# A Line Planning Optimization Model for High-Speed Railway Network Merging Newly-Built Railway Lines 

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#### Abstract

This paper is devoted to developing a line-planning approach for high-speed railway networks merging newly built railway lines, which result in the change of the network's original structure and some passengers' travel routes. In order to exactly describe the choice of time-varying passengers and the operation of the trains, a passenger travel network with time information is constructed based on the pre-generated candidate train set. Following this, a line-planning optimization model for optimizing trains on both the existing railway network and the merged new railway line is established under the considered constraints, such as transportation resources on the network. It does not aim to only provide higher service level for passengers and increase revenue of railway enterprise, but also to ensure the continuity of the existing trains to facilitates passengers and train organization. A framework of the Simulated Annealing Algorithm is designed to solve the proposed model by combining the neighboring solution search strategies with evaluation method based on the allocation of passengers. The case of a partial high-speed railway network in China is studied to test the practicability and validity of the proposed approach.


Keywords: line planning; high-speed railway network; newly built lines; passenger travel network; simulated annealing algorithm

MSC: 90-10; 90B20; 90C11

## 1. Introduction

Line planning is a crucial phase of high-speed railway (HSR) planning. Under the premise of the given HSR network with limited transportation resources and spatiotemporal distribution of origin-destination demand of passengers (OD demand), line planning is for determining the routes, frequencies, speed classes, capacities, and stop patterns of trains on the railway network during a day or the considered period. The line planning provides the basis for subsequent phases in railway planning, such as timetabling, platforming, and crew timetabling. A high-quality line plan can not only provide high-level transport services for passengers, but also increases the operational revenues of railway enterprises [1].

In recent years, with the increasing of total mileage of operational HSR in China, many HSR lines newly built have been integrated into the existing HSR network and been put into operation. In the future, more new HSR lines will be completed and put into operation as shown in Table 1 and more information can be found in the official website of NDRC of China (https:/ / www.ndrc.gov.cn, accessed on 20 August 2022). New HSR lines not only attract a large number of additional passengers to choose HSR trains, but also transform the original structure of the HSR network, causing some passengers to change their travel routes. Therefore, when HSR network merges a new railway line, it is necessary not only to organize the in-line and cross-line trains for the new railway line reasonably, but also to adjust the trains on the existing HSR network properly, so as to adapt to the change
of the travel route of passengers. This problem can be described as the optimization and adjustment of the line plan for an HSR network merging newly built lines, for which an effective solution has pivotal practical significance for optimizing the operational trains on the HSR network and improving services for passengers. There is some research focusing on this issue. Shen [2] constructed a two-stage optimization model of line planning for a new HSR line based on a column generation algorithm. In the first stage, with the aim of minimizing the total cost of a train, candidate train sets were generated for the new railway line and solved by a column generation algorithm. The second stage was a passenger allocation model aiming at maximizing railway operation benefit. However, Shen's paper made the assumption of line closure, ignoring the trains and passengers on the existing HSR network as well as cross-line trains and cross-line OD demand through the new railway line. Li et al. [3] made a detailed analysis of the process of line planning adjustment of the HSR network merging a new railway line, and constructed an assessment indicator system for line planning adjustment. However, the adjustment method in their study is subjective to some extent, and the range of application is limited. The solution of line planning for HSR networks merging new railway lines should not only make the line plan of new railway line match the OD demand, but also make that of the existing railway network fit the change of OD demand, so as to ensure better efficiency of operational trains and higher service level for passengers. Moreover, the adjustment of the existing trains should be moderate, so as to ensure the continuity of the HRS line plan which can facilitate the OD demand and train organization. Nevertheless, the existing research methods are difficult to directly and efficiently apply to this problem on large-scale HSR networks.

Table 1. The development of HSR in China in recent years.

| Year | Total Mileage of Operational <br> HSR Lines (km) | The Number of HSR <br> Lines, Newly Built | The Mileage of HSR Lines <br> Under Construction (km) |
| :---: | :---: | :---: | :---: |
| 2016 | 22,000 | 12 | 1903 |
| 2017 | 25,000 | 10 | 2182 |
| 2018 | 29,000 | 13 | 4100 |
| 2019 | 35,000 | 18 | 5474 |
| 2020 | 38,000 | 18 | 2521 |
| 2021 | 40,000 | 12 | 2168 |

Note: The data above partly come from NBS of China.
Therefore, this paper is devoted to better solving the problem of line-planning optimization for HSR networks merging new railway lines. To meet the demand of time-varying passengers better, an imprecise timetable is introduced to the traditional line-planning problem. A candidate train set composed of trains for adjustment is generated according to the existing trains, and another candidate train set composed of trains for operation is constructed according to the OD demand related to the new line. Following this, passenger travel network with time information is constructed. Then, a model which adapts to solve this problem is established. After that, the solution framework based on the SAA (Simulated Annealing Algorithm) is designed.

The reminder of this paper is organized as follows. Section 2 presents a review of the related line-planning literature, then states the contributions of this paper. Section 3 presents the problem description related to the candidate train set, and analyzes the cost of operational trains. Section 4 constructs a passenger travel network and analyzes the general travel cost of passengers. Section 5 constructs an optimization model. Section 6 designs a solving algorithm under the framework of SAA. Section 7 is devoted to the description and evaluation of the case study. Section 8 presents the major conclusions and policy recommendations.

## 2. Literature Review

Plentiful research has been carried out on the optimization of line plans for passenger trains. From the perspective of railway enterprises, there are some studies aiming to
optimize line plans to minimize the operational costs of railways. For instance, Claessens et al. [4] analyzed the costs of trains operated by railway enterprises, then constructed a nonlinear programming model aimed at minimizing the total cost of railway enterprises by considering the constraints of certain service levels and line capacity. This model did not only determine the routes, frequencies, and types of trains, but also assigned the number of vehicles for each train. Finally, a branch-and-bound algorithm was used to solve the model. Based on [4], two different linearizations and a cut-and-branch algorithm to solve the problem were developed in Bussieck [5]. Another branch-and-cut approach based on the models of [4,5] was proposed in Goossens et al. [6]. A fast procedure for solving the model based on [4] was proposed in Bussieck et al. [7], in which lower bounds for three different linearizations were derived and linear and non-linear programming techniques were combined to solve the model. All the above studies were based on pre-determined stop patterns, pre-given type of trains, and railway networks that were split into several sub-networks based on the process called "system split", which fixed the traveling routes of OD demand before the lines are determined, so that the volume of demand on each track section could be acquired. Later, Goossens [8] discussed an extension named "multi-line planning problem", in which not all operated trains should stop at all stations visited. This problem was modeled as a multi-commodity flow problem in which each type of train was regarded as a flow.

For the purpose of improving the level of railway services, there are also some studies that have been implemented to optimize the line plan from the perspective of passengers' interests. Bussieck et al. [9] derived a mixed integer linear programming model for the optimization of line plans to determine a set of lines of which frequencies satisfy the line-frequency requirement for every track section under the constraint of the railway budget, aiming to maximize the number of direct passengers. Schöbel et al. [10] and Scholl [11] constructed a change-and-go graph to describe the travel behaviors of passengers choosing the shortest travel route. Under the constraint of the railway budget, an integer optimization model of line planning was established with the goal of minimizing the total travel time and transfer times of passengers. Heuristic algorithm, Dantzig-Wolfe decomposition, and branch-and-bound method were used to solve the problem, respectively. Borndörfer et al. [12] and Borndörfer et al. [13] presented a model in which travelers' routes could be freely generated with the purpose of minimizing the riding time of all travelers. Two multi-commodity flow formulae were proposed. The first focused on the situation in which all paths served as lines as an integer programming model in which both the passengers' paths and the routes of the lines were not fixed in advance but determined within the process of optimization. The second set a variable for each potential line and for each potential traveler path, solved by a branch-and-price method. The lines dynamically generated in the second formulation were solved by a column generation approach. Given upper frequency requirements, Puhl et al. [14] introduced a line pool which maximized the number of transported travelers. Klier et al. [15] proposed a model that used frequency-dependent transfer times as input data to maximize the total number of expected passengers.

Additionally, there are some studies considering the interests of railway enterprises and passengers comprehensively. A multi-objective model was presented by Chang et al. [16], in which train stop pattern, frequency, and fleet size are determined, aiming to obtain the best-compromise train service plan for an HSR line. This model is solved by a fuzzy mathematical programming approach. A line-planning model for minimizing the total cost of railway operation and duration of passenger travel time based on two given halting patterns was proposed in Park et al. [17]. Ghoseiri et al. [18] developed a multi-objective optimization model for line planning on a network which included different tracks as well as multiple platforms, aimed at lowering the fuel consumption cost and shortening the total passenger travel time. The solution first determined the Pareto frontier, then improved it gradually by using the distance-based method. Zhou et al. [19] developed an optimization model for line plans to minimize both the expected waiting times for
high-speed trains and the total travel times of high-speed and medium-speed trains. A branch-and-bound algorithm with effective dominance rules was designed to generate Pareto solutions for the model, and a beam search algorithm with utility evaluation rules was used to construct a representative set of non-dominated solutions.

Moreover, there are some studies that apply game theory to line-planning optimization. For example, Schöbel et al. [20] presented a model based on game theory in which lines acted as players and competed to minimize their own delays depending on the volume of demand on each track section and frequencies of line. Using this model, the equilibrium solution of the problem was calculated and analyzed. Kontogiannis et al. [21,22] assumed a potential crowd of line operators with fixed lines that tried to maximize their personal profits, then introduced a network administrator who was in charge of the infrastructure. The target of the network administrator was to achieve a social optimum by maximizing the total profits of the operators. The process by which passengers decide whether to travel or not and choose a transfer plan according to general travel costs based on line plans formulated by railway enterprises that aim to maximize operational revenues under resource constraints was described as a Stackelberg game in Shi et al. [23]. Based on this, a two-layer programming model for line planning was constructed, which combined line planning, transfer network design, and passenger travel route selection. The lower-layer model was based on the line plan generated by the upper layer.

In addition to the research mentioned above, more double-layer programming models to optimize line plans have been constructed. Fu et al. [24] divided stations and trains on an HSR network into two grades, then constructed a bi-level programming model in which the upper-level model minimized the sum of passenger travel time, while the lower-level model maximized the served OD demands. A heuristic algorithm was applied to solve the model: first, higher-grade stop trains were generated based on a possible candidate set; second, lower-grade stop trains were added by adding intermediate stop stations and increasing the frequencies of the existing trains. Szeto et al. [25] proposed a two-stage transit network design problem to optimize the route and frequency of trains: in the first stage, a mixed integer non-linear program to minimize the number of passenger transfers was formulated, and the second stage was the transit assignment process with capacity constraints. A hybrid artificial bee colony algorithm was designed to solve the model. Wang et al. [26] developed a two-layer line planning optimization model, in which the top layer sought an optimal stop pattern with frequencies, and the bottom layer was devoted to passenger flow assignment according to the plan from the top layer.

In terms of OD demand on line planning, early studies are mainly concentrated on the given daily OD demand. However, the OD demand served by HSR has strong elasticity, randomness, and time variability. Therefore, further studies have been conducted on line-planning optimization under different types of OD demand in recent years. They can be divided into elastic OD demand [23,27], dynamic OD demand [28,29], time-varying OD demand [30,31], and uncertain OD demand [32,33]. For instance, Shi et al. [23] analyzed the functional relationship between the OD demand and its influencing factors and concluded that passenger transfer choice was a balanced allocation of users under elastic demand. On this basis, combined with the related costs of passenger trains, a double-layer programming model of line planning under elastic demand was established, and a simulated annealing algorithm (SAA) was designed to solve the problem. Wang [29] proposed an evaluation and adjustment strategy of line planning based on dynamic OD demand, and established a double-layer programming model for the optimization and adjustment of line plans, which was solved based on an SAA framework. The upper layer of the model considered the satisfaction rate of OD demand, the average rate of train occupancy, the matching rate of OD demand, and the direct rate of passengers without transfer. The lower layer model considered the cost and deviation of line planning as the minimum. Based on the given time-varying OD demand, a bi-level model for line planning optimization was formulated, which included the constraints of track section and train capacity and the gap between the number of departing trains and that of arriving trains at a station in [30]. The upper
level aimed to minimize the cost of railway operation and passenger travel. In the lower level, a timetable-based passenger assignment approach, which determined the travel choice of trains for time-varying demands by simulating the ticket-booking process, was used to evaluate the line plan obtained before. Zuo [32] first established a certainty goal programming model to optimize the line plan. Then, under a certain confidence level, an opportunity-constrained objective programming model for line-planning optimization was established by taking the interstation passenger flow in the network as a random parameter. Subsequently, a genetic algorithm based on stochastic simulation was designed to solve the problem.

The comparisons to the existing literature is shown in Table 2, and the main contributions in this paper are stated as follows.

1. The overall impact on passengers of an HSR network merging newly built railway lines is considered, and the coordinated optimization on the line plan of the new railway line and the synergetic adjustment of the line plan of the existing HSR network is realized, which can not only meet the OD demand with high quality, but also maintain the continuity of line plans of the existing HSR to facilitate passengers and railway operational organization.
2. A passenger travel network is designed on the basis of the candidate operational train set of new railway lines and the candidate adjustment train set of the existing HSR network. Next, a cooperative solution model of train optimization on new railway lines and train adjustment on the existing HSR network is established. By introducing the two candidate train sets, the scale of solving the model has been effectively reduced. The relationship between the decision for the candidate train set and passenger travel choice is fully considered in the model.
3. Based on the SAA framework, a collaborative solution algorithm is designed for line-planning optimization for the new HSR line and the line-planning adjustment for the existing HSR network. On the basis of the initial line plan generated for the new HSR network, the current line plan is evaluated by the passenger allocation, which is based on the simulation of the process of passengers purchasing tickets. The current line plan is improved continuously until a satisfactory line plan is obtained.

Table 2. Comparisons of research related to line planning.

| Objective Orientation | Model | Algorithm | Input Demand | Key References |
| :---: | :---: | :---: | :---: | :---: |
| Railway | Mixed integer nonlinear model | Methods of operations research (i.e., branch-and-cut, branch-and-bound) | Each track section demand | [4-7] |
| Railway | Multi-commodity flow model | Commercial solver (CPLEX) | Daily demand | [8] |
| Passenger | Mixed integer linear model | Methods of operations research, heuristic algorithm | Daily demand | [9-11] |
| Railway system | Game theory model | Game theory model method | N.A. | [20-22] |
| Railway and Passenger | Multi-objective model | Fuzzy mathematical approach, heuristic algorithm | Daily demand | [16,17] |
| Railway | Double-layer programming models | Heuristic algorithm (hybrid artificial bee colony) | Daily demand | [25] |
| Railway and passenger | Double-layer programming models | Heuristic algorithm | Daily demand | [24] |
| Railway and passenger | Bi-level programming model | Heuristic algorithm (SAA) | Elastic demand | [23] |
| Railway and passenger | Bi-level programming model | Heuristic algorithm (SAA, GA) | Dynamic demand | [29] |
| Railway and passenger | Chance-constrained goal programming model | Heuristic algorithm based on stochastic simulation | Uncertain demand | [32] |
| Railway and passenger | Bi-level programming model | Heuristic algorithm (SAA, GA) | Time-varying demand | [30,31] |

## 3. Problem Description Based on the Candidate Train Sets

The new HSR network is represented by $W^{\prime}$, which consists of a new HSR line and the existing HSR network. The new HSR line can be denoted as $L=\left(S_{L}, E_{L}\right)$, where $S_{L}=\{i\}$ is the set of stations and $E_{W}=\left\{(i, j) \mid i, j \in S_{W}\right\}$ is the set of track sections, and a track section is the joint between two contiguous stations on a railway track. Similarly, the existing HSR network can be denoted as $W=\left(S_{W}, E_{W}\right)$.

On the new HSR network $W^{\prime}$, there are differences in the OD demand in different periods throughout a day, so denote $q_{r s}^{h}$ as the OD demand, of which the origin is station $r \in S_{W^{\prime}}$ and the destination is station $s \in S_{W^{\prime}}$ on HSR network $W$, and their planned departure time denoted as $t_{r s}^{h}$ is in period $h \in H$, where $h$ is a period of time and $H$ is the time range of railway operation during a day.

The trains operated on the existing HSR network $W$ can be denoted as $\bar{\Psi}_{W}$. The initial station, terminal station, and visiting station set (which includes initial station and terminal station) of any train $\varphi \in \bar{\Psi}_{W}$ can be denoted as $r_{\varphi}, s_{\varphi}$, and $S_{\varphi}$, respectively. The departure time, arrival time, and whether to stop a train $\varphi \in \bar{\Psi}_{W}$ at any station $i \in S_{\varphi}$ can be denoted as $a_{\varphi}^{i}, d_{\varphi}^{i}$, and $s_{\varphi}^{i}$, respectively, where $s_{\varphi}^{i}=1$ refers to the train stops at the station, otherwise $s_{\varphi}^{i}=0$.

The general idea of the two-part candidate train set construction and its function in the proposed approach are shown in Figure 1, which will be explained in more detail later.


Figure 1. Illustration of the construction and function of candidate train set.

### 3.1. The Candidate Train Set

There are a large number of trains running on the HSR network each day, and the combinatorial optimization of their routes and stop patterns is extremely arduous. The corresponding solution space will be enormous and the efficiency of solutions will be poor if the line plan of new HSR lines and the existing HSR network is determined or adjusted directly based on the spatial-temporal distribution of OD demand. In order to improve the efficiency of the solution and ensure the rationality of the line plan obtained, this paper firstly generates a candidate train set (denoted as $\Psi_{\text {adjust }}$ ) composed of trains which may be adjusted on the existing network and another candidate train set (denoted as $\left.\Psi_{\text {new }}\right)$ composed of trains which may be operated on the new railway line. Following this, according to the spatial-temporal distribution of demand, trains are selected from the set $\Psi_{\text {adjust }}$ for adjustment and from the set $\Psi_{\text {new }}$ for operation, and their stop patterns, departure time, and arrival time at each visiting station are optimized simultaneously.

For any candidate train $\varphi \in \Psi_{\text {adjust }}$ its running information including the route, stop pattern, and timetable are known in detail. However, for any candidate train $\varphi \in \Psi_{\text {new }}$, only its route and departure time range are determined when it is generated. Therefore, its stop pattern and exact departure time are unknown and must be optimized. The departure station, arrival station, visiting station set, and departure time range of a train $\varphi \in \Psi_{\text {new }}$ are denoted as $r_{\varphi}, s_{\varphi}, S_{\varphi}$, and $\left[\bar{t}_{\varphi}, \overline{\bar{t}}_{\varphi}\right]$ respectively.

As part of the OD demand on the existing HSR network will switch to the new line, the adjustment strategies of the existing trains include canceling trains, adjusting train stop patterns and departure times, etc., but no new train will be put into operation commonly. Therefore, the trains running on the existing HSR network can be directly taken as the candidate train set for adjustment. Nevertheless, there are some certain trains that cannot be adjusted, such as trains with high profit and hourly trains. Hence, the decision variables of these trains should be constrained in the model.

As for the new railway line, the OD demands that are likely to travel through the new line are obtained first, and they are regarded as the associated OD demand set. All candidate trains that may run on the new line are generated in advance following the principle that provides these associated OD demands with short waiting time, travel time, and service of seamless transfer. In addition, the route and the range of departure time of each candidate train also are determined to form the candidate train set. The detailed generation method is described in Section 6.2.

### 3.2. Problem Analysis

The line planning of HSR usually determines the routes, frequencies, and stop pattern of trains. In order to describe the travel choice of time-varying OD demand better, this paper takes the decision of the departure time of each train as one of the contents of line planning, which is conducive to reflecting passenger departure time preference, to more accurate simulation of passenger travel choice, and to further measuring the service and revenues of line planning effectively. It means that this paper optimizes the arrival and departure time at each visiting station of trains based on the departure time of their initial station. However, the obtained arrival and departure time at each visiting station of train do not necessarily meet the actual operational conditions, which will require further adjusting after considering a variety of safe intervals to generate a feasible train operational diagram that meets the operational requirements.

The original structure is transformed when the existing HSR network merges new railway lines, which results in a change in travel route for some OD demands. Therefore, the trains running on the existing HSR network $W$ need to be adjusted appropriately to match the changes in OD demand. Meanwhile, not only the in-line trains but also the cross-line trains should be rationally organized on the new HSR line L. Under the unified restriction of OD demand and transportation resources of the new HSR network $W^{\prime}$, these three decision-contents are closely related and interact with each other; thus, it is absolutely imperative that the three are synergistically optimized as an inseparable whole.

For any train $\varphi$ in $\Psi_{\text {adjust }}$, the adjustment strategy includes three parts as mentioned above: canceling the train, adjusting its stop, or adjusting its departure time. Therefore, its decision variables are respectively set as follows:

- 0-1 decision variable $\boldsymbol{x}_{\boldsymbol{\varphi}}$ : indicates whether the candidate train $\varphi \in \Psi_{\text {adjust }}$ is canceled; if canceled, $x_{\varphi}=1$; otherwise, $x_{\varphi}=0$.
- 0-1 decision variable $\delta_{\varphi}^{i}$ : indicates whether the stop attribute of the candidate train $\varphi \in \Psi_{\text {adjust }}$ is adjusted when it visits the station $i \in S_{\varphi}$; if it is adjusted from stop to no stop, $\delta_{\varphi}^{i}=-1$. If no stop is adjusted to stop, then $\delta_{\varphi}^{i}=1$. If not adjusted, $\delta_{\varphi}^{i}=0$. The adjusted stop attribute can be denoted as $s_{\varphi}^{i} \leftarrow s_{\varphi}^{i}+\delta_{\varphi}^{i}$.
- Integer decision variable $\Delta t_{\varphi}$ : represents the adjustment amount of the departure time of the candidate train $\varphi \in \Psi_{\text {adjust }}$. If the departure time is advanced, then $\Delta t_{\varphi}<0$;
if the departure time is delayed, then $\Delta t_{\varphi}>0$. Obviously, the adjusted train departure time can be denoted as $d_{\varphi}^{r} \leftarrow d_{\varphi}^{r}+\Delta t_{\varphi}$.
For any train $\varphi$ in $\Psi_{\text {new }}$, the train must first decide whether or not it is operated. Once the train is operated, it must further determine its departure time and whether it stops at each visiting station. Therefore, its decision variables are respectively set as follows:
- 0-1 decision variable $y_{\varphi}$ : indicates whether the candidate train $\varphi \in \Psi_{\text {new }}$ is selected to operate. If so, $y_{\varphi}=1$; otherwise, $y_{\varphi}=0$.
- 0-1 decision variable $s_{\varphi}^{i}$ : indicates whether the candidate train $\varphi \in \Psi_{\text {new }}$ stops in the visiting station $i \in S_{\varphi}$; if stop, $s_{\varphi}^{i}=1$; otherwise, $s_{\varphi}^{i}=0$.
- Integer decision variable $d_{\varphi}^{r}$ : represents the departure time of $\varphi \in \Psi_{\text {new }}$.

For any candidate train $\varphi \in \Psi_{\text {adjust }} \cup \Psi_{\text {new }}$, its arrival and departure time at each visiting station can be calculated according to the average running time of a track section and the average stopping time in a station, based on its departure time $d_{\varphi}^{r}$ and stop variable $s_{\varphi}^{j}$. The arrival and departure time of train $\varphi$ at station $j \in S_{\varphi}$ can be denoted as $a_{\varphi}^{j}$ and $d_{\varphi}^{j}$, respectively, and they satisfy Equations (1)-(4):

$$
\begin{gather*}
a_{\varphi}^{j}=T(i, j) \cdot\left(1-x_{\varphi}\right)+\left(s_{\varphi}^{i}+\delta_{\varphi}^{i}\right) \cdot \bar{T}+\left(s_{\varphi}^{j}+\delta_{\varphi}^{j}\right) \cdot \overline{\bar{T}}-d_{\varphi}^{i}, \forall \varphi \in \Psi_{\text {adjust }} ;(i, j) \in E_{W^{\prime}}  \tag{1}\\
d_{\varphi}^{j}=\left(s_{\varphi}^{i}+\delta_{\varphi}^{i}\right) \cdot T_{j}+a_{\varphi}^{j}, \forall \varphi \in \Psi_{a d j u s t} ; j \in P_{\varphi} \backslash\left\{r_{\varphi}, s_{\varphi}\right\}  \tag{2}\\
a_{\varphi}^{j}=T(i, j) \cdot y_{\varphi}+s_{\varphi}^{i} \cdot \bar{T}+s_{\varphi}^{j} \cdot \overline{\bar{T}}-d_{\varphi}^{i}, \forall \varphi \in \Psi_{\text {new }} ;(i, j) \in E_{W^{\prime}}  \tag{3}\\
d_{\varphi}^{j}=s_{\varphi}^{j} \cdot T_{j}+a_{\varphi}^{j}, \forall \varphi \in \Psi_{\text {new }} ; j \in P_{\varphi} \backslash\left\{r_{\varphi}, s_{\varphi}\right\}, \tag{4}
\end{gather*}
$$

where $T(i, j)$ is the average pure running time of a train on track section $(i, j) ; T_{j}$ is the average stopping time of a train at station $j$, and o $\bar{T}$ and $\overline{\bar{T}}$ are the acceleration and deceleration time of a train, respectively.

### 3.3. Revenue Analysis of Trains in Operation

Different line plans will lead to different railway operational, revenues which is a momentous aspect for railway enterprises to measure the quality of a line plan. Usually, it is described as the differential between operational revenue and operational cost, wherein the former is mainly composed of train ticket revenues and can be calculated as the sum of the product of passenger capacity of a train, track section mileage, and passenger-kilometer fare rate of each train, namely

$$
\begin{equation*}
U=\sum_{\varphi \in \Psi_{\text {adjust }}}\left[\left(\tau \cdot l_{i, j} \cdot Q_{i, j}^{\varphi}\right) \cdot\left(1-x_{\varphi}\right)\right]+\sum_{\varphi \in \Psi_{\text {new }}}\left[\left(\tau \cdot l_{i, j} \cdot Q_{i, j}^{\varphi}\right) \cdot y_{\varphi}\right] \tag{5}
\end{equation*}
$$

where, $\tau$ is the passenger- km fare rate of the train, $l_{i, j}$ is the mileage of track section $(i, j)$, and $Q_{i, j}^{\varphi}$ is the passenger capacity of the train passing through track section $(i, j)$.

The operational cost mainly includes the organization cost of trains and the running cost of trains. The former is the fixed cost required by operational trains, including the related cost of crew and the cost sharing for the purchase of locomotive and rolling stock, and it depends on the number of the operational trains. The running cost of the train is usually the cost of a train running on the railway line, including electricity or fuel costs, line usage fees, depreciation of locomotive and rolling stock, etc., and its value is proportional to the travel time or distance of the train. Denote $\bar{c}_{M}$ as the average organization cost of organizing a train and $\bar{c}_{L}$ as the average operating cost per unit travel time of the train. Then, based on the given adjusted line plan of the existing HSR network and the line plan
of the new line, the total organization $\operatorname{cost} C_{Z}$ and total operating $\operatorname{cost} C_{T}$ of all trains are calculated as follows:

$$
\begin{gather*}
C_{Z}=\bar{c}_{M} \cdot\left[\sum_{\varphi \in \Psi_{\text {adjust }}}\left(1-x_{\varphi}\right)+\sum_{\varphi \in \Psi_{\text {new }}} y_{\varphi}\right]  \tag{6}\\
C_{T}=\bar{c}_{L} \cdot\left[\sum_{\varphi \in \Psi_{\text {adjust }}}\left(1-x_{\varphi}\right) \cdot\left(a_{\varphi}^{s}-d_{\varphi}^{r}\right)+\sum_{\varphi \in \Psi_{\text {new }}} y_{\varphi} \cdot\left(a_{\varphi}^{s}-d_{\varphi}^{r}\right)\right] \tag{7}
\end{gather*}
$$

## 4. Passenger Travel Network and General Travel Cost

Both the adjustment of the existing HSR network line plan and the optimization of the line plan of the new rail line are closely related to passengers' train choice, which directly determines the operational revenues of railway enterprise. In this section, a passenger travel network is constructed and the passenger general travel cost is analyzed to describe passengers' train choice and obtain the passenger volumes of each operational train on the HSR network, which will provide support for the subsequent model and algorithm.

### 4.1. Passenger Travel Network

In order to describe the travel process of passengers including waiting, boarding, riding, transfer, and alighting, a passenger travel network, which is denoted as $G=(N, A)$, is constructed based on the given new HSR network $W^{\prime}$ and the pre-determined set of candidate trains $\Psi=\Psi_{\text {adjust }} \cup \Psi_{\text {new }}$. The nodes of the passenger travel network can be divided into three types: station node, arrival node, and departure node. Thus, the node set of the passenger travel network can be denoted as

$$
N=N_{S} \cup N_{A} \cup N_{D},
$$

where $N_{S}$ is the subset of station node, $N_{A}$ is the subset of arrival node, and $N_{D}$ is the subset of departure node.

The directed arcs of the passenger travel network can be divided into five types: boarding arc, riding arc, staying arc, transfer arcs, and alighting arcs. Therefore, the directed arc set of the passenger travel network can be denoted as

$$
A=A_{S} \cup A_{Q} \cup A_{W} \cup A_{H} \cup A_{X}
$$

where $A_{S}$ is the subset of boarding arc, $A_{Q}$ the subset of riding arc, $A_{W}$ is the subset of staying arc, $A_{H}$ is the subset of transfer arc, and $A_{X}$ is the subset of alighting arc.

The three types of node mentioned above are constructed as follows:

- Station node, which is denoted as $n_{i}$, is generated based on each station $i$ on the HSR network.
- Arrival node and departure node, which are denoted as $n_{\varphi, i}^{a r r}$ and $n_{\varphi, i}^{d e p}$ respectively, and are generated for each candidate train $\varphi \in \Psi$ at each of its visiting stations $i \in P_{\varphi}$.
Based on the three types of node constructed, five types of directed arc can be constructed between the relevant nodes. The specific process of construction is as follows:
- For each station $i$, a boarding $\operatorname{arc}\left(n_{i}, n_{\varphi, i}^{d e p}\right)$ is generated between the station node $n_{i}$ and any departure node $n_{\varphi, i}^{d e p}$ at this station of candidate train $\varphi$.
- For each station $i$, an alighting $\operatorname{arc}\left(n_{\varphi, i}^{a r r}, n_{i}\right)$ is generated between the arrival node $n_{\varphi, i}^{\text {arr }}$ at this station of candidate train $\varphi$ and the station node $n_{i}$.
- For each station $i$, a transfer $\operatorname{arc}\left(n_{\varphi, i}^{a r r}, n_{\varphi^{\prime}, i}^{\text {dep }}\right)$ is generated between the arrival node $n_{\varphi, i}^{a r r}$ and departure node $n_{\varphi^{\prime}, i}^{d e p}$ of any two candidate trains $\varphi$ and $\varphi^{\prime}$ that both visit the station.
- For each candidate train $\varphi$, a staying $\operatorname{arc}\left(n_{\varphi, i}^{a r r}, n_{\varphi, i}^{\text {dep }}\right)$ is generated between the arrival node and departure node of this train at its each visiting station $i$.
- For each candidate train $\varphi$, a riding $\operatorname{arc}\left(n_{\varphi, i}^{d e p}, n_{\varphi, j}^{a r r}\right)$ is generated between the departure node and arrival node of the train at the two adjacent station $i$ and $j$, which are linked by an only track section.
For a given line plan, there may be some passengers who have no suitable train for traveling because all the available trains are fully loaded; these passengers are called detained passengers on the HSR network. In order to describe the detention cost of these passengers, virtual trains (namely, a virtual node set denoted as $N^{\prime}$ and a virtual arc set denoted as $A^{\prime}$ ) are constructed on the basis of the above passenger travel network. Concretely speaking, virtual departure nodes and virtual arrival nodes are respectively constructed for virtual trains at each station on HSR network, and virtual alighting arcs and boarding arcs are constructed for virtual trains at each station on HSR network, accordingly. Also, virtual riding arcs and virtual staying arcs are constructed between each pair of adjacent stations. It should be noted that the transport capacity of all virtual arcs is set to infinity.

It is worth stressing that the nodes and arcs of the passenger travel network are determined and fixed after the candidate train set is generated. The change of the candidate train decision variable only affects the capacity limit of the related directed arc.

Figure 2 shows a passenger travel network constructed based on four stations and two trains, where each letter represents a station and each number represents a generated node. The first train runs from station B to D and stops at station C, and the second train runs from station A to C and stops at station B. Nodes 1, 2, 3, and 4 are station nodes corresponding to station $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D , respectively. Take the first train as an example: its departure nodes at station $B$ and $C$ are nodes 5 and 7 , and its arrival nodes at station $C$ and $D$ are nodes 6 and 8 , respectively. Its riding arc from station $B$ to $C$ is arc ( 5,6 ), its riding arc from station $C$ to $D$ is arc $(7,8)$, and its staying arc at station $C$ is arc $(6,7)$. The alighting and boarding arcs at station $C$ are arc $(6,3)$ and $(3,7)$, respectively. In addition, the transfer arc from the second train to the first train at station B is arc $(10,5)$, and the transfer arc from the first train to the second train at station $C$ is arc $(12,7)$.


Figure 2. Illustration of an example for the passenger travel network.

### 4.2. Passenger General Travel Cost

The general travel cost for passengers is the sum of all the expenses incurred in the process of traveling, which reflects the service level of the line planning to a certain extent. The passenger general travel cost consists of ticket cost, travel time cost, transfer cost, etc., which depend on the corresponding cost of each directional arc in the travel network that passengers select to travel.

As mentioned above, an OD demand can be denoted as $q_{r s}^{h}$, where $r$ and $s$ is its origin and destination, respectively, and $h \in H$ is its planned departure time range. Then, the variable $\delta_{r s}^{h}(m, n)$ is defined to represent the passenger volume of $q_{r s}^{h}$ on directed
$\operatorname{arc}(m, n) \in A$, and $a(m, n)$ and $d(m, n)$ are defined to represent the moment corresponding to the start node and end node of a directed $\operatorname{arc}(m, n) \in A$ respectively, which are based on the arrival and departure time of corresponding train at the station. In particular, the corresponding moments of the start node of boarding arc and the end node of alighting arc depend on the departure and arrival moments of the corresponding train at the station respectively, which also means that there is no time for passengers to spend on the arcs of these two types.

The passengers' ticket costs can be expressed as the sum of the costs that passengers incur based on the riding arc that they choose to travel, namely

$$
\begin{equation*}
C_{F}=\sum_{q_{r s}^{h} \in Q} \sum_{(m, n) \in A_{Q}} \delta_{r s}^{h}(m, n) \cdot \rho(m, n) \tag{8}
\end{equation*}
$$

where $\rho(m, n)$ is the ticket cost (yuan/person) of passengers choosing the riding arc $a \in A_{Q}$, which is obtained by the product of the mileages of track section and the rate of fare.

Passenger travel-time cost can be expressed as the product of passenger unit time value and total travel time, wherein passenger travel time is the sum of the time that a passenger spends on the riding arcs and staying arcs, namely,

$$
\begin{equation*}
C_{Q}=\sum_{q_{r s}^{h} \in Q} \sum_{(m, n) \in A_{Q} \cup A_{W}} \delta_{r s}^{h}(m, n) \cdot[d(m, n)-a(m, n)] \cdot \varepsilon_{1}, \tag{9}
\end{equation*}
$$

where $\varepsilon_{1}$ is the value of passenger travel time (yuan/min).
For the passengers who need to transfer, they may experience extra processes, including alighting, boarding, carrying luggage, and waiting at the transfer station, which will cause loss of time and comfort, and this loss can be quantified as transfer cost. The transfer cost can be divided into fixed transfer cost and variable transfer cost. Fixed transfer cost represents the one-time transfer cost of passengers, which is related to the grade of the transfer station and the conditions of the transfer. Variable transfer cost refers to the time cost required by the passenger due to the transfer, and it is related to the waiting time of passengers at their transfer station. Hence, the transfer costs can be expressed as the sum of the costs that passengers incur due to the transfer arcs that they choose to travel:

$$
\begin{equation*}
C_{H}=\sum_{q_{r s}^{h} \in Q} \sum_{(m, n) \in A_{H}} \delta_{r s}^{h}(m, n) \cdot\left\{[d(m, n)-a(m, n)] \cdot \varepsilon_{2}+G(m, n)\right\} \tag{10}
\end{equation*}
$$

where $\varepsilon_{2}$ is the time value of passengers for transfer (yuan/min); $G(m, n)$ is the fixed transfer fee of a passenger at the transfer arc $(m, n)$ based on the corresponding transfer station.

Passengers on the HSR network are highly sensitive to their departure time and usually have a planned departure time. However, their actual departure time often deviates from their planned departure time, and the deviation can be quantized as the cost of deviation on the departure time. The departure deviation cost corresponding to the time deviation can be expressed as the cost that passengers incur based on the boarding arcs that they choose to travel, namely:

$$
\begin{equation*}
C_{W}=\sum_{q_{r s}^{h} \in Q} \sum_{(m, n) \in A_{S}} \delta_{r s}^{h}(m, n) \cdot t\left(q_{r s}^{h}, \text { deviation }\right) \cdot \varepsilon_{3} \tag{11}
\end{equation*}
$$

where $\varepsilon_{3}$ is the time value of passengers boarding the train in advance or late (yuan/min) and $t\left(q_{r s}^{h}\right.$, deviation $)$ is the deviation of passengers' actual boarding time and planned departure time, which satisfies the following formula:

$$
t\left(q_{r s}^{h}, \text { deviation }\right)=\left\{\begin{array}{cc}
0, & t_{r s}^{h} \in[\bar{h}, \overline{\bar{h}})  \tag{12}\\
\max \left\{\bar{h}-t_{r s}^{h}, t_{r s}^{h}-\overline{\bar{h}}\right\}, & t_{r s}^{h} \notin[\bar{h}, \overline{\bar{h}}) .
\end{array}\right.
$$

As for the detained passengers on the HSR network, they should bear the given cost of detention (i.e., the maximum cost that they can afford in their travel choices).

$$
\begin{equation*}
C_{L}=\sum_{q_{r s}^{h} \in Q} c\left(q_{r s}^{h}\right) \cdot Z\left(q_{r s}^{h}\right), \tag{13}
\end{equation*}
$$

where $c\left(q_{r s}^{h}\right)$ is the maximum cost that $q_{r s}^{h}$ can afford to travel and $Z\left(q_{r s}^{h}\right)$ is the detained passengers' volume of $q_{r s}^{h}$; it can be calculated as follows:

$$
\begin{equation*}
\mathrm{Z}\left(q_{r s}^{h}\right)=\max \left\{\delta_{r s}^{h}(m, n) \mid(m, n) \in A^{\prime}\right\} . \tag{14}
\end{equation*}
$$

## 5. Model Formulation

Both the service level of passengers and the operational revenue of railway enterprise should be taken into account once the line plan of the new railway line is designed or the line plan of the existing network is adjusted. For passengers, the optimization and adjustment of line plan should reduce their general travel cost as much as possible on the premise that their transport demands are satisfied. Nevertheless, for railway enterprises, the optimization and adjustment of line plans should realize cost savings and increase the fare income. Therefore, the objective of the model is to minimize the general travel cost of passengers and to maximize the operational income of railway enterprises, expressed by

$$
\begin{gather*}
\min Z_{1}=C_{F}+C_{Q}+C_{H}+C_{W}+C_{L}  \tag{15}\\
\max Z_{2}=C_{F}-\left(C_{Z}+C_{T}\right) . \tag{16}
\end{gather*}
$$

By introducing an equilibrium parameter $\alpha \in(0,1)$, the above multiple objectives are transformed into the following single objective function:

$$
\begin{equation*}
\min Z=\alpha \cdot Z_{1}-(1-\alpha) \cdot Z_{2} \tag{17}
\end{equation*}
$$

In addition, the model also needs to satisfy the eight sets of constraints as follows. Note that for any adjustment candidate train (i.e., $\varphi \in \Psi_{\text {adjust }}$ ), its adjusted stop variable and adjusted departure time are still denoted as $s_{\varphi}^{i}$ and $d_{\varphi}^{r}$ for the convenience of constrained representation, respectively.

1. Constraints of stop stations for trains

$$
\begin{gather*}
\sum_{i \in S_{\varphi} \backslash\left\{r_{\varphi}, s_{\varphi}\right\}} s_{\varphi}^{i} \leq\left(1-x_{\varphi}\right) \cdot M, \forall \varphi \in \Psi_{\text {adjust }}  \tag{18}\\
\sum_{i \in S_{\varphi} \backslash\left\{r_{\varphi}, s_{\varphi}\right\}} s_{\varphi}^{i} \leq y_{\varphi} \cdot M, \forall \varphi \in \Psi_{\text {new }}  \tag{19}\\
\sum_{i \in\left\{r_{\varphi}, s_{\varphi}\right\}} s_{\varphi}^{i}=2-2 \cdot x_{\varphi}, \forall \varphi \in \Psi_{\text {adjust }}  \tag{20}\\
\sum_{i \in\left\{r_{\varphi}, s_{\varphi}\right\}} s_{\varphi}^{i}=2 \cdot y_{\varphi}, \forall \varphi \in \Psi_{\text {new }}, \tag{21}
\end{gather*}
$$

where $M$ is a sufficiently large value. Inequalities (18) and (19) ensure that the stop variables of the candidate train for adjustment and the candidate train for operation at any intermediate station are 0 when the trains are not operated, respectively. Equations (20) and (21) ensure that the operational train should stop at both the initial and terminal stations.
2. Constraint of the range of estimated departure times at initial station

$$
\begin{equation*}
T_{\varphi}^{\prime} \cdot\left(1-x_{\varphi}\right) \leq \Delta t_{\varphi} \leq T_{\varphi} \cdot\left(1-x_{\varphi}\right), \forall \varphi \in \Psi_{\text {adjust }} \tag{22}
\end{equation*}
$$

$$
\begin{equation*}
\bar{t}_{\varphi} \cdot y_{\varphi} \leq d_{\varphi}^{r} \leq \overline{\bar{t}}_{\varphi} \cdot y_{\varphi}, \forall \varphi \in \Psi_{\text {new }} \tag{23}
\end{equation*}
$$

where $T^{\prime}{ }_{\varphi}$ and $T_{\varphi}$ are the upper and lower bounds of the departure time of the candidate train $\varphi$, respectively. Inequality (22) ensures that the adjusted values of the departure time of the candidate adjustment trains are within the allowed time range. Analogously, Inequality (23) ensures that the departure time of the candidate operational trains must be within the allowed time range.
3. Constraints of train running time and stop time.

For any train in operation, the departure time of any station and the arrival time of the next adjacent station should meet the requirements of the average running time of the corresponding track section; that is, Equations (1) and (3). Further, the times of arriving and departing at the same station that it visits should meet the requirements of the average stopping time of the station, namely Equations (2) and (4).
4. Constraints of track section capacity and train starting/ending capacity of stations

$$
\begin{align*}
& \sum_{\varphi: i_{\varphi}^{i} \in[\overline{\bar{h}}, \overline{\bar{h}})} y_{\varphi}+\left(1-x_{\varphi}\right) \leq C_{i j}, \forall(i, j) \in E_{W^{\prime}} ; \forall h \in H  \tag{24}\\
& \sum_{\varphi: d_{\varphi}^{i} \in\left[\overline{\bar{h}}, \overline{\bar{h}}, r_{\varphi}=i\right.} y_{\varphi}+\left(1-x_{\varphi}\right) \leq C_{i}, i \in S_{W^{\prime}} ; \forall h \in H  \tag{25}\\
& \sum_{\varphi: d_{\varphi}^{i} \in[\bar{h}, \overline{\bar{h}}), s_{\varphi}=i} y_{\varphi}+\left(1-x_{\varphi}\right) \leq C^{\prime}{ }_{i}, i \in S_{W^{\prime}} ; \forall h \in H, \tag{26}
\end{align*}
$$

where $C_{i j}$ is the upper limit of the capacity of the track section $(i, j)$ in each operational period and $C_{i}$ and $C^{\prime}{ }_{i}$ are the upper limit of train starting/ending capacity of station $i$ in each operational period, respectively. Inequality (24) ensures that the number of trains entering any track section $(i, j)$ in each operational period does not exceed the upper limit of capacity of the track section; Inequality (25) and (26) ensure that the number of trains departing from or arriving at station $i$ in each operational period does not exceed the upper limit of the starting or ending capacity of the station, respectively.
5. Constraints of passenger flow balance

$$
\begin{gather*}
\sum_{(m, n) \in A_{k}^{i n}} \delta_{r s}^{h}(m, n)=\sum_{(m, n) \in A_{k}^{\text {out }}} \delta_{r s}^{h}(m, n), \forall k \in N_{A} \cup N_{D} \cup N^{\prime}{ }_{A} \cup N^{\prime}{ }_{D}  \tag{27}\\
\sum_{(m, n) \in A_{S}\left(q_{r s}^{h}\right)} \delta_{r s}^{h}(m, n)=q_{r s}^{h}, \forall q_{r s}^{h} \in Q  \tag{28}\\
\sum_{(m, n) \in A_{X}\left(q_{r s}^{h}\right)} \delta_{r s}^{h}(m, n)=q_{r s}^{h}, \forall q_{r s}^{h} \in Q \tag{29}
\end{gather*}
$$

where $A_{k}^{\text {in }}$ is the arc set, of which its end point is node $k ; A_{k}^{\text {out }}$ is the arc set, of which its start point is node $k ; A_{S}\left(q_{r s}^{h}\right)$ is the boarding arcs set related to OD demand $q_{r s}^{h}$, i.e., the start point $n_{i}$ of the boarding $\operatorname{arc}\left(n_{i}, n_{\varphi, i}^{d e p}\right) \in A_{S}\left(q_{r s}^{h}\right)$ is the initial station $r$ of OD demand $q_{r s}^{h}$; $A_{X}\left(q_{r s}^{h}\right)$ is the set of alighting arcs related to OD demand $q_{r s}^{h}$, i.e., the end point $n_{i}$ of the alighting arc $\left(n_{\varphi, i}^{a r r}, n_{i}\right) \in A_{X}\left(q_{r s}^{h}\right)$ is the terminal station $s$ of OD demand $q_{r s}^{h}$. Equation (27) ensures that for the departure and arrival nodes, their incoming and outgoing passenger flow are both conserved. Equations (28) and (29) ensure that passengers can travel from the initial station to the terminal station to complete their transport process.
6. Capacity constraints of directed arcs in passenger travel network

Since each directed arc in the passenger travel network is based on the structure of candidate trains, it is obvious that when the candidate adjustment train is canceled or the candidate operational train is not operated, the capacity of each corresponding arc is 0 , and no passenger could choose these arcs to travel. Furthermore, the number of passengers choosing the directed arcs for traveling should not exceed the corresponding trains' capacity limit. Therefore, the number of passengers choosing each directional arc should meet the following capacity constraints.
(1) Capacity constraints of boarding arcs

When the train visits and stops at a station, the number of passengers choosing the corresponding boarding arc should not exceed the train capacity $\mathrm{Z}_{\varphi}$. Otherwise, the number of passengers can only be 0 , namely,

$$
\begin{equation*}
\sum_{q_{r s}^{h} \in Q} \delta_{r s}^{h}\left(n_{i}, n_{\varphi, i}^{d e p}\right) \leq s_{\varphi}^{i} \cdot Z_{\varphi}, \forall\left(n_{i}, n_{\varphi, i}^{d e p}\right) \in A_{S} ; i \in P_{\varphi} \backslash\left\{s_{\varphi}\right\} ; \varphi \in \Psi \tag{30}
\end{equation*}
$$

## (2) Capacity constraints of riding arcs

When a train is operated, the passenger choosing riding arcs of the train in each running track section should not exceed the train capacity $Z_{\varphi}$. Otherwise, it is 0 , namely,

$$
\begin{gather*}
\sum_{q_{r s}^{h} \in Q} \delta_{r s}^{h}\left(n_{\varphi, i}^{\text {dep }}, n_{\varphi, j}^{a r r}\right) \leq Z_{\varphi} \cdot\left(1-x_{\varphi}\right), \forall\left(n_{\varphi, i}^{\text {dep }}, n_{\varphi, j}^{a r r}\right) \in A_{S} ;(i, j) \in E_{W^{\prime}} ; \varphi \in \Psi_{\text {adjust }}  \tag{31}\\
\sum_{q_{r s}^{h} \in Q} \delta_{r s}^{h}\left(n_{\varphi, i}^{\text {dep }}, n_{\varphi, j}^{a r r}\right) \leq Z_{\varphi} \cdot y_{\varphi}, \forall\left(n_{\varphi, i}^{\text {dep }}, n_{\varphi, j}^{\text {arr }}\right) \in A_{S} ;(i, j) \in E_{W^{\prime}} ; \varphi \in \Psi_{\text {new }} . \tag{32}
\end{gather*}
$$

(3) Capacity constraints of transfer arcs

When a passenger transfers between any two trains successfully, two conditions must be met: one is that both trains stop at the station; the other is that the time interval between the arrival time of the former train and departure time of the latter train meets the minimum transfer time required by the process of transfer. Only a transfer arc satisfying the above two conditions is called an effective transfer arc, otherwise it is called an invalid transfer arc. A 0-1 intermediate variable denoted as $R\left(n_{\varphi, i}^{a r r}, n_{\varphi^{\prime}, i}^{d e p}\right)$ is introduced to indicate whether the transfer arc is effective. When $R\left(n_{\varphi, i}^{a r r}, n_{\varphi^{\prime}, i}^{\text {dep }}\right)=1$, the transfer arc is effective. Otherwise, it is invalid.

$$
\begin{gather*}
\sum_{q_{r s}^{h} \in Q} \delta_{r s}^{h}\left(n_{\varphi, i}^{a r r}, n_{\phi^{\prime}, i}^{d e p}\right) \leq R\left(n_{\varphi, i}^{a r r}, n_{\phi^{\prime}, i}^{d e p}\right) \cdot M, \forall\left(n_{\varphi, i}^{a r r}, n_{\phi^{\prime}, i}^{d e p}\right) \in A_{H}  \tag{33}\\
R\left(n_{\varphi, i}^{a r r}, n_{\phi^{\prime}, i}^{d e p}\right) \leq \frac{\left(s_{\varphi}^{i}+s_{\varphi^{\prime}}^{i}\right)}{2}, \forall\left(n_{\varphi, i}^{a r r}, n_{\phi^{\prime}, i}^{d e p}\right) \in A_{H}  \tag{34}\\
T_{i}^{\prime}-  \tag{35}\\
{\left[1-R\left(n_{\varphi, i}^{a r r}, n_{\phi^{\prime}, i}^{d e p}\right)\right] \cdot M \leq d_{\phi^{\prime}}^{i}-a_{\varphi}^{i}, \forall\left(n_{\varphi, i}^{a r r}, n_{\varphi^{\prime}, i}^{d e p}\right) \in A_{H} .}
\end{gather*}
$$

Inequality (33) ensures that passengers can choose a transfer arc only when it is effective, meaning that the passenger volume is greater or equal to 0 of the transfer arc, while Inequality (34) and (35) ensure that the two conditions above are met when the transfer arc is effective, respectively.
(4) Capacity constraints of alighting arcs

When a train is operated and stops at the station, the passenger volume of the corresponding alighting arc should not exceed the train capacity $Z_{\varphi}$. Otherwise, the value can only be 0 .

$$
\begin{equation*}
\sum_{q_{s}^{h} \in Q} \delta_{r s}^{h}\left(n_{\varphi, i}^{a r r}, n_{i}\right)=s_{\varphi}^{i} \cdot Z_{\varphi}, \forall\left(n_{\varphi, i}^{a r r}, n_{i}\right) \in A_{X} ; i \in P_{\varphi} \backslash\left\{r_{\varphi}\right\} . \tag{36}
\end{equation*}
$$

## 7. Constraint of train circulation

To minimize the disturbance on the connection relationship of the existing trains is an extremely significant goal while adjusting the original line plan of the existing network. However, the connection relationship can be re-generated after the adjustment and optimization of the line plan. Therefore, the consistent constraint of the number of starting and ending trains at each station should be guaranteed in the adjustment and optimization of the line plan, which is conducive to re-generating a profitable connection relationship of trains.

$$
\begin{equation*}
\sum_{\varphi \in \Psi_{\text {adjust }}} \sigma_{i} \cdot\left(1-x_{\varphi}\right)+\sum_{\varphi \in \Psi_{\text {new }}} \sigma_{i} \cdot y_{\varphi}=\sum_{\varphi \in \Psi_{\text {adjust }}} \sigma_{i}^{\prime} \cdot\left(1-x_{\varphi}\right)+\sum_{\varphi \in \Psi_{\text {new }}} \sigma_{i}^{\prime} \cdot y_{\varphi}, \forall i \in S_{W^{\prime}} \tag{37}
\end{equation*}
$$

where $\sigma_{i}$ is a $0-1$ variable of whether the train $\varphi$ starts from station $i$. If so, $\sigma_{i}=1$. Otherwise, $\sigma_{i}=0$. Similarly, $\sigma_{i}^{\prime}$ is a $0-1$ variable of whether the train $\varphi$ ends at station $i$. If so, $\sigma_{i}^{\prime}=1$. Otherwise, $\sigma_{i}^{\prime}=0$.
8. Constraint of adjustment scope and quantity of the existing trains

In the existing HRS network, passengers tend to choose some operational trains which have high passenger load factor and efficiency, such as hourly trains. If there is no parallel path between existing routes of these trains and the new line, no adjustments will be normally implemented in these trains. Denote $\Psi_{\text {retain }}$ as the set of trains that will not be adjusted during optimization, then each train in $\Psi_{\text {retain }}$ should satisfy:

$$
\begin{gather*}
x_{\varphi}=0, \forall \varphi \in \Psi_{\text {retain }}  \tag{38}\\
\Delta t_{\varphi}=0, \forall \varphi \in \Psi_{\text {retain }}  \tag{39}\\
\delta_{\varphi}^{i}=0, \forall i \in S_{\varphi}, \forall \varphi \in \Psi_{\text {retain }} . \tag{40}
\end{gather*}
$$

In addition, in order to ensure the continuity of the original line plan on the existing network and reduce the adverse impact on the railway organization and passenger travel resulting from adjustment, the quantity of adjusted trains should be controlled within a given allowable number during the adjustment and optimization of the line plan, namely,

$$
\begin{equation*}
\sum_{\varphi \in \Psi_{\text {adjust }}} x_{\varphi} \leq \eta \tag{41}
\end{equation*}
$$

where $\eta$ is the upper limit of the number of trains allowed to be canceled on the existing HSR network.

## 6. Solution Algorithm

### 6.1. Framework for Solution Based on SAA

This section designs a solution algorithm based on a simulated annealing algorithm (SAA) in consideration of the complexity of optimization and adjustment of line plans and the difficulty of the solving method, as well as to synthesize the characteristics of the problem and the model. The algorithm process is shown in Figure 3. The holistic optimization framework is as follows:


Figure 3. Illustration of the adjustment and optimization process.
Step 1 The set of candidate trains for adjustment is constructed according to the trains running on the existing HSR network;

Step 2 The set of candidate trains for operation on the new HSR line is constructed according to the principle of "operating trains based on the OD demands of passengers they serve";

Step 3 An initial line plan of the new HSR network is generated as the current solution;
Step 4 The current solution is evaluated by a process of assignment of passengers for operational trains; then, a feasible new solution is obtained according to the designed search strategies of neighborhood solutions based on the result of the assignment process, and the new solution is evaluated in the same manner;

Step 5 The new solution is judged for acceptance as the current solution and foundbest solution according to simulated annealing conditions. Then Step 4 is repeated, and the current and found-best solution are continuously optimized by minimizing the objective function until a satisfactory solution is obtained.

### 6.2. Generation of the Candidate Train Set

The operational trains on the existing HSR network can be taken as the set of candidate trains for adjustment, namely, $\Psi_{\text {adjust }}=\bar{\Psi}_{W}$. The set of trains that cannot be adjusted (i.e., $\left.\Psi_{\text {retain }}\right)$, which is included by $\Psi_{\text {adjust }}$, can be obtained according to the actual operation of the railway. The upper bound $T_{\varphi}^{\prime}$ and $T_{\varphi}$ of the adjustment value of the candidate train departure time can be comprehensively determined by the departure time and offset of the existing train operational diagram. Obviously, the adjustment on the departure time of the existing trains will not have a fundamental impact on the existing train operational diagram.

The candidate train set for operation on the new line can be divided into two parts: the in-line candidate trains and the cross-line candidate trains. The in-line candidate trains mainly serve the OD demand for which origin and destination both are stations along the
new line, and their frequencies vary greatly in different time periods. Therefore, the in-line candidate trains are constructed according to the principle of "operating the train based on the OD demands of passengers they serve" in each time period. The main procedures are as follows:

Step 1: Look for the stations which can be the initial station or the terminal station for trains on the new HSR line, and construct possible routes for trains between these stations;

Step 2: Take all OD demands in a certain period of time in chronological order and load them onto the new line to obtain passenger volume in each track section of the new line;

Step 3: Generate a train according to the sequences, from long to short, of the train routes. If the load factor of the train meets the minimum passenger load factor threshold of an operational train, the train is regarded as a candidate train for operation. Its starting time range is set as the middle time range of the time period. Then update the passenger volume of each track section on the new line;

Step 4: Repeat Step 3; if no candidate train is generated, return to Step 2 until OD demand of all periods are loaded onto the new line;

Step 5: If there is still a surplus passenger flow at the end, a candidate train is generated based on the longest route, and its departure time range is set as the middle time range of the last time period.

Cross-line candidate trains, which generally have a long running time and low operational frequency, mainly serve cross-line passengers Therefore, when constructing the candidate cross-line train, it should first determine the cross-line passengers served by these trains, then generate suitable routes for these trains, and finally generate the candidate train according to the principle of "operating the train based on the OD demands of passengers they serve". The main procedures are as follows:

Step 1: Determine the main service OD demand of cross-line trains; that is, determine the new cross-line passenger who will be diverted to the new line after the network merging of the new and the cross-line passengers whose origin or destination is the stations along the new line.

Step 2: Generate the operational routes of cross-line candidate trains according to the starting and ending conditions of each station, as well as the OD demand they serve. Meanwhile, eliminate the trains for which routes are obviously circuitous or inappropriate.

Step 3: Generate cross-line candidate trains for the total OD demand of each time period on the new line in accordance with the principle of "operating the train based on the OD demands of passengers they serve"; the departure time of candidate trains is evenly set at equal intervals within the railway operational time range.

The detailed generation method for candidate trains on the new line for operation is described using a case study in Appendix A.

### 6.3. Generation of Initial Line Planning Based on Candidate Train Set

After the two sets of candidate train are generated, the initial values of the decision variables of each candidate train should be determined to form the initial line plan of the new HSR network. For the trains in the candidate adjustment train set $\Psi_{\text {adjust }}$, its initial decision variable is set to be consistent with the existing train set $\bar{\Psi}_{W}$; that is, it is assumed that the trains running on the original network will not be adjusted primitively. For the trains in the candidate operational train set $\Psi_{\text {new }}$, it is assumed that all the candidate trains generated by the method mentioned above are operated, and the start time of each train is set to the middle value of the start time range.

Furthermore, the stop pattern of trains in the set $\Psi_{\text {new }}$ needs to be specified. According to the administrative level of the city where a train station is located, train stations can be divided into three classes, namely provincial stations, municipal stations, and county stations. In general, the higher the level of a station, the greater the number of passengers it serves and the higher the proportion of trains stopping at that station. Thus, a stop pattern can be randomly determined for each train in $\Psi_{\text {new }}$ by the rule that the higher the class of visiting station, the greater the probability of a train stopping. Meanwhile, the minimum
stopping proportion of trains at each station can be determined according to the station class. If the randomly generated stopping patterns do not meet the minimum stopping proportion of a station, the train can be selected from the candidate trains to appropriately increase the stopping at the station.

At this point, the initial line plan of the new HSR network has been generated. If the initial line plan generated by the above methods does not meet the model constraints, it will be improved to a feasible solution by the method in Section 6.6.

### 6.4. Allocation of Passengers for Operational Trains

The purpose of passenger allocation for operational trains is to simulate the process of passengers buying tickets and selecting trains for traveling, so as to obtain the passenger volume on each train that conforms to the actual operational situation of the railway, which provides a basis for evaluating the current line plan. Combining with ticket-selling strategies and historical ticket-booking data of the railway enterprise, the ticket-booking periods can be divided into $k$ periods with the same time range, denoted as $S=\left\{s_{1}, s_{2}, \ldots, s_{k}\right\}$. This means that, in the process of passenger ticket purchase, OD demand in the pre-purchase period $s_{1}$ buys tickets first, and then OD demand in the pre-purchase period $s_{2}$ buys tickets until OD demands in the pre-purchase period $s_{k}$ complete the ticket purchase. For the sake of simplicity, this paper does not consider the behavior of passenger ticket refund; that is, once they purchase tickets, their choice of trains for traveling is determined. After the pre-purchase period is divided, it is necessary to further determine the time-varying OD demand in each pre-purchase period. We assume that the planned departure time of the passenger is completely unrelated to the pre-purchase period. Therefore, the proportion of each OD demand in a certain pre-purchase period can be obtained according to the railway's historical ticket data, and then the total OD demand can be decomposed into each pre-purchase period according to the proportion.

After each pre-purchase period and the corresponding time-varying OD demand are determined and the operational trains of the current line plan are given, the passenger travel choice of trains can be further obtained according to the process of passenger allocation. It is generally recognized that passenger travel choice behavior is not completely rational, as passengers will not always choose the train with the smallest general travel cost due to perceptual bias of general travel cost among different travelers. Therefore, in the process of passenger allocation, suitable operational trains for which general cost for an OD demand does not exceed the highest cost that passengers can afford are selected. Then, passengers are allocated to these trains according to certain principles. The main procedures are as follows:

Step 1: Select a pre-purchase period according to chronological order, and assign an OD demand $q_{r s}^{h}$ in this period randomly;

Step 2: Search all travel paths (i.e., the combination of arcs constructed in the passenger travel network constructed above) that remain capable of transporting the selected OD demand and put these paths into a set denoted as Path;

Step 3: If the general cost of travel path $p \in$ Path, denoted as $c(p)$, is not higher than the highest travel cost (denoted as $c\left(q_{r s}^{h}\right)$ ) that can be accepted by the selected OD demand $q_{r s}^{h}$, then put it into a set denoted as Path'. This step further determines the feasible travel paths of $q_{r s}^{h}$;

Step 4: According to Logit principle and the limit on train capacity, calculate the passenger volume choosing each possible travel path $p \in P a t h^{\prime}$ of the selected OD demand and record their travel choices.

Step 5: Assign passengers to each path $p \in$ Path $^{\prime}$ and update the remaining capacity of each path in the passenger travel network;

Step 6: If the selected OD demand still has passengers who have not completed their travel, return to Step 2 until all passengers of this OD demand have determined their travel path. Note that if any passengers do not have any suitable travel path, or the cost of
their routes are both higher than the passengers' acceptable limit, these passengers will be loaded into the virtual arc of the passenger travel network.

Step 7: Repeat Step 1 to Step 4 until the travel paths of all OD demands in each pre-order period are determined.

### 6.5. Search Strategies for Neighborhood Solutions

After the passenger distributions in each train of the current line plan are obtained, it is necessary to optimize and adjust the line plan to improve the revenue of the railway enterprises and the efficiency of passenger traveling. In this paper, the process of optimization and adjustment is integrated into the search strategies for neighborhood solution based on SAA. This means that the search for a neighborhood solution can be regarded as an adjustment of the current line plan. The search strategies for neighborhood solutions are as follows:

## 1. Cancel operational trains

For all the adjustable operational trains in the current line plan, the passenger load factor denoted as $Z_{\varphi}$ of each train $\varphi$ is calculated and sorted according to the order from small to large. The former $\gamma_{1}$ trains are denoted as $\Omega_{1}$, and $Z_{\max }$ is set as the maximum passenger load factor of trains in $\Omega_{1}$. Second, the probability $P_{\varphi}=\left(Z_{\max }-Z_{\varphi}\right) / \sum_{\varphi^{\prime} \in \Omega_{1}}\left(Z_{\max }-Z_{\varphi^{\prime}}\right)$ that each operational train is selected to be canceled is calculated, and then, $\gamma_{1} \cdot \operatorname{random}(0,1)$ operational trains are selected to be canceled by roulette. Meanwhile, one train with the lowest load factor in the opposite direction of each canceled train will be canceled. The candidate trains in $\Psi_{\text {new }}$ are preferentially selected to be canceled.

## 2. Operate new trains

For the trains that are not operated in the current line plan, the trains that can meet the OD demand of detained passengers who have no trains to choose for traveling should be operated first. If there are no detained passengers, an additional $\gamma_{2} \cdot \operatorname{random}(0,1)$ nonoperational trains with routes that are in accordance with the operational train with a higher load factor are selected to operate along the stations and in time periods permitted by the capacity. The same number of non-operational trains are added to operate in the opposite direction section according to the same principle.

## 3. Cancel the train stops

For any adjustable operational trains in the current line plan, the total number of passengers who board, alight, or transfer at its stop station $k$ is calculated and denoted as $Q_{\varphi, k}$, and the time difference between the former train and the latter train that stop at the station $k$ is calculated and denoted as $T_{\varphi, k}$. The trains which visit and stop at station $k$ are sorted by the time difference $T_{\varphi, k}$ according to the order from small to large. The first $\gamma_{3}$ trains are selected and placed into a set denoted as $\Omega_{2}$; then, the probability $P_{\varphi}=\left(R_{\max }-R_{\varphi, k}\right) / \sum_{\varphi^{\prime} \in \Omega_{2}}\left(Z_{\max }-Z_{\varphi^{\prime}}\right)$ that each operational train's stop is selected to be canceled is calculated. Finally, $\gamma_{3} \cdot \operatorname{random}(0,1)$ operational trains are selected to be canceled for their stop at the station by roulette.

## 4. Add the train stops

For any adjustable operational trains in the current line plan, the total number of passengers who board, alight, or transfer to other trains in the same period at its non-stop station $k$ is calculated. The number is denoted as $Q_{\varphi, k}$, while the time difference between the former train and the latter train which stop at the station $k$ is calculated and denoted as $T_{\varphi, k}$. The trains that visit but do not stop at station $k$ are sorted by the difference according to the value of $R_{\varphi, k}=Q_{\varphi, k} \cdot T_{\varphi, k}$ from small to large. The first $\gamma_{4}$ trains are selected and placed into a set denoted as $\Omega_{3}$; then, the probability $P_{\varphi}=\left(R_{\max }-R_{\varphi, k}\right) / \sum_{\varphi^{\prime} \in \Omega_{2}}\left(Z_{\max }-Z_{\varphi^{\prime}}\right)$ that each train is selected to add the station stop is calculated. Finally, $\gamma_{4} \cdot \operatorname{random}(0,1)$ trains are selected to add the stop at this station by roulette.
5. Adjust the departure time of trains

For any adjustable operational trains in the current line plan, the total advanced departure time denoted as $T_{\varphi}$ and the total delayed departure time $T^{\prime}{ }_{\varphi}$ of the passengers in the train is calculated, as well as the selection probability of these trains, $P_{\varphi}=\left(T_{\varphi}+T_{\varphi}^{\prime}\right) / \sum_{\varphi \in \Psi}\left(T_{\varphi^{\prime}}+T_{\varphi^{\prime}}^{\prime}\right)$. Afterwards, $\gamma_{5} \cdot \operatorname{random}(0,1)$ trains are selected by roulette according to the probability. If the passengers' advanced departure time $T_{\varphi}$ is greater than the delayed departure time $T^{\prime}{ }_{\varphi}$, the departure time of the train $\varphi$ is advanced by a certain value. Otherwise, it is postponed. Note that the advanced or delayed time here should be within the allowed limits.

The $\gamma_{k}(k=1,2 \ldots 5)$ in the strategies mentioned above are all algorithm parameters, which decrease with the decrease in algorithm temperature. The random $(0,1)$ represents a random variable evenly distributed between 0 and 1 . The $\gamma_{k} \cdot \operatorname{random}(0,1)$ means the result of multiplying the two and rounding them to the whole number. No adjustments are applied to the trains in set $\Psi_{\text {retain }}$. Different combinations of the above strategies are employed at different temperatures while searching for the neighborhood solution. If the neighborhood solution obtained does not meet the model constraints, it can be improved by the method described in Section 6.6.

### 6.6. Adjustment Strategies for Infeasible Solutions

The initial line plan generated or the neighboring line plan obtained by the strategies mentioned above may not meet the constraints of the model, so further adjustments should be executed to ensure that the solution meets the conditions of railway operation.

When the line plan does not meet constraints of track section capacity and train starting/ending capacity of stations (i.e., Inequality (24)-(26)), the following steps shall be performed in sequence:

Step 1: Adjusting the departure time of candidate trains is the first to be considered. Select the train with the worst operational efficiency in the time period of which capacity exceeds the upper limit. If the train visits this track section or the station in the first half of the unsatisfied time period, move its departure time forward within the allowed adjustment range; otherwise, move its departure time within the allowable adjustment range.

Step 2: If the constraints are still not satisfied, try to adjust the train stops within a certain range. If the train visits the unsatisfied track section or station in the first half of the time period, randomly cancel stops that this train previously visited, according to the principle that the higher the station class is, the lower the probability of canceling the stop is. Otherwise, the same rules are applied to adding a stop at a visiting station prior to the unsatisfied station.

Step 3: If the constraints are still not satisfied, the train with the second-lowest operating efficiency shall be adjusted as above.

Step 4: If the constraints are still not met, the train with the lowest operating efficiency shall be canceled, and a train with the same route as the canceled train shall be operated at other time periods for which capacity has not reached saturation; if the capacity has reached saturation in all periods, the operational train will be directly canceled.

When the line planning does not meet the constraints of train circulation (i.e., Equation (37)), the following steps shall be applied in sequence:

Step 1: Select a train that can meet the constraint after being operated from the candidate trains that have not been operated. If more than one train can meet the constraint, the train that belongs to the period with maximum passenger volume and has capacity that has not reached saturation is selected to put into operation.

Step 2: If the constraint is still not satisfied, single out the train that can meet the constraint after it is canceled and has the worst effectiveness from the operational trains, then cancel it.

When the line planning does not meet the constraints of the adjustment quantity of existing trains (i.e., Equation (41)), the following steps shall be carried out in sequence:

Step 1: Select the train $\varphi$ with the highest effectiveness among the operational trains in $\Psi_{\text {adjust }}$, then randomly select any train with the same route as train $\varphi$ among the trains which are not operated in $\Psi_{\text {adjust }}$ to run.

Step 2: Select the train $\varphi$ with the highest effectiveness among the operational trains in $\Psi_{\text {adjust }}$, then randomly select any train with the departure time range as train $\varphi$ among the trains which are not operated in $\Psi_{\text {adjust }}$ to run.

## 7. Case Study

### 7.1. The Basic Input Data and Parameters

To evaluate the experimental results of the proposed line-planning approach, a case study on a partial Chinese HSR network is presented. Figure 4 displays the structural framework of the studied HSR network and main junctions on each HSR line (also called HSR corridor). There are 514 stations and 1106 track sections with a combined length of $22,918 \mathrm{~km}$ in this HSR network, including the newly built Shangqiu-Hefei-Hangzhou HSR line (SHH), which is shown in Figure 5. There are 1746 trains running on the existing network. The total number of passengers used in this experiment was $2,160,782$, including 147,439 new passengers attracted by the new line and $2,013,343$ on the existing network. The time-varying demand intensity functions and ticket-booking intensity functions were generated for each OD demand from the historical operation of the railway. Detailed information on the studied case is given in Appendix B.


Figure 4. Illustration of the HSR network studied.


Figure 5. Illustration of the new HSR line studied.
The parameters were set as follows: the railway operational period is from 07:00 to 23:00, and is divided into 16 sub-periods with duration of 1 h . Stations are divided into three classes according to the political and economic level of the cities at which they are located, with train stop times of $5 \mathrm{~min}, 3 \mathrm{~min}$, and 2 min , respectively. The capacity of starting/ending trains at each station is 10 trains each hour. The acceleration and deceleration time of trains at all stations are both 1 min , namely $\bar{T}=\overline{\bar{T}}=1$, and the capacity of each train is 1100 persons, namely $Z_{\varphi}=1100$. Other parameters in the model are $\bar{c}_{M}=5000$ yuan $/$ train, $\bar{c}_{L}=300$ yuan $/ \mathrm{h}, \varepsilon_{1}=18$ yuan $/ \mathrm{h}, \varepsilon_{2}=54$ yuan $/ \mathrm{h}$, $\varepsilon_{3}=36$ yuan $/$ h, $\alpha=0.5, \eta=26$.

### 7.2. Performance Analysis

### 7.2.1. Performance Analysis on the Obtained Line Plan of HSR

The proposed line-planning method was conducted in Microsoft Visual Studio 2022 on an Intel i5 3.1 GHZ with 8 GB RAM in the environment of Microsoft Win10. Figure 6 shows the objective convergence processes of the network plan and line sub-plans. The line plan objectives all show a tendency of decreasing convergence, where the objective function of the HSR network, cross-line trains, and the new line have a faster decline rate, resulting from the major optimization and adjustment implemented on the trains that visit the stations along the new line. Moreover, the objective function of the existing HSR line also shows a trend of decreasing convergence with some fluctuations, demonstrating the validity of the proposed model and methods which can take the optimization efficiency of both the existing lines and the new line into account and can reach an integral optimization of the HSR system.

Table 3 shows the comparison of railway operational cost and passenger general travel cost, which are both important indexes to evaluate the line plan, between solved results and original results on partial corridors. It demonstrates that the model and algorithm coordinate the train running on each corridor from the perspective of the network when designing the line plan for the new line, which not only reduces the operational cost of railway origination, but also reduces the general travel cost of passengers and has reached a systematic optimization.


Figure 6. Illustration of the objective convergence processes of network plan and corridor plan.
(a) HSR network. (b) cross-line trains. (c) SHH. (d) JH. (e) JG-GS. (f) HK.

Table 3. Line plan of corridor comparisons between solved results and original results.

| Name | The Proportion of <br> Objective Improvement <br> after Optimization (\%) | The Proportion of <br> Objective Improvement <br> after Optimization (\%) | The Proportion of <br> Objective Improvement <br> after Optimization (\%) | The Proportion of <br> Objective Improvement <br> after Optimization (\%) |
| :---: | :---: | :---: | :---: | :---: |
| JH | $3.232 / 2.909$ | $4.289 / 4.168$ | $3.760 / 3.496$ | 7.03 |
| JG-GS | $5.356 / 5.086$ | $5.978 / 5.843$ | $5.667 / 5.373$ | 5.19 |
| HFS | $4.409 / 4.200$ | $5.505 / 5.255$ | $4.957 / 4.674$ | 5.71 |
| HK | $4.747 / 4.267$ | $5.955 / 5.621$ | $5.351 / 4.795$ | 10.39 |
| HHR | $4.028 / 3.824$ | $5.297 / 5.155$ | $4.663 / 4.401$ | 5.60 |
| GK | $0.770 / 0.726$ | $0.936 / 0.905$ | $0.853 / 0.809$ | 5.16 |
| GG | $0.529 / 0.474$ | $0.596 / 0.569$ | $0.563 / 0.527$ | 6.39 |
| DX | $0.447 / 0.421$ | $0.585 / 0.559$ | $0.516 / 0.492$ | 4.65 |
| QT | $0.613 / 0.582$ | $1.000 / 0.906$ | $0.807 / 0.735$ | 8.92 |

### 7.2.2. Performance Analysis of Line Plans on the New Railway Lines

Figure 7 shows the train plan diagram of SHH, where every line represents a train and every point indicates the arrival time and departure time of train at each station it visited. As shown in Figure 7, there are a total of 143 trains running on the SHH, including 15 in-line trains and 57 cross-line trains operated in the up direction and 14 in-line trains and 57 crossline trains operated in the down direction (only the train plan diagram related to SHH of cross-line trains is shown in Figure 7). The peak period of departure of passengers in the up direction is around 12:00 to 21:00, and that of passengers in the down direction is around from 10:00 to 18:00, demonstrated by the dense train line during these periods. Many operated trains on SHH are cross-line trains with a long-distance route, as they can not only serve cross-line OD demand, but also serve partial in-line OD demand, which results in their high load factor. In the time interval between two cross-line trains, in-line trains with short-distance routes are operated as complementary to enrich railway operation products, which can provide passengers more choices and reduce travel deviation time for passengers.


Figure 7. Train plan diagram of SHH.
Figure $8 \mathrm{a}, \mathrm{b}$ show the number of train services and the total number of alighting and boarding passengers at each station along SHH in the up and down direction, respectively. As a general trend, there is a greater number of passengers in higher-grade stations with higher-frequency train services, which means that the operated trains adapt the OD demand
properly. However, due to the limitation of train capacity and passengers boarding in earlier visited stations, the frequency of train service at a station is not entirely determined by the number of boarding and alighting passengers, which results in the deviation between train services and volume of passengers, such as at Station Wuhu and Station Hefei.

The relationship between passenger and train service in each period is shown in Figure 8 c , which shows that the number of train stop services fluctuate with the change of passengers in each period so that the operated trains can satisfy the time-varying OD demand better to provide higher-quality service. Within the railway operational time ranges (such as 7:00-8:00, 22:00-23:00), the main OD demand consists of long-distance passengers departing from or arriving at high-grade stations, resulting in low frequency of operated trains with long routes and high load factor.

Figure 8 d specifically shows the passenger and train service in each period at three different grades stations (namely Hangzhoudong, Fuyangxi, Langxinan). It indicates that the OD demand at high-grade stations is relatively well-distributed in each period, while the OD demand at two other stations shows differences in each period, with obvious difference between high-peak and low-peak periods. The frequency of train service at each station in each period can also suit this pattern well, as shown in the figure.


Figure 8. Illustration of the served stations and passenger distribution. (a) Up direction. (b) Down direction. (c) Passenger and train service in each period. (d) Passenger and train service at three stations.

### 7.2.3. Performance Analysis on the Impact of New Line Plan on the Existing Trains

As shown in Figure 4, when the existing HSR network merges with the SHH line, the travel distance of OD demand between the stations located in the northwest of Shangqiu and the stations located in the southeast of Hangzhoudong can be shortened to a certain
extent, and these OD demand will gradually transfer to the new line. Meanwhile, the SHH can share the partial OD demand of the parallel line (namely JH-NH). Based on the above analysis, the corresponding candidate trains for operation are constructed, and the information on trains which are canceled or operated after optimization is shown in Table 4. Note that there is a correspondence of the canceled or added trains in up and down directions; thus, Table 4 just shows the information of trains in the down direction in the interest of brevity.

Table 4. The information of the existing trains and newly operated trains in solution.

| No. | Origin | Destination | Adjust | Num. | The Main Visited HSR Line of the Train Route | The Main OD Demand Served |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Lanzhou | Nanchang | Cancel | 1 | XL, JG, HK | T1 |
| 2 | Lanzhou | Hangzhou | Cancel | 1 | XL, JG, HHR, NH | T1 |
| 3 | Lanzhou | Hefei | Cancel | 1 | XL, JH, HB | T1 |
| 4 | Xi'an | Shanghai | Cancel | 1 | XL, JH | T1 |
| 5 | Xi'an | Hangzhou | Cancel | 2 | XL, JG, HHR, NH | T1 |
| 6 | Xi'an | Hefei | Cancel | 1 | XL, JH, HB | T1 |
| 7 | Xi'an | Fuzhou | Cancel | 1 | XL, JH, HB, HF | T1 |
| 8 | Zhengzhou | Hangzhou | Cancel | 1 | XL, JG, NH | T1 |
| 9 | Zhengzhou | Hefei | Cancel | 1 | JG, HHR | T1 |
| 10 | Wuhan | Hangzhou | Cancel | 1 | HHR, NH | T1 |
| 11 | Hefei | Hangzhou | Cancel | 1 | HHR, NH | T1 |
| 12 | Lanzhou | Nanchang | Operate | 2 | XL, SHH, HF, HK | T1, T2-1 |
| 13 | Lanzhou | Hangzhou | Operate | 2 | XL, SHH | T1, T2-2, T2-5 |
| 14 | Lanzhou | Hefei | Operate | 1 | XL, SHH | T1, T2-3, T2-6 |
| 15 | Lanzhou | Shanghai | Operate | 2 | XL, SHH, HK | T1, T2-4 |
| 16 | Xi'an | Shanghai | Operate | 2 | XL, SHH, HK | T1, T2-4 |
| 17 | Xi'an | Hangzhou | Operate | 2 | XL, SHH | T1, T2-5 |
| 18 | Xi'an | Hefei | Operate | 1 | SL, SHH | T1, T2-6 |
| 19 | Xi'an | Fuzhou | Operate | 1 | XL, SHH, HB, HF | T1, T2-7 |
| 20 | Zhengzhou | Hangzhou | Operate | 2 | XL, SHH | T1, T2-8, T2-9 |
| 21 | Shijiazhuang | Ningbo | Operate | 2 | JG, XL, HHR, SHH, HFS | T1, T2-10 |
| 22 | Wuhan | Ningbo | Operate | 1 | HHR, SHH, HFS | T1, T2-10 |
| 23 | Hefei | Wenzhou | Operate | 1 | SHH, HK, JW | T1, T2-11 |
| 24 | Beijing | Bozhou | Operate | 1 | JG, XL, SHH | T1, T3 |
| 25 | Beijing | Jinhua | Operate | 1 | JG, XL, SHH, HK | T1, T3 |
| 26 | Beijing | Wuhu | Operate | 2 | JG, HB, SHH | T1, T3 |
| 27 | Bozhou | Ningbo | Operate | 1 | SHH, HFS | T1, T3 |
| 28 | Fuyang | Ningbo | Operate | 1 | SHH, HFS | T1, T3 |
| 29 | Fuyang | Jinhua | Operate | 1 | SHH, HK | T1, T3 |
| 30 | Xuancheng | Guangzhou | Operate | 1 | SHH, HK, JG | T1, T3 |
| 31 | Wuhu | Guiyang | Operate | 1 | SHH, HK | T1, T3 |
| 32 | Chongqing | Ningbo | Operate | 1 | HHR, SHH, HFS | T1, T3 |
| 33 | Bozhou | Shenzhen | Operate | 2 | SHH, HK, JG | T1, T3 |
| 34 | Xi'an | Jinhua | Operate | 1 | XL, SHH, HK | T1, T3 |
| 35 | Taiyuan | Ningbo | Operate | 2 | JG, XL, SHH, HFS | T1, T3 |
| 36 | Taiyuan | Huangshan | Operate | 2 | JG, XL, SHH, HF | T1, T3 |
| 37 | Chengdu | Hangzhou | Operate | 2 | HHR, SHH | T1, T3 |
| 38 | Chongqing | Hangzhou | Operate | 1 | HHR, SHH | T1, T3 |
| 39 | Bozhou | Hangzhou | Operate | 14 | SHH | T2-11, T3 |

Note: T1 represents the existing OD demand between the stations along the route of trains; T2- $\times$ represents the transfer OD demand from train Number $\times$ partly; T3 represents the new attracted OD demand between the stations along the route of trains.

There are 24 existing trains that were canceled and 106 newly operated trains that all visit the new line on the HSR network, including 28 in-line trains operated on SHH. The canceled trains were replaced by the trains with the same initial station and terminal station but with shorter route (such as train 1 and train 12). Similarly, the OD demand served by the canceled trains mainly transfers to the newly operated trains with less travel cost. The
other newly operated trains mainly serve the added OD demand attracted by the new line. However, in order to maintain the existing train operational rules, some trains which have shorter routes were still retained-nonetheless, these trains will be gradually canceled in the future for further optimization.

### 7.3. Sensitive Analysis

### 7.3.1. Sensitive Analysis on the Equilibrium Parameter $\alpha$

Figure 9 shows the calculated relevant indicators with different values of parameter $\alpha$. It indicates that both the total travel cost of passengers and the average departure deviation time of passengers are reduced with the increase of parameter $\alpha$, resulting in the general increase of operational trains and operational cost for railway enterprises. Thus, railway enterprises need to comprehensively consider the operational profit and the social commonweal nature of passenger service to decide this parameter.


Figure 9. Sensitive analysis on the equilibrium parameter $\alpha$.

### 7.3.2. Sensitive Analysis on the Time Value of Passengers

Due to the small number of transfer passengers in the HSR network, the effect of the value of parameter $\varepsilon_{2}$ on the result is negligible; hence, the influence of parameter values $\varepsilon_{1}$ and $\varepsilon_{3}$ on the results is mainly analyzed. Along with the increase of the two parameters, the travel cost of passengers will also rise gradually, which is equivalent to increasing parameter $\alpha$. Therefore, the value of $\varepsilon_{1}$ is fixed as 1 and the ratio of $\varepsilon_{1}$ and $\varepsilon_{3}$ is changed. As shown in Table 5, when $\varepsilon_{3} / \varepsilon_{1}=0$; that is, the departure time of the passenger is not considered, average departure deviation time will be as high as 294 min , which means that passengers may not have trains to choose at the time that they expected and have to delay or advance their departure time with a high probability. Along with the growth of the ratio of $\varepsilon_{1}$ and $\varepsilon_{3}$, the total number of train stop stations is incremental, which leads to increased railway costs for operating trains with fluctuation, while there is a significant reduction in average departure deviation time of passengers. However, when $\varepsilon_{3} / \varepsilon_{1}=2.5$, there is a sudden drop in the railway cost of operating trains with unaltered deviation time of passengers; then, it increased as the rule mentioned above. Thus, this ratio can be apply to case studies.

Table 5. Line plan of corridors comparisons between solved results and original results.

| $\varepsilon_{3} / \boldsymbol{\varepsilon}_{\mathbf{1}}$ | The Total Operational <br> Time of Trains (min) | The Total Number of Train <br> Stop Stations <br> (Station) | The Railway Cost of <br> Operating Trains <br> $\left(\times \mathbf{1 0}^{7}\right.$ yuan) | Average Departure <br> Deviation Time of <br> Passengers (min) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 18,172 | 943 | 1.1886 | 293 |
| 0.5 | 18,657 | 1055 | 1.2129 | 68 |
| 1.0 | 19,010 | 1142 | 1.2305 | 55 |
| 1.5 | 19,109 | 1042 | 1.2455 | 52 |
| 2.0 | 19,263 | 1078 | 1.2532 | 48 |
| 2.5 | 18,892 | 1114 | 1.2246 | 48 |
| 3.0 | 19,046 | 1025 | 1.2423 | 46 |
| 3.5 | 19,398 | 1113 | 1.2599 | 47 |
| 4.0 | 19,568 | 1155 | 1.2684 | 44 |

### 7.3.3. Sensitive Analysis on the Parameter $\eta$ of Adjustment Limit

The sensitive analysis of parameter $\eta$ is carried out on a localized network, and the result is as shown in Figure 10. It indicates that the number of actual canceled trains rises with the increased parameter $\eta$, causing improvement in the objective value at first. However, when $\eta$ is large enough, the objective value and the canceled trains will not change, which means that the most satisfactory feasible solution is found in the absence of adjustment limitations. The value of $\eta$ needs to be carefully considered to ensure that the number of canceled trains stays within a certain range to maintain the existing law of running trains, and to ensure that the trains running on the HSR network meet the OD demand after the new line is put into operation.


Figure 10. Sensitive analysis on the parameter $\eta$.

## 8. Conclusions and Policy Recommendations

### 8.1. Conclusions and Further Research

This paper proposes a line-planning approach for HSR networks merging new lines, in which the route, stop patterns, arrival, and departure time at each visiting station of trains are optimized synthetically based on pre-generated candidate train sets. An optimization model for operating trains on the new line and adjusting the trains on the existing network synergistically was formulated and solved under the SAA framework, by which the solution was improved gradually by the search and evaluation strategies proposed. A case study of a Chinese HSR network is presented, and it indicates that the optimized line plan can not only adapt to the changes of OD demand and ensure the service
level, but also can increase the operational revenue of railway enterprises. Furthermore, the optimized line plan maintains the regularity of the existing train organization of the network before merging a new line, which can facilitate passenger travel and railway operational management and realize the collaborative optimization of new lines and the existing networks.

Different from the traditional line-planning problem [1,4-33], the problem studied in this paper is a specific condition in which new trains should be operated for the new railway lines and the existing trains on the railway network should be adjusted properly from the perspective of the entire railway system; meanwhile, the obtained line plan should maintain the continuity of the existing line plan to facilitate passenger travel and railway operation, to which the existing method cannot be directly applied $[2,3]$. For the optimization model, this paper constructed a candidate train set to narrow the scope of solutions due to the complexity of the problem, as noted in [30,31]; however, the decision variables of the generated candidate set were redesigned and restricted according to the characteristics of the problem. Similar to recent research [30,31], the planned departure time of passengers and that of trains was introduced, aiming to enhance the adaptability of the obtained line plan to time-varying passengers in the proposed model. An improved passenger travel network from [23] was constructed that can represent the operation of trains and the travel choices of passengers to facilitate the expression of the model. For the algorithm design, similar to most research $[23,29]$, this paper adopted a heuristic algorithm based on SAA, which can search for satisfactory solutions in a reasonable time, and designed corresponding neighborhood solution search strategies according to the characteristics of the studied problem. The generation approach of candidate trains for newly built railway lines was improved from [31], which can better adapt to time-varying requirements and screen out reasonable trains. Different from [23,30,31], a passenger flow allocation method based on Logit principle under the constraint of the maximum travel cost of passengers was applied in this paper, which better considers the imperfect rationality of passenger travel choice.

However, there are some limitations and constraints in this paper, as follows: (1) the passenger allocation method to simulate passenger travel behaviors is not completely consistent with the actual travel choice of passengers; (2) although the time-varying OD demands were considered, a fixed total OD demand was used in the studied periods, ignoring the relationship between OD demands and operated trains; (3) while the candidate train set is determined, the search range of the solution is determined; that is, the quality of the generated candidate train set has a great influence on the subsequent solution.

In future work, we intend to focus on extensions of this research as follows: (1) proposal of a more accurate method for HSR passenger allocation to describe passenger travel behaviors more precisely; (2) consideration of the elastic passenger transport demand, which varies with the change of general travel costs, to flexibly design line plans; (3) extension of research findings to the integrated optimization of line planning and timetabling that involves conflicts and connection relationship among trains.

### 8.2. Policy Recommendations

Generally speaking, the newly built HSR lines will have an impact in three aspects after they are merged by the existing HSR network. From the perspective of the overall social and economic interest and the operational efficiency of the whole railway system, the change in the network structure leads to adjustment on the railway transportation organization and changes on passenger travel, which are closely connected with each other, and their mutual influence is intricate. It is extremely difficult to quantify the impact of the merged railway lines on the existing line plan so that the line plan of the new HSR network should be redesigned to achieve the optimum of the whole system. However, to redesign the line plan of the new HSR network and implement the plan is unrealistic in actual railway operation. From the point of view of railway operational enterprise, a new line plan will bring a subversive impact on the railway operation organization, such that a new timetable
for each train should be prepared, a new motor train-set plan should be determined, and a new crew plan should be made. This requires a great deal of manpower and material resources and is extremely difficult to implement. Therefore, railway enterprises usually adjust the existing trains based on the original line plan, including canceling the trains with low efficiency and detours, so as to improve the efficiency of train operation. Additionally, railway enterprises will organize the trains related to the new railway line to satisfy the attracted OD demand, aiming to increase railway revenue. From the perspective of passengers, the new railway line and added trains can shorten the travel distance of some passengers and reduce their travel time, provide HSR services for the attracted passengers along the new railway line, and serve some transfer passengers with additional choices. The adjustment of the existing trains provides passengers with higher-quality train choices, but may change the travel habits of some passengers. For example, the passengers who planned to travel in the morning can only choose trains that leave in the afternoon after adjustment, which will bring inconvenience to a small number of passengers, but the overall passenger service level can be improved.

In conclusion, the optimization of the line plan of HSR networks merging newly-built railway lines needs to comprehensively consider the social benefits, the interests of the operating enterprises, and the passengers, as considered in this paper. First of all, the in-line and cross-line trains running on the new HSR line should be organized felicitously to meet the attracted time-varying OD demands. Second, the existing trains with obvious detours and low operating efficiency should be considered for cancelation or adjusted to improve train efficiency. Third, adjustment and optimization of the existing trains should consider the perspective of the HSR network and limit the range and scope of adjustment to prevent fundamental damage to the order and law of the existing railway operation and attain coordination with the newly operated trains. Finally, the optimization of the line plan of the new HSR network should be regarded as a long-term dynamic process, where railway enterprises should adjust the operated train in different terms step by step, so as to reduce the impact of the adjustment on railway operation and passenger travel and aim to maximize the overall benefit.

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## Appendix A. Illustration of Candidate Trains Generation in Initial Solution

As shown in Figure A1, there are five stations denoted as A, B, C, D, and E on the new high-speed railway line, wherein stations A, C, and E are capable of starting and ending trains. The mileage of track section between each pair of adjacent stations is shown in the figure. It assumes that the minimum load factor of a train is $\bar{\sigma}=0.65$, the capacity of a train is Cap $=500$, and the size of the train's departure time range is $T_{f}=30 \mathrm{~min}$. Now the candidate in-line trains in the downward direction are generated by the following process:

Step 1: Generate feasible routes for train between stations A, C, and E. As shown in Figure A1a, there are three feasible routes for the train, which are sorted from the longest to the shortest as $l_{1}: \mathrm{A} \rightarrow \mathrm{B} \rightarrow \mathrm{C} \rightarrow \mathrm{D} \rightarrow \mathrm{E}, l_{2}: \mathrm{C} \rightarrow \mathrm{D} \rightarrow \mathrm{E}, l_{3}: \mathrm{A} \rightarrow \mathrm{B} \rightarrow \mathrm{C}$.

Step 2: Allocate the OD demand of a certain period $h$ (such as 7:00-8:00) to each track section in chronological order, as shown in Figure A1b, and add the period of $h$ into the set denoted as $H^{\prime}$. Assume that a train with route is $l_{1}$ is operated, then calculate its load factor: $\sigma=\frac{50 \times 60+80 \times 70+50 \times 60}{500 \times(60+50+70+60)} \approx 0.097<0.65$. Thus, this candidate train fails to meet operational requirements. Assume that a train with route is $l_{2}$ is operated, then calculate its load factor and calculate its load factor: $\sigma=\frac{70 \times 80+60 \times 50}{500 \times(70+60)} \approx 0.132<0.65$. Thus, this candidate train fails to meet operational requirements. Assume that a train with route is $l_{3}$ is operated, then calculate its load factor and calculate its load factor: $\sigma=\frac{60 \times 50}{500 \times(50+60)} \approx 0.054<0.65$. Thus, this candidate train also fails to meet operational requirements.

Step 3: Based on the remaining OD demand in each track section, load the OD demand in period $h+1$ (such as 8:00-9:00) to each track section as shown in Figure A1c, and add the period $h+1$ to $H^{\prime}$. Suppose that a train with route $l_{1}$ is operated, then calculate its load factor: $\sigma=\frac{500 \times 60+400 \times 50+500 \times 70+500 \times 60}{500 \times(60+50+70+60)} \approx 0.95>0.65$. Thus, a candidate train1 with route is $l_{1}$ is generated and added to the set of operational trains denoted as $\Psi_{t}$.

Step 4: Update passenger flow in each track section, as shown in A1d. Assume that a train with a route of $l_{1}$ is operated, then calculate its load factor: $\sigma \approx 0.48<0.65$. Assume a train with a route of $l_{2}$, then calculate its load factor: $\sigma \approx 0.75>0.65$. Thus, a candidate train2 is generated and added to the $\Psi_{t}$ with route $l_{2}$.

Step 5: Update passenger flow in each track section, as shown in A1e. Assume that trains with routes $l_{1}, l_{2}$, and $l_{3}$ is operated in sequence, then calculate their load factor respectively by the method mentioned above. However, all the case trains here do not satisfy the minimum occupancy requirements. So go to Step 6.

Step 6: Now there are two candidate trains with initial stations A and C. Set the departure time range (i.e., $7: 45-8: 15$ ) of the two candidate trains to the middle time range in $H^{\prime}$ (i.e., 7:00-9:00) with time range $T_{f}$, then add the candidate trains to the $\Psi_{b}$ and set $\Psi_{t}=\varnothing, H^{\prime}=\varnothing$.

Step 7: On the basis of the remaining passengers in each track section, allocate passengers for the next period. Repeat Step 2 to Step 5 until candidate trains are generated for all OD demands. Then output the candidate train set $\Psi_{b}$.


Figure A1. Illustration of generating in-line candidate trains. (a) Railway line and candidate train routes. (b) Load OD demand in period $h$ to track section. (c) Load OD demand in period $h+1$ to track section and generate a candidate train. (d) Update the OD demand in each track section and generate a new candidate train. (e) Update the OD demand in each track section again.

The process of generating cross-line candidate trains is similar to that of generating in-line candidate trains. The primary difference is that candidate trains are generated after
all OD demands among all periods are loaded into each track section. Furthermore, there are particular restrictions on train route generation.

As shown in Figure A2, there are six stations denoted as A, B, C, D, E, and F located in a small HSR network which consists of a new line and an existing line, wherein stations A, C, D, and F are capable of starting and ending trains. The figure assumes that the passengers between stations $B$ and $F$ are sufficient for operation of a train with a route that is determined by the following procedure:

Step 1: generate a route for train between station B and F by the shortest route, as route 1 shown in Figure A2.

Step 2: Check whether the initial station and terminal station of the route both are capable of starting or ending trains. As station B does not meet this condition, route 1 is extended to the nearest station, which satisfies the above conditions, to form a new route 2 .

Step 3: Check whether the mileage of route 2 is longer than the longest route permitted. If so, route 1 is shortened to the second nearest station to form a new route 3 .

Step 4: Check whether route 3 satisfies the constraint of the circuities of cross-line train routes that prevent trains from taking detours. In other words, calculate the quotient of the mileage of the route and the Euclidean distance between station C and F and judge whether the value exceeds the threshold.

Step 5: If all the above conditions are satisfied, a feasible train path is generated.


Figure A2. Illustration of generating routes of cross-line candidate trains.

## Appendix B. Detailed Information on Railway Lines Studied in the Case Study

Table A1. Information on HSR lines in the studied network.

| No. | Name | Origin | Terminal | Mileage <br> $\mathbf{( k m})$ | Number of <br> Stations | Number of <br> Existing Trains | Number of <br> Passengers |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | JH | Beijing | Shanghai | 1318 | 23 | 228 | 302,332 |
| 2 | JG | Beijing | Guangzhou | 2298 | 38 | 218 | 291,259 |
| 3 | GS | Guangzhou | Shenzhen | 102 | 5 | 248 | 274,959 |
| 4 | HFS | Hangzhou | Shenzhen | 1464 | 52 | 57,820 |  |
| 5 | QT | Qingdao | Taiyuan | 857 | 23 | 56 | 31,112 |
| 6 | XL | Xuzhou | Lanzhou | 1434 | 29 | 22 |  |
| 7 | XLY | Xuzhou | Lianyungang | 185 | 5 | 20 |  |

Table A1. Cont.

| No. | Name | Origin | Terminal | Mileage (km) | Number of Stations | Number of Existing Trains | Number of Passengers |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | HK | Shanghai | Kunming | 2252 | 53 | 202 | 224,195 |
| 9 | HHR | Shanghai | Chengdu | 1985 | 38 | 160 | 201,161 |
| 10 | GK | Guangzhou | Kunming | 1287 | 28 | 40 | 53,647 |
| 11 | GG | Guiyang | Guangzhou | 857 | 23 | 24 | 34,348 |
| 12 | HL | Hengyang | Nanning | 721 | 14 | 16 | 16,857 |
| 13 | CG | Chengdu | Guiyang | 648 | 24 | 10 | 11,290 |
| 14 | YG | Chongqing | Guiyang | 346 | 12 | 12 | 15,359 |
| 15 | XC | Xi'an | Chengdu | 658 | 22 | 14 | 22,020 |
| 16 | LY | Lanzhou | Chongqing | 862 | 16 | 16 | 17,876 |
| 17 | DX | Datong | Xi'an | 859 | 25 | 30 | 36,062 |
| 18 | WJ | Wuhan | Jiujiang | 223 | 16 | 42 | 29,561 |
| 19 | CJ | Nanchang | Jiujiang | 138 | 6 |  |  |
| 20 | QJ | Jiujiang | Quzhou | 334 | 10 |  |  |
| 21 | CG | Nanchang | Ganzhou | 385 | 12 |  |  |
| 22 | CF | Nanchang | Fuzhou | 547 | 11 | 6 | 6727 |
| 23 | GLC | Ganzhou | Xiamen | 442 | 13 | 4 | 7233 |
| 24 | NL | Yanping | Longyan | 247 | 9 | 4 | 7074 |
| 25 | JW | Jinhua | Wenzhou | 183 | 8 | 4 | 5659 |
| 26 | QY | Qingdao | Lianyungang | 253 | 8 | 4 | 4912 |
| 27 | NH | Nanjing | Hangzhou | 256 | 11 | 20 | 27,514 |
| 28 | HB | Hefei | Bengbu | 132 | 5 | 28 | 36,842 |
| 29 | HF | Hefei | Fuzhou | 850 | 22 |  |  |
| 30 | SHH | Shangqiu | Hangzhou | 795 | 28 | N.A. | 147,439 |
| 31 | Cross-line | N.A. | N.A. | N.A. | N.A. | 338 | 287,524 |

Note: HSR lines 2 and 3, lines 6 and 7 , line 18, 19, 20, 21, 28, and 29 are usually regarded as a whole when organizing operational trains; therefore, the total number of passengers and the total number of existing trains on them are given in the table. The number of passengers on line 30 (namely the new line) includes the new passengers which are attracted by the line and the cross-line passengers whose origin or destination is a station along the new line. N.A. means not available or not applicable.

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