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An Improved Optimization Algorithm for Aeronautical Maintenance and Repair Task Scheduling Problem

Changjiu Li [1], Yong Zhang [1,2], Xichao Su [1,*] and Xinwei Wang [3,*]

[1] School of Basic Science for Aviation, Naval Aviation University, Yantai 264001, China
[2] Department of Precision Instrument, Tsinghua University, Beijing 100084, China
[3] State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, China
* Correspondence: suxich@126.com (X.S.); wangxinwei@dlut.edu.cn (X.W.)

Abstract: The maintenance of carrier-based aircraft is a critical factor restricting the availability of aircraft fleets and their capacity to sortie and operate. In this study, an aeronautical maintenance and repair task scheduling problem for carrier-based aircraft fleets in hangar bays is investigated to improve the maintenance efficiency of aircraft carrier hangar bays. First, the operational process of scheduling aeronautical maintenance tasks is systematically analyzed. Based on maintenance resource constraints and actual maintenance task requirements, a wave availability index and load balance index for the maintenance personnel are proposed for optimization. An aeronautical maintenance task scheduling model is formulated for carrier-based aircraft fleets. Second, model abstraction is performed to simulate a multi-skill resource-constrained project scheduling problem, and an improved teaching-learning-based optimization algorithm is proposed. The algorithm utilizes a serial scheduling generation scheme based on resource constraint advancement. Finally, the feasibility and effectiveness of the modeling and algorithm are verified by using simulation cases and algorithm comparisons. The improved teaching-learning-based optimization algorithm exhibits improved solution stability and optimization performance. This method provides theoretical support for deterministic aeronautical maintenance scheduling planning and reduces the burden associated with manual scheduling and planning.

Keywords: carrier-based aircraft; maintenance scheduling; resource-constrained; teaching-learning-based optimization; scheduling optimization

MSC: 90-10; 90B25

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1. Introduction

As the core combat unit of an aircraft carrier formation, carrier-based aircraft play an essential role in air control, air-to-submarine defense, electronic countermeasures, and strikes against ships. Aeronautical maintenance is necessary and indispensable in military operations to restore the fleet to excellent technical conditions and provide flight safety guarantees for various combat and training missions [1]. The efficiency of aeronautical maintenance significantly affects the availability and sustained combat capabilities of a fleet. As the scale of combat or training increases, the impact of such constraints becomes more prominent. A hangar bay is required to execute an efficient scheduling scheme for maintenance tasks to shorten the time for maintaining the fleet. Compared with land-based maintenance workshops, hangar bays have the following characteristics: (i) a smaller workspace and complex environment; (ii) complicated processes for fleet-aeronautical maintenance tasks; (iii) the need for a high degree of coordination among maintenance personnel; (iv) limited resources for maintenance personnel, equipment, and workshops; and (v) strict requirements for task timelines. These characteristics make maintenance tasks challenging to execute. In this context, meeting reliability and timeliness requirements using a manual scheduling scheme based on experience is difficult. Based on the environmental characteristics of the hangar bay and the actual needs of combat and training missions, a scientific time-series scheduling scheme and resource allocation approach for limited maintenance personnel, equipment, and workshops can shorten the duration of operations and enable the fleet to quickly return to a usable and combat-ready state. This is a critical and urgent issue that needs to be addressed to maintain the availability of the fleet. It is also a core consideration restricting the overall effectiveness of maritime operations, an essential aspect of future military efforts. This is vital to the evolution of future warfare.

For several years, the process scheduling of carrier-based aircraft has relied heavily on manual empirical approaches for formulating plans. The U.S. Navy has evolved to use an aviation data management and control system [2], known as the "electronic Ouija board", to simulate the locations and statuses of carrier-based aircraft and associated maintenance personnel. Compared with the old Ouija board, this system incorporates multiple interaction modes, and the operator interacts with the system by pointing at the aircraft and gesturing to make decisions. However, a considerable amount of the corresponding work [3] relies on manual completion because a mechanism that can generate autonomous and intelligent scheduling solutions is lacking. Ryan et al. [4] developed a decision system for carrier deck operations based on the concept of human-computer interaction and designed a set of experiments for comparing automatic planning algorithms with manual empirical decisions [5]; this enabled the development of automatic scenario planning and rapid decision-making capabilities for carrier-based aircraft scheduling. However, current research on maintenance scheduling remains limited. A lack of transparency within naval fleet aviation maintenance, complex constraints, special research areas, and the confidentiality of information or data [6] cause challenges to the development of this field.

By contrast, research on civil aeronautical maintenance and management is more established and centered on aircraft maintenance planning (AMP) problems [7]. In the context of optimization problems, AMP is a complex decision problem [8] that involves resource allocation to maintenance tasks but may also involve distributed maintenance area selection. For maintenance tasks in a certain area, the scheduling allocation problem involves assigning maintenance operations to the maintenance equipment/workshops performing the task, assigning maintenance personnel to the tasks of the corresponding operation, and determining the start and end times of the operation [9]. The shortcomings arise from the limitations in maintenance resources or the number of tasks that can be performed simultaneously. The focus of maintenance assurance tasks in naval aircraft fleet aviation differs from that in civil aviation in the following ways:

i. Slack of distributed constraints. Commercial airlines must manage a complex network of routes and the complex coupling between distributed workshops and routes, whereas the majority of military aeronautical maintenance tasks are concentrated in ship-based hanging bays on large sea platforms;
ii. Differences in the maintenance cycles, civil aeronautical maintenance optimization models, and methods applied to solve problems in fleet decision optimization. A commercial fleet is highly stable and has longer maintenance cycle intervals than a military naval aircraft fleet. The carrier-based aircraft fleet aeronautical maintenance tasks investigated in this study involve military tasks with urgent task requirements [10];
iii. Differences in maintenance goals. The literature on civil aeronautical maintenance mainly focuses on profitability, and the optimizations mainly consider economic benefits [11], such as balancing the maintenance cost of the fleet with the amount of hangar resources [12] or the labor costs of maintenance personnel [13]. By contrast, military aviation maintenance tasks are optimized to avoid delaying military response and to ensure appropriate conduct in both combat and training tasks. In other words, the goal is to positively impact operational effectiveness and subsequent warfare.

These differences make the direct application of civil aviation maintenance mission scheduling models to the maritime military domain difficult. Moreover, because of the characteristics of a cluster wave sortie in fleet combat and training missions, the downtime caused by preventive maintenance and failure repairs within a specified flight interval can significantly impact a wave sortie mission [14], thus requiring the redesign of models and optimization requirements applicable to fleet aviation maintenance assurances. The following attempts have been made in the military domain to address these issues:

i. Mission maintenance aspects: Han et al. [15] simulated mission maintenance for deck crews, with the number of aircraft ranging from five to nine. However, they considered only a single maintenance mode and not multimode/hybrid situations, such as preventative maintenance and failure repairs, and realistic constraints, such as maintenance coverage, parallel maintenance capacity, and maintenance workstation space. Thus, the simulation differs substantially from an actual task;
ii. Optimization/scheduling of fleet maintenance tasks: Most studies on optimizing fleet maintenance tasks have focused on minimizing the maintenance completion time [16]. However, Raju et al. [17] defined a military aircraft availability index for fleet wave sortie availability; the index comprised the ratio of the number of aircraft in mission-capable states to the total number of aircraft in the fleet at a given time point. The military maintenance and operational characteristics of naval aircraft were used for closer integration by the index;
iii. Optimization/Scheduling of resources: The main considerations in terms of resources have involved personnel and personnel scheduling strategies [18], resource constraints for maintenance personnel [19], and maintenance personnel time balancing [20]. No studies have been conducted to integrate limited maintenance resources, such as maintenance equipment, workshops, and space, in the models.

Moreover, differences exist in the selection of optimization models in previous studies, in which maintenance scheduling planning was typically treated as a mixed integer linear program (MILP) [21]. The scheduling problem in naval aircraft fleet maintenance involves the coordination of related personnel, equipment, and workshops; in addition, it incorporates complex and highly constrained operational processes and resources and a long makespan. This is the core of the challenge in scheduling the entire process of aircraft carriers and amphibious ships. It is also a typical resource-constrained project scheduling problem (RCPSP) [20]. The RCPSP differs from the MILP in that it emphasizes resource constraints [19]. The RCPSP can be studied to combine the classical RCPSP with the maintenance scheduling task, particularly for the scheduling of resources, such as multi-skill personnel or multifunctional equipment. Based on this feature, the maintenance scheduling problem has been classified as a multi-skill resource-constrained project scheduling problem (MS-RCPSP) to better approximate the actual situation [22]. The MS-RCPSP rationalizes scheduling in terms of time and resources to optimize the objectives while optimizing the use of skills and resources. Because the RCPSP has been proven to be an NP-hard problem, exact algorithms, such as the branch-and-bound method [19] and linear programming [23], treat the maintenance state or the working state of the object as the decision variable, and the value of the variable is usually 0 or 1. These approaches can precisely and efficiently find the optimal solution for a small-scale RCPSP within a reasonable time frame [24,25]. However, most mathematical models are oversimplified, less scalable, and still have limitations in solving large-scale problems. In addition, the solutions for integer decision variables and linear constraints rely heavily on optimization solvers, such as CPLEX [26]. However, as the problem scale increases, the complexity of the corresponding solution space increases significantly. Hence, an exact algorithm cannot complete the solution within an acceptable time frame [27]. Studies have shown that the current best exact methods can solve instances with up to only 60 activities and low resource constraints. As real projects often exceed this size and usually require fast scheduling solutions, exact algorithms are not suitable [28]. Common optimization methods, such as sequential games [29] and multi-agent approaches [18], involve the same issue. Recent developments to improve the solutions for large-scale MS-RCPSPs have been based on classical mathematical techniques. Peschiera [30] proposed a new approach based on a new mixed integer program, highlighting a good trade-off between optimality and infeasibility degradation in the performance search process. Another approach is the use of metaheuristic algorithms. Metaheuristic algorithms have been widely used to quickly obtain approximate optimal solutions for project scheduling, as they achieve the best trade-offs between accuracy, computation time, ease of implementation, and flexibility [31]. The literature is more abundant in this regard. Teaching-learning-based optimization (TLBO) is a population-based algorithm that is similar to the genetic algorithm, particle swarm optimization (PSO), and differential evolution (DE) algorithms [32]. However, TLBO differs from other algorithms in that it does not require algorithm-specific parameter settings. This avoids different optimization effects owing to different parameter settings. The algorithm has been successfully applied to problems such as flow shop scheduling [33], job shop scheduling [34], steelmaking-continuous casting scheduling [35], and RCPSP [36,37], showing good optimization performance and problem adaptability.

Overall, the current research on naval aeronautical maintenance and repair tasks is limited owing to the unique characteristics of the field, such as its complexity and confidentiality.

Above all, the unique characteristics of the naval aeronautical maintenance and repair tasks, such as complexity and confidentiality, make current research on it quite limited. In contrast, the research on civil aviation maintenance and management is more established. However, because the maintenance and repair tasks in naval aeronautics differ from those in civil aviation, the models and optimization objectives need to be redesigned. Some research attempts have been made in the field of military maintenance, but some shortcomings still remain. Moreover, various characteristics of the scheduling problem in naval fleet maintenance are consistent with those of the RCPSP. Therefore, to solve the aeronautical maintenance and repair task scheduling problem (AMRSP), we propose a mathematical formulation model based on the RCPSP using the currently popular metaheuristic algorithm.

The contributions of this study are as follows. First, a comprehensive mathematical model is proposed for the AMRSP of a carrier-based aircraft fleet for the requirements of carrier-based aircraft wave sorties. This model considers constraints regarding personnel, equipment, workshop, workspace, and operational processes, allowing the model to approximate situations in the military aviation maintenance field. Second, an improved teaching-learning-based optimization algorithm with a serial scheduling generation scheme (ITLBO-S) is proposed for solving the model. The algorithm includes a new assistant teaching phase and serial scheduling generation scheme (SSGS) based on resource constraint advancement. Third, simulation cases for method comparisons are used to verify the feasibility and effectiveness of the model and algorithm for large-scale tasks and highly resource-constrained conditions, thereby providing a scheduling scheme for the maintenance process, personnel, and equipment/workshop.

The remainder of this paper is outlined as follows. Section 2 describes the AMRSP. Section 3 presents the mathematical model of the AMRSP. Section 4 describes the process and improvement measures of the ITLBO-S. Section 5 presents a case analysis and details of the simulations. Finally, Section 6 provides conclusions and suggestions for future studies. The research content and framework of this study are shown in Figure 1.

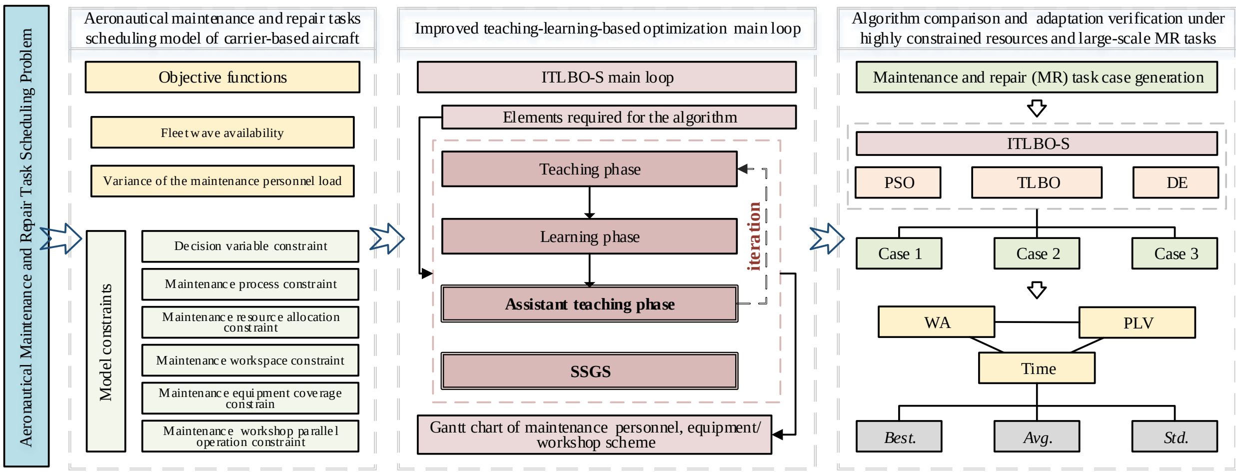


Figure 1. Research content and framework.

2. Problem Statement

The most common military aeronautical maintenance tasks include preventive maintenance, failure repair, and overhaul. Overhaul is the detailed inspection of airborne equipment and accessories of carrier-based aircraft. Generally, this task must be transferred to a land-based repair workshop for standard land-level maintenance. The ship-based hangar bay is responsible for carrier-based aircraft maintenance and repair (MR) tasks. These two types of tasks are the main focus of this study and are collectively called MR tasks. Carrier-based aircraft must be tested for failure before and after a mission. If losses are detected, the aircraft must be recovered to the hanging bay and queued for repair after entering the parking spots. After a repair, failures are fixed within certain limits, and the structural shape and performance are restored. In addition to unplanned repair tasks, both scheduled and preventive maintenance must be completed. Maintenance activities are typically conducted after an aircraft has been operational for a certain number of flight hours. Carrier-based aircraft maintenance activities are also performed after an aircraft has been operational for a certain number of flight hours [38]. Periodic inspections are usually conducted after 25, 50, and 100 flight hours, seven days, three months, and six months, respectively. The hangar is subject to extensive MR tasks to maintain high fleet availability. Figure 2 shows the maintenance resources and environment of the Kuznetsov aircraft carrier hangar bay. The involved constraints are described in more detail below.

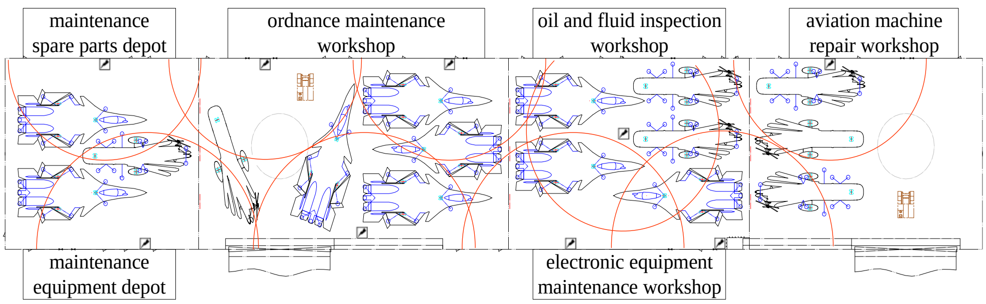


Figure 2. Maintenance resources and environment of the Kuznetsov aircraft carrier hangar bay.

2.1. Maintenance Process

The MR task operations of carrier-based aircraft have a precedence-based logical relations constraint. For the repair tasks of carrier-based aircraft, the sequential order of operations is as follows: failure location, failure repair, and re-inspection. For preventive maintenance tasks, the operations are in networked precedence relations, and any immediately preceding operation is not unique. In the case of the RCPSP, the maintenance task of a single aircraft can be regarded as a project, and the precedence constraints of the maintenance operations can be described by the activity-on-node (AoN) network node, where an operation is represented by a node, and the precedence constraints between activities are denoted by the arcs. I represents a set of carrier-based aircraft, $I = \{1, 2, \cdots, i, \cdots, |I|\}$. J_i represents the set of all maintenance operations of the ith carrier-based aircraft, $J_i = \{1, 2, \cdots, j, \cdots, |J_i|\}$, and J represents the set of all maintenance operations of the fleet, $J = \{(i, j) | i \in I, j \in J_i\}$. A maintenance operation starts after its tethering completion time Ex_i in parking spot p_i. An AoN diagram for preventive maintenance operations is shown in Figure 3. O_{ij} represents the jth maintenance operation of the ith aircraft in the fleet to be maintained. O_S and O_E refer to the virtual start and end of the virtual operation, respectively; these do not consume any resources, have zero operation durations, and serve to integrate all maintenance operations. O_S has no immediately preceding operations, and O_E has no immediately subsequent operations. The dotted line represents the process and virtual process connections.

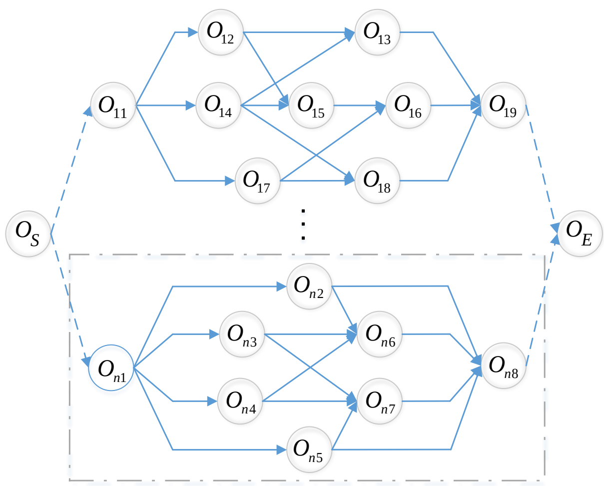


Figure 3. Activity-on-node (AoN) diagram of preventive maintenance operations.

2.2. Maintenance Personnel and Skills

In the AMRSP, the maintenance skills represent the direct operational demands, and the operations correspond to specific skill categories. As the number of maintenance personnel is a constraint, it is common to allocate personnel with multiple skills to enhance the flexibility of task execution. In other words, a set of maintenance personnel is established as a flexible resource with multiple skills. Each person is equipped to perform cross-professional maintenance work in a compatible manner. Different maintenance operations usually require different skills, and a competent professional is identified according to their maintenance skills to complete the task. Lp indicates the set of maintenance personnel. Kc indicates the set of skill categories of the maintenance personnel, $Kc = \{1, 2, \cdots, |Kc|\}$. Figure 4 shows the matching relationship between maintenance operations, skills, and personnel. It represents a further refinement of the maintenance skills and personnel requirements corresponding to the operations.

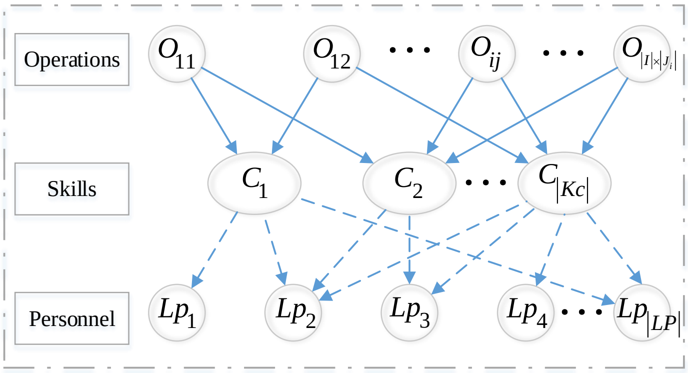


Figure 4. Matching relationship between maintenance operations, skills, and personnel.

2.3. Maintenance Equipment, Workshop, and Workspace

The maintenance equipment of hangar bays can be divided into fixed resource stations and maintenance workshops. Fixed resource stations support the tasks of carrier-based aircraft at the parking spots within the coverage area. Workshops are distributed around the hanging bay and provide regular maintenance and off-site repair of aviation components. This study focuses on a power supply station among the fixed resource stations, as represented by the red line in Figure 2. Maintenance workshops are also located around the hangar bay. They are used to provide scheduled maintenance and off-site repair for aviation components. These workshops consist of aeronautical machine repair, oil and fluid inspection, ordnance, and electronic equipment maintenance workshops. Le indicates a set of maintenance equipment or workshops. Ke indicates the set of skill categories for a piece of maintenance equipment/workshop, and $Ke = \{1, 2, \cdots, |Ke|\}$.

Given the aeronautical MR process for carrier-based aircraft, owing to space constraints, some operations, such as cockpit operations, can accommodate only a certain number of personnel for parallel operations. Ks indicates the set of skill categories for the maintenance workspace, and $Ks = \{1, 2, \cdots, |Ks|\}$.

3. Mathematical Model for Aeronautical Maintenance and Repair Task Scheduling Problem (AMRSP)

3.1. Problem Assumptions

The AMRSP mathematical modeling includes the following simplifications.

i. The MR tasks are known with certainty and do not consider the interference of dynamic factors.
ii. The MR process cannot be preempted or interrupted once started.
iii. The maintenance skills are adapted to each aircraft's MR task mode.
iv. The transit time in the hangar bay is ignored.
v. An adequate reserve of fixed-resource station resources is available.

3.2. Constraints

The related notations and descriptions of the AMRSP mathematical modeling are formulated in Table 1.

Table 1. Related notations and description of the AMRSP mathematical model.

Notations	Descriptions
I	The set of carrier-based aircraft, $I = \{1, 2, \cdots, i, \cdots, \|I\|\}$.
p_i	The parking spot of the ith carrier-based aircraft.
J_i	The set of maintenance operations of the ith carrier-based aircraft, $J_i = \{1, 2, \cdots, j, \cdots, \|J_i\|\}$.
J	The set of all maintenance operations of the fleet, $J = \{(i, j) \| i \in I, j \in J_i\}$.
A_t	The set of all maintenance operations of the fleet in the execution state at time point t.
A_{it}	The set of maintenance operations of the ith carrier-based aircraft in the execution state at time point t.
O_{ij}	The jth maintenance operation of the ith carrier-based aircraft.
Ps_{ij}	The set of immediately preceding operations of O_{ij}.
Ex_i	The tethering completion time of the ith carrier-based aircraft.
d_{ij}	The operation duration of O_{ij}.
BM	A sufficiently large real number.
Lp	The set of maintenance personnel.
Le	The set of maintenance equipment/workshops.
Kc	The set of skill categories of the maintenance personnel, $Kc = \{1, 2, \cdots, \|Kc\|\}$.
Ke	The set of skill categories for maintenance equipment/workshops, $Ke = \{1, 2, \cdots, \|Ke\|\}$.
Ks	The set of skill categories at the maintenance workspace, $Ks = \{1, 2, \cdots, \|Ks\|\}$.
rc_{ijk}	An indicator variable valued 0 or 1, where 1 indicates that O_{ij} has a demand for the kth skill category, whereas 0 indicates otherwise.
re_{ijk}	An indicator variable valued 0 or 1, where 1 indicates that O_{ij} has a demand for the kth maintenance equipment/workshop category, whereas 0 indicates otherwise.
rs_{ijk}	An indicator variable valued 0 or 1, where 1 indicates that O_{ij} has a demand for the kth maintenance workspace, whereas 0 indicates otherwise.
λ_{kl}^{p}	An indicator variable valued 0 or 1, where 1 indicates that the lth of the kth maintenance equipment/workshop category has a reachability relation with p, whereas 0 indicates otherwise.
ns_{ik}	The number of personnel who can work in parallel with the ith carrier-based aircraft kth workspace category.
Ne_{kl}	The number of operations that can be accommodated in parallel in the lth workshop of the kth category.
Sm_{ij}	A decision variable indicating the start time of O_{ij}.
Em_{ij}	A decision variable indicating the end time of O_{ij}.
Xp_{ijkl}	A decision variable valued 0 or 1, where 1 indicates that O_{ij} is assigned to the lth maintenance personnel using the kth skill category, whereas 0 indicates otherwise.
Xe_{ijkl}	A decision variable valued 0 or 1, where 1 indicates that O_{ij} is assigned to the lth of the kth maintenance equipment/workshop category, whereas 0 indicates otherwise.
Yp_{ijeg}	A decision variable valued 0 or 1, where 1 indicates that O_{ij} is assigned to the same maintenance personnel as O_{eg}, and O_{ij} is prioritized over O_{eg}, whereas 0 indicates otherwise.
Ye_{ijeg}	A decision variable valued 0 or 1, where 1 indicates that O_{ij} is assigned to the same maintenance equipment/workshop as O_{eg}, and O_{ij} is prioritized over O_{eg}, whereas 0 indicates otherwise.

Constraints:

The first constraint concerns the starting time sequence for an MR task operation after tethering in the parking spot is completed. Sm_{i1} is the maintenance start time of the first maintenance operation of aircraft $i(i \in I)$. Aircraft i must start the first maintenance operation after the tethering completion time Ex_i. This constraint is expressed as follows:

$$Sm_{i1} \geq Ex_i, \forall i \in I \tag{1}$$

The MR task process for each aircraft must be performed sequentially by following the established workflow and precedence relations. Em_{ih} denotes the end time of maintenance operation O_{ih}, where $(i, h) \in Ps_{ij}$; Ps_{ij} denotes the set of processes immediately preceding maintenance operation O_{ij}. This constraint is expressed as follows:

$$Sm_{ij} \geq Em_{ih}, \forall (i, h) \in Ps_{ij}, \forall (i, j) \in J \tag{2}$$

When different operations require the same resources and because the number of maintenance personnel and maintenance equipment/workshops are limited, it is necessary to determine the order of maintenance according to priority. BM denotes a sufficiently large positive number; d_{ih} denotes the operation duration of a maintenance operation O_{ij}. $Yp_{ijeg} = 1$ indicates that the maintenance operation O_{ij} is assigned to the same maintenance personnel as O_{eg} and that O_{ij} takes priority over O_{eg}. $Ye_{ijeg} = 1$ indicates that the maintenance operation O_{ij} is assigned to the same maintenance equipment/workshop as O_{eg} and that O_{ij} takes priority over O_{eg}. This constraint is expressed as follows:

$$Sm_{ij} + d_{ij} \leq Sm_{eg} + BM \cdot (1 - Yp_{ijeg}), \forall (i,j), (e,g) \in J \tag{3}$$

$$Sm_{ij} \leq Sm_{eg} + BM \cdot (1 - Ye_{ijeg}), \forall (i,j), (e,g) \in J \tag{4}$$

Skills are direct demand resources for MR task operations. The number of skills demanded by any MR task operation should match the number of personnel assigned to that operation. $rc_{ijk} = 1$ indicates that the maintenance operation O_{ij} has a demand for the kth category skill. $Xp_{ijkl} = 1$ indicates that the maintenance operation O_{ij} is assigned to the lth$(l \in Lp)$ personnel and that the personnel performs the operation using the kth$(k \in Kc)$ skill category. This constraint is expressed as follows:

$$\sum_{l \in Lp} Xp_{ijkl} = rc_{ijk}, \forall (i,j) \in J, \forall k \in Kc \tag{5}$$

The demand for various types of maintenance equipment/workshops should match the number of resources assigned to that operation. $re_{ijk} = 1$ indicates that O_{ij} has a demand for the kth maintenance equipment/workshop category (which can accommodate a certain number of parallel operations). $Xe_{ijkl} = 1$ indicates that O_{ij} is assigned to the lth of the kth maintenance equipment/workshop category. The constraint is expressed as follows:

$$\sum_{l \in Le} Xe_{ijkl} = re_{ijk}, \forall (i,j) \in J, \forall k \in Ke \tag{6}$$

Each person uses at most one skill for any operation. This constraint is expressed as follows:

$$\sum_{k \in Kc} Xp_{ijkl} \leq 1, \forall (i,j) \in J, \forall l \in Lp \tag{7}$$

Constraint (8) represents the coverage of the fixed resource stations. $\lambda_{kl}^{p} = 1$ indicates that the maintenance equipment/workshop has a reachability relationship with p. Constraint (8) is expressed as follows:

$$\sum_{(i,j) \in J} \sum_{k \in Ke} \sum_{l \in Le} Xe_{ijkl} \cdot \left(1 - \lambda_{kl}^{p_i}\right) = 0, \forall (i,j) \in J \tag{8}$$

Constraint (9) concerns the number of resources in a parallel workspace, and constraint (10) is used for the maintenance workshop. A_{it} indicates the set of maintenance operations of the ith carrier-based aircraft in the execution state at time point t. A_t indicates the set of all maintenance operations when the fleet is in the execution state at time point t. $\mathrm{rs}_{ijk} = 1$ indicates that O_{ij} has a demand for the kth maintenance workspace. ns_{ik} indicates the number of personnel that can work in parallel with the ith carrier-based aircraft kth category workspace. Ne_{kl} indicates the number of operations that can be accommodated in parallel in the lth workshop in the kth category. This constraint is expressed as follows:

$$\sum_{j \in A_{it}} rs_{ijk} \leq ns_{ik}, \forall i \in I, \forall k \in Ks, \forall t > 0 \tag{9}$$

$$\sum_{j \in A_t} re_{ijk} \cdot Xe_{ijkl} \leq Ne_{kl}, \forall i \in I, \forall k \in Ke, \forall l \in Le, \forall t > 0 \tag{10}$$

Constraint (11) states that Xp_{ijkl}, $Xe_{ijk'l'}$, Yp_{ijeg}, and Ye_{ijeg} are Boolean variables.

$$\begin{gathered} Xp_{ijkl}, Xe_{ijk'l'}, Yp_{ijeg}, Ye_{ijeg} \in \{0,1\}, \\ \forall k \in Kc, \forall l \in Lp, \forall k' \in Ke, \forall l' \in Le, \forall (i,j), (e,g) \in J \end{gathered} \tag{11}$$

3.3. Objective Function

The optimization objectives for fleet wave availability and maintenance personnel load variance are constructed based on the requirements of aircraft fleet combat and training missions. The optimization objectives for the maintenance of existing equipment or support mission studies are usually set to minimize the maximum makespan (min$C_{\max}$). However, owing to the operational characteristics of carrier-based aircraft (which usually attack in clusters), the aircraft sorties are mainly focused on fleet wave sorties with the prerequisite of maximizing the number of available fleets in the sortie plan. After a command to launch the fleet is received, if the number of aircraft in good condition is insufficient for the wave, there will be aircraft with incomplete preventive maintenance or failure repair tasks. This situation can severely affect operational effectiveness if the available fleet cannot be replenished in time. In this study, the wave availability index is defined as the weighted availability of the fleet before each subsequent wave. The increased wave availability means that MR tasks can provide more intact aircraft for each wave. That is, MR tasks can meet the numbers for the wave sorties' requirements. Another consideration is to minimize the load variance of the maintenance personnel to increase the sustainability of personnel operations.

(1) Maximizing fleet-wave availability (WA)

$$\max\text{WA} = \sum_{w \in W} v_w \frac{Nm - \sum\limits_{i \in I} \text{pcf}(ET_i - SW_w)}{Nm} \tag{12}$$

Here, W denotes the set of wave sorties, SW_w the start time of the waves, v_w the weight for wave availability, and ET_i the makespan of the maintenance of carrier-based aircraft i. Moreover, $\text{pcf}(\cdot)$ is an indicator function, where $\text{pcf}(x) = 1$ when $x > 0$, and $\text{pcf}(x) = 0$ when $x \leq 0$. The purpose of the WA function is to maximize the sum of the weighted availability in the set of waves.

(2) Minimizing the personnel load variance (PLV) results in

$$\min\text{PLV} = \frac{\sum\limits_{l \in Lp} \left(TB_l - \overline{TB}\right)^2}{|Lp|} \tag{13}$$

Here, TB_l denotes the total number of task hours spent by the $l(l \in Lp)$ maintenance personnel, and $\overline{TB}$ represents the mean value of the maintenance task hours for all personnel. PLV defines the personnel load variance in hours, and the objective is to minimize it.

4. Algorithm for AMRSP

4.1. Encoding and Serial Scheduling Generation Scheme (SSGS)

The encoding strategy is an essential factor affecting the effectiveness and efficiency of an algorithm search. The primary encoding strategies for solving an RCPSP problem include the task list, random number, and priority rules. Owing to the precedence relations between the operations of a task, the encoding forms from the task list and priority rules may be used to obtain combinations of operations that do not conform to the relations in the next crossover and mutation; therefore, random number encoding is used. The Gth generation population $\mathbf{P}_G$,$\mathbf{P}_G = [\mathbf{X}_{1,G}, \mathbf{X}_{2,G}, \cdots, \mathbf{X}_{n,G}], n = 1, 2, \cdots, Np$, where Np denotes the number of individuals in the population and $\mathbf{X}_{n,G}$ denotes the nth individual code of the Gth generation, can be defined as $\mathbf{X}_{n,G} = \left[x_{11,n,G}, x_{12,n,G}, \cdots, x_{ij,n,G}, \cdots\right]$, $\forall i \in I, j \in J_i$. Here, $x_{ij,n,G}$ denotes the priority number of the jth maintenance operation of the ith carrier-based aircraft in the nth individual of the Gth generation. Each aircraft is sequentially arranged in order of operation and assigned a random priority number in the interval $(0, 1)$. The smaller the priority number, the higher the priority of the corresponding operation. Integration and stitching form an individual encoding matrix with $\sum_{i \in I} |I| \times |J_i|$ dimensions for each $\mathbf{X}_{n,G}$. A schematic representation of the encoding structure is shown in Figure 5. The discrete encoding matrix avoids illegal generation in subsequent operations.

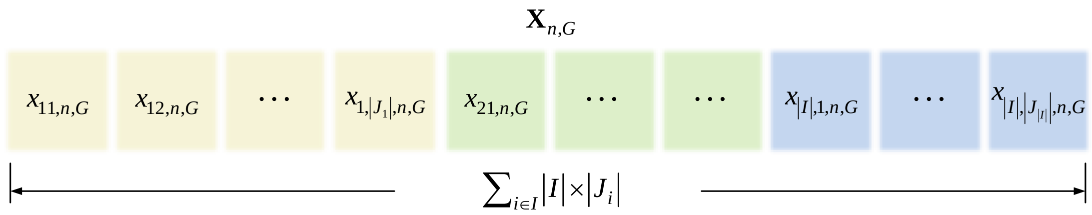


Figure 5. Schematic diagram of the encoding structure.

The schedule generation scheme (SGS) is at the core of most RCPSP metaheuristic algorithms. The SGS can generate a feasible scheduling scheme by incrementally extending the partial schedule from the start of the project. A partial schedule for a project with J tasks contains only l ($l < J$) tasks. Depending on the generated method, the SGS can be classified into task-based and time-based phase variables [39]. The SGS with task-based phase variables is also called a serial schedule generation scheme (SSGS), whereas that with time-based phase variables is called a parallel schedule generation scheme (PSGS). Hartmann [40] pointed out that the search space of the PSGS is a subset of the solution space, and using the PSGS can find a better solution in a short time, but it may not contain the optimal solution. Therefore, using an SSGS remains the optimal choice.

Unlike the conventional SSGS, to address the MS-RCPSP, the ITLBO-S is used for any waiting scheduling maintenance operation O_{ij}. The search phase of the schedule advancement, which includes constraints on the maintenance personnel, equipment, and workspace requirements, has an embedded function for matching the skills required for O_{ij} with suitable personnel. In other words, the allocation of personnel and selection of skills occur simultaneously during the progressive expansion of the schedule. A set of scheduled operations S_g is defined, along with a set of schedulable operations D_g. In the scheduling generation scheme, operation O_{ij} is selected from D_g according to the precedence relations, and the start time for O_{ij} is equal to the tethering completion time. D_g is determined by the sequence constraints and precedence relations from the AoN diagram. Next, O_{ij} is selected from D_g, and resources, such as personnel, equipment, and workshops, are then allocated to O_{ij}. After the allocation is completed, O_{ij} is added to set S_g. The iteration moves to the next selection stage and gradually expands the scheduling scheme until all operations are scheduled. Figure 6 shows the flowchart of the SSGS.

The following heuristic rules are added to the SSGS. First, considering that multi-skilled personnel are more flexible than regular personnel, priority is given to personnel with fewer skills to improve the scheduling scheme's robustness and ensure that the availability of the skills required for subsequent maintenance processes is maximized. Second, a tie-breaking priority rule is added; if the same numbers of skills are available, the personnel with fewer accumulated work hours are assigned to perform the task first to maintain the load balance. Third, the minimum total processing time remaining in the covering area rule is used for the allocation of the maintenance equipment. Fourth, in assigning maintenance workshops, a resource concentration rule is used, which assigns the tasks to the workshops with the highest number of maintenance operations in execution.

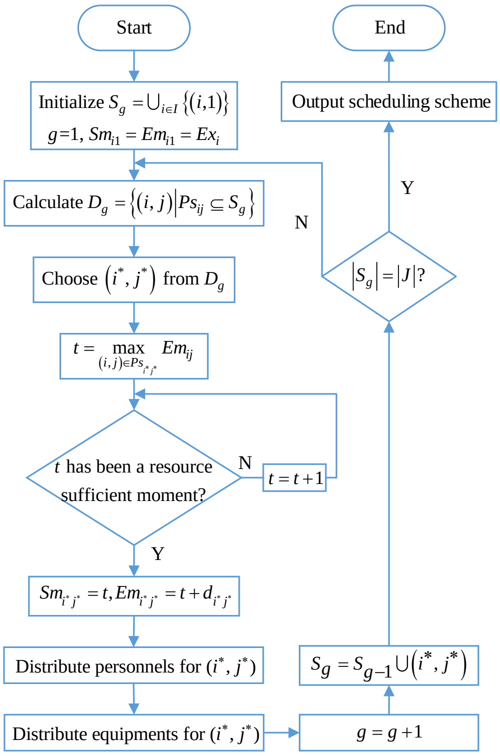


Figure 6. Flowchart of serial scheduling generation scheme (SSGS).

A decoding process is used to facilitate individual evaluation. For simplicity, this study utilizes a weighted sum approach [41] to construct the fitness function f. Several objectives are multiplied, according to their importance, by a set of weight coefficients $\alpha_1, \alpha_2, \ldots, \alpha_n$, which are then summed as the final objective function, thus simplifying the multi-objective problem to a single-objective problem. This significantly simplifies the computational process. The objective functions WA and PLV are combined linearly with weight coefficients as a single fitness function. Thus, f is formulated as follows:

$$\min f = \alpha_1 \cdot (1 - \mathrm{WA}) + \alpha_2 \cdot \mathrm{LBM} \tag{14}$$

In the above, α_1 and α_2 are the weight coefficients, and the weights can be adjusted according to the task requirements. According to this method, a set of single-objective optimization subproblems can be constructed, and the smaller the result, the better the individual fitness function.

By integrating the above preparations, a solution strategy for the algorithm is proposed, where the encoding of each individual in the population represents the order priority of all maintenance operations, and the operation of the SSGS maps this encoding to the actual operation order. In this process, a judgment that includes all types of resources (personnel, equipment, space) and the logical relations of the precedence operation order (AoN diagram) is required. This is because although some operations of a high priority level should be prioritized, if they do not meet the constraints, they are held back until the resources meet the conditions and then prioritized if they are still of a high priority. After applying the SSGS for all individuals, we obtained a population with a variety of operational orderings that meet the constraints. The quality of these operational sequences can be good or bad. We need to evaluate and evolve the population such that the quality of the operational order improves until the set conditions are met; these steps are achieved by the ITLBO main loop.

4.2. Improved Teaching-Learning-Based Optimization (ITLBO) Main Loop

Similar to other nature-inspired algorithms, TLBO is a population-based approach that uses population evolution for globally optimal solutions [31]. It simulates a teacher-student teaching-learning process in the classroom, and the optimization process consists of a teaching phase and a learning phase. The teaching phase refers to learning from the teacher, and the learning phase refers to learning through student interaction. The individual with the optimal fitness value in the population is the teacher (i.e., the optimal individual in the population). The other individuals in the population are students. The TLBO uses an objective function to evaluate an individual's performance and to determine a solution for the individual's optimal global performance. The learning results will increase in "fitness", similar to other population-based optimization algorithms.

Traditional TLBOs used in optimization problems have the following disadvantages. In the teaching phase, all individuals gain knowledge from the teacher based on the difference between the teacher and the individuals' overall average position. Therefore, in the teaching phase, all individuals gather around the teacher, ensuring a quick convergence. However, as the teacher approaches the local optimal solution, the population inevitably converges early. In the learning phase, individuals learn from each other to escape the local optima [42]. However, without the injection of new knowledge, the search space in the learning phase remains limited, and the diversity of the population can hardly increase further.

Therefore, the TLBO is still very likely to fall into a local convergence phenomenon when dealing with complex optimization problems [43]. To solve this problem, this study proposes the ITLBO-S, which is based on the TLBO, with an assistant teaching phase based on the optimal fitness-distance ranking ratio. This guides all individuals to learn from the teachers and assistant teachers according to the differential law. The exploitation and exploration abilities of the TLBO are improved by the differential knowledge between students and teachers and between students and assistant teachers. A balance between local and global exploitation is achieved, improving the performance of the algorithm [44]. Meanwhile, to capture the characteristics of the AMRSP, an SSGS based on resource constraint advancement is added to the algorithm to enable it to solve the MS-RCPSP.

4.2.1. Teaching Phase

In the teaching phase, the individual with the best fitness from the Gth generation population is selected as the individual teacher $\mathbf{X}_{t,G}$. According to the instructional guidance mechanism, all individuals (students) $\mathbf{X}_{n,G}$ learn from the Gth generation population and teacher $\mathbf{X}_{t,G}$ (their encoding numbers will be closer to the teacher's encoding number). Each individual produces a new individual $\mathbf{X}_{n,G}^{\text{new}}$ after the teaching phase, as shown in Equation (15).

$$\begin{cases} \mathbf{X}_{n,G}^{\text{new}} = \mathbf{X}_{n,G} + r_n \cdot (\mathbf{X}_{t,G} - T_F \cdot \mathbf{M}_G) \\ T_F = \text{round}(1 + \text{rand}(0\,,\ 1)) \end{cases} \tag{15}$$

Here, $\mathbf{M}_G$ represents the average encoding matrix for the Gth generation of individuals, r_n denotes a random number between $(0\,,\ 1)$, and T_F denotes the learning weight. From Equation (15), it can be seen that T_F takes a value of 1 or 2. The two random parameters r_n and T_F perform the teaching phase randomization. Comparing $\mathbf{X}_{n,G}$ with $\mathbf{X}_{n,G}^{\text{new}}$ in the one-to-one method for adaptation evaluation, the individual with the better adaptation is selected to update and replace the original individual. Updating is performed using Equation (16).

$$\mathbf{X}_{n,G} = \begin{cases} \mathbf{X}_{n,G}, & if\ f(\mathbf{X}_{n,G}) \leq f\left(\mathbf{X}_{n,G}^{\text{new}}\right) \\ \mathbf{X}_{n,G}^{\text{new}}, & otherwise \end{cases} \tag{16}$$

4.2.2. Learning Phase

In the learning phase, new individuals $\mathbf{X}_{n,G}^{\text{new}}$ are generated among the students in the Gth generation population through mutual learning according to the learning guidance mechanism described by Equation (17), as follows:

$$\mathbf{X}_{n,G}^{\text{new}} = \begin{cases} \mathbf{X}_{n_1,G} + r_{n_1} \cdot \left(\mathbf{X}_{n_1,G} - \mathbf{X}_{n_2,G}\right), & if\ f\left(\mathbf{X}_{n_1,G}\right) \le f\left(\mathbf{X}_{n_2,G}\right) \\ \mathbf{X}_{n_1,G} + r_{n_1} \cdot \left(\mathbf{X}_{n_2,G} - \mathbf{X}_{n_1,G}\right), & otherwise \end{cases} \tag{17}$$

In the above, r_{n_1} denotes a random number between $(0\ ,\ 1)$, $\mathbf{X}_{n_1,G}$ and $\mathbf{X}_{n_2,G}$ are two randomly selected student individuals in the current generation population, and $n_1 \neq n_2$. After the learning phase has generated the new individual $\mathbf{X}_{n,G}^{\text{new}}$, the same adaptation assessment is performed for $\mathbf{X}_{n,G}$ and $\mathbf{X}_{n,G}^{\text{new}}$. An individual with better adaptation is selected to update and replace the original individual. The updating method is shown in Equation (16).

4.2.3. Assistant Teaching Phase

In the teaching phase, because the individual population learns from the teacher, the algorithm achieves better convergence. However, when the individual teacher is located near the local optimal solution, it causes all individuals to move closer to the local optimal solution position, leading to the premature convergence of the algorithm. A subsequent learning phase in which students learn from each other can be used to prevent the population from falling into the local optimum. However, this is limited by the inherent knowledge of students in the population, which leads to unsatisfactory results from the algorithm exploration in the learning phase and makes it challenging to jump out of the local optimum.

To solve these problems, an assistant teacher teaching phase is proposed to guide students to learn from both the teacher and the assistant teacher. The assistant-teaching phase is based on an optimal fitness-distance ranking ratio method. Subsequently, the algorithm can balance the local exploitation and global exploration abilities of the solution space in the assistant-teaching phase.

(1) Fitness-distance ranking ratio

In any Gth generation, a fitness sorting matrix $\mathbf{F} = [F_1\ ,\ F_2\ ,\ \ldots\ ,\ F_n]$ is defined. $\mathbf{F}$ is obtained using $[f(\mathbf{X}_{1,G})\ ,\ f(\mathbf{X}_{2,G})\ ,\ \cdots\ ,\ f(\mathbf{X}_{n,G})]$ after sorting the fitness values $f(\mathbf{X}_{n,G})$ of the individuals $\mathbf{X}_{n,G}$ from best to worst. F_n corresponds to the index value of $[f(\mathbf{X}_{1,G})\ ,\ f(\mathbf{X}_{2,G})\ ,\ \cdots\ ,\ f(\mathbf{X}_{n,G})]$ after sorting so that the individual $\mathbf{X}_{n,G}$ with the best fitness is used as the Gth generation teacher; that is, the individual $\mathbf{X}_{n,G}$ corresponding to $F_n = 1$ is used as the individual teacher $\mathbf{X}_{t,G}$.

The Euclidean distance sorting matrix is defined as $\mathbf{D} = [D_1\ ,\ D_2\ ,\ \cdots\ ,\ D_n]$. $\mathbf{D}$ is obtained after sorting in ascending order using $[E(\mathbf{X}_{1,G})\ ,\ E(\mathbf{X}_{2,G})\ ,\ \cdots\ ,\ E(\mathbf{X}_{n,G})]$ (Euclidean distance from near to far), where $E(\mathbf{X}_{n,G})$ is the Euclidean distance between an individual $\mathbf{X}_{n,G}$ and teacher individual $\mathbf{X}_{t,G}$. D_n corresponds to the index value of $[E(\mathbf{X}_{1,G})\ ,\ E(\mathbf{X}_{2,G})\ ,\ \cdots\ ,\ E(\mathbf{X}_{n,G})]$ after sorting. The fitness-distance ranking ratio is defined as $\mathbf{FD} = [FD_1\ ,\ FD_2\ ,\ \cdots\ ,\ FD_n]$, $FD_n = \frac{F_n}{D_n}$. The sorting ratio FD_n of the smallest fitness distance corresponds to the individual $\mathbf{X}_{n,G}$ as the Gth generation of assistant teacher individual $\mathbf{X}_{a,G}$. In other words, the individual $\mathbf{X}_{n,G}$ corresponding to the minor value FD_n in $\mathbf{FD}$ is $\mathbf{X}_{a,G}$. To illustrate the relative positions, fitness levels, and distance distributions of the teachers and assistant teachers, a two-dimensional population with 20 individuals in the Gth generation is selected as an example. Figure 7a shows a schematic of the distribution of the individual positions. Figure 7b shows the fitness levels of the population individuals and distances between the population individuals and teacher individuals; these can be used to determine the individuals with the best fitness $\mathbf{X}_{4,G}$ as $\mathbf{X}_{t,G}$. The sorting matrices $\mathbf{F}$ and $\mathbf{D}$ can be obtained after sorting the information in Figure 7b, and the fitness-distance ranking ratio $\mathbf{FD}$ can then be calculated. From this, the individual $\mathbf{X}_{17,G}$ can be determined as the assistant teacher's individual $\mathbf{X}_{a,G}$.

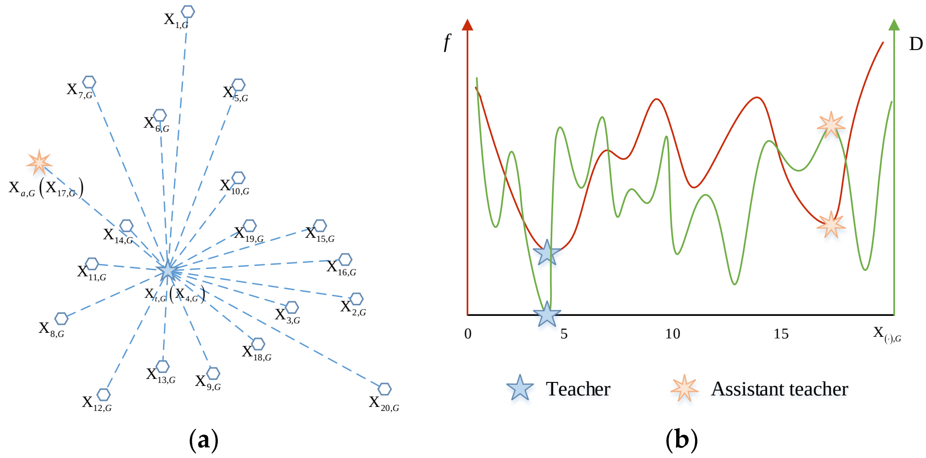


Figure 7. Example of fitness-distance ranking ratio. (**a**) Individual position distribution; (**b**) Fitness and distance distribution between teachers and assistant teachers.

(2) Assistant Teacher Teaching

In this phase, based on the difference operators $r_{1,n} \cdot (\mathbf{X}_{t,G} - \mathbf{X}_{n,G})$ and $r_{2,n} \cdot (\mathbf{X}_{a,G} - \mathbf{X}_{n,G})$, the individual $\mathbf{X}_{n,G}$ learns from teachers and assistant teachers to form new individuals. Equation (18) represents the generation of new individuals, as follows:

$$\mathbf{X}_{n,G}^{\text{new}} = \mathbf{X}_{n,G} + r_{1,n} \cdot (\mathbf{X}_{t,G} - \mathbf{X}_{n,G}) + r_{2,n} \cdot (\mathbf{X}_{a,G} - \mathbf{X}_{n,G}) \quad (18)$$

where $r_{1,n}$ and $r_{2,n}$ are random numbers between $(0, 1)$. When $r_{1,n} \geq r_{2,n}$, the position of the new individual $\mathbf{X}_{n,G}^{\text{new}}$ leans toward the teacher $\mathbf{X}_{t,G}$ to improve the algorithm's local search capability. When $r_{1,l} < r_{2,l}$, the position of the new individual $\mathbf{X}_{n,G}^{\text{new}}$ leans toward the assistant teacher $\mathbf{X}_{a,G}$ to improve the algorithm's global search capability. Taking the two-dimensional space as an example, a new individual $\mathbf{X}_{n,G}^{\text{new}}$ is generated, as shown in Figure 8.

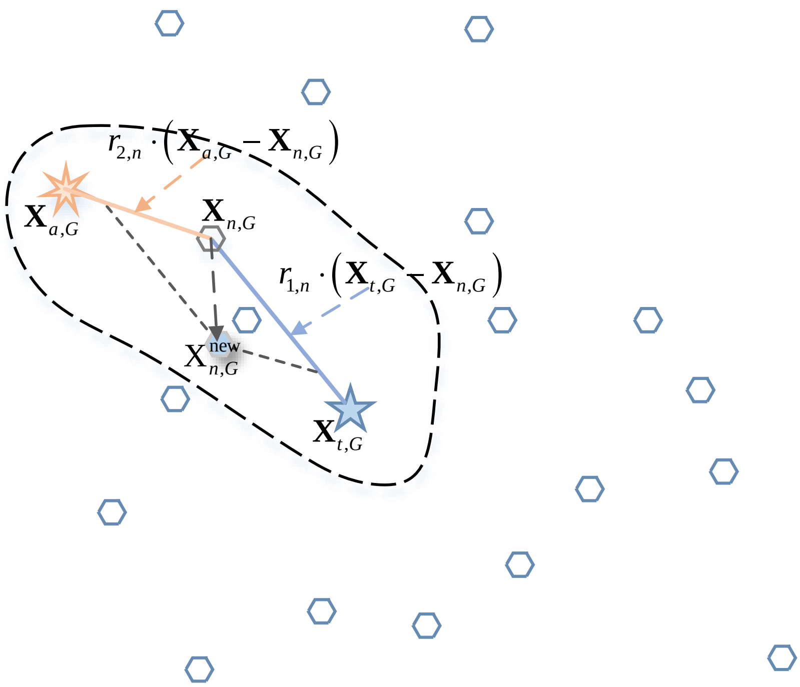


Figure 8. Schematic diagram of the two-dimensional new individual generation.

The Gth assistant teaching phase is defined to generate population $\mathbf{P}_G^{\text{new}}$ and $\mathbf{P}_G^{\text{new}} = \left[\mathbf{X}_{1,G}^{\text{new}}, \mathbf{X}_{2,G}^{\text{new}}, \cdots, \mathbf{X}_{n,G}^{\text{new}}\right]$. After the assistant teaching phase, the population generation is $G+1$. NP individuals with optimal fitness from $\{\mathbf{P}_G \cup \mathbf{P}_G^{\text{new}}\}$ are selected as the new population $\mathbf{P}_{G+1}$. A flowchart for the ITLBO-S is shown in Figure 9.

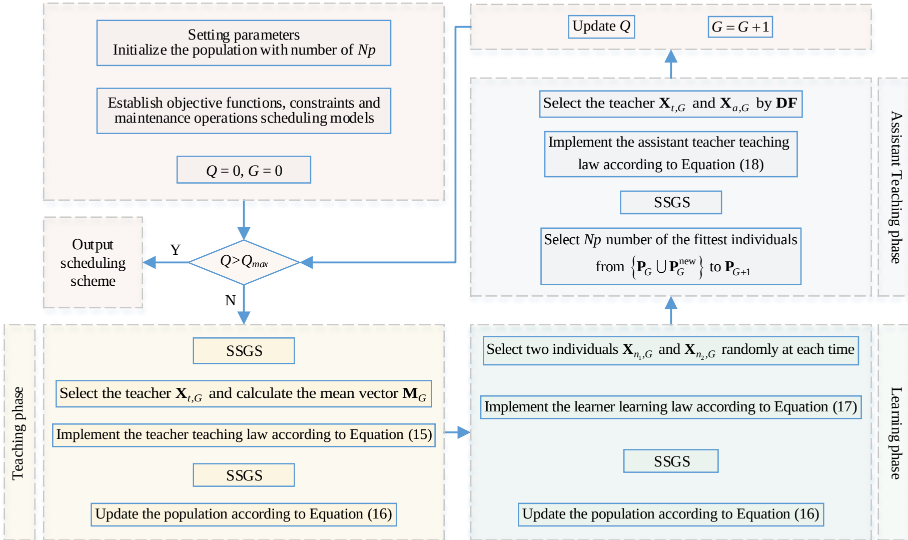


Figure 9. Flowchart for improved teaching-learning-based optimization algorithm with a serial scheduling generation scheme (ITLBO-S).

4.3. Complexity Analysis

The complexity of the ITLBO-S is reflected in two aspects. First, it is reflected in the teaching phase, learning phase, and assistant teaching phase, where each individual is coded with the dimension $\sum_{i \in I} |I| \times |J_i|$, and the complexity of the teaching phase is $O(Np \times \sum_{i \in I} |I| \times |J_i|)$. This indicates that Np individuals learn from the teacher. The complexity of the learning phase is also $O(Np \times \sum_{i \in I} |I| \times |J_i|)$. This indicates that Np individuals learn from each other. The complexity of the assistant teaching phase is $O(Np \times \sum_{i \in I} |I| \times |J_i|) + O(Np \times \log_2(2Np))$, indicating that Np individuals learn simultaneously from the teacher and assistant teacher and that Np individuals are selected from $2Np$ individuals as the next generation of the population.

However, the computational complexity of the ITLBO-S is also reflected in the SSGS process. According to the literature [39,45], the complexity of an SSGS is $O(|J|^2 R)$, where R is the number of reproducible resource types. In the AMRSP, three resource states must be considered when finding feasible resources, and the complexities of finding the spaces for the maintenance personnel, maintenance equipment/workshops, and maintenance station space are $O(|J|^2|Lp|)$, $O(|J|^2|Le|)$, and $O(|J|^2 \times |Ks|)$, respectively.

5. Simulation Case Analysis

5.1. Maintenance and Repair (MR) Task Case Generation

The simulation case in this study is based on the hangar-bay environment shown in Figure 2. Multimode mixed situations, such as preventive maintenance and failure repair, were considered, and 10, 12, and 14 carrier-based aircraft numbered A–N were set for the MR tasks. The task settings for the fleet MR are shown in Table 2. In the table, maintenance modes 1–6 correspond to six maintenance modes: mechanical failure, avionics system failure, special equipment failure, and maintenance after 25, 50, and 100 h flight hours. The maintenance process of the AoN diagram for preventive maintenance is shown in Figure 10. The subsequent outgoing wave was set as $|W| = 3$. The earlier the wave is deployed, the more critical the impact on the battlefield and the greater its importance. The wave weight was set as $[v_1,\ v_2,\ v_3] = [0.5,\ 0.3,\ 0.2]$. The wave interval period was 100 min; that is, the starting times were $[SW_1,\ SW_2,\ SW_3] = [100,\ 200,\ 300]$ min.

Table 2. MR tasks for fleet.

P.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
MR Tasks	Carrier-based aircraft no.; Ex (min) MR tasks modes													
Case 1	A; 2 2	B; 8 3	C; 0 5	D; 0 4	E; 9 1	F; 16 1	G; 0 6	H; 0 5	I; 3 1	J; 15 2	-	-	-	-
Case 2	A; 2 2	B; 8 3	C; 0 5	D; 0 4	E; 9 1	F; 16 1	G; 0 6	H; 0 5	I; 3 1	J; 15 2	K; 21 5	L; 22 3	-	-
Case 3	A; 2 2	B; 8 3	C; 0 5	D; 0 4	E; 9 1	F; 16 1	G; 0 6	H; 0 5	I; 3 1	J; 15 2	K; 21 3	L; 22 4	M; 27 4	N; 29 4

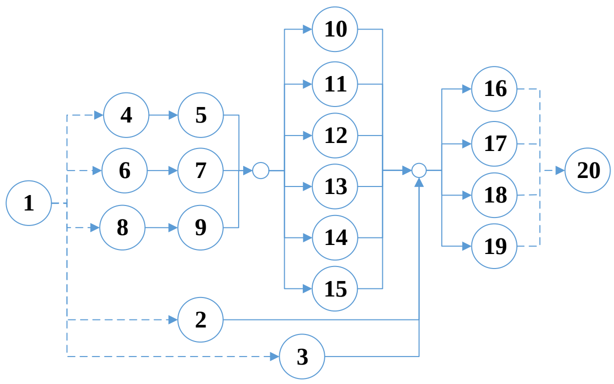


Figure 10. Process AoN diagram for preventive maintenance.

The reachability relation between parking spots (*P.*) and equipment is shown in Table 3. In Table 3, Ke_1 indicates the type of power supply station. As for the configuration of the maintenance workshops, owing to the space limitations of the compartment around the hangar bay, each workshop is equipped with one maintenance workshop covering the entire hangar bay. $Ke_{2–5}$ denote the aeronautical machine repair, oil and fluid inspection, ordnance maintenance, and electronic equipment maintenance workshops, respectively, and the number of parallel operations are $[Ne_1,\ Ne_2,\ Ne_3,\ Ne_4,\ Ne_5] = [\infty,\ 3,\ 2,\ 1,\ 4]$ for each workshop. The number of resources in this category usable for the parking space is indicated by [.]. The operation duration, resources, and skills required for each maintenance operation are shown in Table 4. In Table 4, $Kc_{1–4}$ denote special equipment, avionics, ordnance, and machinery specialties, respectively. In addition, the number of personnel is configured as $[5,\ 6,\ 4,\ 10]$. Special equipment is set to be compatible with avionics, ordnance, and machinery. The first four personnel in each profession have corresponding and compatible skills. Bold numbers indicate that the operation needs two personnel. The workstation space constraint *Ks* considers the cockpit space; "1" indicates that the number of personnel able to work in parallel is one, and "-" indicates that there is no demand for such resources.

Table 3. Reachability relation between parking spots and equipment.

P.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ke_1	[3]	[4]	[3,5]	[5]	[9]	[10]	[9]	[10]	[6]	[1]	[3]	[7]	[2]	[8]
$Ke_{2–5}$	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]

Table 4. Duration and requirements of maintenance and repair operations for carrier-based aircraft.

MR Task Modes		Operation No.																			
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
		Operation Duration (min)																			
Mechanical failure		0	0	0	0	24	0	0	0	0	44	0	0	0	0	0	12	0	0	0	0
Avionics system failure		0	0	0	0	19	0	0	0	0	53	0	0	0	0	0	22	0	0	0	0
Special equipment failure		0	0	0	0	26	0	0	0	0	47	0	0	0	0	0	17	0	0	0	0
Maintenance after 25 flight hours		0	18	30	8	6	8	10	6	8	15	20	0	16	18	0	3	10	8	6	0
Maintenance after 50 flight hours		0	25	45	8	8	8	12	6	8	30	30	26	26	28	16	8	18	10	10	0
Maintenance after 100 flight hours		0	34	66	10	12	10	15	10	12	48	40	45	33	44	46	16	26	18	14	0
Required resource type	*Kc*	-	4	**4**	3	1,2,3,4	2	2	1	1	1,2,4	4	**4**	**2**	**1**	**3**	1,2,4	2	1	3	-
	Ke	-	-	-	-	1	-	1	-	1	2,5	3	3	5	5	4	1	1	1	1	-
	Ks	-	-	-	-	1	-	1	-	1	-	-	-	-	-	-	1	1	1	1	-

5.2. Simulation Comparison Analysis

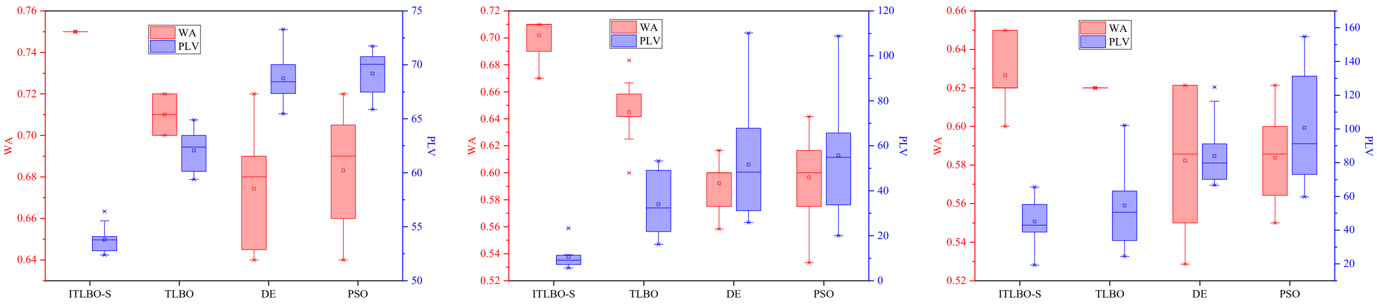
5.2.1. Algorithm Comparison

To verify the effectiveness of the proposed ITLBO-S algorithm and its performance in solving the AMRSP, the TLBO, DE, and PSO were selected for the performance comparison. The parameters of each algorithm were set as follows. An $Np = 30$ was selected as the population size for both the ITLBO-S and the TLBO. In the PSO, the number of particles was set as $N = 30$; the learning factors were $c_1 = 2$ and $c_2 = 2$; and the linear decreasing weight strategy was $\omega = (\omega_{ini} - \omega_{end})(Q - q)/Q + \omega_{\text{end}}$, where ω indicates the variable inertia weight, Q indicates the maximum number of evaluations, q is the current number of evaluations, ω_{ini} is the initial weight, and ω_{end} represents the end-of-iteration weight. The ω_{ini} and ω_{end} were 1.2 and 0.1, respectively. In the DE, the population size was set as $Np = 30$, the crossover rate as $cr = 0.1$, and the mutation probability as $F = 0.1$. A weight coefficient of $\alpha = 10^{-6}$ was used for the variance of the maintenance personnel load in the above algorithm fitness function f. Because the MR task demand prioritizes the number of intact aircraft provided for each sortie wave, the weight coefficients were selected as $\alpha_1 = 1$ and $\alpha_2 = 10^{-6}$. In all methods, an evaluation number of $Q = 3000$ was used to mark the end of the iteration.

Each algorithm was programmed using MATLAB 2020a and a personal computer (Windows 7 64-bit operating system, Intel Xeon Gold 5122 CPU @ 3.60 GHz, 32G of RAM). Each algorithm was run 15 times independently, and the results were recorded. After the optimization simulation, a statistical comparison between the algorithms of the optimization functions, WA and PLV, for the three groups of hangar MR task scheduling cases was conducted, and the results are listed in Table 5. The evaluation indicators were the optimal solution (*Best.*), average solution (*Avg.*), and standard deviation (*Std.*). In Table 5, the bold numbers indicate the optimal solutions for the algorithm comparison. A box plot of the distribution of the solutions for the repeated calculations is shown in Figure 11. The convergence trend of each algorithm is shown in Figure 12 (the result of one iteration when the WA achieves the optimal value in Case 1 is considered).

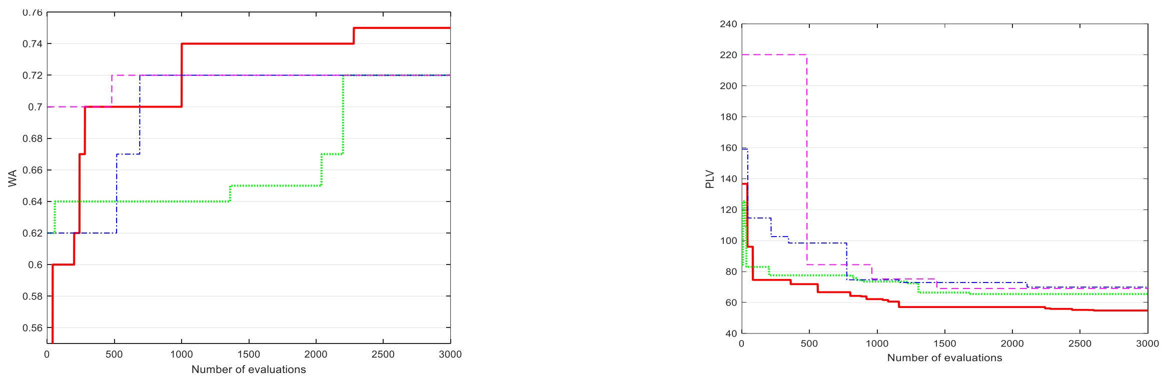
Table 5. Statistical comparison of algorithmic results.

Cases	Objective Functions	Evaluating Indicators	Algorithms			
			ITLBO-S	TLBO	DE	PSO
Case 1	WA	*Best.*	**0.750**	0.720	0.720	0.720
		Avg.	**0.750**	0.709	0.714	0.713
		Std.	**0**	0.010	0.008	0.009
	PLV	*Best.*	**52.382**	59.380	66.107	65.866
		Avg.	**53.720**	64.663	69.068	69.424
		Std.	**1.264**	3.383	1.999	1.802
Case 2	WA	*Best.*	**0.710**	0.690	0.600	0.620
		Avg.	**0.702**	0.648	0.592	0.598
		Std.	**0.012**	0.021	0.019	0.028
	PLV	*Best.*	**5.626**	16.186	25.946	20.026
		Avg.	**10.612**	33.983	51.695	55.754
		Std.	**5.113**	13.326	21.837	26.416
Case 3	WA	*Best.*	**0.650**	0.620	0.600	0.600
		Avg.	**0.627**	0.620	0.582	0.584
		Std.	0.015	**0**	0.031	0.018
	PLV	*Best.*	**19.280**	24.480	66.582	60.720
		Avg.	**45.088**	54.560	83.964	100.624
		Std.	**12.503**	21.056	16.966	31.059



(**a**) Case 1 (**b**) Case 2 (**c**) Case 3

Figure 11. Box plot of the repeated calculation results.



: ITLBO-S; : TLBO; : DE; : PSO.

Figure 12. Change trend of wave availability (WA) and personnel load variance (PLV).

The shapes of this Pareto front are presented in Figure 13 to validate the feasibility of the solutions' distribution. The distribution of the solutions shows that the feasible solutions are scattered throughout the two-dimensional plane and that the number of points gathered in the Pareto front is small. Meanwhile, note that for the case set in this paper, the area of greatest concern is the feasible solution at the bottom right corner of Figure 13 (the maximum WA and the minimum PLV); therefore, the other Pareto front points are not of high priority.

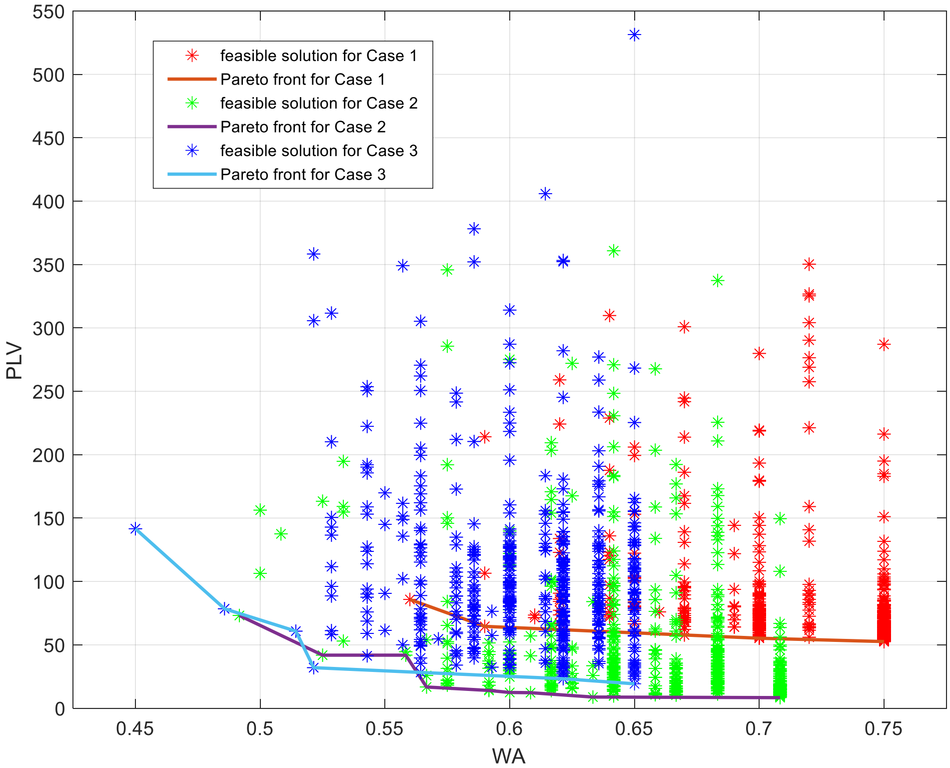
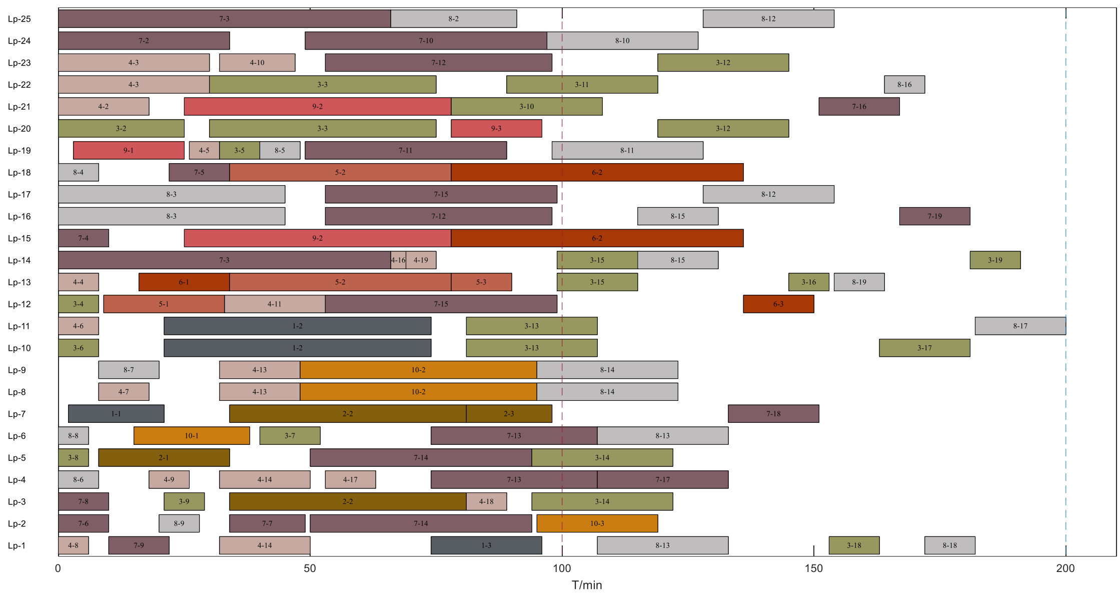


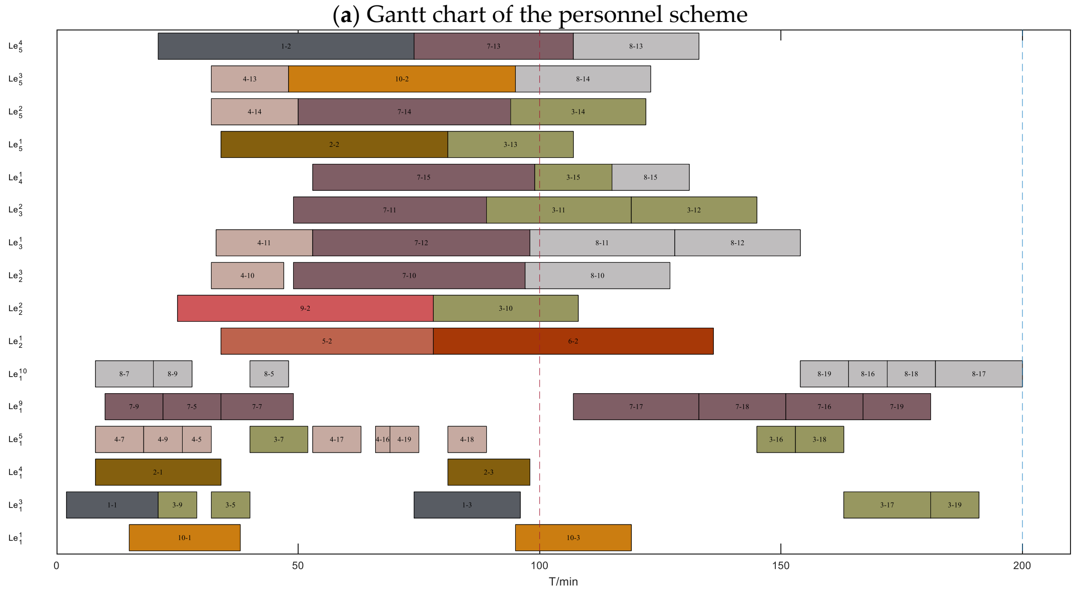
Figure 13. Pareto front and feasible solution for ITLBO-S.

First, the results in Table 5 show that based on the quality of the solutions, the ITLBO-S exhibits the best performance among the four algorithms in the three sets of experiments comprising three evaluation indicators and two objective functions. Notably, in Case 1, the ITLBO-S has the best convergence effect, and the indicators for WA converge to 0.750 for the 15 independent operations. By contrast, the other algorithms do not have sufficient search depths and are unable to search for a better solution. The box plot in Figure 11 shows the stability of the distribution of the observed results; with regard to the WA and PLV, the ITLBO-S reaches the highest median, upper quartile, and maximum value for all three sets of simulations compared to the other three classical metaheuristic algorithms while showing strong stability. By contrast, the DE and PSO perform poorly in the three sets of cases, either failing to find the optimal result or resulting in the WA having a more scattered distribution of solutions, indicating that the DE and PSO are less adaptable in finding the solution to this problem. In Case 3, the indicator WA for the TLBO converges to 0.620 for the 15 independent operations, whereas the optimal value of the ITLBO-S reaches 0.650. This is the most intuitive and evident indication that the conventional TLBO falls into a local optimum in the process of solving the problem. Owing to the enhanced global and local search operations in the assistant teaching phase, the ITLBO-S can explore better solutions in the local search process. This improves the diversity of populations and the accuracy and stability of the local optimal solutions, and the optimization effect of the algorithm is improved. However, one problem reflected in the results is that the stability of the algorithm tends to decrease as the problem scale increases; therefore, it is necessary to continue to test the adaptability of the algorithm for increasing problem scales. In summary, the ITLBO-S proposed herein is the optimal solution compared with other optimization algorithms, such as the TLBO, under the premise of considering the quality and stability of the solution.

The optimal Gantt charts for scheduling maintenance personnel and maintenance equipment/workshops, obtained from the Case 1 results, are shown in Figure 14. In Figure 14a, the vertical coordinate “$Lp - l$” indicates the lth personnel, and the order is numbered according to the special equipment, avionics, ordnance, and machinery specialties. The maintenance operations are indicated on the Gantt chart bars, where $i - j$ (hyphen in the Figure) represents the maintenance operationof O_{ij}. In Figure 14b, when the vertical coordinate is $k = 1$, Le_1^l refers to the lth power supply station. For consistency, aeronautical machine repair, oil and fluid inspection, ordnance, and electronic equipment maintenance workshops are indicated by $k = 2 - 5$, respectively. Le_{2-5}^l denotes the lth parallel maintenance operation line for the corresponding workshop. Owing to length constraints, the maintenance scheduling schemes for Cases 2 and 3 are not provided. After testing, the scheduling schemes shown in each Gantt chart were proven to satisfy all constraints, thereby verifying the correctness of the proposed model and scheduling method.



(**a**) Gantt chart of the personnel scheme



(**b**) Gantt chart of the equipment/workshop scheme

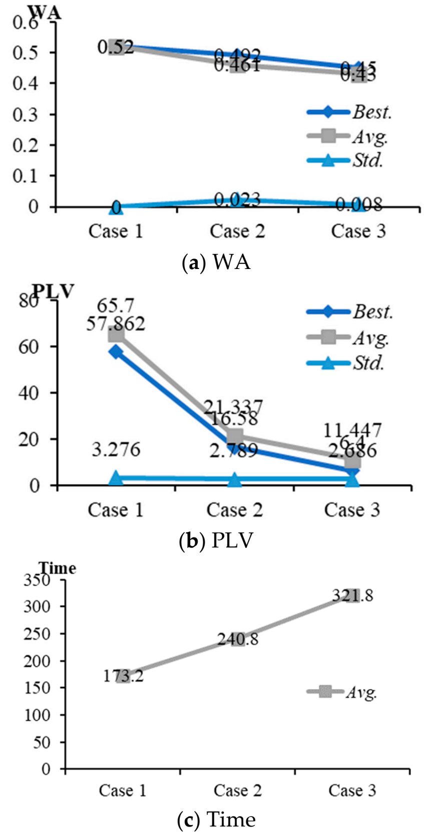
Figure 14. Gantt chart of maintenance and repair (MR) task scheduling schemes.

5.2.2. Adaptation Verification of Algorithms

To verify the performance of the ITLBO-S for highly constrained resources and large-scale fleet MR tasks, the sortie wave interval was shortened to 80 min, and a test simulation was conducted with the other algorithm parameters kept constant. The algorithm was independently run 15 times, resulting in the statistics shown in Table 6.

Table 6. Statistics of high-intensity task simulation results.

Objective Functions	Evaluating Indicators	MR Tasks		
		Case 1	Case 2	Case 3
WA	*Best.*	0.520	0.492	0.450
	Avg.	0.520	0.461	0.430
	Std.	0	0.023	0.008
PLV	*Best.*	57.862	16.580	6.400
	Avg.	65.7	21.337	11.447
	Std.	3.276	2.789	2.686
Time/s	*Avg.*	173.2	240.8	321.8



(**a**) WA
(**b**) PLV
(**c**) Time

The comparison results from the simulation reveal the following three conclusions. First, after the wave interval period is shortened, the WA gradually decreases, and the PLV gradually decreases with an increase in the fleet scale. This is because completing these MR tasks within the specified time is difficult, and the ratio of a sufficient number of aircraft at the start time of the wave gradually decreases owing to the highly constrained resource situation of tight MR tasks. Furthermore, the large number of MR tasks completed by the personnel results in inter-task idle times, a short gap between the work hours of the maintenance personnel, and a minor load variance. The stability aspect was considered under index *Std.* The results in Table 6 show that increasing the scale of the problem can improve the stability of the algorithm. However, the results also indicate that the algorithm's adaptability to the problem is an advantage. The computation time of the algorithm for solving the scheduling tasks gradually increases as the problem scale increases but remains within the acceptable solution time. In summary, the proposed ITLBO-S algorithm performs well in solving the AMRSP under high resource constraints.

6. Conclusions and Future Work

In this study, for the AMRSP, we first analyzed the maintenance process, personnel, equipment, workshop, workspace, skills, and other constraints in MR task scheduling. Next, using the WA and PLV as optimization indexes, we constructed a mathematical model for aeronautical MR task scheduling problems in carrier-based aircraft fleets. An

ITLBO-S algorithm was proposed to solve the model. Finally, after case simulations and comparative experiments were performed, an optimal scheduling scheme was provided for maintenance personnel and equipment/workshops. After verification, the scheduling scheme obtained by employing the ITLBO-S algorithm was proven to comply with the constraints of the model. The improved algorithm shows advantages in terms of the quality and stability of the solution. In other words, the algorithm has strong adaptability in solving large-scale scheduling problems.

However, in this study, the model and optimization of carrier-based aircraft MR tasks were applied only to deterministic tasks. The assumption that the interference of unexpected factors, such as task changes, can be excluded is inconsistent with an actual and complex maintenance environment. Failures in aircraft systems or components often appear to be random, and maintenance activities are tightly coupled in a sequential manner. Any delay in performing a task may have a snowball effect on subsequent maintenance activities, eventually leading to maintenance delays [46]. With appropriate modifications, this model can be used for a dynamic MR system or to optimize other factors.

Subsequent research will improve the algorithm to achieve dynamic scheduling and to cope with unforeseen situations, unpredictability, and different organizational scenarios, thereby making it more relevant to the AMRSP. Moreover, according to the "no free lunch" theory [47], each algorithm has its applicable problem scope. The applicable scope is related to the characteristics of the algorithm. In this context, by considering the scale of the problem, we can select the best algorithm based on its actual performance on a particular problem.

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