mathematics

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Sustainable Fuzzy Portfolio Selection Concerning Multi-Objective Risk Attitudes in Group Decision

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Abstract: Fuzzy portfolio selection has resulted in many researchers to focus on this field. Based on the risk attitudes, this study discusses the risk attitudes in a decision group for portfolio selection. Therefore, we adopt the risk attitudes to describe the experts' risk preferences and subjective judgments, and then we suppose that the risk seeker considers a higher return for an excess investment based on the selected guaranteed rate of return; the risk averter considers a shortage in investment for the securities whose return rates are smaller than the selected guaranteed rate of return; and finally, the risk neutral pursues the regular return rate. In order to solve the multi-objective return rate functions under the corresponding investment risks, the SMART-ROC weighting method is used to hybridize the multi-objective programming model to a linear programming model for solving the portfolio selection. Finally, we illustrate a numerical example and two risk scenarios to show the optimal portfolio selection under different investment risks. The results show that the proposed model can obtain a more robust portfolio than the compared models under different risk priorities in a decision group.

Keywords: fuzzy portfolio model; guaranteed return rates; risk-attitude; multi-objective function; SMART-ROC

MSC: 90B50; 90B60

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Citation: Huang, Y.-Y.; Tsaur, R.-C.; Huang, N.-C. Sustainable Fuzzy Portfolio Selection Concerning Multi-Objective Risk Attitudes in Group Decision. *Mathematics* **2022**, *10*, 3304. https://doi.org/10.3390/math10183304

Academic Editor: Mar Arenas-Parra

Received: 19 August 2022
Accepted: 7 September 2022
Published: 12 September 2022

1. Introduction

Portfolio selection is formulated as a mean–variance model in random variables with Gaussian distribution to maximize the expected return of financial assets under a constrained risk [1]. Researchers have applied the mean–variance portfolio selection method to develop their model in portfolio selections [2–7]. Most of the above portfolio selection models are solved by linear programming model and/or probability theory based on historical data securities markets. However, the uncertainty from an event is not always evaluated by probability theory; by contrast, we might consider the portfolio selection by the possibility distribution in fuzzy random variables to maximize the expected return of financial assets under a constrained risk. For example, COVID-19 is a new pandemic disease that spread at the end of 2019 for which most countries or regions around the world have been in lockdown without an efficient curing method, which significantly affected to the global economy in business transactions, manufactures, transportations and tourism industry, etc. The economic losses of various industries are significant and have been difficult to estimate. Therefore, each country needs to reduce the loss resulting from the COVID-19 pandemic and propose economic stimulus or relief programs. In spite of the stock market during the COVID-19 pandemic period returning significant profit for most investors, it is still difficult to make portfolio selection based on past securities data. So, most investors cannot derive the precise probability distributions of the returns on risky assets from uncertain factors, such as economics, pandemic disease, policies, regulations, and psychological behaviors. In order to cope with the research gap of portfolio selection in the uncertain factors in some specific period and the uncertainty evaluations for the above factors, human subjective opinions from experts should be taken into consideration where fuzzy approaches are more appropriate than probabilistic approaches [8–12].

In a continuous fluctuation dynamic process with vagueness information, the investment strategy varies depending on the business cycle, regulations, risk attitudes, multi-period portfolio selections, emergency in pandemic disease, etc. Therefore, the aim of this article is to suggest that investors summarize investment information from a group of experts whose risk preference and subjective judgment are used to select assets, including demand deposits, fixed deposits, bonds, fixed foreign currency deposits, futures, ETFs, stocks and real estate. Based on the concept of multi-objectives in environmental, social, and governance (ESG) and judging the priority of the objectives [13], we can solve the fuzzy portfolio the same as sustainable portfolio selection. However, the risk attitudes in the invited experts are different, including risk averse, risk neutral, and risk seeking. The most important investment criterion in an investor's portfolio is the rate of return, and most investors believe that higher risk implies higher return rate, and vice versa. Therefore, an expert in risk seeking tries to invest higher risk securities for higher return; one that is risk averse tries to avoid risk securities in their investment; and one that is risk neutral tries to pursue the regular return rates in the securities selection process. Therefore, during the COVID-19 pandemic period with vagueness investment information, we adopt the risk attitudes concept to describe the experts' risk preferences and subjective judgments [14], and then we suppose that the risk seeker considers higher return for the excess investment based on the selected guaranteed rate of return [15], the risk averter considers the adjustment investment [16], and finally the risk neutral pursues the regular return rate [14]. In order to solve the multi-objective return rate functions under the corresponding risk attitudes, a weighting method adopts from SMART-ROC [17] is considered from sustainable portfolio selection to solve the fuzzy portfolio selection based on the multi-objective programming model. The main contributions of this article are to consider a positive evaluation of the group of experts to justify their importance and appropriateness in terms of their risk preferences and subjective judgments, which can assist the investor to make a portfolio selection model with vagueness information. Next, we use the SMART-ROC method to weigh the multi-objective functions of the rate of returns into a linear programming model, and then we can easily solve the weighted fuzzy selected model. Finally, we conduct a sensitivity analysis for different risk priorities of different experts for testing the portfolio selection, which shows that the risk priority of the expert will affect the result of the portfolio selection with different rate of return and constrained risk.

This paper is organized as follows. Section 2 provides a literature review for the fuzzy portfolio selection. In Section 3, a group decision in the fuzzy portfolio model with hybrid return rates and investment risks is given. In Section 4, an illustration with sensitivity analysis and discussions are presented. Finally, the conclusion is discussed in Section 5.

2. Literature Reviews

The fuzzy portfolio model extends traditional probability measures into fuzzy possibilities to solve the fuzzy portfolio selection problem. Many researchers have devoted themselves to the field of fuzzy portfolio models [18–23]. In addition, for multi-objective portfolio selections, Guo et al. [24] considered a fuzzy multi-period portfolio selection problem with V-shaped transaction costs, and then designed a fuzzy simulation-based genetic algorithm for illustration. Tsaur et al. [15] proposed a fuzzy return function and considered excess investment using guaranteed return rates for the selecting securities, and then efficient portfolios could be obtained under different levels of investment risk. Tsaur et al. [23] revised Chen and Tsaur's fuzzy portfolio model [22] for the COVID-19 pandemic, which has greatly influenced the global economy, by using the fuzzy goal programming model with different linguistic descriptions for the imprecise goal of expected returns and the future stock market, the optimal portfolio selection that can be solved under different investment risks. Tsaur [14] considered different risk attitudes for investors and extended the MV models to assist investors in optimizing their investment strategies. García et al. [25] proposed a mean–semivariance multiobjective credibilistic portfolio selection model with the price-to-earnings ratio to measure the portfolio performance. García et al. [26] extended the stochastic mean–variance model to a credibilistic multiobjective model in which the semivariance and the CVaR are used to measure portfolio performance and the non-dominated sorting genetic algorithm II is applied to solve efficient portfolios in the fuzzy return–risk–liquidity trade-off to create the efficient frontier. Mehlawat et al. [27] proposed multiobjective functions with variance and CVaR as risk measures for performance evaluation in the fuzzy portfolio selection models in which the inherent uncertainty of the investment market was incorporated through trapezoidal fuzzy returns using the credibility theory.

From the above discussion, the research studies about fuzzy portfolio models have made great progress, and most of the existing models deal with risk constraints, economy issues, and investor behaviors. With the advancement of renewable energy, we can consider applying the fuzzy portfolio analysis to analyze renewable energy power generation for the aim of ESG. Wang and Singh [28] investigated the trade-off effect of wind power intermittency on system risk and total generating costs. Lee and Chen [29], dealing with wind and PV generation system mix, investigated the optimal contract capacities and installed capacities. They found that the energy cost is the substantial factor influencing profitability. The capital cost becomes the influential factor of optimal installed capacity. Chang [30] applied the MCGP model to select the optimal location of renewable energy facilities in Taiwan. Ervural et al. [13] formulated a MODM model under several constraints for sustainable energy investment planning. In this era, most countries are heavily reliant on the energy sector, and the global energy demand is growing by the day. Despite the fact that fossil fuels provide a significant portion of energy [31], increasing worries about energy supply, climate change, political and social pressure to reduce greenhouse gas pollution, high and uncertain oil prices, and high reliance on international energy sources has intensified the shift from nonrenewable to renewable sources [32]. Therefore, electricity generation from clean energy sources is increasingly becoming a significant part of most countries' national energy mix [33]. Many countries are now increasing their investments in renewable energy (RE) sectors and assist with different national policies [34]. Therefore, the energy portfolio should be flexible and adjustable for different periods of challenge, for example, the portfolio scenario for different periods, the risk for different energy portfolios, and the different demand and price for different energy portfolios under different energy risk. From the point of the mean–variance portfolio model, we suppose optimizing the energy portfolio to obtain the optimal strategies of energy resource allocations and achieve the efficient usages and productions of energy. According to the literature on electricity generation planning, many studies have developed multi-objective algorithms to solve the energy portfolio. Energy planning usually needs discussion in a group of experts about their opinions to collect valuable sources of information and knowledge from the future energy markets, potential new technology, energy policy trends, and analyses. Therefore, based on the triple bottom line (under economic, environmental, and social), we can employ fuzzy set theory into analyzing the energy systems to consider all information regarding the linguistic variables related to both perception and cognition collected from the experts. At this stage, fuzzy set theory allows vague human assessments to be included in the analysis process by using membership functions. The membership functions of fuzzy parameters are commonly constructed based on available expert opinions. In many research fields, the fuzzy method is applied separately to conduct an uncertainty analysis, and thus the fuzzy portfolio model can be used to analyze the energy portfolio to provide useful information for the government policy decision. Therefore, in theory and practical, our proposed fuzzy portfolio selection can be supported and logically applied to sustainable portfolio management.

3. Fuzzy Portfolio Selection for Different Risk Attitudes

Let x_j be the proportion of investment and let the return rate of security j be a triangular fuzzy number, $\widetilde{r}_j = (r_j,\ c_j,\ d_j)$, $j = 1, \ldots, n$, where r_j is its central value, and c_j, d_j are its left and right spread values, respectively. Assuming that the membership function of the fuzzy return rate is selected as $k = 1$ for risk neutral [14], the expected fuzzy returns can be obtained as follows:

$$\widetilde{R} = \sum_{j=1}^{n} x_j \widetilde{r}_j, \tag{1}$$

Analogous to Markowitz's mean–variance portfolio model, the fuzzy portfolio model under a risk-neutral attitude can be formulated by the maximum of the possibilistic mean value for investment return and the possibilistic standard deviation for risk constrained by the upper bound of the desired values of an investor when $k = 1$ [14]. Therefore, the fuzzy portfolio model under a risk-neutral attitude can be formulated as follows:

$$\begin{aligned} & Max \sum_{j=1}^{n} [r_j x_j + \tfrac{1}{6}(d_j - c_j)x_j] \\ & s.t. \tfrac{1}{2\sqrt{6}} \left[\sum_{j=1}^{n} x_j (d_j + c_j) \right] \le \sigma \\ & \sum_{j=1}^{n} x_j = 1 \\ & l_j \le x_j \le u_j, \forall\, j = 1,\ 2, \ldots, n. \end{aligned} \tag{2}$$

Next, Tsaur et al. [15] assumed that higher risk usually comes with higher returns, and the investors usually can realize unexpected returns from these higher return securities. The guaranteed return rate $\widetilde{p}_k\ \forall k = 1,\ 2, \ldots,\ m$ is selected by the risk seeker, and we assume that the ordering of the guaranteed return rate is $\widetilde{p}_1 < \widetilde{p}_2 < \ldots < \widetilde{p}_m$ (i.e. $R(\widetilde{p}_1) < R(\widetilde{p}_2) < \ldots < R(\widetilde{p}_m)$). If the fuzzy return rate $\widetilde{r}_j = (r_j,\ c_j,\ d_j)$, $j = 1, \ldots, n$, is larger than $\widetilde{p}_k = (p_k,\ e_k,\ f_k)$, and $R(\widetilde{r}_j) > R(\widetilde{p}_k)$ [35], then excess investment will be made on security j; otherwise, no excess investment will be made. For an investor with a risk-seeking attitude in investment, the investment strategy is to seek the risk with the higher returns. For the risk-seeking attitude, the expected fuzzy returns can be obtained as follows:

$$\widetilde{R} = \sum_{j=1}^{n} x_j \widetilde{r}_j + \sum_{j=1}^{n} \sum_{k=1}^{m} x_j (|\widetilde{r}_j - \widetilde{p}_k| + \widetilde{r}_j - \widetilde{p}_k) \big/ 2 \tag{3}$$

To solve Equation (3), if the fuzzy return rate $\widetilde{r}_j = (r_j,\ c_j,\ d_j)$, $j = 1, \ldots, n$, is larger than $\widetilde{p}_k = (p_k,\ e_k,\ f_k)$, and $R(\widetilde{r}_j) > R(\widetilde{p}_k)$, then excess investment can be made on security j; otherwise, no excess investment will be made. This concept can be formulated as follows:

$$(|\widetilde{r}_j - \widetilde{p}_k| + \widetilde{r}_j - \widetilde{p}_k) \big/ 2 = \begin{cases} \widetilde{r}_j - \widetilde{p}_k & \text{if } R(\widetilde{r}_j) > R(\widetilde{p}_k) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Then, the combination of the possibilistic mean value, which is larger than the guaranteed rate of return, can be obtained as follows:

$$\begin{aligned} & M[\sum_{j=1}^{n} x_j \widetilde{r}_j + \sum_{j=1}^{n} \sum_{k=1}^{m} x_j (\widetilde{r}_j - \widetilde{p}_k)] = \sum_{j=1}^{n} x_j M(\widetilde{r}_j) + \sum_{j=1}^{n} \sum_{k=1}^{m} x_j M(\widetilde{r}_j - \widetilde{p}_k) \\ & = \sum_{j=1}^{n} x_j [r_j + \tfrac{8}{15}(d_j - c_j)] + \sum_{j=1}^{n} \sum_{k=1}^{m} x_j [(r_j - p_k) + \tfrac{8}{15}[(d_j + e_k) - (c_j + f_k)] \end{aligned} \tag{5}$$

Then, the lower and upper possibilistic variances are derived as follows:

$$Var_*[\sum_{j=1}^{n} x_j \widetilde{r}_j + \sum_{j=1}^{n} \sum_{k=1}^{m} x_j (\widetilde{r}_j - \widetilde{p}_k)] = \frac{\sqrt{11}}{15} [\sum_{j=1}^{n} x_j c_j + \sum_{j=1}^{n} \sum_{k=1}^{m} x_j (c_j + f_k)]^2,\ \forall k > 0. \tag{6}$$

$$Var^*[\sum_{j=1}^{n} x_j\widetilde{r}_j + \sum_{j=1}^{n}\sum_{k=1}^{m} x_j(\widetilde{r}_j - \widetilde{p}_k)] = \frac{\sqrt{11}}{15}[\sum_{j=1}^{n} x_j d_j + \sum_{j=1}^{n}\sum_{k=1}^{m} x_j(d_j + e_k)]^2,\ \forall k > 0. \quad (7)$$

Then, its possibilistic standard deviation is defined as follows:

$$SD(\sum_{j=1}^{n} x_j\widetilde{r}_j + \sum_{j=1}^{n}\sum_{k=1}^{m} x_j(\widetilde{r}_j - \widetilde{p}_k)) = \frac{\sqrt{11}}{30}\left[\sum_{j=1}^{n}(c_j + d_j)x_j + \sum_{j=1}^{n}\sum_{k=1}^{m}(c_j + f_k + d_j + e_k)x_j\right]. \quad (8)$$

Therefore, the fuzzy portfolio model under risk seeking is formulated as follows:

$$\begin{aligned}
&Max \sum_{j=1}^{n} x_j[r_j + \tfrac{8}{15}(d_j - c_j)] + \sum_{j=1}^{n}\sum_{k=1}^{m} x_j[(r_j - p_k) + \tfrac{8}{15}[(d_j + e_k) - (c_j + f_k)]\\
&s.t. \tfrac{\sqrt{11}}{30}\left[\sum_{j=1}^{n}(c_j + d_j)x_j + \sum_{j=1}^{n}\sum_{k=1}^{m}(c_j + f_k + d_j + e_k)x_j\right] \le \sigma\\
&\quad \sum_{j=1}^{n} x_j = 1\\
&\quad l_j \le x_j \le u_j, \forall\, j = 1, 2, \ldots, n.
\end{aligned} \quad (9)$$

For an investor with a risk-averse attitude in investment, the investment strategy is to avoid the higher risk with the higher returns. For an investor with a risk-averse attitude, the membership function of the fuzzy return rate $\widetilde{r}_j = (r_j,\ c_j,\ d_j)$ is selected as k = 0.5 [16]. The securities whose return rates are lower than the guaranteed rates of return $\widetilde{p}_k\ \forall k = 1, 2, \ldots, m$ are avoided by the risk averters, and excess investment for securities who return rates are higher than the guaranteed return rate; therefore, the expected fuzzy returns can be obtained as follows:

$$\widetilde{R} = \sum_{j=1}^{n} x_j\widetilde{r}_j - \sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j|\widetilde{r}_j - \widetilde{p}_k| + \sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j|\widetilde{r}_j - \widetilde{p}_k| \quad (10)$$

Then, the possibilistic mean value of formula (10) with respect to all securities in the portfolio selection is obtained as follows:

$$\begin{aligned}
&M\left[\sum_{j=1}^{n} x_j\widetilde{r}_j - \sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j(\widetilde{p}_k - \widetilde{r}_j) + \sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j(\widetilde{r}_j - \widetilde{p}_k)\right]\\
&= \sum_{j=1}^{n} x_j M(\widetilde{r}_j) - \sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j M(\widetilde{p}_k - \widetilde{r}_j) + \sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j M(\widetilde{r}_j - \widetilde{p}_k)\\
&= \sum_{j=1}^{n} x_j[r_j + \tfrac{1}{6}(d_j - c_j)] - \sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j[(p_k - r_j) + \tfrac{1}{6}[(e_k + d_j) - (f_k + c_j)]\\
&+ \sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j[(r_j - p_k) + \tfrac{1}{6}[(d_j + e_k) - (c_j + f_k)]
\end{aligned} \quad (11)$$

Then, the lower and upper possibilistic variances are derived as follows:

$$\begin{aligned}
&Var_*[\sum_{j=1}^{n} x_j\widetilde{r}_j - \sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j(\widetilde{p}_k - \widetilde{r}_j) + \sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j(\widetilde{r}_j - \widetilde{p}_k)]\\
&= \tfrac{1}{36}[\sum_{j=1}^{n} x_j c_j - \sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j(c_j + f_k) + \sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j(c_j + f_k)]2,\ \forall k > 0.
\end{aligned} \quad (12)$$

$$\begin{aligned}
&Var^*[\sum_{j=1}^{n} x_j\widetilde{r}_j - \sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j(\widetilde{p}_k - \widetilde{r}_j) + \sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j(\widetilde{r}_j - \widetilde{p}_k)]\\
&= \tfrac{1}{36}[\sum_{j=1}^{n} x_j d_j - \sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j(d_j + e_k) + \sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j(d_j + e_k)]2,\ \forall k > 0.
\end{aligned} \quad (13)$$

Then, the possibilistic standard deviation is obtained as follows:

$$\begin{aligned}
&SD\left[\sum_{j=1}^{n} x_j\widetilde{r}_j-\sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j\left(\widetilde{p}_k-\widetilde{r}_j\right)+\sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j\left(\widetilde{r}_j-\widetilde{p}_k\right)\right] \\
&=\sum_{j=1}^{n} x_j\tfrac{c_j+d_j}{12}-\sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j\tfrac{\left(c_j+f_k\right)+\left(d_j+e_k\right)}{12}+\sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j\tfrac{\left(c_j+f_k\right)+\left(d_j+e_k\right)}{12}.
\end{aligned} \tag{14}$$

Therefore, the fuzzy portfolio model with risk averse attitude can be formulated as follows:

$$\begin{aligned}
&Max\sum_{j=1}^{n} x_j[r_j+\tfrac{1}{6}(d_j-c_j)]-\sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j[(p_k-r_j)+\tfrac{1}{6}[(e_k+d_j)-(f_k+c_j)] \\
&+\sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j[(r_j-p_k)+\tfrac{1}{6}[(d_j+e_k)-(c_j+f_k)] \\
&s.t.\sum_{j=1}^{n} x_j\tfrac{c_j+d_j}{12}-\sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j\tfrac{\left(c_j+f_k\right)+\left(d_j+e_k\right)}{12}+\sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j\tfrac{\left(c_j+f_k\right)+\left(d_j+e_k\right)}{12}\le\sigma \\
&\quad\sum_{j=1}^{n} x_j\le 1 \\
&\quad l_j\le x_j\le u_j,\forall\, j=1,\ 2,\ldots,n.
\end{aligned} \tag{15}$$

From the above description, we can understand that different risk attitudes can obtain different portfolio selection in different investment strategies. In an investment scenario, we suppose that an investor would like to make returns robustly, and thus he tries to organize a group of consultants for gathering more information in this portfolio selection. In those consultants, the risk attitudes can be clustered as risk neutral, risk seeking, or risk averse. Based on the above modeling process, the objectives of the possibilistic mean values of investment return for risk neutral, risk seeking, and risk averse are defined as Ob_1, Ob_2, and Ob_3, respectively, as follows:

$$\mathrm{Ob}_1=\sum_{j=1}^{n}[r_jx_j+\frac{1}{6}(d_j-c_j)x_j] \tag{16}$$

$$\mathrm{Ob}_2=\sum_{j=1}^{n} x_j[r_j+\frac{8}{15}(d_j-c_j)]+\sum_{j=1}^{n}\sum_{k=1}^{m} x_j[(r_j-p_k)+\frac{8}{15}[(d_j+e_k)-(c_j+f_k)] \tag{17}$$

$$\begin{aligned}
&\mathrm{Ob}_3=\sum_{j=1}^{n} x_j[r_j+\tfrac{1}{6}(d_j-c_j)]-\sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j[(p_k-r_j)+\tfrac{1}{6}[(e_k+d_j)-(f_k+c_j)] \\
&+\sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j[(r_j-p_k)+\tfrac{1}{6}[(d_j+e_k)-(c_j+f_k)]
\end{aligned} \tag{18}$$

Next, the possibilistic standard deviations for risk neutral, risk seeking, and risk averse are defined as follows:

$$\mathrm{Risk}_1=\frac{1}{2\sqrt{6}}\left[\sum_{j=1}^{n} x_j(d_j+c_j)\right] \tag{19}$$

$$\mathrm{Risk}_2=\frac{\sqrt{11}}{30}\left[\sum_{j=1}^{n}(c_j+d_j)x_j+\sum_{j=1}^{n}\sum_{k=1}^{m}(c_j+f_k+d_j+e_k)x_j\right] \tag{20}$$

$$\mathrm{Risk}_3=\sum_{j=1}^{n} x_j\frac{c_j+d_j}{12}-\sum_{j=1}^{n_1}\sum_{k=1}^{m} x_j\frac{\left(c_j+f_k\right)+\left(d_j+e_k\right)}{12}+\sum_{j=n_1+1}^{n}\sum_{k=1}^{m} x_j\frac{\left(c_j+f_k\right)+\left(d_j+e_k\right)}{12} \tag{21}$$

After the information received from the investment consultants, the decision maker considers the weights of expected return objective functions by the past investment performance of the consultants with different risk attitudes. Therefore, the hybrid multi-objective functions from Equations (16)–(18) are the weight of w_1, w_2, and w_3, respectively. In order to rank the priorities of the past investment performance of the consultants, we use the SMART–ROC method [36] to evaluate the weights through the gravity method defined as $w_i=\frac{1}{n}\sum_{k=i}^{n}\frac{1}{k}$, and thus the weights of the risk attitudes of consultants can be used for solving the proposed model. In addition, the possibilistic standard deviations from

Equations (19)–(21) are also hybridized and their weight is based on the past investment performance. Therefore, for the fuzzy portfolio model under the consideration of different risk attitudes, we maximize the hybrid possibilistic mean values and the constrained possibilistic standard deviations as follows:

$$\begin{aligned} & Max w_1\,\mathrm{Ob}_1 + w_2\,\mathrm{Ob}_2 + w_3\,\mathrm{Ob}_3 \\ & s.t. w_1\,\mathrm{Risk}_1 + w_2\,\mathrm{Risk}_2 + w_3\,\mathrm{Risk}_3 \le \sigma \\ & \quad \sum_{j=1}^{n} x_j = 1 \\ & \quad l_j \le x_j \le u_j, \forall\, j = 1,\ 2, \ldots, n. \end{aligned} \tag{22}$$

4. Illustrations and Results

In this section, we use an illustration and its sensitivity analysis to test the effect of the guaranteed return rates and the priority to the risk attitudes of the consultants in the group decision for the proposed fuzzy portfolio selection model. This illustration is secondary data collected by Zhang [37], where five securities were obtained from the weekly closed prices from April 2002 to January 2004, and the portfolio sample is based on the historical data, the corporations' financial reports, and future information from the Shanghai Stock Exchange. Therefore, the possibility distribution for each security is estimated as follows: $\widetilde{r}_1$ = (0.073, 0.054, 0.087), $\widetilde{r}_2$ = (0. 105, 0.075, 0.102), $\widetilde{r}_3$ = (0.138, 0.096, 0.123), $\widetilde{r}_4$ = (0.168, 0.126, 0.162), $\widetilde{r}_5$ = (0.208, 0.168, 0.213). In addition, we set the lower and upper bounds of investment proportion x_j for security j, given by (l_1, l_2, l_3, l_4, l_5) = (0.1, 0.1, 0.1, 0.1, 0.1), and (u_1, u_2, u_3, u_4, u_5) = (0.4, 0.4, 0.4, 0.5, 0.6). The threshold values for the guaranteed return rates are selected as $\widetilde{p}_1 = (0.1,\ 0.05,\ 0.05)$, higher than $\widetilde{r}_1$ to relax securities 2–5 for excess investment; $\widetilde{p}_2 = (0.15,\ 0.1,\ 0.1)$ is selected based on the average of the highest fuzzy return $\widetilde{r}_1$ and the lowest return $\widetilde{r}_5$ to relax securities 4, and 5 for excess investment; and $\widetilde{p}_3 = (0.2,\ 0.1,\ 0.15)$ is selected to be lower than $\widetilde{r}_5$ to relax security 5 for excess investment. The risk attitudes of those consultants are selected as three types, as risk neutral, risk seeking, or risk averse, in the modeling process.

4.1. Proposed Modeling Results and Discussions

Step 1: Rank the security fuzzy returns and selected guaranteed return rate.

We defuzzify all the fuzzy returns and guaranteed return rates as $R(\widetilde{r}_1) = 0.4236$, $R(\widetilde{r}_2) = 0.4302$, $R(\widetilde{r}_3) = 0.4394$, $R(\widetilde{r}_4) = 0.4500$, and $R(\widetilde{r}_5) = 0.4665$; $R(\widetilde{p}_1) = 0.4283$, $R(\widetilde{p}_2) = 0.4421$, and $R(\widetilde{p}_3) = 0.4647$. Therefore, the ranked fuzzy numbers are $\widetilde{r}_1 < \widetilde{p}_1 < \widetilde{r}_2 < \widetilde{r}_3 < \widetilde{p}_2 < \widetilde{r}_4 < \widetilde{p}_3 < \widetilde{r}_5$.

Step 2: Formulate the objective functions and constrained risks.

In this step, we first select the guaranteed return rate $\widetilde{p}_1 = (0.1,\ 0.05,\ 0.05)$ to deal with the proposed model for portfolio selection. Then, security 1 is not permitted for excess investment, and securities 2–5 can be excess investments because $R(\widetilde{p}_1)$ is lower than $R(\widetilde{r}_2)$,$R(\widetilde{r}_3)$,$R(\widetilde{r}_4)$, and $R(\widetilde{r}_5)$. Therefore, we obtain the expected fuzzy returns for the risk neutral, risk seeking, or risk averse, which can be obtained as follows:

$\mathrm{Ob}_1 = 0.0785\, x_1 + 0.1905\, x_2 + 0.1425\, x_3 + 0.174\, x_4 + 0.2155\, x_5$

$\mathrm{Ob}_2 = 0.0906\, x_1 + 0.1388\, x_2 + 0.2048\, x_3 + 0.2744\, x_4 + 0.3640\, x_5$

$\mathrm{Ob}_3 = 0.046\, x_1 + 0.119\, x_2 + 0.185\, x_3 + 0.248\, x_4 + 0.331\, x_5$

The investment risks for risk neutral, risk seeking, or risk averse are obtained as follows:

$\mathrm{Risk}_1 = 0.028782\, x_1 + 0.036130\, x_2 + 0.044703\, x_3 + 0.058788\, x_4 + 0.077771\, x_5$

$\mathrm{Risk}_2 = 0.015588\, x_1 + 0.050192\, x_2 + 0.059478\, x_3 + 0.074735\, x_4 + 0.095298\, x_5$

$\mathrm{Risk}_3 = -0.00833\, x_1 + 0.037833\, x_2 + 0.044833\, x_3 + 0.056333\, x_4 + 0.071833\, x_5$

Finally, the investor can evaluate the past performance of the experts in the decision group and judge the priority of their decision power to the objective functions. Without loss of its generality, we supposed that in this investment, the priority of the risk attitudes for the experts who are risk seeking is prior to those that are risk neutral and risk averse, and thus the expected return rates are ranked as $\mathrm{Ob}_2 > \mathrm{Ob}_1 > \mathrm{Ob}_3$. According to the

SMART-ROC weighting method, we can obtain the weights of the objectives and risks as $w_1 = 0.2778$, $w_2 = 0.6111$, $w_3 = 0.1111$.

Therefore, we can formulate the fuzzy portfolio model as follows:

$$\begin{array}{c} Max\ 0.082362\,x_1 + 0.128570\,x_2 + 0.185436\,x_3 + 0.243750\,x_4 + 0.319296\,x_5 \\ s.t.\ 0.016624\,x_1 + 0.044948\,x_2 + 0.053791\,x_3 + 0.068319\,x_4 + 0.0879\,x_5 \leq \sigma \\ x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\ 0.1 \leq x_1 \leq 0.4, 0.1 \leq x_2 \leq 0.4, 0.1 \leq x_3 \leq 0.4, 0.1 \leq x_4 \leq 0.5, 0.1 \leq x_5 \leq 0.5 \end{array} \tag{23}$$

Step 3: Solving the proposed illustration.

In this step, the investment risks are ranged from 4% to 9%, and we use model (23) to obtain the portfolio selections shown in Table 1. We can find that investment proportion for security 1 decreases from 0.396 to 0.1 because the increase in the bound of the investment risk makes the risky security 5 increase from 0.3039 to 0.6 because the return rate of security 5 is higher than the selected guaranteed rate of return $\widetilde{p}_1$ and contributes to the investor making an excess investment. In the other, the investor focuses on stationary investment when the investment risk exceeds or equals 8%, with $x_1 = 0.1$, $x_2 = 0.1$, $x_3 = 0.1$, $x_4 = 0.1$, and $x_5 = 0.6$, with the expected return rate of 25.5589%.

Table 1. The portfolio selection with different risk in a guaranteed return rate $\widetilde{p}_1$.

Risk Proportion	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**
x_1	Infeasible	0.3961	0.2558	0.1155	0.1	0.1
x_2	Infeasible	0.1	0.1	0.1	0.1	0.1
x_3	Infeasible	0.1	0.1	0.1	0.1	0.1
x_4	Infeasible	0.1	0.1	0.1	0.1	0.1
x_5	Infeasible	0.3039	0.4442	0.5845	0.6	0.6
Expected Return Rate	Infeasible	0.185422	0.218664	0.251906	0.255589	0.255589

Step 4: Sensitivity analysis for the guaranteed return rates.

In this step, we select the guaranteed return rates $\widetilde{p}_2$ and $\widetilde{p}_3$ for portfolio analysis and the results are shown in Tables 2 and 3, respectively. In Table 2, the ranking of the fuzzy numbers is as follows: $\widetilde{r}_1 < \widetilde{r}_2 < \widetilde{r}_3 < \widetilde{p}_2 < \widetilde{r}_4 < \widetilde{r}_5$. Therefore, securities 4 and 5 are assumed to be excess investments, and securities 1–3 are assumed to be regular investments. We determine the investment risks in this step to be ranging from 3% to 8%; we find that the investment proportion for securities 2 and 3 decrease from 0.2394 and 0.4 to 0.1, and securities 1 and 3 are all 0.1 in different investment risks. This is because the increase in investment risks pushes the investment to the securities whose return rates are higher than the selected guaranteed rate of return $\widetilde{p}_2$. For the trade-off of securities 4 and 5 in return rates, security 5 increases from 0.398 to 0.6 because the return rates of securities 4 and 5 are higher than the selected guaranteed rate of return $\widetilde{p}_2$ and contribute to the investor making an excess investment. Therefore, when the investment risk exceeds or is equal to 8%, implying that the investor focuses on excess investment, we can suggest that securities 4 and 5 be the major investment with the proportions 0.1606 and 0.6. In the other, the investor focuses on optimal investment when the investment risk exceeds or equals 7%, with $x_1 = 0.1$, $x_2 = 0.1$, $x_3 = 0.1$, $x_4 = 0.1$, and $x_5 = 0.6$, with the expected return rate of 22.4396%.

The guaranteed return rate $\widetilde{p}_3$ is used for the portfolio analysis, while securities 1–4 are allowed for regular investments and excess investment is for security 5 because the ranking of the fuzzy returns is $\widetilde{r}_1 < \widetilde{r}_2 < \widetilde{r}_3 < \widetilde{r}_4 < \widetilde{p}_3 < \widetilde{r}_5$. In Table 3, we define the investment risks to be ranging from 3% to 8% because most of securities are a feasible solution in this range, and the investment proportions for securities 1 and 2 are always 0.1; securities 3 and 4 are decreased from 0.1825 and 0.5, respectively, to 0.1 for both. Next, we can find that the investment proportion of security 5 increases from 0.1175 to 0.5 to make the excess investment because the return rate of security 5 is larger than the selected guaranteed return rate $\widetilde{p}_3$. In the other, the investor focuses on optimal investment when the investment risk exceeds or equals 7%, with x_1 = 0.1, x_2 = 0.1, x_3 = 0.1, x_4 = 0.1, and x_5 = 0.6, with the expected return rate of 19.1169%.

Table 2. The portfolio selection with different risk in a guaranteed return rate $\widetilde{p}_2$.

Risk Proportion	4%	5%	6%	7%	8%	9%
x_1	0.1	0.1	0.1	0.1	0.1	0.1
x_2	0.2524	0.1318	0.1	0.1	0.1	0.1
x_3	0.4	0.4	0.3051	0.1763	0.1	0.1
x_4	0.1	0.1	0.1	0.1	0.1	0.1
x_5	0.1476	0.2682	0.3949	0.5237	0.6	0.6
Expected Return Rate	0.156702	0.177665	0.196101	0.213663	0.224058	0.224058

Table 3. The portfolio selection with different risk in a guaranteed return rate $\widetilde{p}_3$.

Risk Proportion	4%	5%	6%	7%	8%	9%
x_1	0.1	0.1	0.1	0.1	0.1	0.1
x_2	0.1	0.1	0.1	0.1	0.1	0.1
x_3	0.1496	0.1	0.1	0.1	0.1	0.1
x_4	0.5	0.4182	0.2810	0.1438	0.1	0.1
x_5	0.1504	0.2818	0.4190	0.5562	0.6	0.6
Expected Return Rate	0.163093	0.171601	0.178587	0.185572	0.187801	0.187801

From the above analysis, as shown in Figure 1, we investigate the expected return rate of a portfolio with a lower guaranteed return rate being higher in different investment risk levels because a higher guaranteed return rate makes some securities have smaller investments and thus obtain a lower expected return rate. Therefore, in order to obtain higher investment returns, we suggest that an investor should not select too high of a guaranteed rate of return. In addition, the proportion of security 5 is always higher than the other securities when the investment risk is higher, and thus it is the most important security in the investment.

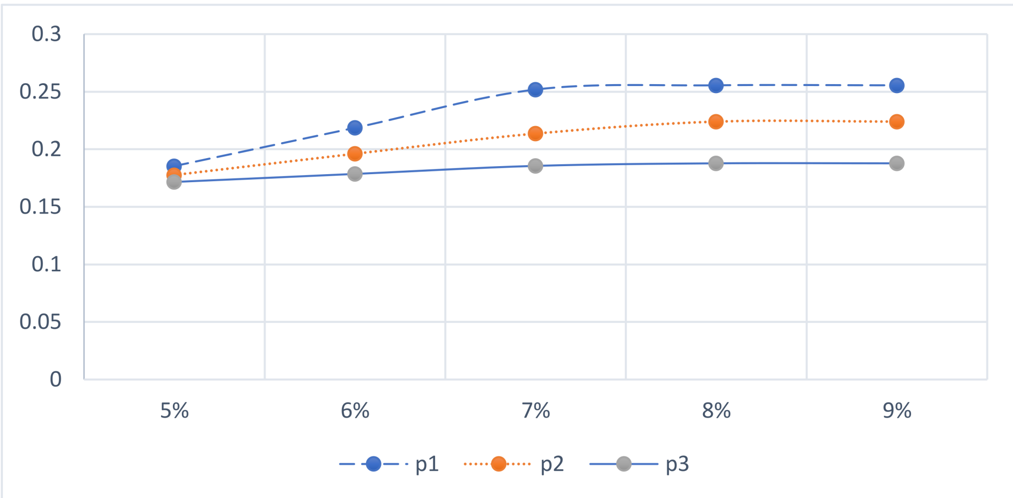

Figure 1. The comparisons between different guaranteed return rates.

4.2. The Portfolio Selection in Different Decision Priority

In this subsection, we assume two scenarios for the decision priorities in the decision group for portfolio selection. The first condition assumes that the portfolio selection is decided by the decision group, where the member with a risk-averse attitude has the major decision power, the next one is the risk-neutral member and the last one is the risk-seeking member. The next scenario is that the portfolio selection is decided by the decision group where the member with a risk-averse attitude has the major decision power, the next one is the risk-neutral member and the last one is the risk-seeking member.

Condition 1: averse > neutral > seeking

In this scenario, the investor can evaluate the past performance of the experts in investment and judge the priority of the objectives. Since the expert that is risk averse is better than the risk-neutral and risk-seeking ones, thus we assume $Ob_3 > Ob_1 > Ob_2$, and thus, according to the SMART-ROC weighting methods, we can obtain the weights of the objectives and risks as $w_1 = 0.2778$, $w_2 = 0.1111$, and $w_3 = 0.6111$. In this step, we first select the guaranteed return rate $\widetilde{p}_1 = (0.1,\ 0.05,\ 0.05)$ to deal with the adjustable security proportion investment in the portfolio. Finally, we can formulate the fuzzy portfolio model as follows:

$$\begin{array}{c} Max\ 0.060062\ x_1 + 0.11867\ x_2 + 0.175536\ x_3 + 0.23055\ x_4 + 0.302796\ x_5 \\ s.t.\ 0.004664\ x_1 + 0.038769\ x_2 + 0.046469\ x_3 + 0.059118\ x_4 + 0.076168\ x_5 \le \sigma \\ x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\ 0.1 \le x_1 \le 0.4, 0.1 \le x_2 \le 0.4, 0.1 \le x_3 \le 0.4, 0.1 \le x_4 \le 0.5, 0.1 \le x_5 \le 0.5 \end{array} \tag{24}$$

By solving the above model, the investment risks are ranged from 4% to 9%, and then we obtain the portfolio results as shown in Table 4, where the investment proportion for security 1 decreases from 0.4 to 0.1 because the increase in the bound of the investment risk makes the risky security 5 increase from 0.3119 to 0.6 because the return rate of security 5 is higher than the selected guaranteed rate of return $\widetilde{p}_1$, which contributes to the investor making an excess investment. In the other, the investor focuses on the optimal investment when the investment risk exceeds or equals 7%, with $x_1 = 0.1$, $x_2 = 0.1$, $x_3 = 0.1$, $x_4 = 0.1$, and $x_5 = 0.6$, with the expected return rate of 24.0159%.

Table 4. The portfolio selection in condition 1 with a guaranteed return rate $\widetilde{p}_1$.

Risk Proportion	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**
x_1	0.3881	0.2483	0.1084	0.1	0.1	0.1
x_2	0.1	0.1	0.1	0.1	0.1	0.1
x_3	0.1	0.1	0.1	0.1	0.1	0.1
x_4	0.1	0.1	0.1	0.1	0.1	0.1
x_5	0.3119	0.4517	0.5916	0.6	0.6	0.6
Expected Return Rate	0.170219	0.204166	0.238113	0.240159	0.240159	0.240159

Next, we select the guaranteed return rates $\widetilde{p}_2$ and $\widetilde{p}_3$ for portfolio analysis, respectively, and the results are shown in Tables 5 and 6, respectively. Compared to Tables 1–3 and Tables 4–6, we can find that when the decision priority is majored on being risk averse, with the same risk, the corresponding return is lower than the decision priority majored on risk seeking. Therefore, there are different decision priorities in the portfolio selection, and the derived expected return is also different.

Table 5. The portfolio selection in condition 1 with a guaranteed return rate $\widetilde{p}_2$.

Risk Proportion	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**
x_1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_2	0.2464	0.1640	0.1	0.1	0.1	0.1	0.1
x_3	0.4	0.4	0.3812	0.2969	0.2125	0.1281	0.1
x_4	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_5	0.1536	0.2360	0.3188	0.4031	0.4875	0.5719	0.6
Expected Return Rate	0.136647	0.151984	0.166406	0.177631	0.188855	0.20008	0.203818

Table 6. The portfolio selection in condition 1 with a guaranteed return rate $\widetilde{p}_3$.

Risk Proportion	3%	4%	5%	6%	7%	8%	9%
x_1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_2	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_3	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_4	0.4856	0.4	0.3145	0.2289	0.1434	0.1	0.1
x_5	0.2144	0.3	0.3855	0.4711	0.5566	0.6	0.6
Expected Return Rate	0.140921	0.146483	0.152044	0.157605	0.163166	0.165988	0.165988

Condition 2: neutral > seeking > averse

In this scenario, the investor can evaluate the past performance of the experts in investment and judge the priority of the objectives. Since the expert that is risk averse is better than the risk-neutral and risk-seeking ones, thus we assume $\text{Ob}_1 > \text{Ob}_2 > \text{Ob}_3$, and thus, according to the SMART-ROC weighting methods, we can obtain the weights of the objectives and risks as $w_1 = 0.6111$, $w_2 = 0.2778$, and $w_3 = 0.1111$. In this step, we first select the guaranteed return rate $\widetilde{p}_1 = (0.1,\ 0.05,\ 0.05)$ to deal with the adjustable security proportion investment in the portfolio. Finally, we can formulate the fuzzy portfolio model as follows:

$$\begin{array}{c} Max\ 0.078341\ x_1 + 0.118834\ x_2 + 0.164733\ x_3 + 0.210387\ x_4 + 0.269949\ x_5 \\ s.t.\ 0.021009\ x_1 + 0.040276\ x_2 + 0.048882\ x_3 + 0.06302\ x_4 + 0.082076\ x_5 \le \sigma \\ x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\ 0.1 \le x_1 \le 0.4,\ 0.1 \le x_2 \le 0.4,\ 0.1 \le x_3 \le 0.4,\ 0.1 \le x_4 \le 0.5,\ 0.1 \le x_5 \le 0.5 \end{array} \tag{25}$$

By solving the above model, the investment risks are ranged from 4% to 9%, and then we obtain the portfolio results as shown in Table 4, where the investment proportion for security 1 decreases from 0.4 to 0.1 because the increase in bound of the investment risk makes the risky security 5 increase from 0.1 to 0.6 because the return rate of security 5 is higher than the selected guaranteed rate of return $\widetilde{p}_1$, and contributes to the investor making an excess investment. In the other, the investor focuses on optimal investment when the investment risk exceeds or equals 7%, with $x_1 = 0.1$, $x_2 = 0.1$, $x_3 = 0.1$, $x_4 = 0.1$, and $x_5 = 0.6$, with the expected return rate of 21.9199%.

Next, we select the guaranteed return rates $\widetilde{p}_2$ and $\widetilde{p}_3$ for portfolio analysis, respectively, and the results are shown in Tables 5 and 6, respectively. Compared to Tables 1–3 and Tables 7–9, we can find that when the decision priority is majored on being risk neutral with the same risk, the corresponding return is lower than the decision priority majored on risk seeking. Because different decision priorities in the portfolio selection derive different expected returns, the decision group that takes a specific decision priority in risk attitudes can derive a different portfolio selection and expected return rates.

Table 7. The portfolio selection in condition 2 with a guaranteed return rate $\widetilde{p}_1$.

Risk Proportion	4%	5%	6%	7%	8%	9%
x_1	0.4	0.2675	0.1	0.1	0.1	0.1
x_2	0.2865	0.1	0.1	0.1	0.1	0.1
x_3	0.1135	0.1	0.1	0.1	0.1	0.1
x_4	0.1	0.4325	0.4445	0.1	0.1	0.1
x_5	0.1	0.1	0.2555	0.6	0.6	0.6
Expected Return Rate	0.132111	0.167302	0.198681	0.219199	0.219199	0.219199

Table 8. The portfolio selection in condition 2 with a guaranteed return rate $\widetilde{p}_2$.

Risk Proportion	4%	5%	6%	7%	8%	9%
x_1	0.1	0.1	0.1	0.1	0.1	0.1
x_2	0.2857	0.1383	0.1	0.1	0.1	0.1
x_3	0.4	0.4	0.2793	0.1162	0.1	0.1
x_4	0.1	0.1	0.1	0.1	0.1	0.1
x_5	0.1143	0.2617	0.4207	0.5838	0.6	0.6
Expected Return Rate	0.142961	0.164122	0.182521	0.199952	0.201684	0.201684

Table 9. The portfolio selection in condition 2 with a guaranteed return rate $\widetilde{p}_3$.

Risk Proportion	4%	5%	6%	7%	8%	9%
x_1	0.1	0.1	0.1	0.1	0.1	0.1
x_2	0.1	0.1	0.1	0.1	0.1	0.1
x_3	0.3478	0.1	0.1	0.1	0.1	0.1
x_4	0.3522	0.4604	0.2703	0.1	0.1	0.1
x_5	0.1	0.2396	0.4297	0. 6	0.6	0.6
Expected Return Rate	0. 149103	0.164634	0.173769	0.181949	0.181949	0.181949

4.3. The Comparisons among Fuzzy Portfolio Models

Zhang [37] discussed fuzzy portfolio selection for bounded assets, and the upper possibilistic means and variances method can derive the higher expected return. Tsaur et al. [15] proposed a fuzzy portfolio model to discuss fuzzy portfolio selection under a selected guaranteed return rate. In this subsection, we compare the proposed portfolio model to those of Zhang [37] and Tsaur et al. [15], whose results are shown in Table 10. In our proposed model, we illustrate that the decision group depends on the expert being risk seeking rather than being risk neutral and risk averse. The comparison results show that the stationary portfolio selection is $x_1 = x_2 = x_3 = x_4 = 0.1$, $x_5 = 0.6$ among the three models. First, the differences to the expected return rates in different investment risks are higher than Zhang [37]. Second, we also find that our proposed model can obtain the stationary portfolio selection compared to the model of Tsaur et al. [15] under a lower investment risk. Therefore, the risk attitudes for the group decision when making the portfolio selection can select a proper portfolio with lower investment risk and obtain higher expected return rate.

Table 10. The comparison among fuzzy portfolio models.

	σ (%)	*M* (%)	x_1	x_2	x_3	x_4	x_5	$\sum_{j=1}^{5} x_j$
Zhang [37]	4%	22.55%	0.1	0.1	0.1611	0.1	0.5389	1
	5%	23.16%	0. 1	0.1	0.1	0.1	0.6	1
	6%	23.16%	0. 1	0.1	0.1	0.1	0.6	1
	7%	23.16%	0. 1	0.1	0.1	0.1	0.6	1
	8%	23.16%	0. 1	0.1	0.1	0.1	0.6	1
Tsaur et al. model [15] with a guaranteed return rate $\widetilde{p}_1$	5%	10.50%	0.2127	0.1	0.1	0.1	0.1	0.6127
	8%	17.46%	0.4	0.1	0.1	0.1	0.2659	0.9659
	10%	21.59%	0.2811	0.1	0.1	0.1	0.4189	1
	12%	25.63%	0.121	0.1	0.1	0.1	0.5790	1
	15%	26.17%	0.1	0.1	0.1	0.1	0.6	1
Proposed model with a guaranteed return rate $\widetilde{p}_1$	4%	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	
	5%	18.54%	0.3961	0.1	0.1	0.1	0.3039	1
	6%	21.87%	0.2558	0.1	0.1	0.1	0.4442	1
	7%	25.19%	0.1155	0.1	0.1	0.1	0.5845	1
	8%	25.56%	0.1	0.1	0.1	0.1	0.6	1
	9%	25.56%	0.1	0.1	0.1	0.1	0.6	1

4.4. Discussion

Based on the results in this illustration, we find that our proposed fuzzy portfolio model can be applied to solve the portfolio by an investor who received the decision information from a group of experts under uncertain factors. Next, by the sensitivity analysis, we can find different risk preference priorities in the experts of a group, and the different portfolios are solved. As we discussed in the section of the literature review, energy is also very important for most oil-poor countries. In the sustainable energy field [38,39], there are three objectives related to sustainability for the analysis: economic, environmental and social impacts are formulated. Economic impacts are of great significance in sustainable performance evaluation; environmental impacts have gained strong interests in recent years for sustainable development; and social attitudes are influential in the energy selection decision-making process. The brief explanations of these objectives are as follows:

(1) For economic objective, we focus on the issue of maximizing total revenue from different sources of energy. In addition, we assume some sources of energy whose production costs are lower and supposed to have excess production for the supply of more electricity.
(2) For environmental objective on the sustainability of the climate change, we focus on the issue of greenhouse gas of CH_4 and CO_2 emissions reduction. Therefore, the emission performance of each kind of energy source depends not only on the performance of energy technology, but on the emissions of the electricity system. Therefore, this objective is set to minimize the emission risk of the greenhouse gas for the selected energy portfolio.
(3) For the social objective, we focus on the public opinions for the preference and degree of acceptance of the types of power plants. Therefore, this objective is set to minimize the turnover rate of the public opinion for the policy of each energy resource.

As Barykin et al. [40] stated, the goals of real business in the digital transformation have necessitated the application of a scientifical and theoretical approach to formalize the description of what is acceptable for predictive planning based on leading indicators. Additionally, Khalid and Naumova [41] stated that to face the changing business environment and multiple changing objectives, we should consider altering the supply chain, including the reduction in product design and a manufacturing period, and faster delivery of products to customers. Therefore, a scientific, theoretical approach and multiple objectives are important issues in the digital transformation process, and thus we suggest the proposed theory of the multi-objective fuzzy portfolio model to be applied to the sustainable fuzzy portfolio model with the objective functions of the Triple Bottom Line. Then, the oil-poor governments can use our proposed model to optimize the energy portfolio by the objectives of the Triple Bottom Line.

5. Conclusions

The scientific novelty of the research focuses on the risk attitudes of a decision group and guaranteed return rates to propose a fuzzy portfolio model for considering the effect of risk in portfolio selections. Most importantly, if the fuzzy portfolio model for the risk priority in the decision group meets expectations, this approach will be useful to obtain higher expected return rates under smaller risk for the selected securities. The conclusion section can be extended as follows:

(1) The numerical results obtained in this paper: In our proposed model, an investor uses the past investment performance of the experts to decide the weights of their objective functions of the investment return rates and investment risk, and then transforms the multi-objective programming model into a weighted linear programming model. The numerical results obtained in this paper show that the selected lower guarantee return rate derived a higher fuzzy return rate than the higher guarantee return rates. By contrast, risk attitudes affect the expected return rates under the constrained investment risk; when the decision priority in the group decision is majored on risk seeking, the expected return rate under the same risk is higher than the decision priority majored on being risk neutral or averse.
(2) Limitation of this research: Because different decision priorities in the portfolio selection derive different expected returns, the group decision taking a specific decision

priority in the risk attitudes is important work for the investor. Therefore, the group of experts selected by the investor is very important. The limitation of this study should be considered for excellent experts. Finally, we suggest that the investor considers the past investment performance for each member in the decision group and takes the risk priority and the heuristic experience and selects the order of risk attitudes to make the portfolio selection.

(3) The future research should focus on (1) considering the number of group experts for the selected guarantee return rates and formulating the objective functions; (2) establishing comprehensive threshold values and decision criteria according to the guaranteed return rates [42]; (3) conducting a sensitivity analysis to test the weights of each objective function.

Author Contributions: Formal analysis, Y.-Y.H.; Methodology, R.-C.T.; Software, N.-C.H.; Supervision, R.-C.T.; Validation, Y.-Y.H.; Writing—original draft, N.-C.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the financial support of National Science and Technology with project No. MOST 110-2221-E-032-033-.

Data Availability Statement: Not applicable.

Acknowledgments: The authors would like to thank referees for their helpful comments.

Conflicts of Interest: The authors declare no conflict of interest.

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