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A Novel Driving-Strategy Generating Method of Collision Avoidance for Unmanned Ships Based on Extensive-Form Game Model with Fuzzy Credibility Numbers

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Abstract: This study aims to solve the problem of intelligent collision avoidance of unmanned ships at sea, and it proposes a novel driving strategy generating method of collision avoidance based on an extensive-form game mode with fuzzy credibility numbers. The innovation of this study is to propose an extensive-form game model of unmanned ships under the situation of two-sides clamping and verify the validity by fuzzy credibility. Firstly, this study divides the head-on situation of the ship at sea quantitatively to help the unmanned ship take targeted measures when making collision avoidance decisions. Secondly, this study adopts an extensive-form game model to model the problem of collision avoidance of an unmanned ship in the case of clamping on two sides. Thirdly, the extensive-form game model is organically combined with the fuzzy credibility degree to judge whether the collision avoidance game of unmanned ship achieves the optimal collision avoidance result. The effectiveness of the introduced game model is verified by case analysis and simulation. Finally, an illustrative example shows that the proposed mathematical model can better help unmanned ships make real-time game decisions at sea in the scenario of two-sides clamping effectively.

Keywords: collision avoidance; encounter situation; fuzzy credibility numbers; intelligent unmanned ships; extensive-form game model

MSC: 90C70

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Citation: Cui, H.; Zhang, F.; Li, M.; Cui, Y.; Wang, R. A Novel Driving-Strategy Generating Method of Collision Avoidance for Unmanned Ships Based on Extensive-Form Game Model with Fuzzy Credibility Numbers. *Mathematics* **2022**, *10*, 3316. https://doi.org/10.3390/math10183316

Academic Editor: Mariano Luque

Received: 2 August 2022
Accepted: 5 September 2022
Published: 13 September 2022

1. Introduction

In actual maritime navigation, the entire collision avoidance operation of unmanned ships revolves around the three stages of "observation, judgment and decision making" [1]. At the same time, the specific water environment and different encounter states will also affect the collision avoidance decision-making process of the unmanned ship. Under the above background, to help unmanned ships take targeted measures to avoid collision decisions, this study analyzes the situation of ships under the condition of both sides.

The two-sides clamping scenario is a condition in which a ship sails between two ships while at sea. Investigation illustrates that it is dangerous when a ship is trapped in this two-sides clamping situation. Considering that the collision avoidance operations of unmanned ships is a game process, this study proposes an anti-collision decision model for unmanned ships based on extensive-form game model [2].

1.1. Literature Review

According to investigation, the issues of unmanned ship collision avoidance in a two-side scenario is focused on. At present, scholars' research on ship collision avoidance mostly focuses on three aspects: strategies for avoiding ship collisions, application of game theory to ships, and practical application of fuzzy credibility numbers.

In the past five years, avoiding collision problems have been mainly studied from the viewpoints of risk assessment, variable distribution, safety domain, etc. Scholars have performed research on strategies for ships avoiding collisions. In the study by Li et al. [3], by balancing the safety and economy of ship collision avoidance, the avoidance angle and the time to the action point are used as the variables encoded by the algorithm, and the fuzzy ship domain is used to calculate the collision avoidance risk to achieve collision avoidance. Thereafter, Lee et al. [4] proposed a heuristic search technology for collision avoidance operations for autonomous ships. Based on the multi-vessel collision avoidance problem, Wang et al. [5] researched the decision-making for obstacle avoidance based on deep reinforcement learning to solve the problem of intelligent collision avoidance for unmanned ships in unknown environments. Based on the mathematical model group's ship motion mathematical model, Xing et al. [6] proposed an open sea ship collision prevention approach to enhance the prediction of ship collision risk and the real-time and dependability of collision avoidance method.

At present, the application fields of extensive-form game mode are concluding containing transportation; Lisowski [7] introduced the application of the game control processes in marine navigation. The control goal has been defined first. Then, the approximated models of multi-stage positional game and multi-step matrix game of the safe ship steering in a collision situation has been presented. Subsequently, Lisowski et al. [8] described six methods of optimal and game theory and artificial neural network for synthesis of safe control in collision situations at sea. The optimal control algorithm and game control algorithm were used to determine the safe track. Afterwards, Zou et al. [9] identified the safety evaluation indicator system and evaluation standards and established an after-collisions safety evaluation model of maritime ships based on the extension cloud theory. Considering the defects of the classic extensive game method in ship collision avoidance decision-making, Tu et al. [10] proposed the improved extensive game method based on the velocity obstacle method.

Up to now, fuzzy credibility numbers were mainly used to solve decision making problem, project scheduling problem, multi-objective fuzzy-interval credibility-constrained non-linear programming, etc. Ran et al. [11] aimed at the problems of inaccurate evaluation results caused by experts in the process of simulation credibility evaluation based on traditional fuzzy comprehensive evaluation according to personal preferences or expectations, and unreasonable selection of fuzzy synthetic calculations, and a simulation credibility evaluation method based on improved fuzzy comprehensive evaluation was proposed. Moreover, Ye et al. [12] proposed the concept of a fuzzy credibility number as a new extension of the fuzzy concept. Thereafter, Vercher et al. [13] presented a new forecasting scheme based on the credibility distribution of fuzzy events. In the same year, Zhou et al. [14] proposed a decision support model for USVs to improve the accuracy of collision avoidance decision-making.

Based on the aforementioned analysis, the collision avoidance of unmanned ships is studied. The main innovation of this study is combining extensive-form game model and FCN together. Specifically, by using the extensive-form game model, the collision avoidance strategy of unmanned ships is studied for the special situation between the two-sides. By using FCN, the danger of collision is quantified.

1.2. Goals and Contributions

The purpose of this study is to explore the decision-making problem of collision avoidance for unmanned ships at sea under the two-sides clamp scenario. In response to the aforementioned problems, this study establishes an extensive-form game model based on the two-sides clamping scenario and applies it to solve the specific collision avoidance problem.

The contribution of this study is as follows. Firstly, based on the extensive-form game model, this study establishes a description of the ship collision avoidance structure under the situation of two-sides clamping. Secondly, this study chooses driving strategy following a priority principle on ship collision avoidance and introduces a utility function to describe it. By combining this utility function and the extensive-form game model, a set of utilities of the own ship and the target ship are collected and compared to find the optimal collision avoidance decision. Thirdly, this study establishes a ship collision risk fuzzy credibility operator to judge whether the ship has escaped from collision danger.

The rest of this study is organized as follows. Section 2 clarifies the research basis. Section 3 proposes the driving-strategy generating method for collision avoidance. Section 4 carries out simulation verification for the proposed method. Section 5 summarizes and points out possible future work. The structure of this study is shown in Figure 1.

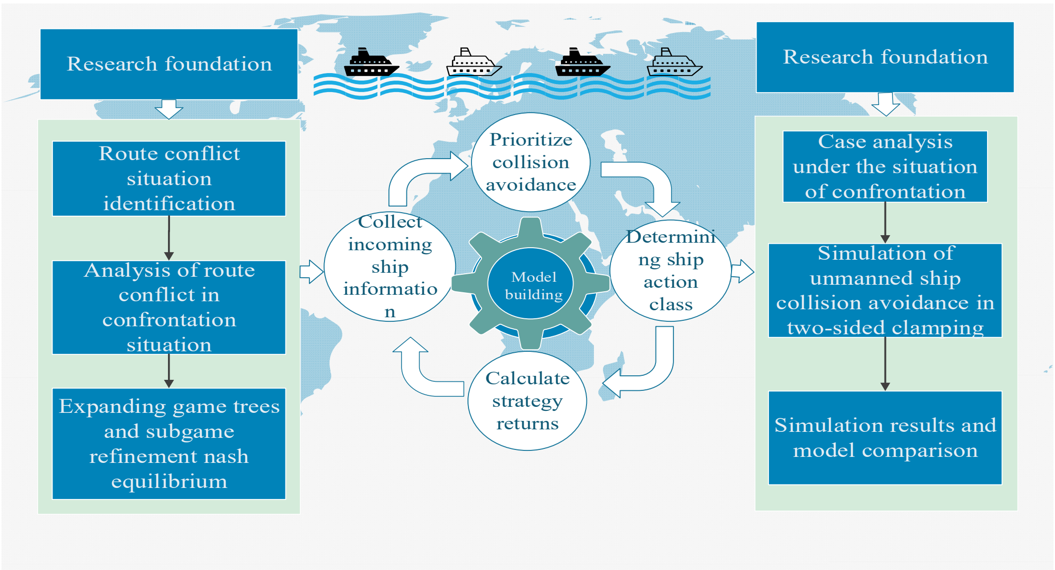

Figure 1. Research process.

2. Research Basis

This part mainly introduces the conflict identification of the ships' encounter situation at sea, which quantitatively analyzes the ship's encounter situation, and introduces the relevant knowledge of extensive-form game tree and sub-game refinement Nash equilibrium.

2.1. Route Conflict Situation Identification

The identification of the conflict situation on the route and the division of ship responsibilities are based on the 1972 International Regulations for Preventing Collisions at Sea, namely COLREGS. In actual navigation, the collision avoidance measures taken by unmanned ships are based on the collision avoidance rules listed in COLREGS combined with various ship identification devices for automatic collision avoidance [15]. The uncoordinated collision avoidance measures may lead to the uncoordinated collision avoidance process of the entire ship so that the best avoidance opportunity is missed [16]. According to the different angles of encounter of ships, the encounter situation will be divided into three types: head-on situation, overtaking situation, and cross encounter situation. Head-on situation is the situation that ships often encounter at sea, and it is also the main situation that causes the ship to be in imminent danger or to collide. Therefore, this study researches the collision avoidance strategy of ships in the confrontation situation.

2.2. Judgment of Head-On Ship Situation

The "International Regulations for Preventing Collisions at Sea" has the following four points to judge the head-on situation of ship [17]. Firstly, both ships must be motorized ships. Secondly, the sailing directions of the two ships are in an opposite or almost opposite confrontation on the route. Thirdly, one motorized ship is sailing directly in front of or nearly in front of the other. Finally, the two ships are seeing each other and constitute a collision hazard. Therein, the heading angle of B the two ships in the confrontation situation ΔC is the relative azimuth. The heading opposite or close to the opposite means that the heading difference between the two ships is within $174^{\circ} \leq \Delta C \leq 186^{\circ}$ the range. From the point of view of encountering the relative orientation of the two ships, the heading of the two ships is close to the opposite, which means that one ship is located within 6° on the left and right in front of the other ship. Therefore, the relative orientation of the confronting situation should satisfy $B \leq 005^{\circ}$ or $B \geq 351^{\circ}$; the specific details are shown in Figure 2.

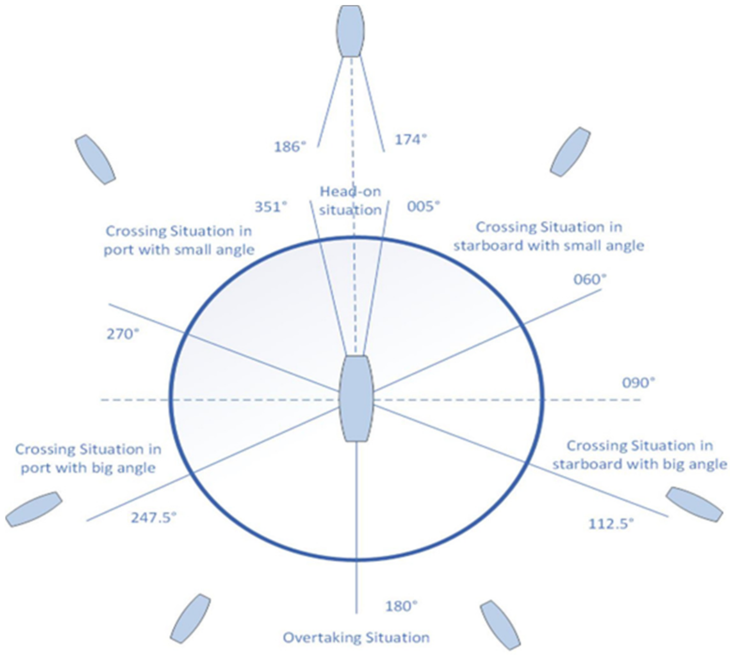

Figure 2. Schematic diagram of Head-on situation.

2.3. Extensive-Form Game Model Tree

The extensive-form game is dynamic. The difference between it and the static game is that the extensive-form dynamic game needs to determine the order of actions [18]. Each knot on the "game tree" represents a player's decision point, and this point is said to belong to the player acting at that point [19]. The branches represent the possible actions of the players, and each branch connects two knots, which has a direction from one knot to the other. Each branch of the game tree may or may not be expanded. Meanwhile, each branch in the game tree can be regarded as a new game tree, called a sub-tree, as shown in Figure 3. The part of A is the sub-game of B, and A is also the sub-game of the whole game. The nodes are expanded outward, as shown in Figure 4.

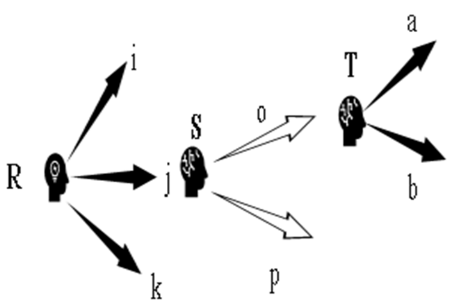

Figure 3. Extensive-form game model tree.

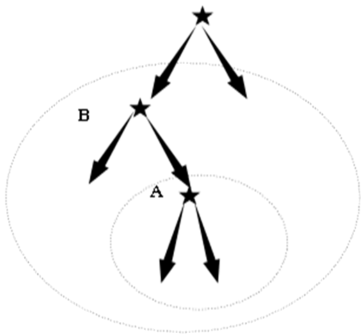

Figure 4. The sub-tree of the game tree.

2.4. Sub-Game Refinement Nash Equilibrium

The Nash equilibrium strategy is that all players in the game adopt the best strategy that is beneficial to them [20]. In the whole process of the game, the players of each game are rational and intelligent. The combination of actions taken in each game is the optimal strategy, and the sub-game developed by the game tree is also the optimal solution. The combination of action strategies taken in the game process conforms to the Nash equilibrium strategy. Sub-game refined Nash equilibrium is the most effective tool for analyzing perfect information dynamic games in the game theory [21].

3. Unmanned Ship Collision Avoidance Model in Two-Sides Clamp Scenario

This subsection adopts the fuzzy mathematics method, which organically combines the extensive-form game with the collision risk fuzzy credibility numbers. This study analyzes the collision avoidance game problem of route conflict in the situation where the unmanned ship is under two-sides clamping situation particularly. In this model, the fuzzy confidence degree of collision risk is used to calculate whether the ship escapes from the collision risk after the collision avoidance game, so as to judge whether the ship adopts the optimal collision avoidance strategy.

3.1. A Novel Ship Collision Avoidance Model

In this subsection, a ship collision avoidance model is proposed in two-sides clamp scenario. Specific steps are as follows.

Step 1: Determination of priority. When ships encounter emergency and dangerous situations in the course of navigation, if they want to recognize each other's game information through various ship identification equipment on unmanned ships, they also need to play sequential dynamic game on ships. To determine the action sequence of the players in the game process, this study proposes a ship priority function. This study makes two assumptions about the gross tonnage of the ship and the sailing speed of the ship regarding the actual sailing experience. The larger the gross tonnage of the ship in the voyage, the higher the priority in the game situation. Then, it is assumed that the higher the speed of the ship during the voyage, the higher the priority in the ship game. The following formula is given for the aforementioned two assumptions:

$$p_i = w_1(G_i / \sum_{i=1}^{n} G_i) + w_2(V_i / \sum_{i=1}^{n} V_i). \tag{1}$$

In Equation (1), it is noteworthy that p_i represents the priority index of player i in the game, G_i represents the total tonnage of player i in the game, and V_i represents i the speed of the player in the game. Among them w_1 and w_2 represents the weight of the gross tonnage of the ship and the speed, at the same time $w_1 + w_2 = 1$. After p_i has been determined, players p_i alternately make action decisions based on the magnitude of the index.

Step 2: Action space (Action set). After obtaining the corresponding action sequence based on the ship collision avoidance priority in step 1, it is assumed that ship i start to act. The set of game decision it makes in the current situation is called the action set of the ship i. In this set of action strategies, the number of action strategies made by ship i is related to the complexity of the game situation; the number of action strategies made by the ship is related to the complexity of the game situation. The more complex the game model, the more actions can be made, the more combinations of actions, and the longer the solution process will take. This study only adopts steering avoidance as a collision avoidance measure to simplify the development space of the game and reduce the time required for the game-solving process. In sailing practice, the steering angle is too large, which will cause inconvenience in resuming the voyage. Therefore, the upper and lower limits of steering are 30° in this study, and each turn is 10° as an action strategy, then the action set of the ship i can be represented as $A_i = \{-30^\circ, -20^\circ, -10^\circ, 0^\circ, 10^\circ, 20^\circ, 30^\circ\}$.

Step 3: Profit function. After determining the ships' collision avoidance priority and the ships' decision-making action set, this study only considers the ships' offset as a profit on the premise of ensuring that the ship can sail safely and establishes a profit function. In collision avoidance, the lower the ships' drift, the lower the ships' cost, and the more the ship benefits throughout the game process. Set the initial position of the ship as $, x_0, y_0,$, the speed of the ship as v, the heading angle as ψ, and the time interval of the ship game as t. This study only studies a series of games between our ship and the other two ships under the special situation of the two-sides. It is assumed that one of the ships is an environmental variable, that is, the ship does not take any steering measures to maintain direction and speed. If the planned course is sailing at a constant speed, t is the displacement increments of the abscissa, and the increment of the x_l ordinate of the ship in time y_l are:

$$x_l = \begin{cases} vt\sin(\psi), & 0^\circ \le \psi \le 90^\circ; \\ vt\cos(\psi - 90^\circ), & 90^\circ < \psi \le 180^\circ; \\ -vt\sin(\psi - 180^\circ), & 180^\circ < \psi \le 270^\circ; \\ -vt\cos(\psi - 270^\circ), & 270^\circ < \psi < 360^\circ. \end{cases} \tag{2}$$

$$y_l = \begin{cases} vt\cos(\psi), & 0^\circ \le \psi \le 90^\circ; \\ -vt\sin(\psi - 90^\circ), & 90^\circ < \psi \le 180^\circ; \\ -vt\cos(\psi - 180^\circ), & 180^\circ < \psi \le 270^\circ; \\ vt\sin(\psi - 270^\circ), & 270^\circ < \psi < 360^\circ. \end{cases} \tag{3}$$

After the i-th decision is made, the coordinates where the ship arrives (x_p, y_p) according to the planned course and constant speed, it gets:

$$x_p = x_0 + ix, y_p = y_0 + iy_l. \tag{4}$$

During the actual ship's action, the ship's expected position (x_i, y_i) will be affected by the last decision. If the ship's position after making a decision is (x_{i-1}, y_{i-1}), then:

$$x_i = x_{i-1} + x_m, y_i = y_{i-1} + y_m \tag{5}$$

where ψ_i represents the new heading angle of the ship after the i-th decision is executed:

$$\psi_i = \begin{cases} \psi_i & 0^\circ \le \psi_i < 360^\circ \\ \psi_i - 360^\circ & \psi_i \ge 360^\circ \\ \psi_i + 360^\circ & \psi_i < 0^\circ \end{cases}. \tag{6}$$

However, environmental variables should be taken into account when considering collision avoidance strategies. Therefore, the relevant distance variable is introduced in combination with the collision risk μ. The unmanned ship will take measures to avoid the ship when it encounters the nearest distance. The influence of bump measure on revenue function is as follows:

$$\mu = \frac{1}{2} - \frac{1}{2}\sin\left[\frac{\pi}{d_2 - d_1}\left(\omega - \frac{d_1 + d_2}{2}\right)\right]. \tag{7}$$

Among them, d_1 and d_2 are the safety field value of the ship and the safe passing distance of the ship, respectively, and the distance ω between our ship and the environmental variable ship.

To sum up, it can be extracted that the ships' offset S in the i-th decision of the player S is shown in Equation (8):

$$S = \begin{cases} \sqrt{\begin{matrix} [x_0 + vt\sin(\psi)(i-1) + vt\sin(\psi_i) - (x_0 + vt\sin(\psi)i)]^2 \\ + \\ [y_0 + vt\cos(\psi)(i-1) + vt\cos(\psi_i) - (y_0 + vt\cos(\psi)i)]^2 \end{matrix}} + \mu, 0^{\circ} \leq \psi \leq 90^{\circ}; \\ \sqrt{\begin{matrix} \left[x_0 + vt\cos(\psi - \frac{\pi}{2}), i-1, +vt\cos(\psi_i - \frac{\pi}{2}) - \left(x_0 + vt\cos(\psi - \frac{\pi}{2})i\right)\right]^2 \\ + \\ \left[y_0 - vt\sin(\psi - 90^{\circ})(i-1) - vt\sin(\psi_i - \frac{\pi}{2}) - \left(y_0 - vt\sin(\psi - \frac{\pi}{2})i\right)\right]^2 \end{matrix}} + \mu, 90^{\circ} < \psi \leq 180 \\ \sqrt{\begin{matrix} [x_0 - vt\sin(\psi - \pi), i-1, -vt\sin(\psi_i - \pi) - (x_0 - vt\sin(\psi - \pi)i)]^2 \\ + \\ [y_0 - vt\cos(\psi - \pi)(i-1) - vt\cos(\psi_i - \pi) - (y_0 - vt\cos(\psi - \pi)i)]^2 \end{matrix}} + \mu, 180^{\circ} < \psi \leq 270 \\ \sqrt{\begin{matrix} \left[x_0 - vt\cos(\psi - \frac{3\pi}{2}), i-1, -vt\cos(\psi_i - \frac{3\pi}{2}) - \left(x_0 - vt\cos(\psi - \frac{3\pi}{2})i\right)\right]^2 \\ + \\ \left[y_0 + vt\sin(\psi - 270^{\circ})(i-1) + vt\sin(\psi_i - \frac{3\pi}{2}) - \left(y_0 + vt\sin(\psi - \frac{3\pi}{2})i\right)\right]^2 \end{matrix}} + \mu, 270^{\circ} < \psi < 36 \end{cases} \tag{8}$$

Step 4: Collision avoidance decision. In the dynamic game with complete information, the reverse solution from the final decision position is the most effective method to solve Nash equilibrium [22]. In order to facilitate understanding, the following complete information dynamic game is taken as an example to analyze.

Suppose there are two ships No. 1 and No. 2, in which ship No. 1 can choose an action a_1 from the action set A_1 and ship No. 2 can choose an action a_2 from the action set A_2. At the same time, $U_1(a_1, a_2)$ and $U_2(a_1, a_2)$ represent the value of the ship's profit of No. 1 and the ship's profit of No. 2, respectively. Based on the principle of the inverse solution method, it is assumed that ship No.1 in this example makes an action decision first, so the analysis starts from ship No. 2. Assuming that ship No. 1 is selected from the action set first a_1, then ship No. 2 needs to choose an action from its own action set that is the most profitable for itself in the environment affected by the decision of ship No. 1. Therefore, ship No. 2 faces that the decision problem is denoted as $\max U_2(a_1, a_2), a_2 \in A_2, \forall a_1 \in A_1$, the optimal strategy made by ship No. 2 after ship No. 1 makes the action decision is denoted by $F_2(a_1)$, and there is one and only one optimal strategy.

When inferring the decision made by ship No. 1 in the process of reverse solving, ship No. 1 predicts that ship No. 2 will take the next action according to its decision. Therefore, ship No. 1 only needs to arbitrarily find an action that can maximize its benefits in its own action set. So, the decision-making problem of ship No. 1 is written as follows. At this time, (a_1, a_2) represents the maximum value of ship No. 1 and ship No. 2 which are the best combination of actions.

In summary, the choice of ship collision avoidance strategy based on perfect information game is mainly divided into four steps. Firstly, the surrounding environment of the ship is checked during the voyage. Secondly, the occurrence of collision risk is judged in the encounter situation. Thirdly, priority action sequence is taken into account. Finally, the optimal strategy to play the game is calculated according to the action sequence. Specifically, the ship collision avoidance strategy and flow chart of the perfect information game are shown in Figure 5.

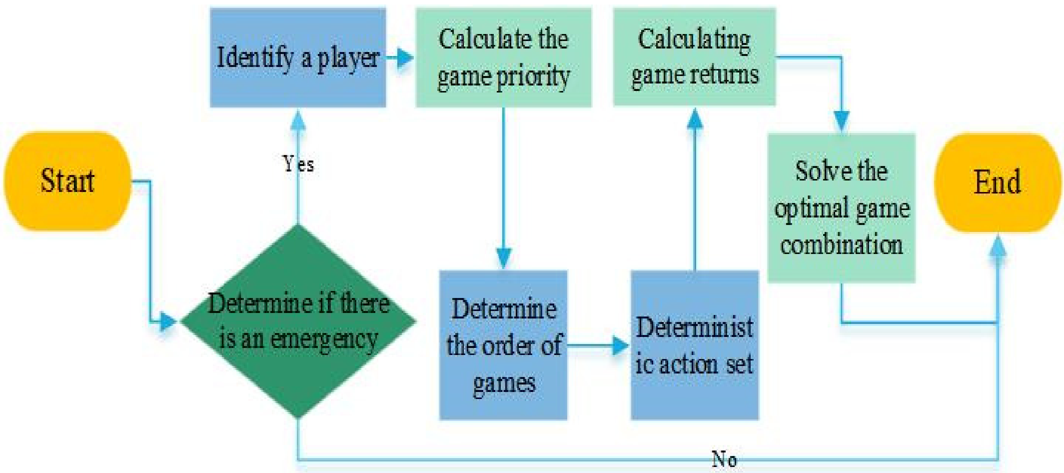

Figure 5. Flow chart of ship collision avoidance strategy in the perfect information game.

3.2. Expansion of Unmanned Ship Collision Avoidance Game Tree

The unmanned ship collision avoidance game model constructed in the previous section is the process of game tree expansion. The game tree designed in this study is a breadth search tree [23]. The nodes in the state space of the whole game tree can be divided into three categories: UNSEARCH nodes, OPEN node sets, and CLOSE node sets. Taking the game expansion tree with game round 3 as an example, node 1 is the head node, which contains the heading angle, offset, and collision risk of the unmanned ship in the current encounter situation. Node 1 is expanded to generate sub-nodes 2, 3, and 4. The three sub-nodes are respected in the new ship state formed by the combination of different actions taken by the ship in the situation. The aforementioned three nodes (including all the information in the new state) are initialized, listed in sequence after the head node, and pointed the parent pointer to node 1. After node 1 is expanded, the next node is sequentially expanded in the queue, namely node 2. Then, node 2 becomes the current node, and then it expands based on the space state of node 2. If the collision risk degree in the space state of node 3 is greater than 0.5, there is a possibility of collision risk if the node in this space state is expanded. So, node 3 is skipped and node 4 becomes the current node [24]. By analogy, until the end of the game round, the schematic diagram of the game algorithm is shown in Figure 6.

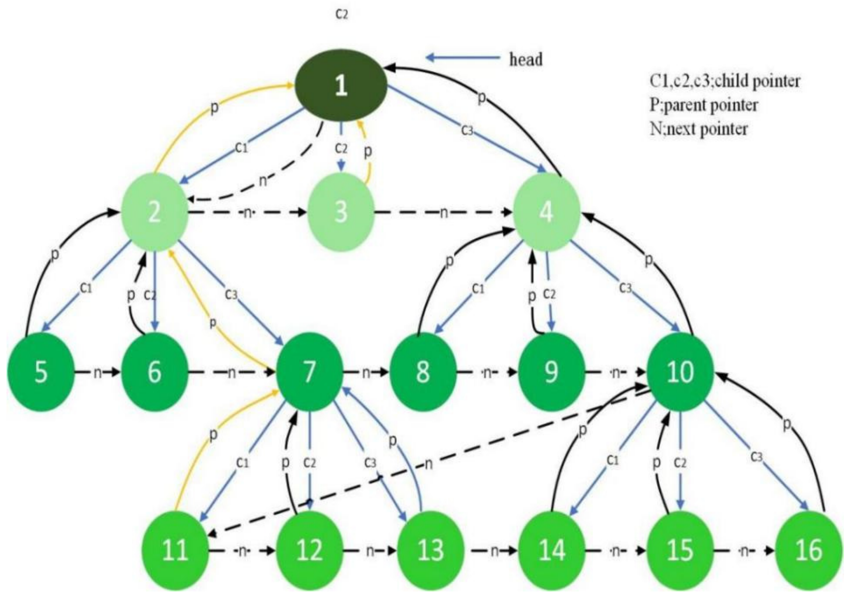

Figure 6. Game expansion tree of ship A and ship B.

The process of solving is to find the node with the largest profit in the last layer of nodes, that is, the node with the smallest sum of the offset of the two ships, in which the value of the collision risk of the node members must be less than 0.5. After finding the node with the greatest profit, it can follow its parent pointer for a reverse solution until the root node of the entire extended game tree is found, and the final optimal solution is the action combination information contained in the game strategy combination sequence node of the two ships.

3.3. Collision Risk Fuzzy Credibility Number

After the collision avoidance game, the ship collision risk can be determined by using the fuzzy credibility number of the ship collision risk [25]. There are many methods to calculate ship collision risk: fuzzy mathematical calculation method, BP neural network method, hazard mode immune control algorithm, bacterial foraging algorithm, and so on. The fuzzy mathematical method has high calculation accuracy. BP neural network method has strong self-learning ability, small calculation error, but high failure probability and long calculation time. Therefore, this study uses the fuzzy mathematics method to measure the ship collision risk.

In the introduced encounter situation, the judgment of whether there is a danger of collision between ships mainly depends on the distance to the closest point of approach D_{CPA}, the time to the closest point of approach value between the ships T_{CPA}, the ship speed ratio between the ships K, the distance between the ships D, the azimuth angle of the target ship relative to the own ship θ, and other related factors. In this study, the method of fuzzy mathematics is used to calculate the collision risk index (*CRI*) [26]. When $CRI = 0$, it means that there is no danger of collision between two ships. When $CRI = 1$, it means that the collision cannot be avoided and $CRI = 1$. Let U_{DCPA}, U_{TCPA}, U_θ, U_D, U_K be the D_{CPA}, the T_{CPA}, the azimuth angle between two ships, D between the two ships, and the risk membership degree of the shipping speed ratio K, respectively, and its belong to $[0, 1]$. Then, it gets:

$$\begin{aligned} CRI = \; & a\left\{\tfrac{1}{2} - \tfrac{1}{2}\sin\left[\tfrac{\pi}{d_2-d_1}\left(D_{CPA} - \tfrac{d_1+d_2}{2}\right)\right]\right\} \\ & +b\left[\left(\tfrac{t_2-|T_{CPA}|}{t_2-t_1}\right)^2\right] \\ & +c\left\{\tfrac{1}{2}\left[\cos(\theta-19^\circ) + \sqrt{\tfrac{440}{289}+\cos^2(\theta-19^\circ)}\right] - \tfrac{5}{17}\right\} \\ & +d\left[\left(\frac{H_1\cdot H_2\cdot 1.7\cos(\theta-19^\circ)+\sqrt{4.4+2.89\cos^2(\theta-19^\circ)}-D}{\left[H_1\cdot H_2\cdot 1.7\cos(\theta-19^\circ)+\sqrt{4.4+2.89\cos^2(\theta-19^\circ)}\right]-(H_1\cdot H_2\cdot DLA)}\right)^2\right] \\ & +\frac{e}{1+\frac{2}{K\sqrt{K^2+1+2K|\sin(|\psi_t-\psi_o|)|}}}. \end{aligned} \tag{9}$$

Among the d_1 and d_2 are the value of the safety field of the ship safety threshold and the safe passing distance of the ship, respectively. At the same time $a + b + c + d + e = 1$. Then, the aforementioned ship collision time t_1 and ship attention time t_2 are obtained as:

$$t_1 = \begin{cases} \dfrac{\sqrt{{D_1}^2 - {D_{CPA}}^2}}{V_r}, D_{CPA} \le D_1, \\ \dfrac{D_1 - D_{CPA}}{V_r}, D_{CPA} > D_1, \end{cases} \tag{10}$$

and:

$$t_2 = \frac{\sqrt{12^2 - {D_{CPA}}^2}}{V_r}. \tag{11}$$

It is noteworthy that, in Equations (10) and (11), D_1 represents the closest avoidance removal and D_2 represents the distance at which the approaching ship should take avoidance actions. V_r is defined as the velocity vector of the incoming ship relative to the present ship. Meanwhile, the schematic diagram of the latest avoidance distance D_1 is shown in Figure 7, where D_{LA} is defined as the latest distance to turn the rudder. Here, the value of D_{LA} is valued as 12 times the length of the ship for convenience [27]. Especially, on the conditions that $D_{CPA} \leq d_1$, $0 \leq |T_{CPA}| \leq t_1$ and $D \leq D_1$, the value of U_{DCPA}, U_D, U_{TCPA}, and CRI are all obtained as 1. In this situation, the ship is collided. Meanwhile, on the conditions that $d_2 < D_{CPA}$ and $D_2 \leq D$, it gets the value of U_{DCPA}, U_D, and U_{TCPA} which are all 0, which means there is no danger of collision between the two ships.

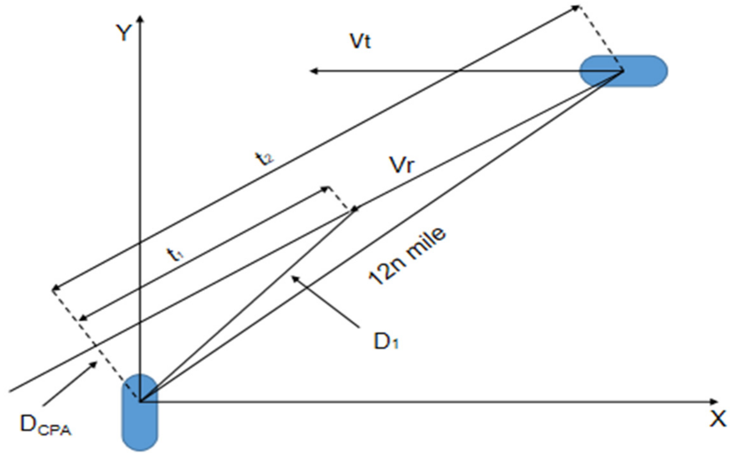

Figure 7. Geometric diagram of the latest avoidance distance.

4. Illustrative Example

To explain and verify the aforementioned extensive-form game model, an illustrative example is given as follows.

4.1. Problem Introduction

On 26 March 2019, the Xinde Maritime Network released news that on the 24th local time in the port of Fujairah, the United Arab Emirates, a tragic and incredible ship collision accident occurred. An exceptionally large tanker collided with another LNG carrier. The accident is a typical conflict scenario where the two ships sail down, as shown in Figure 8.

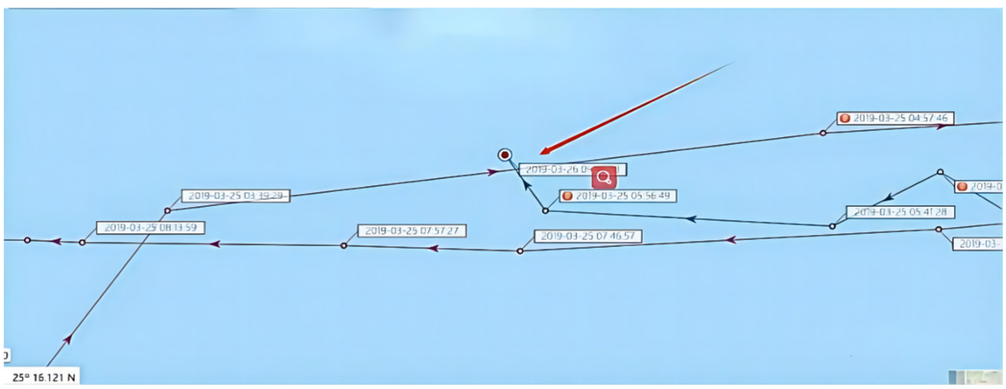

Figure 8. Course conflict scenario between two ships.

In such a situation where the two-sides are clamped, the ships can judge by the conflict of the routes during the encounter: the ships in the blue route are the right-give-way vessels, the purple-route vessels under the right-give-way vessels are the left-give-way vessels, and the vessels located in the right-give-way vessels are the left-give-way vessels. The pink route ships aforementioned are treated as environmental parameter variables in the whole game situation. In this collision avoidance game, the action set of the ship in the green route is $\{10^{\circ}, 20^{\circ}, 30^{\circ}\}$, the action set of the ship in the purple route is $\{-30^{\circ}, -20^{\circ}, -10^{\circ}\}$, and then the action combination of the two ships is $\{(10^{\circ}, -30^{\circ}), (10^{\circ}, -20^{\circ}), \ldots, (30^{\circ}, -20^{\circ}), (30^{\circ}, -10^{\circ})\}$. The next action combination will change, and the action set will change. Otherwise, the ship will be greatly offset, which is not in line with the benefits.

Consider the two-sides clamping scenario combine with the head-on situation, the target ships on two-sides of my ship approached at the $174^{\circ} \leq \Delta C \leq 186^{\circ}$ relative course of my ship. At this point, the ship is in the head-on sides of the two-sides clamping scenario. The schematic diagram of the head on scenario analysis is shown in Figure 9.

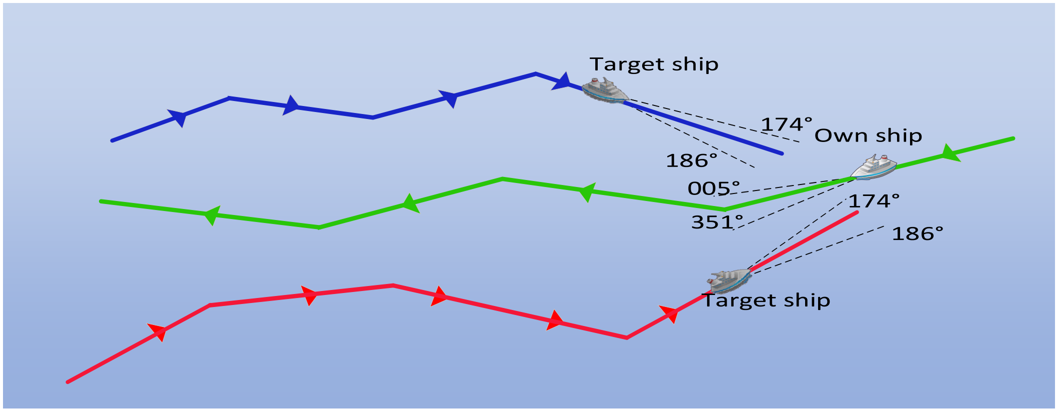

Figure 9. Head-on situation of two-sides clipping scenario schematic diagram.

4.2. Simulation Process and Analysis

In this section, two ships, i.e., "Own Ship" and the "Target Ship" are taken into account in this simulation sample, where the ship length of "Own Ship" is 105 m, the maximum speed of "Own Ship" is 18 kn, and the gross tonnage of "Own Ship" is 6000 tons, whereas the ship length of "Target Ship" is 139.8 m, the maximum speed of "Target Ship" is 13.5 kn, and the gross tonnage of "Target Ship "is 6000 tons.

For convenience, the game round is valued as 3. According to the relevant parameters of the two ships, the position of them is initialized. According to the 1972 International Collision Avoidance Regulations, "the two ships should each take a right turn to avoid collision" in encounter situation, which makes ship A as its own ship. In this case, the relevant parameter variables are obtained as in Table 1. By using Equation (9), the original collision risk between the two ships is 0.5911. Then, each ship starts to make a collision avoidance decision at 0 s [28]. In the first round, the own ship takes a 10° right turn to avoid collision. The target ship takes a 20° right turn to avoid collision. At the time node of 300 s in the second round, the own ship takes a 10° right turn to avoid collision, the target ship takes a 20° left turn to avoid collision, and the collision risk is 0.4635. At the time node of 600 s in the third round, the own ship takes a 10° left turn to avoid collision, the target ship takes a 10° left turn to avoid collision, and the collision risk is 0.4329. In the third round, because all ships completed the collision avoidance operation and there is no risk of subsequent collision, the course is readjusted, and the original course is restored. The simulation results of the confrontation situation based on the aforementioned are shown in Table 2. The optimal collision avoidance sequence combination composed of the obtained sub-game Nash equilibrium is $\{(10^{\circ}, 20^{\circ}), (0^{\circ}, 0^{\circ}), (0^{\circ}, 0^{\circ})\}$. All collision avoidance behaviors are consistent with COLREGS.

Table 1. Related parameter variables.

Ship Parameters							
V_r	24 kn	t_2	1800 s	D	6 n mile	U_θ	0.9558
φ_r	180°	D_{CPA}	0 n mile	D_1	0.9057 n mile	U_D	0
d_1	1.12 n mile	T_{CPA}	900 s	D_2	4.278 n mile	U_K	0.4143
d_2	2.21 n mile	U_{DCPA}	1	t_1	135.874 s	CRI	0.5912
θ	0°	U_{TCPA}	0.2926				

Table 2. Simulation results of encounter situation.

Time/s	Vessel	Decision	Course Angle	The Abscissa (Nautical Miles)	Y-Coordinate (Nautical Miles)	Offset (Nautical Miles)	Sum of Offset	Risk Collision Index
initial time	A		0°	5	1	0	0	0.5911
	B		180°	5	7	0		
0	A	Turn right 10 °	10°	5	1	0	0	0.5911
	B	Turn right 20°	200°	5	7	0		
300	A	Turn right 10°	20°	5.1736	1.9848	0.1743	0.5216	0.4635
	B	Turn left 10°	190°	4.658	6.0603	0.3473		
600	A	Turn left 10°	10°	5.5157	2.9245	0.5212	1.0423	0.4329
	B	Turn left 10°	190°	4.4843	5.755	0.5212		
900	A	Restore the course	10°	5.6893	3.9093	0.6953	1.2164	0.4803
	B	Restore the course	180°	4.4843	4.0755	0.5212		

5. Conclusions

This study proposes a decision-making problem on the collision avoidance of unmanned ships at sea in the situation of two-sides clamping. This study introduces the decision process of collision avoidance of unmanned ships at sea based on the extensive-form game model and verifies the effectiveness of collision avoidance by using fuzzy credibility numbers. Specifically, the main innovations of this study are concluded as follows.

Firstly, this study proposes a two-sides clamping intelligent collision avoidance strategy for unmanned ships. This strategy can provide real-time collision avoidance measures for unmanned ships at sea. The example analysis shows that this strategy can effectively improve the efficiency of collision avoidance of unmanned ships.

Secondly, the simulation experiment is carried out with the navigation simulator to realize the ship's extended game collision avoidance decision-making system. The simulation of two unmanned ships is carried out in the situation where two unmanned ships in the case of clamping on two-sides. Aiming at the intelligent collision avoidance problem of unmanned ships in the situation of two-sides, this study establishes a dynamic collision avoidance game model for ships based on the extensive-form game model. The unmanned ship can make the optimal collision avoidance action in the situation of being clamped on two sides.

Thirdly, a novel collision risk fuzzy credibility number is used to calculate the ship collision risk at the comprehensive fuzzy assessment based on the same time. The evaluation indicators include D_{CPA}, T_{CPA}, the distance between the two ships, the relative orientation between the two ships, the speed ratio of the two ships, and other factors.

Moreover, by using fuzzy credibility numbers, the decision-making efficiency on collision avoidance of ships in uncertain environments can be improved. In future work, the fuzzy credibility numbers can be considered in more decision-making situations in shipping management. Furthermore, the Fermatean fuzzy sets [29] is applied to the collision avoidance process of unmanned ships. Considering the multiple fuzzy factors that affect the collision avoidance of unmanned ships at sea, this study combines Fermatean fuzzy sets and links it with extensive-form game to provide support for the intelligent collision avoidance of unmanned ships at sea.

Author Contributions: Conceptualization, F.Z.; Data curation, M.L.; Formal analysis, R.W.; Investigation, Y.C.; Writing—original draft, H.C. All authors have read and agreed to the published version of the manuscript.

Funding: The work is partially supported by Shanghai Pujiang Program (2019PJC062), the Natural Science Foundation of Shandong Province (ZR2021MG003), the Research Project on Undergraduate Teaching Reform of Higher Education in Shandong Province (No. Z2021046), the National Natural Science Foundation of China (51508319), and the Nature and Science Fund from Zhejiang Province Ministry of Education (Y201327642).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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