitable distribution of educational achievement (see Coughlan [25]). It has many similarities with other Western nations' education systems; however, the country has unique cultural, linguistic, and historical attributes.

Card, Dooley, and Payne [26] estimate positive significant effects of Catholic school attendance on the governmental test scores of elementary school students from the Province of Ontario where parents have the choice between two fully funded public systems. One is open to all and the other is restricted to students with a Catholic ancestry (although school boards do admit non-Catholic students if there are an adequate number of spaces). Furthermore, using longitudinal administrative data from the elementary school system of British Columbia (a western province of Canada), Azimil, Friesen, and Woodcock [27] find that independent schools (secular or confessional) lead to significantly higher standardized test results in reading and mathematics.

Some studies have provided evidence that attending independent schools in Québec causally (average treatment effect on the treated) improves scores in international cognitive tests such as PISA (see Lefebvre [28,29]) or other math tests (see Lefebvre, Merrigan, and Verstraete [13]). This paper takes these studies a step further to answer some additional questions.

Most studies based on PISA (see Vandenberghe and Robin [30]) do not find significant effects of independent schools on standardized scores. Canada, in these studies, is sometimes classified as a country with low independent-sector attendance (8%). (Frenette and Chan [31] analyze the differences between students from both sectors in many provinces in terms of a few indicators of academic performance (PISA 2000 scores, high school grad-

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uation, postsecondary enrollment and graduation) with a very particular sample that is not representative of the Québec education system (see Lefebvre, 2015). Our study focuses exclusively on Québec, where a much larger proportion of students attend private schools and where the treatment effect is better understood as 5 years in an independent high school). This is certainly not the case for the province of Québec, where, from administrative data, we observe that the percentage of students in independent high schools was around 20% in the mid-2000s (see Table A2).

Differences in postsecondary enrollment, and university attendance and graduation may be linked to differences in the academic achievement of students based on their socioeconomic status in high school [32–34]. Some recent studies demonstrate that skills acquired during adolescence, notably mathematical competencies, are stronger predictors of educational completion than measures of non-cognitive abilities. A rise of one standard deviation in mathematics scores is thus associated with multiple years of additional education [35–38]. The evidence suggests that differences in cognitive competencies acquired early and linked to income and family education are probably important mechanisms through which socioeconomic status is transmitted between generations. Because of the evidence that independent schools in Québec increase test scores, it is reasonable to ask whether they causally improve educational attainment.

The structure of this paper is as follows: following the Introduction, Section 2 presents the methods and data. Section 3 presents the results, with a discussion of the latter given in Section 4, followed by a very brief conclusion.

## 2. Materials and Methods

In Appendix A, we describe at length Québec's independent-school system, showing that it is well established in the province at the secondary level with a substantial proportion of students attending numerous establishments. Most importantly, we explain why schools' selection of students on the basis of admission tests should not be of great concern in our analysis. Also, it must be stressed again that Québec's educational system is non-confessional and that independent schools admit individuals from all backgrounds. Therefore, we do not believe that controls for religion, either faith or intensity of worship, would impact the results we present later in Section 3.

Our methodology can be described as follows: formally, let D denote a binary indicator of treatment, Y the outcome, and X a vector of observable covariates. The ATT compares the average outcome of the treated group (D=1) to that of a sample of non-treated individuals with the same distribution of X as the treated. These estimators assume that the potential outcome in the non-treated state is independent of treatment conditional on X. There are many types of estimators proposed in the literature on treatment effects (see Imbens [39]). Many exploit the conditional probability of treatment (P(D=1|X)), also known as propensity scores. These types of ATT estimators can be semiparametric or nonparametric and use propensity scores in a matching procedure. Other types of nonparametric estimators directly employ X to determine the weight assigned to each untreated observation without estimating propensity scores.

EB constructs the counterfactual mean outcome of the treated if untreated (Hainmueller [40] and Hainmueller and Xu [41]). This method seeks to balance covariates between groups with a maximal entropy weighting scheme. (Frölich, Hubert, and Wiesenfarth [42] analyze the performance of a large group of treatment effect estimators, both semiparametric and non-parametric, on the basis of mean quadratic error. The EB estimators are part of the group with the best performance.) When successful, the reweighting of untreated subjects offers an exact balancing of the specified moments of the distribution of every element of X. The weights are found using a numerical optimization procedure that produces weights as a solution. Instead of relying on a propensity score model, EB uses analyst-supplied base (initial) sampling weights. Re-estimated weights are then calculated in order to minimize the Kullback–Leibler divergence from the initial weights, subject to balancing constraints. Hainmueller [40] draws attention to the fact that the estimator could

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have a large variance when a few observations receive large weights. The same can be said of estimators based on inverse propensity score weighting. These extreme weights could be caused by a significant difference in the covariate distributions between groups.

Formally, the procedure is the following:

$$\min_{\omega_i} \sum_{\{i:D_i=0\}} h(\omega_i) \tag{1}$$

Subject to the balancing constraint in (3),

$$\sum_{\{i:D_i=0\}} \omega_i X_i = \frac{1}{N_1} \sum_{\{i:D_i=1\}} X_i, \text{ for all } X$$
 (2)

And normalization constraints,

$$\sum_{\{i:D_i=0\}} \omega_i = 1 \text{ and } \omega_i \ge 0, \ \forall i, \ D_i=0, (4)$$
(3)

where  $\omega_i$  is the weight to be estimated for individual i, h(.) is a distance metric, X is a covariate, and D is the treatment indicator. Hainmueller [43] suggests the Kullback–Leibler function  $h(\omega_i) = \omega_i \log\left(\frac{\omega_i}{q_i}\right)$  as the loss function  $h(\omega_i) = \omega_i \log\left(\frac{\omega_i}{q_i}\right)$ . It measures the difference between the distribution of the estimated weights  $\omega_1, \ldots, \omega_{iN_0}$  and the base weights, in our case, sample probability weights supplied by Statistics Canada.

For the first stage of the estimations presented in this paper, weights are generated with EB in order to balance the selected moments of the chosen covariates between the two groups while minimizing the distance from the base sampling weights supplied by Statistics Canada. Therefore, the algorithm finds weights that will ensure equality between the sample-weighted moments of the treated Xs and the EB-weighted moments of the untreated. In the second stage, a weighted linear regression of Y on D is performed to estimate the treatment effect (ATT). For almost all of the covariates, which are dummy variables, we require only that the first moment of the distribution be balanced. In the case of family income, equality of the second moment is also imposed for this specific covariate.

Specifically, the data used for estimations are provided by the Statistics Canada Youth in Transition Survey (YITS) cohort A, cycles 1–4 (2000–2006). The sample used for the estimations was restricted to individuals residing in Québec in cycle 1 and students who are in grades 9 or 10. The second restriction is imposed because almost no student in independent schools repeated a grade, and because we are estimating the ATT effect of independent schooling (students repeating a grade are very poor matches for independent-school students and are in a grade lower than the ninth grade).

For each estimate, all original respondents present in the cycle of interest (i.e., those for which the outcome is available) are included, without consideration of their potential future nonresponse in later waves. Sample and replicate weights used for inference are those provided by Statistics Canada, adjusted for nonresponse in each cycle.

Probit estimations were performed to ascertain which factors are susceptible to affect nonresponse probabilities. Results show that the only available variable that is statistically significant is family income, which positively affects the probability of response. As such, the sample is likely to be biased in favor of individuals coming from higher income families in cycles following cycle 1. Given the strong correlation between income and educational attainment, the results obtained from the study's estimates are very likely to be conservative. Individuals from lower socioeconomic backgrounds appear to be under-sampled in later cycles and would have negatively affected public-school average results had there been no attrition.

The sensitivity to omitted variables analysis method (SENSATT available with STATA software) employed in this paper is based on the hypothesis that the CIA holds when a

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binary confounding factor U is taken into account, alongside the observable variables W, and  $Y_0$ ,  $Y_1$ , the potential outcomes, i.e., the outcome we would observe if an individual is assigned to treatment 1, as well as the one observed if the assigned treatment is 0. So,

$$Pr(T = 1|Y_0, Y_1, W) \neq Pr(T = 1|W)$$
 (4)

while:

$$Pr(T = 1|Y_0, Y_1, W, U) = Pr(T = 1|W, U).$$
(5)

The distribution of the unobserved binary confounding factor U is characterized by the parameters

$$p_{ij} \equiv \Pr(U = 1 | T = i, Y = j, W) = \Pr(U = 1 | T = 1, Y = j)$$
 (6)

with  $i,j \in \{0,1\}$  giving the probability that U = 1 in each of the four groups defined by the treatment and outcome status.

This method has several advantages. First, the hypothetical link between Y, U, and T is stated in proportions characterizing the distribution of  $U \mid T$ , Y, W, thus avoiding an invalid parametric specification of  $Y \mid T$ , U, W. Second, the parameters  $p_{ij}$  can be specified in a way as to mimic the distribution of some observed binary covariate, allowing the econometrician to determine the robustness of the initial estimates to deviations from the CIA. Third, one can set up the parameters  $p_{ij}$  in a way that drives the ATT down to zero, and then assess the plausibility of their distribution. This exercise seeks to determine the likelihood of killer unobserved confounders. These correspond to a simulated omitted variable that will render our treatment effect insignificant statistically. Once a killer confounder is found, odd ratios for the outcome and treatment equations for this confounder are estimated by using a matching method. When very high odd ratios relative to observed covariate odd ratios are necessary to "kill" the significant treatment effect, we conclude that the estimated treatment effect is robust. Lastly, the SENSATT method can be employed regardless of the algorithm used to match the observations, when matching methods are used for estimation.

What are the unobserved variables that may cause our estimation method to produce a biased estimate of independent schooling? Potential candidates include unobserved ability, or unobserved characteristics of the parents that might increase an individual's human capital, such as stimulating his or her interest in reading, sciences, or mathematics. These factors would, of course, increase both the probability a child graduates from high school or a postsecondary institution and the probability he or she attends an independent school. Parents who observe a child's ability in grade school and perceive that he or she is academically gifted may be induced to send him or her to a high school where most of the students are skillful. In addition, parents who send their child to an independent school possibly value education and thus probably spend more money on goods and services that will increase the human capital of their child. We therefore assume that this omitted variable is strongly positively correlated with both independent schooling and the outcome. The simulated variable is calibrated using the observed correlation between income, independent schooling, and outcomes in cohort A as it is highly correlated with the latter two variables. The SENSATT procedure is performed with a kernel matching procedure; however, because this procedure produces results very similar to EB, we believe it is an appropriate sensitivity exercise for the EB estimates.

Table A5, constructed from official administrative data on all students in Québec entering high school, displays the graduation rate (high school diploma and other qualifications), by time span in years since entering high school (5, 6, or 7), gender, type of school (independent and public), and language of instruction (French/English). For each cohort (2001 to 2007), graduation by the number of years since entering high school is computed by the Ministry of Education, for 5, 6, and 7 years (5 and 6 years respectively for the 2008 and 2009 cohorts). These administrative statistics indicate very important differences between independent and public system graduation rates, showing much larger rates in the independent system.

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Table 1 presents descriptive statistics on high school graduation rates, both for an unrestricted and a restricted sample (the latter for estimation). The computed rates with YITS data sets in the unrestricted sample are consistent with population data of the later cohorts in Table A5, with large gaps between independent and public schools. Québec's Ministry of Education does not provide longitudinal administrative data on educational trajectories of youths leaving high school (graduated or not). The usual annual province-wide statistics on enrollment and graduation rates in postsecondary institutions cannot identify these trajectories according to the characteristics of high school students (cohort, gender, language of instruction, or type of school). Table 2 computed with YITS data sets offers unique statistics on these trajectories. Significant gaps between independent and public-school students in favor of independent-school students are observed for postsecondary enrollment.

**Table 1.** High school graduation rate by sex and type of school, cohort A of the YITS.

		All			Male			Female			
	All	Ind.	Public	All	Ind.	Public	All	Ind.	Public		
						Cohort A n	o restriction				
%	66.3	84.7	62.6	60.7	81.6	56.2	72.2	88.4	69.2		
S-D	0.8	1.5	0.9	1.2	2.2	1.3	1.1	2	1.3		
N	4043	722	3321	2090	394	1696	1953	328	1625		
						Cohort A wi	th restriction				
%	73.9	86	71.2	69.5	83.8	66	78.3	88.5	76.2		
S-D	0.8	1.5	0.9	1.2	2.1	1.4	1.1	2	1.2		
N	3270	629	2641	1639	346	1293	1630	293	1337		

Notes: The restriction excludes students who have repeated grades during their studies; Ind: Independent; S-D: standard deviation. Source: Authors' computations from cycles 1–3 YITS weighted data sets.

**Table 2.** Enrollment and graduation rate by sex, cycle, level of studies, type of high school, and difference by school type, percentages, and number of observations.

		All			Public	2	In	depend	lent	Indep	endent-	Public
Level of studies	All	M	F	All	M	F	All	M	F	All	M	F
					Coho	rt A cyc	le 2 (17	years (	old)			
Graduation high school	74	70	78	71	66	76	86	84	89	15	18	13
C					Coho	rt A cyc	le 3 (19	years	old)			
Enrollment CEGEP/university	71	65	77	67	61	73	88	82	95	21	21	22
Enrollment prof. program	14	24	10	12	21	9	19	29	12	7	8	3
1 1 0					Coho	rt A cyc	le 4 (21	years	old)			
Enrollment CEGEP/university	72	66	78	68	61	$74^{\circ}$	91	85	97	23	24	23
Enrollment university	33	25	40	28	20	35	54	46	65	26	26	30
Enrollment prof. program	17	26	12	13	17	12	25	41	13	12	24	1

Notes: Enrollment prof. program is enrollment in a professional program at the university level. M: male; F: female. Independent-Public: difference between independent and public-school rates.

The Cohort A respondents are aged 15 in cycle 1 (2000) of the YITS-PISA survey. Four outcomes of interest are identified for estimation: 1. Graduated from high school 5 years and 7 years after entering high school, respectively, computed in cycles 2 (2002) and 3 (2004); 2. Enrolled in CEGEP (acronym for "Collège d'enseignement géneral et professionnel") or university at ages 19 and 21, respectively (at cycles 3 and 4, 2004 and 2006). Graduating from CEGEP is an almost necessary pathway to university. The two-year CEGEP programs are geared towards admission to universities, while three-year programs lead to a technical degree. Several students stop schooling once they have received a three-year technical degree (e.g., police officers, lab technicians, and nursing auxiliaries); 3. Attended university at age 21, as in general, students have graduated from CEGEP by this age (cycle 4, 2006); 4. Enrolled in a program leading to an occupation regulated by professional orders in Québec (medicine, law, engineering, etc.) in cycle 4 (for a sample

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of students attending university). For respondents born after September 30, cycle 3 was used in order to determine whether high school graduation occurred at the expected time as their admission to kindergarten was delayed by one year.

For each of these outcomes, all models are estimated using the same set of covariates with their cycle 1 values. The following covariates are used in the analysis: the student's gender; his or her age in months; the presence in the household of a family member born outside of Canada; his or her mother's education level (no diploma; high school diploma; college diploma; or university degree); and language spoken at home (French, English, or other). Our measure of the child's socioeconomic status is the parents' highest international socioeconomic index (ISEI) as measured by PISA analysts. This measure, frequently used in sociological analysis, attributes a score of between 11 and 90 to different occupations based on professional characteristics, such as the required level of education and associated income. The index's creators (Ganzeboom et al. [43]) aimed to improve the measure of socioeconomic status for research purposes. The index has been intensively used in the literature on socioeconomic gradients The values regroup individuals according to different professions; levels 11-20 include individuals working in service sectors and unskilled workers, while levels 80–90 include highly qualified professions such as physicians, judges, or CEOs. Values are grouped by quintiles for our empirical analysis. We add the following as covariates: household income in thousands of 1999 Canadian dollars; the number of books at home (widely considered as an acceptable proxy for the importance granted to education by the parents along with the student's access to cultural possessions); family status (nuclear or other) along with the number of siblings (none, one, two, or more); and finally parental expectations about the educational attainment of their child. Each of these variables can potentially affect the choice of school and educational attainment.

Table 3 presents means and percentages for the control variables used in the EB procedure. There are striking differences between independent and public-school samples. First, students from independent schools are from wealthier families. The majority are in upper quintiles of the ISEI index (International Socio-Economic Index of Occupational Status). Their households contain on average many more books. The percentage of mothers with a university-level education is more than twice the equivalent percentage for public-school students. A much larger percentage of parents expect their child to obtain a university diploma. Finally, the percentage of children in immigrant households is also much larger for students in independent schools. Not surprisingly, independent-school children are from privileged households and this, as we shall see, explains a large proportion of the gaps in high school graduation rates and postsecondary enrollment.

**Table 3.** Mean characteristics of respondents by type of high school, cohort A cycles 1–2.

High Cahool True	All	Public	Independent
High School Type	100%	80.10%	19.90%
Family income			
Mean	\$64,999	\$61,031	\$82,941
S-D	\$52,263	\$48,594	\$63,384
Male %	49.7	48.6	54.6
Age in months	186	186	186
S-D	3	3	3
Immigrant %	15.8	12.2	33
English %	11	9	15
French %	84	86	73
Two-parent %	72	71	77
Quintiles ISEI %			
1	20	22.4	8.5
2	20	21.3	11.9
3	20	21.2	14.9
4	20	19.0	27.7
5	20	16.2	37.0

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Table 3. Cont.

High School Type	All	Public	Independent
High School Type	100%	80.10%	19.90%
Siblings %			
0	10.7	8.1	10.7
1	47.1	46	47.1
2 or more	42.3	45.9	42.3
Number of books at home %			
Books 1 (0–10)	9.6	10.1	7.3
Books 2 (11–50)	23.2	24.8	16
Books 3 (51–100)	22.6	23.5	18.3
Books 4 (101–250)	22	21.3	25.2
Books 5 (>250)	22.6	20.3	33.3
Mother's education level %			
Less than high school	17.3	19.3	8.3
High school	33.2	35.2	23.9
>High school and <university< td=""><td>27.4</td><td>27.2</td><td>28.5</td></university<>	27.4	27.2	28.5
University diploma or more	22	18.3	39.4
Parental education expectation %			
High school diploma	9.7	11.1	3.2
Postsecondary studies	26.4	29	14.7
University diploma or more	63.9	59.9	82
N	3270	2641	629

Note: Quintiles of occupation values are constructed from values of parental ISEI. Source: Authors' computations from cycles 1–2 of YITS weighted data.

#### 3. Results

## 3.1. General Remarks and Assessments of Robustness

The estimates utilizing the later outcomes related to postsecondary schooling are performed using a smaller number of observations than for secondary-school graduation because of attrition. Tables 4 and 5. present the estimated ATT (multiplied by 100 to give the estimated difference in percentage points) by outcome, cycle, and sample (all observations, males, and females). The first column (EB-NC, or NC for those with no covariates in the post-balancing regression) presents our main findings, which are the ATT estimates obtained with the maximal entropy balancing scheme. The next four columns are estimates obtained for robustness checks.

We perform four exercises to assess robustness with the full sample. The first (column EB-WC, or WC for those with covariates) is simply a weighted regression of outcomes on the treatment dummy and the covariates used for the EB procedure. As the results show, these estimates are nearly identical to the estimates without the covariates, showing the almost perfect balance between samples. Second, we perform the regressions excluding any observations for which the assigned entropy weight is over the 99th percentile of the weight distribution (EB-99). Again, the results are very similar to those including all observations. Third, we estimate the ATT effects using a kernel propensity score matching procedure. Again, these results are generally similar.

Finally, we perform a simulation in which the sensitivity of estimates to omitted variables is assessed using STATA's SENSATT package, written by Nannicini [10]. Despite the use of EB, as for all models in which selection is based on observables, the ATT estimator is biased if a variable that is correlated with both the outcome and independent-school attendance but which is not dependent on the treatment is omitted from the model. In this case, the conditional independence assumption (CIA) is violated. The simulation of a confounding variable is thus employed to assess the robustness of our estimates to omitted variable bias. The method consists in including a generated binary variable in the group of Xs, which is both linked to treatment assignment and to the dependent variable of interest. The ATT is then re-estimated for each simulation of the confounder. A comparison between the estimates without the confounder and the ones obtained by using

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the simulation provides a measure of the robustness of the matching estimator [44,45]. The complex sampling structure of the survey imposes the use of replicated weights supplied by Statistics Canada for inference.

**Table 4.** Estimated effects of independent high school on high school graduation by sex and estimation methods, cohort A cycles 2 and 3, YITS.

Level of Studies	С	Sex	Param.	EB-NC	EB-WC	EB-99	Kernel	Sensatt	Δ%
Graduation high school	2	All	Coef.	0.078 (+++)	0.080 (+++)	0.070 (+++)	0.099 (†††)	0.071 (++)	-28.3
Č			S-D	0.018	0.018	0.017	0.026	0.033	
			N	3.270	3.270	3.251	3.270	3.270	
Graduation high school	2	M	Coef.	0.090 (†††)	0.093 (†††)	0.078 (+++)			
_			S-D	0.028	0.028	0.024			
			N	1.644	1.644	1.634			
Graduation high school	2	F	Coef.	0.066 (†††)	0.071 (+++)	0.077 (+++)			
			S-D	0.021	0.021	0.023			
			N	1.626	1.626	1.615			
Graduation high school	3	All	Coef.	0.036 (†††)	0.036 (†††)	0.033 (†††)	0.019	0.023	+21.1
<u> </u>			S-D	0.012	0.011	0.011	0.017	0.019	
			N	2.817	2.817	2.792	2.817	2.817	
Graduation high school	3	M	Coef.	0.038 (†)	0.039 (††)	0.032 (†)			
<u> </u>			S-D	0.020	0.019	0.018			
			N	1.400	1.400	1.386			
Graduation high school	3	F	Coef.	0.037 (†††)	0.037 (+++)	0.039 (†††)			
			S-D	0.007	0.008	0.009			
			N	1.417	1.417	1.403			
Enrollment CEGEP/university	3	All	Coef.	0.079 (+++)	0.084 (†††)	0.075 (+++)	0.057 (††)	0.056 (†)	-1.75
•			S-D	0.020	0.020	0.021	0.026	0.032	
			N	2.817	2.817	2.792	2.817	2.817	
Enrollment CEGEP/university	3	M	Coef.	0.075 (††)	0.078 (†††)	0.057 (††)			
•			S-D	0.030	0.029	0.027			
			N	1.400	1.400	1.386			
Enrollment CEGEP/university	3	F	Coef.	0.091 (†††)	0.096 (†††)	0.095 (†††)			
•			S-D	0.023	0.023	0.023			
			N	1.417	1.417	1.403			

Notes: C: cycles 2 (17-year-olds), 3 (19-year-olds); M: male; F: female; Param.: Estimated parameter; EB-NC: Entropy balancing no controls; EB-WC: Entropy balancing with controls; EB-99: Entropy balancing removing individuals who have a weight higher than the 99th percentile of the weight distribution; Kernel: kernel matching with propensity scores; SENSATT: estimated with simulation of confounding variable;  $\Delta$ %: difference in percentage between coefficient estimated by kernel matching and SENSATT. †: 90% statistical significance, ††: 95%., †††: 99%.

### 3.2. Estimated Effects of Independent Schooling

Most estimated effects are large and statistically significant (p < 0.01). The first two panels of Table 4 show the results for high school graduation in five and seven years after entry, measured in cycles 2 and 3, respectively. Large and statistically significant (p < 0.01) effects are estimated for graduating in 5 years. The ATT is estimated to be 7.8 percentage points (pp) for the whole sample. Restrictions on gender provide estimates of 9 pp for male and 6.6 pp for female students. The kernel estimate is slightly higher, whilst the effect of the simulated regressor on the estimate is relatively strong, but the ATT remains large and significant at 7.1 pp for this case.

The effect on high school graduation is also estimated in cycle 3, when the respondents are 19 years old, i.e., 7 years after they started high school. The coefficients are much smaller and the estimated gap between treatment and counterfactual narrows substantially as compared to graduation after 5 years. Estimates are 3.6 pp for the whole sample and are practically the same for males and females. For the kernel estimate, the effect is very small and statistically not significant. Therefore, we can surmise that the overall high school graduation rate of the treated is not greatly changed by independent-school attendance.

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**Table 5.** Estimated effects of independent high school on CEGEP or university enrollment by sex and estimation methods, cohort A cycles 3 and 4.

<b>Level of Studies</b>	C	Sex	Param.	EB-NC	EB-WC	EB-99	Kernel	Sensatt	$\Delta\%$
Enrollment CEGEP/university	4	All	Coef.	0.103 (†††)	0.107 (†††)	0.095 (†††)	0.083 (†††)	0.069 (††)	-16.9
			S-D	0.020	0.020	0.021	0.026	0.032	
			N	2.388	2.388	2.366	2.388	2.388	
Enrollment CEGEP/university	4	M	Coef.	0.103 (†††)	0.104 (†††)	0.081 (†††)			
			S-D	0.033	0.033	0.030			
			N	1.185	1.185	1.174			
Enrollment CEGEP/university	4	F	Coef.	0.107 (†††)	0.113 (†††)	0.114 (†††)			
			S-D	0.017	0.017	0.017			
			N	1.203	1.203	1.191			
Enrollment university	4	All	Coef.	0.140 (†††)	0.146 (†††)	0.137 (†††)	0.175 (†††)	0.116 (††)	-33.7
			S-D	0.030	0.031	0.031	0.037	0.046	
			N	2.388	2.388	2.366	2.388	2.388	
Enrollment university	4	M	Coef.	0.152 (†††)	0.151 (†††)	0.134 (†††)			
			S-D	0.042	0.042	0.038			
			N	1.185	1.185	1.174			
Enrollment university	4	F	Coef.	0.132 (††)	0.146 (††)	0.140 (†††)			
			S-D	0.058	0.062	0.050			
			N	1.203	1.203	1.152			
Enrollment prof. program	4	All	Coef.	0.094 (†)	0.093 (†)	0.103 (†)	0.099	0.105	+6.1
			S-D	0.053	0.054	0.055	0.068	0.061	
			N	327	327	324	327	327	
Enrollment prof. program	4	M	Coef.	0.166 (†)	0.171 (†)	0.134			
			S-D	0.099	0.098	0.134			
			N	118	118	117			
Enrollment prof. program	4	F	Coef.	0.035	0.029	0.030			
			S-D	0.045	0.048	0.048			
			N	209	209	207			

Notes: C: cycles 3 (19 years old), 4 (21 years old); M: male; F: female; Param.: estimated parameter; EB-NC: Entropy balancing no control; EB-WC: Entropy balancing with control; EB-99: Entropy balancing removing individuals who have a weight higher than the 99th percentile of the weight distribution; Kernel: kernel matching with propensity scores; Sensatt: estimated simulation with confounding variable;  $\Delta$ %: difference in percentage between coefficient estimated by kernel matching and Sensatt. †: 90% statistical significance, ††: 95%., †††: 99%.

The second outcome evaluated in cycle 3 is attending a community college (CEGEP) or university enrollment. All estimates present in the last panel of Table 4 are significant at the 1% level. The estimated ATT is 7.9 pp for the whole sample, 7.5 pp for males, and 9.1 pp for females. The kernel estimate is smaller but robust to omitted variable simulation.

The first outcome (Table 5) evaluated in cycle 4 is attending a community college (CEGEP) or being enrolled in a university (same as the last in Table 1, but 2 years later). The whole-sample estimate is 10.3 pp, while it is the same for males and 10.7 pp for females. The kernel estimate is smaller and less robust to the simulated omitted variable. However, even with the omitted variable, the effect is rather large at 6.9 pp.

Table 5 starts with the same outcome as the last panel of Table 4 (enrollment in a community college or university), but examining cycle 4 data. The effects are found to be larger than in Table 4. The next outcome is university enrollment. Once again, the estimated

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coefficients (panel 2 of Table 5) are significant at the 1% level in all cases. The estimated effects are 14 pp for the whole sample, 15.2 pp for males, and 13.2 pp for females. In this case, the kernel estimate is slightly higher, but the introduction of the cofounder has a very strong effect on the estimate. However, even for this latter case, the effect is very high, close to 10 pp.

The last outcome analyzed in cycle 4 is professional program enrollment among university students, with results presented in the last panel of Table 5. In the case of the base model, the ATT for the whole sample is 9.4 pp; for males it is 16.6 pp and for females it is 3.5 pp. These are statistically significant at conventional levels for the first two cases, but not significant for females. The kernel estimate is in the same range, and is robust to omitted variable bias but not significant.

This last result is certainly intriguing: independent-school attendance has a large and significant effect on enrollment in professional programs for male university students, while it is null for females. This would imply that the effect operates on a different level depending on gender, as it appears that it encourages male students to pursue a specific academic path, which is to say that mediating factors might differ across genders.

We end this section by discussing killer confounders. Our simulations for graduation in 5 years and university enrollment demonstrate that for all cases, the killer confounder must, ceteris paribus, almost double the probability of both the outcome and the treatment to render the treatment effects not significant. (We do not perform the killer confounder simulations for the outcome of graduating in 7 years from high school. The effect in this case seems to be much weaker and less robust.) Given the number of crucial covariates in the set of controls (income, social class, parental education, and parental expectations), we feel quite confident that independent schooling matters for educational achievement. Therefore, the evidence that independent-school attendance has a strong effect on graduating high school in the required amount of time and on postsecondary enrollment is strong and robust.

#### 4. Discussion

## 4.1. Explaining the Results

The magnitude of the estimated effects of independent-school attendance on various educational outcomes is large in many cases. The effect of independent schooling on the probability of graduating in 5 years is large and rather robust at 7.8 points. The role of observables is extremely important as the raw difference between independent-school and public-school attendees for graduating in 5 years is over 25 points, and the same is true for university attendance. However, the effect on the overall high school graduation rate is very small. This is an interesting result, as the effect on postsecondary attendance is robust at close to 8 points in cycle 3, rising to 10 points in cycle 4. Graduating in 5 years is therefore a key indicator for progression to postsecondary study. The increase of the effect in cycle 4 is partly due to a lower postsecondary retention rate for the public-school students. Therefore, mechanisms that are driving the effect on graduating in 5 years are also certain to be playing some role in postsecondary attendance. A large majority of the public-school comparison group eventually passes the exams to graduate from high school, but several need 6 or 7 years to achieve this goal. Therefore, the important ATT effect of independent schooling is on postsecondary attendance, because the impact on high school graduation, once a respondent is 19 years old, is very small and possibly not significant.

Graduating high school on time, i.e., within a five-year span, might also play a more important role in the province of Québec than elsewhere in Canada. To be admitted to a university, a student must graduate with a CEGEP degree, which takes a minimum of 2 years. Therefore, if a student graduates from high school at age 19 in seven years, he or she will start university at age 21, which could be experienced as very late for students with high discount rates. The CEGEP, in this case, becomes some kind of obstacle for students who ultimately seek a university degree.

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As a final exercise, we estimate the effects for the full sample, with tests scores at age 15 added to the set of balancing covariates, one at a time, and report the ATT estimates for the full sample. Unfortunately, we cannot include all test results as covariates in the estimation as the first 2000 PISA wave focuses on one particular subject, in this case reading. Math and science test scores are reported for half the students, as the students who did not take the math test took the science test and vice versa, with selection being random. Note that the estimates for independent schooling are no longer causal because test scores depend on type of schooling, but the exercise can partly reveal the role of test scores as a mediator of independent-school effects.

The results appear in Tables 6 and 7. As expected, the estimate of the independentschooling effect was reduced for all outcomes and in most cases by between 25 and 30 percent. The largest impact on the independent-school coefficient following the addition of test scores within the set of covariates is for university attendance in the case of men. In this case, adding math scores reduces the independent-schooling coefficient by 40 percent, but for girls it reduces it only by 9 percent. Therefore, the impact of independent schooling on math scores for boys may be crucial for their future. This relationship between independent schooling, math scores, and university attendance for boys is certainly intriguing. Although attendance in technical or hard sciences programs is rapidly increasing for women, they were male-dominated fields in the early-millennium years. According to our computations from YITS data, 25.3% of male university students are enrolled in such programs, while the same statistic is only 3.8% for female university students. (We selected from within the YITS the following degrees as technical or "hard" science degrees: Computer and Information Sciences and Support Services, Engineering, Mathematics and Statistics, Mathematics and Computer Science, Astronomy and Astrophysics, Atmospheric Sciences and Meteorology, Chemistry, and Geological and Earth Sciences/Geosciences and Physics.) These programs are in general costly, as they require labs and equipment spaces. Most universities are therefore forced to highly constrain the number of spaces in these areas (see Fortin [46]). This link between math and university attendance through independent schooling is an important result, as university attendance is crucial for a well-paid job and long-term success in the labor market. Thus, the mechanisms that are driving the effects of independent schooling on educational attainment are linked to those that have an impact on test scores, in particular math scores for males. Math is generally the topic that students find the hardest. To obtain strong scores in math tests, usually, one must be disciplined and do homework regularly. Therefore, by offering support to perform better in math, independent schools develop skills necessary for higher education. Also, obtaining higher scores in math could provide confidence in oneself to apply and attend universities. Finally, certain degrees necessitate high math marks simply to be admitted, so that independent schooling may have an effect through that channel as well. Clearly, our results demonstrate that test scores mediate the impact of independent schooling on educational attainment, and, more precisely, mediate these impacts more for males than for females at university level.

Although math scores cannot be considered a confounding variable in the same way as the others as they are impacted by the treatment, they may also be considered a (partially) confounding variable. Math abilities are, in part, predetermined before high school. If one assumes that the level of PISA scores is entirely selection driven, it is logical to control for that hypothetical selection by including them within the EB algorithm. Then, if one still obtains a positive and statistically significant independent-school effect on educational attainment, our main results concerning the effects of independent schools would become even stronger.

It is instructive to compare our results with those of Laliberté [47]. Using administrative data on almost all schools and five cohorts of students entering first grade chosen between 1995 and 2001 in the city of Montreal, for which the full education history is known up until university enrollment (and, for earlier cohorts, graduation from university), Laliberté manages to separate quality of school effects from neighborhood effects.

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To identify this decomposition, he uses the open enrollment policy in place as well as the default option that guarantees a student a space in their neighborhood public school. Very briefly, after constructing a quality measure of a school based on its propensity to enroll students in a university program, he finds, using an RD-IV design, that quality has a very strong impact on the probability of enrolling in a university program. He computes the difference in mean quality value between independent and public schools to be 0.31 (all quality values are constrained to be between 0 and 1). This difference is estimated to have an effect of between 0.19 and 0.26 on the probability of enrolling in a university. (Although these estimates are not included in his paper, Laliberté graciously computed them for us. We are very grateful for his time and trouble.) His lower bound is higher than our estimates, which range between 0.10 and 0.17. However, it is important to note that his estimates provide evidence that independent schools are of higher quality than public schools on average and that quality has substantial effects on university enrollment.

**Table 6.** Estimated effects of independent high school on academic outcomes by sex and PISA scores as additional matching covariate, cohort A cycles 2–3.

Outcome	Cycle	Sex	Para.	Math Subsample	PISA- Math	Δ Math	Read Subsample	PISA Read	Δ Read	Science Subsample	PISA- Science	Δ Science
Graduation high school	2	A	Coef. S-D	0.069 (†††) 0.021	0.051 (†††) 0.019	-26.1%	0.078 (+++) 0.018	0.053 (+++) 0.017	-32.1%	0.100 (†††) 0.026	0.071 (†††) 0.024	-29.0%
			N	1.826	1.826		3.27	3.27		1.801	1.801	
		M	Coef.	0.084 (††)	0.057 (††)	-32.1%	0.089 (†††)	0.065 (†††)	-27.0%	0.113 (†††)	0.087 (††)	-23.0%
			S-D	0.033	0.029		0.028	0.023		0.042	0.037	
			N	937	937		1.644	1.664		919	919	
		F	Coef.	0.069 (†††)	0.059 (††)	-14.5%	0.066 (†††)	0.042 (†††)	-36.4%	0.093 (+++)	0.060 (†††)	-23.0%
			S-D	0.026	0.029		0.021	0.020		0.023	0.022	
			N	889	889		1.626	1.626		882	882	
Graduation	3	A	Coef.	0.031 (+++)	0.022	-29.0%	0.035 (†††)	0.026 (††)	-25.7%	0.027 (†)	0.017	-37.0%
high school			S-D	0.017	0.016		0.011	0.010		0.016	0.015	
-			N	1.575	1.575		2.817	2.817		1.551	1.551	
		M	Coef.	0.036	0.020	-44.0%	0.038 (†)	0.028	-26.3%	0.026	0.018	-30.7%
			S-D	0.031	0.029		0.020	0.017		0.029	0.025	
			N	791	791		1.400	1.400		786	786	
		F	Coef.	0.032 (†††)	0.029 (†††)	-9.4%	0.037 (+++)	0.029 (†††)	-21.6%	0.041 (†††)	0.033 (†††)	-19.5%
			S-D	0.012	0.011		0.007	0.007		0.010	0.010	
			N	784	7844		1.417	1.417		765	765	
Enrollment	3	A	Coef.	0.072 (†††)	0.055 (††)	-23.6%	0.081 (†††)	0.062 (†††)	-23.4%	0.077 (+++)	0.055 (††)	-28.5%
CEGEP/			S-D	0.024	0.024		0.020	0.020		0.023	0.021	
university			N	1.575	1.575		2.817	2.817		1.551	1.551	
•		M	Coef.	0.061(†)	0.028	-54.1%	0.075 (††)	0.055(†)	-26.6%	0.069(†)	0.054	-21.7%
			S-D	0.034	0.034		0.030	0.028		0.041	0.034	
			N	791	791		1.4	1.4		786	786	
		F	Coef.	0.081 (†††)	0.076 (†††)	-6.2%	0.091 (+++)	0.077 (†††)	-15.4%	0.095 (†††)	0.069 †††)	-27.4%
			S-D	0.031	0.028		0.023	0.021		0.028	0.029	
			N	784	784		1.417	1.417		765	765	

Notes: Param.: estimated parameter; Math/Read/Science subsample: EB estimate for respondents who took math/read/science test; PISA-Math/Read/Science:EB estimate for respondents who took math/read/science test including PISA score as a control;  $\Delta$  math/read/science: difference in estimate between original estimate and estimate with PISA score as control in percent.  $\pm$ : 90% statistical significance,  $\pm$ : 95%.,  $\pm$ 1: 99%.

We provide some explanations as to why our estimates are lower. First our sample is different as we use students from all regions in Québec rather than strictly from the island of Montreal. Second and most importantly, we are estimating ATT effects. Hence, our control group students possibly attend better public schools than the average public school as our matched public-school sample comes from higher income and better educated families than the average public-school family. These two factors could explain why our estimates are lower than Laliberté's. However, we feel that the numbers from the Laliberté paper add considerably to the credibility of our estimates.

## 4.2. Public Policy Implications

The evidence in this paper, and drawn indirectly from Laliberté [47], shows that independent high school education in Québec has a strong positive effect on the educational attainment of those who attend independent schools. The strategy of subsidizing independent schools should therefore increase the aggregate stock of human capital in

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the province, unless the policy considerably reduces the human capital of individuals educated in the public sector. To our knowledge, no evidence of such an effect has been produced in Québec. In fact, public-sector students perform better on math tests than their counterparts in the rest of Canada [see [13]). Unfortunately, data sets in Canada do not permit an estimation strategy that could identify the key factors producing these strong effects. We can, however, suggest some factors that may be in play.

**Table 7.** Estimated effects of independent high school on academic outcomes by sex and PISA scores as additional matching covariate, cohort A cycle 4.

Outcome	Cycle	Sex	Para.	Math Subsample	PISA- Math	Δ Math	Read. Subsample	PISA Read	Δ Read	Science Subsample	PISA- Science	Δ Science
Enrollment university	4	A	Coef. S-D N	0.133 (†††) 0.038 1.344	0.095 (††) 0.040 1.344	-28.6%	0.145 (†††) 0.031 2.388	0.114 (†††) 0.031 2.388	-21.4%	0.158 (+++) 0.052 1.309	0.120 (††) 0.053 1.309	-24.1%
		M	Coef. S-D N	0.101 (††) 0.041 672	0.048 0.042 672	-52.5%	0.152 (†††) 0.042 1.185	0.125 (+++) 0.037 1.185	-17.8%	0.143 (††) 0.069 666	0.126 (††) 0.063 666	-13.5%
		F	Coef. S-D N	0.145 (††) 0.067 672	0.131 (†††) 0.070 672	-9.7%	0.131 (††) 0.058 1.203	0.099 (†) 0.057 1.203	-24.4%	0.170 (+++) 0.023 643	0.136 (†††) 0.027 643	-20.0%
Enrollment CEGEP/ university	4	A	Coef. S-D N	0.096 (†††) 0.024 1.344	0.070 (+++) 0.019 1.344	-27.1%	0.105 (†††) 0.020 2.388	0.081 (†††) 0.018 2.388	-22.8%	0.107 (+++) 0.027 1.309	0.082 (†††) 0.023 1.309	-23.4%
,		M	Coef. S-D N	0.103 (+++) 0.035 672	0.059 (†) 0.031 672	-42.7%	0.102 (†††) 0.033 1.185	0.078 (†††) 0.029 1.185	-23.5%	0.082 0.053 666	0.069 0.043 666	-15.9%
		F	Coef. S-D N	0.069 (††) 0.028 672	0.059 (†††) 0.022 672	-14.5%	0.107 (†††) 0.017 1.203	0.086 (††) 0.015 1.203	-19.6%	0.138 (††) 0.017 643	0.099 0.018 643	-28.3%

Notes: Param.: estimated parameter; Math/Read/Science subsample: EB estimate for respondents who took math/read/science test; PISA-Math/Read/Science: EB estimate for respondents who took math/read/science test including PISA score as a control;  $\Delta$  math/read/science: difference in estimate between original estimate and estimate with PISA score as control in percent.  $\pm$  90% statistical significance,  $\pm$  95%.,  $\pm$  99%.

To explain the positive effects of independent schools on educational attainment, several mechanisms can be invoked, such as smaller class sizes, peer effects, teaching methods, autonomy of schools, or the quality of teachers. We do not think that class size plays a role. Independent schools must run a tight ship as the fees they can charge cannot surpass the amount spent per pupil (financed by fees and government subsidies) in public schools, which strongly reduces the possibility of using smaller class sizes to achieve good results. We could not find in the empirical literature or official statistics any evidence that class sizes in private schools in Québec are smaller than those in public schools. However, some statistics are available Canada-wide, and class sizes are slightly larger on average in independent schools (see [31]). Also, the empirical literature on the effects of class size on achievement is rather mixed and the evidence that smaller classes improve educational outcomes is far from overwhelming (see [48]).

Independent-sector teachers are not particularly different from their public-school counterparts; they graduate from the same university programs, are often members of the same union, and have similar working conditions. Unfortunately, there is a lack of information regarding teacher quality in Québec (and Canada). American research, equipped with longitudinal data regarding teachers, along with primary and high school students and their results, shows that teacher quality is probably a school's most valuable asset (including good principals) [49,50]. However, certain mechanisms may favor independent schools in Québec for their choice of teachers. Independent schools select their teachers without interference from school boards who allocate teachers across public schools. Therefore, they have much more discretion in hiring, enabling them to find teachers who are a better fit to their curricula or to the type of programs found in their schools (arts, music, sports, etc.). Obviously, if good teachers are attracted to schools with good students, or without problematic students, this will also be beneficial to students in independent schools. As with the issues in reducing class sizes, it is difficult to use higher wages to attract better teachers due to financial constraints. As for teaching approaches, private

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schools are autonomous. Whilst they are still constrained by the fact that all students must receive a passing grade in province-wide examinations (mathematics, French, etc.) in order to receive their high school degree, examinations with questions that can be closely tied to certain teaching methods, teachers are free to use their own approach to prepare students for these examinations. This could attract teachers who are willing to be more innovative.

Peer effects can also increase cognitive and non-cognitive skills leading to higher educational achievement. Boucher et al. [44] estimate peer effects using administrative data from the province of Québec and find that they are strong and can be precisely estimated for math scores in province-wide exams. Lefebvre et al. [13], using panel data from the Province of Québec, observe math scores for children before they start high school and find that children who end up in independent schools have higher mean math scores than those choosing public schools. Since there are more children of higher ability in independent high schools, and peer effects are positive for math, the latter may contribute to part of the success of independent schools in Québec. Furthermore, given our finding that ability in math is important for university attendance, in particular for boys who favor STEM fields, these peer effects may be more important for boys.

Finally, anecdotal evidence shows that independent school are on average extremely well organized, have a zero-toleration policy for delinquent behavior, can more easily expel very disruptive students, and closely follow the academic results of their students. For example, we know of a school that has set up an application whereby parents are immediately informed of their child's poor result in an examination. Students are expected to work hard and perform well.

Faced with the evidence, it is challenging to draw conclusions that could offer potential solutions for public policy to improve students' educational results and competencies. Measures that would simultaneously improve both types of students (low and high social status) would not reduce the dispersion of student abilities. Conversely, cutting back or eliminating Québec's independent-school subsidies, as suggested by proponents of public education, would only cause a decline in abilities and competencies for students that attend independent schools. A voucher system increasing enrollment in independent schools for the less advantaged should be seriously considered given the effects found in this paper. An interesting policy alternative would be to pursue research in this area in order to identify the mediators that explain the effect of independent schools on attainment and then to apply these findings to public schools.

To summarize, Québec's educational policy is founded on offering parents a choice of high school for their children at a reasonable cost. Another policy pursued in British Columbia and Québec is the introduction of "open enrollment", which allows children to attend a school outside of their regular attendance zone. This policy provides an opportunity to estimate the extent to which increased choice of public schools affects student achievement, concentrates minority students in enclaves, and induces cream skimming. According to Friesen, Cerf, Harris, and Woodcock [45], greater school choice has improved the reading and numeracy scores of grade-4 students in some areas of British Columbia.

#### 5. Conclusions

Recent studies have sought to establish a causal link between type of high school establishment, i.e., independent or public, and students' educational performance, based on pupils' performances on standardized tests. These studies include those pursued with the National Longitudinal Study of Children and Youth (NLSCY) led by Statistics Canada from 1994 to 1995 through 2008 to 2009 (see [13] and Haeck et al. [51]), and those produced with international surveys such as PISA [28] or with administrative data [27]. Results from these estimates lead to a common conclusion: the average treatment effect on the treated of independent schools on achievement is positive and significant.

Future research on the mediating factors that might explain the success of independent schools is crucial for the implementation of policies that could improve public schools or

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increase students' chances of being admitted to an independent school. Administrative data, which has very fine-grained information regarding all students and teachers, are very difficult to obtain due to privacy concerns, but could be very helpful for untangling the key factors that explain the differences between both types of schools.

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**Data Availability Statement:** The STATA code used to produce the tables in the paper can be made available to all who wish to obtain it. The data used in this paper cannot be made available publicly. However, it can be made accessible to researchers who apply to Statistics Canada and who are willing to work at the Research Data Centres operated by Statistics Canada in collaboration with universities across Canada.

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### Appendix A

In 1961, a Ministry of Education was created in the province of Québec. Over the following years, it adopted many significant changes: a kindergarten level, a K-11 system, the creation of public community colleges (CEGEP; see Section 2) leading to university programs or technical occupations, and finally a new network of public universities. Hereafter, the education system functioned with a secular linguistic structure.

In 1982, to preserve schooling options for parents, Québec adopted a new subsidy program for independent schools. First, abstracting from expenditures in infrastructure and equipment, the annual subsidies for operation costs were fixed along the same lines as the financial aid given to public schools. Second, the percentage of costs covered by public subsidies was reduced from 80% to 50% of direct subsidies paid to public schools for all levels (kindergarten, primary, secondary, and CEGEP) (see below).

Table A2 presents the evolution of student enrollment in both public and independent sectors by level, for selected years,. Demographic statistics (not shown) demonstrate that the decreasing enrollment in public schools is explained by a large decline in the fertility rate over the years (with a small increase at the end of the nineties). Conversely, for independent schools, there is a modest positive upwards trend at the primary level and a substantial increase at the secondary level, which contains 70 percent of independent-sector students. There are 128 independent schools at the primary level, 124 at the secondary level, with 68 offering school services at both levels. In general, independent schools offering only the primary level of schooling are much smaller than those offering a secondary level of education.

Table A3 presents the subsidies in Canadian dollars (approximately 1CAN\$ exchanged for US\$0.70–0.90 over this period) for selected years, in particular for the early 2000s, the period of our analysis. A government regulation requires that the fees of subsidized independent schools do not exceed their public subsidy per student. Table A4 shows the

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authorized maximum fee according to provincial regulations and the actual maximum fee charged by independent schools by schooling level. Very few independent schools charge the allowed maximum. On average, the independent-school admission fee is 68.3 percent of the authorized maximum fee, and this gap varies by region (from 39% to 84.3% in Montreal), which suggests that families are price sensitive and that the market is competitive. These low prices provide a strong incentive for middle-class families who prefer independent schooling for their children. For public schools, 90 percent of revenues are obtained by direct transfers from the provincial government (74%) and property school taxes (16%), with the rest acquired through related activities.

All independent schools in Québec must have a permit issued by the Ministry of Education in order to operate legally. To be eligible for subsidies from the government, the school must operate as a not-for-profit organization and under an approved curriculum. Their students must pass the same final state-wide exams in French or English, History, Mathematics, and Sciences as those in public schools in grades 10 and 11, the last two years of high school in the province. It is necessary to obtain a passing grade in these exams to graduate from high school. Therefore, the requirements to graduate from high school are the same for both independent and public systems.

At the primary and secondary levels, almost 90 percent of independent schools are subsidized, while a small number of students in independent schools are enrolled in "elite" schools with no subsidies. Most of these are English-speaking with bilingual teaching, some with students of the same gender, very high pedagogical supervision, and smaller class sizes, and are located in Montreal (the largest city in Québec). These schools charge much higher fees than the average subsidized independent school. Finally, 12 schools are specialized for handicapped youths, and 20 schools offer trade or vocational training.

It is also important in the context of this paper to provide a proper description of the admission process in independent schools. In a 2006 document on admissions, the Federation of Independent Schools and Institutions reports that "70.0% of students who took an admission exam for grade one in secondary school were admitted, 17.6% had their application rejected because of space limitations, and 5.4% were turned away because the school did not have the specialized human resources to respond to the special needs of these students" (page 3). Therefore, very few are turned away because of a lack of basic skills. Even those turned away can end up in an independent school as they can apply to several schools. Selection is used when applications are higher than available spaces, a rare case.

The most recent information can be retrieved from the FEEP. Their web site provides information on membership for 171 schools along with their admission process and schedule for autumn 2017. From this list, we can identify three distinct school policies: (1) for 120 schools (70%), a student is admitted and enrolled after he or she applies to the school; (2) among secondary schools only, 22 (13%) have a sorting categorization exam; (3) again for secondary schools only, 20 (12%) have an admission exam, which also serves as a sorting exam. The latter schools are almost all located in the two largest cities (Québec and Montreal), where there is an excess demand for spaces in schools considered excellent in most unofficial rankings. Because very few students applying to independent schools are not admitted into the system, bias due to selection via admissions tests should not be of great concern in our analysis.

**Table A1.** Top ten mean country scores—PISA 2018—Reading.

Country or Province	Mean Score	95% Confidence Interval
BSJZ-China	555	550–561
Singapour	549	546–553
Alberta-Canada	532	523–540
Macao-China	525	523–528

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Table A1. Cont.

<b>Country or Province</b>	Mean Score	95% Confidence Interva
Hong Kong-China	524	519–530
Ontario-Canada	524	517–531
Estonia	523	516–525
Canada	520	517–524
Finland	520	516–525
Québec-Canada	519	513–526
Top ten r	nean country scores—PISA 2	018—Math
BSJZ-China	591	586–596
Singapour	569	566–572
Macao-China	558	555–561
Hong Kong-China	551	545–557
Québec-Canada	532	525–539
Taipei	531	525–537
Japan	527	522–532
Korea	526	520-532
Estonia	523	520–527
Netherlands	519	514–524
Top ten m	ean country scores—PISA 20	018—Science
BSJZ-China	590	585–596
Singapour	551	548–554
Macao-China	544	541–546
Alberta-Canada	534	525–542
Estonia	530	526–534
Japan	529	524–534
Finland	522	517–527
Québec-Canada	522	514–529
Korea	519	514–525
Ontario	519	511–526

Source: O'Grady et al. (2019).

Table A2. Number of students by school level and type, selected years 1994–1995 to 2013–2014, Québec.

School Year		Public Schools		Independent Sc	Independent Schools (Ratio Independent/Public)				
School Ical	Kindergarten	Primary	Secondary	Kindergarten	Primary	Secondary			
1994–1995	86,091	522,714	421,467	3821 (4,2)	24,681 (4.5)	76,839 (15.4)			
1997-1998	91,001	531,816	404,333	3098 (3.3)	25,350 (4.5)	73,806 (15.4)			
2000-2001	83,073	546,444	373,504	4010 (4.6)	27,831 (4.8)	73,343 (16.4)			
2001-2002	80,006	543.546	370,197	4362 (5.2)	28,995 (5.1)	74,964 (16.8)			
2002-2003	76,421	533,276	376,409	4303 (5.3)	29,462 (5.2)	77,913 (17.1)			
2003-2004	72,223	517,996	385,139	4372 (5.7)	29,473 (5.4)	81,310 (17.4)			
2006-2007	69,043	460,502	402,946	4776 (6.5)	31,101 (6.3)	88,203 (18.0)			
2009-2010	70,319	429,950	369,759	4968 (6.6)	32,136 (7.0)	88,779 (19.4)			
2012-2013	98,561	438,711	327,216	5414 (5.2)	32,688 (6.9)	86,181 (20.8)			
2013–2014	102,415	449,352	318,132	5484 (5.1)	32,898 (6.8)	84,898 (21.6)			

Source: Statistics of education 2015, Ministry of Education, Leisure and Sports (MELS).

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**Table A3.** Public subsidy per student paid to Québec's independent schools by schooling level, selected years, in Canadian dollars.

Year	Kindergarten Schools	<b>Primary Schools</b>	Secondary Schools
1997–1998	2275 + 82	2092 + 82	2919 + 122
1998-1999	2297 + 82	2108 + 82	2944 + 122
2000-2001	2496 + 85	2292 + 85	3179 + 122
2002-2003	2807 + 88	2421 + 88	3331 + 131
2004-2005	3006 + 93	2582 + 93	3556 + 139
2005–2006	3064 + 95	2808 + 95	3612 + 142
2016–2017	3875 + 35	3515 + 35	4512 + 157

Note: The subsidy is based on teaching and non-teaching personnel, as well as other costs and location value of premises. Source: Ministry of Education, Leisure and Sports (MELS), Budgetary Rules for Schools under Agreement, annual.

**Table A4.** Authorized and actual maximum schooling fees charged by Québec's independent schools, by education level, selected school years 2003–2004 and 2004–2005.

Level	Year	Maximum Fee According to Regulation	Average Fee Asked	Number of Schools with Maximum Fee			
Kindergarten	2003–2004 2004–2005	\$2886 \$2924	\$1650 \$1679	1 out of 48			
Primary	2003–2004	\$2488	\$1724	2 out of 72			
	2004–2005	\$2518	\$1813	2 out 01 72			
Secondary	2003–2004	\$3421 \$2464	\$2122 \$2210	2 out of 142			
•	2004–2005	\$3464	\$2219				

Source: Ministry of Education, Leisure and Sports (MELS), Financial Reports of Subsidized Independent Schools, and Budgetary Rules for Schools under Agreement, annual.

**Table A5.** High school graduation rate by cohort, number of years since entrance in high school, sex, schooling system, and language of instruction, cohort 2001 to cohort 2009, Québec.

Cohort Year	Cohort 2001				Cohort 2002						Col		Cohort 2004							
<b>Duration in Years</b>	5	6		7		5	6		7		5	6		7		5	6		7	-
Coverage	A	A	A	M	F	A	A	A	M	F	A	A	A	M	F	A	A	A	M	F
All	61	69	72	66	80	59	68	72	66	78	61	69	72	66	79	61	70	73	68	80
Public schools	56	65	69	62	76	54	64	68	61	75	55	64	68	61	75	56	65	69	63	76
Independent schools	83	88	89	86	93	84	89	90	87	94	85	89	91	88	94	85	90	91	88	94
Independent-Public	27	23	20	24	17	30	25	22	26	19	30	25	23	27	19	29	25	22	25	18
French	60	68	72	65	78	59	68	71	65	78	60	68	72	65	79	60	69	73	67	79
English	72	79	81	76	86	70	77	80	76	84	69	77	79	75	84	72	79	82	78	86
Cohort year		Col	hort 2	2005			Col	nort 2	006			Col	nort 2	.007		C	Cohor	t 200	8	2009
Duration year	5	6		7		5	6		7		5	6		7		5		6		5
Coverage	A	A	A	M	F	A	A	A	M	F	A	A	A	M	F	A	A	M	F	A
All	63	72	75	70	81	64	72	76	71	81	65	74	78	73	83	66	75	69	81	67
Public schools	58	67	71	65	77	58	68	72	66	78	60	69	74	68	80	60	70	64	77	62
Independent schools	86	90	92	89	95	87	91	92	90	95	87	91	93	90	95	87	92	89	94	88
Independent-Public	28	23	21	24	18	29	23	20	24	17	27	22	19	22	15	27	22	25	17	26
French	62	71	74	69	80	63	75	75	81	81	64	73	77	72	83	65	74	68	80	67
English	74	80	82	78	87	76	85	85	88	88	76	82	85	80	89	76	82	78	86	76

Notes: A = All. M = Male. F = Female. The high school graduation rate presented is the proportion of students who received a high school diploma or a GED within the cohort. The public network is formed by 72 school boards excluding native nation boards. Duration in years measures the number of years since the cohort entered in secondary school. Independent-Public: Independent rate minus the Public rate. Main language of instruction, F: French, E: English. Source: "High school certificate and qualifications by school boards: 2015 Edition", Ministry of Education, Québec.

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