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Editorial

Geometrical Theory of Analytic Functions

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1. Introduction

This Special Issue, devoted to the topic of the "Geometric Theory of Analytic Functions", aims to bring together the newest research achievements of scholars studying the complex-valued functions of one variable. The Special Issue was intended to cover all aspects of this topic, starting with special classes of univalent functions and encompassing operator-related results, studies using the theories of differential subordination and superordination, and other techniques which can be applied in the field of complex analysis and its applications.

The editor of this Special Issue was pleased to invite the authors to submit their original results related to analytic functions and even the results of studies related to non-analytic functions, in cases where research was conducted that compared these two types of complex-valued functions. The latest results submitted for publication by the researchers relate to classic differential subordination and superordination, strong differential subordination and superordination, and fuzzy differential subordination and superordination. The editor believes that these researchers are eager to examine how differential, integral, and differential–integral operators are used for the special classes of univalent functions that are already known or have been newly introduced and to define their importance to the field.

This Special Issue contains contributions related exclusively to complex analysis and, hopefully, from the results published, new approaches to geometric function theory can be found, which may inspire further achievements in the field.

2. Overview of the Published Papers

The present Special Issue gathers 15 papers, which were accepted for publication after a thorough reviewing process.

The research conducted by Rabha W. Ibrahim, Rafida M. Elobaid, and Suzan J. Obaiys in [1] provides a study of a symmetric conformable derivative operator in connection to the geometric function theory. The authors' contribution involved imposing two classes of symmetric differential operators in the open unit disk and describing the further development of these operators by introducing convex linear symmetric operators. In addition, by applying these symmetric conformable derivative operators to the class of univalent functions, a set of sub-classes of analytic functions with geometric properties, such as starlikeness and convexity, was defined. Moreover, by using the symmetric conformable derivative operator, a generalized class of Briot–Bouquet differential equations was defined in order to introduce what is known as the symmetric conformable Briot–Bouquet differential class of equations. The upper bound of this class was proven to be symmetric in the open unit disk.

Sheza M. El-Deeb, Teodor Bulboacă, and Bassant M. El-Matary published their research based on quantum calculus and bi-univalent functions in [2]. A new subclass of the bi-univalent functions, defined in the open unit disk and connected with a q-analogue derivative, was introduced and studied in regard to coefficient estimates of the first two Taylor–Maclaurin coefficients for the functions in this subclass. An estimation of the Fekete–Szegő problem of this function class was also obtained. Estimations of the coefficients $|a_2|$, $|a_3|$, and the Fekete–Szegő problem for the function class were not sharply defined. The

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identification of the sharp upper bounds of the abovementioned Fekete–Szegő problem is still an interesting and open problem, as is that of $|a_n|,\ n \geq 4$.

Paper [3] investigates centered polygonal lacunary functions restricted on the unit disk and in the symmetry angle space, which is defined by the symmetry the angles of a given centered polygonal lacunary function. The periodicity of the p-sequences and the existence of a convergent subsequence provided a framework for the decomposition of the centered polygonal lacunary functions. This decomposition could potentially be useful in renormalization procedures as one approaches the natural boundary. The authors, Leah K. Mork, Keith Sullivan, and Darin J. Ulness, hope that this work will provide useful insight into the nature of the natural boundary of centered polygonal lacunary functions, both on the full unit disk and also when restricted to the symmetry angle space. Statistical mechanics provide the most promising link between this work and physics.

In paper [4], Mugur Acu and Gheorghe Oros introduce a new differential–integral operator using the Sălăgean differential operator and Alexander integral operator. Using this operator, a new integral operator was defined and its starlikeness was proven by means of differential subordination theory. The starlikeness of the certain order $0 \leq \alpha < 1$ could also be further studied. As in the case of most operators, special classes of univalent functions could be introduced using this operator, which is subject to further studies.

The topic of the introduction of subclasses of bi-univalent studies is considered again in relation to the results presented in [5]. The authors, Ibtisam Aldawish, Tariq Al-Hawary, and B. A. Frasin, consider bi-univalent functions defined in the open unit and the Frasin differential operator in order to define and investigate two new subclasses of bi-univalent functions associated with a new differential operator of analytic functions, involving a binomial series. Furthermore, estimations of the coefficients $|a_2|$, $|a_3|$ for the functions in these new subclasses are obtained, but they are not sharply defined, and the identification of the sharp upper bounds for the abovementioned estimations remains an interesting and open problem, as does that for $|a_n|,\ n \geq 4$.

Certain applications of third-order differential subordination for a newly defined linear operator that includes ξ-generalized Hurwitz–Lerch Zeta functions are considered by the authors Hiba Al-Janaby, Firas Ghanim, and Maslina Darus in [6]. A new generalized Noor-type linear integral operator for the class of p-valent functions is defined, and several outcomes concerning the use of third-order differential subordination for multivalent functions, including this operator, are presented.

The aim of paper [7] is to identify several new criteria for the univalence, strong starlikeness, and strong close-to-convexity of functions for the normalized analytic function in the open unit disk U and the meromorphic strong starlikeness in the punctured open unit disk, using a well-known lemma provided by Nunokawa. Authors Ali Ebadian, Nak Eun Cho, Ebrahim Analouei Adegani, and Sibel Yalçın also compare the results presented in this paper to previous outcomes in this area.

In paper [8], the authors Ágnes Orsolya Páll-Szabó and Georgia Irina Oros reconsider the popular topic of the introduction of subclasses of bi-univalent functions, for which the coefficient estimates and Fekete–Szegő problem are discussed. Using the recently introduced Sălăgean integro-differential operator, three new classes of bi-univalent functions are introduced in this paper. Estimates of the first two Taylor–Maclaurin coefficients are usually provided, but in this paper, the bounds of the first three coefficients, $|a_2|$, $|a_3|$, and $|a_4|$, of the functions in the newly defined class are provided. The results of this paper could also inspire further research on integro-differential operators used to introduce new classes of bi-univalent functions.

The main contribution of article [9] is its definition of a family of starlike functions associated with a cosine hyperbolic function. The authors, Abdullah Alotaibi, Muhammad Arif, Mohammed A. Alghamdi, and Shehzad Hussain, obtained the convolution conditions, integral preserving properties, and coefficient sufficiency criteria of this family. Differential subordination problems related to the Janowski and cosine hyperbolic functions were also studied. Furthermore, these results were used to introduce a new class of univalent functions, for which sufficient conditions for the starlike functions, connected with the cosine hyperbolic function, were obtained. Other problems such as the coefficient bounds, Hankel determinant, partial sum inequalities of this class, and many more, may be addressed in future research.

Article [10] presents the quantum calculus aspects integral to geometric function theory. Rizwan Salim Badar and Khalida Inayat Noor introduce a q-generalized linear operator and investigate its application in the introduction of two new classes of analytic bounded functions of a complex order. Certain properties are defined for these classes, and certain results involving differential subordination theory are also obtained. The integral inclusion of the classes related to the q-Bernardi operator is also proven.

The work presented in [11] investigates a special class of generalized Painlevé differential equations in a complex domain. Rabha W. Ibrahim, Rafida M. Elobaid, and Suzan J. Obaiys study the asymptotic expansion solution, univalent solution, and approximate solution of this class from the perspective of the geometric function theory. Painlevé differential equations are formulated as a boundary value problem in terms of the connection estimates. The consequence is a univalent solution with a geometric illustration. The novelty of this work is its analytical study of a class of Painlevé differential equations. The outcomes of this study are based on geometric function theory, used to describe the geometric behavior of these solutions. The upper bound of these solutions is indicated using Janowski formula. Finally, the symmetric solution is constructed using a convex function in the open unit disk.

In the study presented in paper [12], S. Melike Aydoğan and Zeliha Karahüseyin constructed a new subclass of bi-univalent functions with respect to the symmetric conjugate points in the open disk described by the Horadam polynomials. For this subclass, initial Maclaurin coefficient bounds were acquired, and the Fekete–Szegö problem of this subclass was also considered.

Paper [13] examines the differential superordination of harmonic complex-valued functions, a concept introduced recently as a dual method of differential subordinations of harmonic complex-valued functions. The theory of differential subordinations was extended from the analytic functions to include the harmonic complex-valued functions in 2015. In this article, Georgia Irina Oros obtains the conditions for a harmonic complex-valued function p as the best subordinant of a differential superordination for harmonic complex-valued functions. Finding the best subordinant of a differential superordination is among the main goals of this research field. Examples are also provided to show how the theoretical findings can be used and to prove the connection with the results obtained in 2015.

In the analysis conducted by Firas Ghanim, Khalifa Al-Shaqsi, Maslina Darus, and Hiba Fawzi Al-Janaby in [14], a new convolution complex operator defined for meromorphic functions, related to the Hurwitz–Lerch-Zeta-type functions and Kummer functions, is considered. The Hurwitz–Lerch Zeta type functions, as part of special function theory, are significant for the development and provision of further new studies. Certain sufficient stipulations are provided for several formulas of this defining operator so as to attain subordination. These outcomes are an extension of the known outcomes of starlikeness, convexity, and close-to-convexity.

Article [15] describes and defines the key terms and elements required for identifying the subordinants of a system of simultaneous second-order differential inclusions. José A. Antonino and Sanford S. Miller identified the conditions for finding subordinants for some special cases of such systems. An interesting and open problem is addressed in the conclusion of this article, regarding the general case of a system of simultaneous second-order differential inclusions.

3. Conclusions

The 15 papers published as part of this Special Issue, “Geometrical Theory of Analytic Functions”, are available as a printed book with the same title. The subjects under discussion in the papers published as part of this project are quite varied. Hence, researchers of geometric function theory and related topics should be able find something of interest in this Special Issue. The follow-up of this Special Issue is entitled "New Trends in Complex Analysis Researches". Scholars interested in the field are welcome to visit the webpage for the Special Issue before 25 November 2022 in order to find out more about the proposed topics and possibly contribute to the success of this new project by submitting their own research outcomes.

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