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A Multi-Period Vehicle Routing Problem for Emergency Perishable Materials under Uncertain Demand Based on an Improved Whale Optimization Algorithm

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Abstract: The distribution of emergency perishable materials after a disaster, such as an earthquake, is an essential part of emergency resource dispatching. However, the traditional single-period distribution model can hardly solve this problem because of incomplete demand information for emergency perishable materials in affected sites. Therefore, for such problems we firstly construct a multi-period vehicle path distribution optimization model with the dual objectives of minimizing the cost penalty of distribution delay and the total corruption during delivery, and minimizing the total amount of demand that is not met, by applying the interval boundary and most likely value weighting method to make uncertain demand clear. Then, we formulate the differential evolutionary whale optimization algorithm (DE-WOA) combing the differential evolutionary algorithm with the whale algorithm to solve the constructed model, which is an up-and-coming algorithm for solving this type of problem. Finally, to validate the feasibility and practicality of the proposed model and the novel algorithm, a comparison between the proposed model and the standard whale optimization algorithm is performed on a numerical instance. The result indicates the proposed model converges faster and the overall optimization effect is improved by 23%, which further verifies that the improved whale optimization algorithm has better performance.

Keywords: emergency material distribution; multi-period; uncertain demand; perishable materials; whale optimization algorithm; differential evolution algorithm

MSC: 90B06

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1. Introduction

Large-scale sudden natural or man-made disasters occur frequently around the world every year, posing serious threats and impacts on society, human production, and life [1]. How to respond quickly effectively to these unpredictable emergencies has attracted much attention from governments and management at all levels, and has also placed high requirements on them from all aspects [2].

A scientific distribution and reasonable delivery of emergency relief materials, a key aspect of emergency relief work, can reduce the damage to property and casualties caused by disasters, improve the efficiency of rescue work and release the psychological pressure on the victims [3–5]. Due to the suddenness of disasters and the urgency of rescues, the demand for perishable emergency supplies for affected locations is often vague [6]. In addition, the longer the transport time, the more serious the spoilage phenomenon. Currently, this problem can be solved by a single-period delivery model. However, in this case, plenty of restrictions influence the solution's precision. For example, the actual demand for one disaster site is much greater than the maximum loading capacity of a vehicle, and the number of vehicles is limited. Thus, the single-period delivery cannot satisfactorily solve this problem. To more efficiently solve this problem, we consider a multi-period distribution model. Given the situation of sufficient supplies in the distribution center, one can use it to make decisions on the distribution vehicle's path and the distribution quantity in each period to minimize the cost penalty of distribution delay and total corruption during delivery, and minimize the total amount of demand that is not met, which is worthy of studying to improve relief work's efficiency and reduce losses in disaster areas.

The remainder of this paper is organized as follows. Section 2 performs literature reviews relevant to this study. Section 3 constructs the optimization model with the bi-objective and multi-period vehicle path distribution, and proposes the improved differential whale optimization algorithm, which is a novel algorithm for solving the vehicle path problem with multi-objective optimization. Section 4 presents a practical example to verify the validity of the proposed model and algorithm. A comparison of the solution results of the algorithm before and after the improvement reveals that the improved differential evolutionary whale optimization algorithm optimizes better regarding the two objectives of minimizing the distribution delay penalty and corruption cost, and minimizing the unsatisfied degree of demand. Finally, conclusions and possible future research are given in Section 5.

2. Literature Review

The vehicle routing problem (VRP), as a classical problem in the field of operations research and combinatorial optimization, has been widely studied and played a significant role in transportation, logistics production and emergency rescue since its introduction in 1959 by Dantzig and Ramser [7]. A large number of experts and scholars have conducted in-depth research and analysis on it so far. Many variants of the VRP problem have been derived, and related theories and models have become relatively mature, among which, the multi-period vehicle path problem (PVRP) is one. Traditional vehicle paths and their derivatives are mostly deterministic vehicle path optimization problems, where the relevant variables are known in advance. However, in practice, uncertain information abounds whether in production transport or emergency relief, including demands, road conditions, casualties and so on. It can be divided into fuzzy information, random information and dynamic information concerning the properties of uncertain information. Therefore, the analysis and research of uncertain vehicle path optimization problems have become the focuses of experts and scholars. With the increasing development of intelligent optimization algorithms in recent years, a good research foundation has been laid for solving such problems.

The problem of optimizing the routes of emergency material distribution vehicles is a typical VRP problem in which the distribution center provides materials to some demand points with different quantities of materials, and vehicles are assigned to appropriate routes which form closed loops such that departure and final return are both the distribution center, so that the demand points' needs are met. Such goals of minimum transport costs, shortest driving distance and time spent under certain constraints should also be accounted for. Given the condition of the demand of distribution being known, the shortest driving distance is used as a goal to indicate the shortest resource allocation time, and a suitable distribution path is selected for a vehicle to satisfy the distribution demand of each affected location. Zhou et al. constructed a heterogeneous vehicle path optimization model for the vehicle path problem in which the pre-emergency transporters' vehicles are insufficient; the maximum system satisfaction and the minimum total time and the total cost were considered as the goals [8]. Li Zhuo et al., focusing on different interests of demand points and transporters, developed a multi-objective hybrid vehicle path optimization model with a soft time window, and a non-dominated sorting ant colony algorithm was proposed to solve this model. An arithmetic case analysis indicated the effectiveness of the modified algorithm [9].

For the multi-site, open-emergency material distribution vehicle path problem considering secondary disasters, with the objective of shortest transportation time, Tan Jie et al. established two types of mathematical planning models that, under the one-sided fuzzy soft time window and fuzzy demand constraints, consider the risk of random failure at supply points, and designed an improved variable neighborhood search algorithm to solve the problem [10].

To solve the site-path problem of post-disaster emergency relief, different objectives and models were developed by scholars. With the objectives of maximizing rescue efficiency and minimizing the total cost, Gao Xinyu et al. developed a multi-stage site-path optimization model under the constraint of demand uncertainty and proposed an improved fast non-dominated genetic algorithm [11]. To maximize the matching degree of emergency demand at each dispatch point in the current stage, minimize the variance of the average matching degree of emergency demand at the previous k stages of dispatch and minimize the total travel time of the dispatch path, Liu Yang et al. constructed a multi-stage distribution and dispatching model for emergency relief supplies based on the historical travel time functions of road sections to portray the dynamics of the traffic on a road network [12]. In addition, an integrated optimization algorithm and coding adjustment strategy were made for the solution of multi-stage distribution and dispatching of disaster relief supplies. With the objective of minimizing the maximum distribution time, Zhou Yufeng et al. formulated an emergency facility siting-allocation model applicable to the initial post-earthquake relief phase by considering the phase characteristics, facility disruption scenarios, multi-species uncertain demand, facility capacity limitations, etc. The defuzzification of uncertain demand was processed through the expectation value formula of interval boundary, and on this basis, the result could be obtained by the proposed hybrid integer coding genetic algorithm [13].

The period vehicle routing problem (PVRP) was firstly proposed by Bekrami and Bodin in 1974 [14], arguing that different customers have different access frequencies for the recycling of industrial waste in New York City. Christofides et al. (1984) initially constructed a mathematical model of PVRP [15]. After nearly forty-five years of development, PVRP has been further extended in practical applications, such as the period vehicle routing problem with time window (PVRPTW), multidepot and periodic vehicle routing problems (MAPVRP) and the dynamic multi-period vehicle routing problem (DPVRPD) [16–19], and other existing studies mostly used heuristic algorithms to solve the extended PVRP model.

Wang et al. (2019) put forward a multi-stage model for distributing emergency supplies to multiple affected locations with the objectives of minimizing losses caused by shortages, total fixed costs of transportation and distribution costs. They designed and constructed a nonlinear utility function to reflect the negative utility caused by a lack of funding, and experiment results proved the feasibility of this model [20]. With the objectives of minimizing total delay time and total system loss for distribution of emergency supplies, Wang Yanyan et al. developed a dynamic distribution optimization decision model that uses fuzzy information conditions with multiple demand points, multiple distribution centers, multiple supplies, multiple periods and multiple objectives. After analyzing the clarification methods of the interval objective function, interval fuzzy constraints and triangular fuzzy constraints, they designed a two-dimensional Euclidean distance-based objective empowerment fuzzy algorithm to solve the model [21]. With the dual objectives of minimizing the risk of sending unsatisfying amounts of resources and minimizing the risk of resources not reaching disaster areas, Zhou et al. considered of the inherent nature of the multi-period dynamic emergency resource scheduling (ERS) problem to establish a multi-objective optimization model for the multi-period dynamic emergency resource scheduling (ERS) problem, and a decomposition-based multi-objective evolutionary algorithm (MOEA/D) was made to achieve great performance [22].

The vehicle routing problem for perishable goods (VRPFPG) is one of the vehicle routing problems (VRP) [23]. Large quantities of perishable goods around the world are transported from suppliers to consignees on a daily basis. Perishable goods, such as food and pharmaceuticals, require special handling during transportation due to their limited lifespans, and they must be transported as fast as possible before they deteriorate. Besides transport time constraints, the high frequency of transport can generate high transportation costs, which makes the optimization of perishable materials particularly vital. With the multiple objectives of minimizing operational costs, spoilage costs and carbon emissions, and maximizing customer satisfaction, Zulvia et al. paid attention to time windows, different travel times during peak and off-peak hours and working hours to develop a green VRP model and design a multi-objective gradient evolution (MOGE) algorithm whose results showed great performance [24].

To solve the perishable with uncertain demand material distribution vehicle path problem, researchers constructed various models with different objectives. With minimum total cost, maximum product freshness and minimum carbon emission as objectives, Qian Zhang et al. established a multi-objective optimization model for distribution path planning, and designed the main objective method and fruit fly algorithm based on robust optimization to deal with the uncertainty problem [25]. With the objective of minimizing the operating cost and emission cost, Babagolzadeh et al. constructed a two-stage stochastic planning model to determine the optimal replenishment strategy and transportation plan in the presence of carbon tax controls and uncertain demand, and an improved result was obtained by the proposed mathematical algorithm with respect to iterative local search (ILS) algorithm and mixed integer programming [26]. With the objectives of minimizing costs, minimizing environmental impacts and maximizing customer service levels, Talouki et al. formulated a dynamic green vehicle path model for perishable material under green transportation conditions in view of time window implementation, and then designed an algorithm based on a new augmented-constrained heuristic for solutions [27]. With the goal of profit maximization, Wu et al. developed a variable fractional inequality distribution path optimization model considering the uncertainty of perishable food demand for high speed rail and designed an augmented Lagrangian with the Euler algorithm based on the pairwise algorithm [28].

For the problem of uncertainty in demand and return of perishable goods with different periods, with the objective of minimizing the total cost of the system, Guo Jiangyan et al. constructed a multi-period closed-loop logistics network for perishable goods and figured out a mixed-integer linear programming (MILP) model solving by a proposed genetic algorithm [29].

For the problems of high-frequency distribution, uncertain demand and return of fresh goods due to perishability, with the objective of minimizing the total cost of the system, Yang et al. constructed the corresponding fuzzy mixed-integer linear programming (FMILP) model for the system and designed genetic algorithm (GA) and particle swarm optimization (PSO) algorithms to solve it [30].

The vehicle path problem is considered as an NP-hard problem, so it may be time-consuming and ineffective to use ordinary mathematical methods, such as exact algorithms, to deal with it. Most scholars nowadays use intelligent optimization algorithms for solving such problems. The whale optimization algorithm (WOA) is a biomimetic metaheuristic algorithm developed by Mirjalili et al. in 2016 to simulate the feeding mode of whales [31]. In recent years, it has been successfully applied to some large-scale optimization problems with the advantages of few artificial parameters and simple operation, such as resource scheduling problems [32], workflow planning for construction sites, site selection and path planning [33] and neural network training [34]. However, because the traditional WOA has the disadvantages of slow late convergence and easily falling into a local optima, some scholars have combined other algorithms with it to improve its performance in operation speed. Rohit Salgotra et al. addressed the problems of poor search performance and easily falling into a local optimum of the WOA algorithm. Three different improved versions, including WOA-adversarial-based learning, exponentially reduced parameters and worst-particle elimination and reinitialization methods, have been proposed to improve its exploratory capabilities. These properties have been exploited to improve the exploration capabilities of WOA by maintaining the diversity among search agents [35]. Shang Mang et al. proposed a WOA-based vehicle path optimization method for the distribution logistics of the VRP problem; modified the WOA algorithm using random inertia weights and a non-uniform variation strategy; and verified the effectiveness of the improved algorithm by testing functions. The verification results showed that the improved whale optimization algorithm can efficiently optimize the distribution path for vehicles and reduce the distribution cost of logistics [36].

As a novel algorithm, the WOA algorithm has attracted extensive attention from scholars in various fields since a basis has been built for the research, development and improvement of the algorithm, and application studies have been conducted regarding engineering, scheduling, optimization and site selection. Additionally, there is a richness in algorithm improvement. However, there are fewer applications in vehicle path research, so further development and utilization are needed.

A great deal of research has been carried out in the existing literature on the optimization of vehicle paths for the distribution of emergency and perishable materials. In addition to large demands for emergency supplies, such as communication equipment, quilts and tents, in the early stage of post-disaster relief, there also would be large demands for life-saving and living emergency supplies, such as medicines and foodstuffs. As for the perishable characteristics of these emergency supplies, along with the likelihood of severe damage to some roads, there is often uncertainty about the needs of the affected sites, making it difficult for these emergency perishable supplies to be delivered quickly and meet demand requirements at once. Therefore, in order to improve the optimization efficiency, this paper combines the differential evolutionary algorithm with the whale optimization algorithm to solve the vehicle path problem for the distribution of emergency perishable materials with dual objectives, which is rarely studied at present. Finally, further verification of the effectiveness of the improved whale optimization algorithm at solving the realistic vehicle path problem through examples shows convincing performance.

3. Multi-Period Optimization Model and DE-WOA Algorithm

The distribution vehicle path problem for emergency perishable materials has special characteristics compared to the same problem for general emergency materials, which negatively affect the solving process. The standard whale optimization algorithm greatly improves the operation efficiency of the algorithm because of the relatively simpler process and searching mechanism. Thus, it is suitable for solving the problem of emergency perishable material distribution optimization. This sub-section, while taking the uncertain demand situation into account, analyses the situation of adequate supplies in distribution centers and constructs a multi-period vehicle path distribution optimization model with the dual objectives of minimizing the cost penalty of distribution delay and total corruption during delivery, and minimizing the total amount of demand that is not met. The improved differential evolutionary whale algorithm is designed to solve the model by combining the features of the differential evolutionary algorithm with the standard whale optimization algorithm with strong global search capability.

3.1. Description of the Problem

Given a simple discrete undirected road traffic network $G = (V, E)$, where $V = (v_0, v_1, v_2, \ldots, v_n)$ is the set of nodes and v_0 denotes the distribution center for emergency perishable relief supplies, $v_1, v_2, \ldots, v_n$ denotes the affected point and $E = \{e(v_i, v_j) | v_i, v_j \in V\}$ is the set of edges. Assume that the supplies are sufficient. The demand for emergency perishable supplies at the affected point v_i is represented by interval boundary $\widetilde{D}_i (i = 1, 2, \ldots, n)$. The distribution center v_0 possesses a sufficient emergency perishable supply, and the total quantity is S. The spoilage rate of emergency perishable supplies during the distribution process will linearly change along with the distribution time, and it is θt_j. The demand for emergency perishables at each site can be hardly met at once due to uncertain information on demand and limited vehicle capacity.

v_0: The distribution center now has k vehicles available with a maximum capacity of R for each; d_{ij} represents the distance between any two points; v_{ij}, depending on the road conditions, represents the actual speed of the vehicle on edge (i,j) during transportation, and $\overline{v}_{ij}$ represents the average speed of the vehicle so that the actual time for the vehicle to reach the disaster site j is $t_j = d_{ij}/v_{ij}$ and the ideal time is $\overline{t}_j = d_{ij}/\overline{v}_{ij}$. c_j is defined as the delay penalty, relying on the demand and the degree of damage at the disaster site. The distribution service will deliver emergency perishable materials to each disaster site and back to the distribution center until all the needs of the disaster sites are met. The most probable value weighting method is used to identify the uncertain demand, and the distribution route and the amount of each demand point in each period are decided with the dual objectives of minimizing the cost penalty of distribution delay and the total corruption during delivery, and minimizing the total amount of demand that is not met.

The model was constructed based on the following assumptions.

(1) The demand points' locations and total amounts are known.
(2) A tour of one vehicle is a closed loop such that its departure and final return are both the distribution center.
(3) The condition of the roads and the extent of damage to the affected sites are known for each period, so vehicles' ideal and actual speeds can be calculated.

3.2. Model Building and Notation Definition

The symbols and parameters used in this model are defined in Table 1. Decision variables are identified in Table 2.

Table 1. The symbols and parameters used in the model.

P	Collection of distribution periods
Z	The set of all nodes
R	The maximum load capacity per vehicle
$\widetilde{D}_{ip}$	Disaster site i demand for emergency perishable goods for period p
S	Total amount of material in distribution center
t_0	The ideal arrival time of vehicles
t_j	The actual time of arrival of the vehicle
c	Cost of delay penalties per unit of vehicle delivery time
k	The number of vehicles that can be arranged
δ	The minimum permissible rate of spoilage of material during vehicle transport
ω	Vehicle utilization

Table 2. Decision variables.

x_{ijk}^{p}	1, if vehicle transports material from point i to point j in period p 0, otherwise
y_{ik}^{p}	1, if the task at point i is performed by vehicle k 0, otherwise
d_{ipk}	Volume of emergency perishable materials provided by vehicle k to disaster site i in period p

Taking into account all the objectives and constraints, the model is developed.

$$\min \sum_{i \in Z} \sum_{p \in P} (t_j - t_0) c_{1j} x_{ijk}^{p} + c_{2j} d_{ipk} \theta t_j,\ while\ t_j \leq t_0,\ t_j - t_0 = 0 \tag{1}$$

$$\min \left\{ \sum_{p \in P} 1 - d_{ipk} (1 - \widetilde{\theta} t_j) / \widetilde{D}_{ip},\ i \in Z,\ k \in K \right\} \tag{2}$$

$$s.t. \sum_{i \in Z} d_{ipk} y_{ik}^{p} \leq R,\ k \in K,\ p \in P; \tag{3}$$

$$\sum_{i \in Z} \sum_{k \in K} d_{ipk} \leq S,\ p \in P; \tag{4}$$

$$0 \leq \sum_{k \in K} d_{ipk} \theta t_j \leq \widetilde{D}_{ip},\ i \in Z,\ p \in P; \tag{5}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk}^{p} \leq |S| - 1,\ k \in K,\ p \in P; \tag{6}$$

$$\sum_{j \in Z} x_{ojk}^{p} = \sum_{j \in Z} x_{jok}^{p} \leq 1,\ i \in Z,\ k \in K, p \in P; \tag{7}$$

$$\sum_{j \in Z, i \neq j} x_{ijk}^{p} \leq 1,\ i \in Z,\ k \in K, p \in P; \tag{8}$$

$$x_{ijk}^{p} \leq y_{ik}^{p}; \tag{9}$$

$$0 \leq \frac{d_{ipk} \theta t_j}{d_{ipk}} \leq \delta; \tag{10}$$

$$\frac{d_{ipk}}{R_k} \geq \omega; \tag{11}$$

$$x_{ijk}^{p} = 0\ or\ 1,\ y_{ik}^{p} = 0\ or\ 1,\ (i,j) \in Z,\ p \in P; \tag{12}$$

$$d_{ipk} \geq 0; \tag{13}$$

Equations (1) and (2), respectively, represent the dual objectives that minimize the cost penalty of distribution delay and the total corruption during delivery and minimize the total amount of demand that is not met. Equation (3) guarantees the load amount of each vehicle does not exceed the maximum capacity per vehicle. Equation (4) ensures that the total amount of distribution in each period is less than the available amount in the distribution center. Equation (5) is aimed at restricting the amount of emergency perishable supplies delivered to the disaster site in each period that does not exceed its ideal demand. Equation (6) indicates that the sub-loop in the distribution process is broken. Equation (7) guarantees each vehicle starts and ends transportation at the distribution center. Equation (8) presents a vehicle does not pass through the same path twice or more in any period. Equation (9) ensures that the vehicle serves a disaster site before passing through. Equation (10) indicates the degree of spoilage of emergency perishable materials during distribution should be less than a given rate. Equation (11) represents that each vehicle's utilization rate for each period should be more than a given rate. Equation (12) is related to the integer variable constraint. Equation(13) represents the non-negative constraint.

Where $\widetilde{D}_{ip} = \widetilde{D}_{ip} + |\widetilde{D}_{i(p-1)} - \sum d_{ik(p-1)}|$ when $p \geq 2$.

3.3. Clarity of Ambiguous Needs

In this paper, uncertain demand is expressed as interval boundary:

$$\widetilde{D}_{ip} = [q_{1i}, q_{2i}, q_{3i}],\ q_{1i} \leq q_{2i} \leq q_{3i} \tag{14}$$

The affiliation function is:

$$\mu_{\widetilde{D}_i}(x) = \begin{cases} 0 & x \leq q_{1i}, x \geq q_{3i} \\ (x - q_{1i})/(q_{2i} - q_{1i}) & q_{1i} < x < q_{2i} \\ (q_{3i} - x)/(q_{3i} - q_{2i}) & q_{2i} < x < q_{3i} \end{cases} \tag{15}$$

where $q_1 i, q_2 i$ and $q_3 i$ represent the left boundary, the point with affiliation 1 (most likely value) and the right boundary of the interval boundary, respectively. The interval boundary is constant with the weights given by experts or decision-makers. $\widetilde{D}_{ip} = [q_{1i}, q_{2i}, q_{3i}]$ is expressed by the Equation (16)

$$\widetilde{D}_{ip} = \omega_1 q_{1i} + \omega_2 q_{2i} + \omega_3 q_{3i}. \tag{16}$$

ω_1 is the weight of the lower boundary, ω_2 is the weight of the most probable value and ω_3 is the weight of the upper boundary.

Such methods that determine weights by experience and knowledge of experts or decision makers are relatively subjective. The results thus are influenced by strong human elements. Some more objective methods to identify fuzzy weights were developed, such as the same weight method and hierarchical analysis. The most common is the most likely method. The most likely value of the interval boundary is given the highest weight, as it is most accurate. The value of boundary is less accurate; thus, they are assigned smaller weights.

To indicate differences between the three estimates $q_{1,q}$ and q_3, the weights of them are consequently determined by $\omega_1 = \omega_3 = 1/6$ and $\omega_2 = 4/6$. Therefore, Equation (15) can be converted into Equation (16).

$$\widetilde{D}_{ip} = \frac{q_{1i} + 4q_{2i} + q_{3i}}{6} \tag{17}$$

After replacing Equation (5) with Equation (17), the updated constraint is shown as Equation (18):

$$0 \leq \sum_{k \in K} d_{ipk} \leq \frac{1}{6}q_{1i} + \frac{4}{6}q_{2i} + \frac{1}{6}q_{3i},\ i \in Z,\ p \in P \tag{18}$$

$$\min\left\{\sum_{p \in P} 1 - d_{ikp}(1 - \widetilde{\theta} t_j) / \frac{1}{6}q_{1i} + \frac{4}{6}q_{2i} + \frac{1}{6}q_{3i},\ i \in Z,\ k \in K\right\} \tag{19}$$

3.4. Handling of Dual Targets

The ϵ conventional method aims to convert a muti-objective problem into a single-objective optimization problem by linear weighting. However, because of the non-uniformity of the objective magnitude, the solution of the original problem and that of the converted problem are not in simple one-to-one correspondence. The weights of each objective may largely affect the accuracy of solutions. This paper takes advantage of the idea of the constraint method (Haimes et al. 1971), combining it with an improved differential evolutionary whale algorithm.

In this case, two single-objective optimization problems are solved by converting one of the dual objectives into the other's constraints based on the importance of the objectives in each period in turn and solving them separately to obtain the Pareto solution set of the model.

1. Construct a single objective optimization problem with objective A and objective B, respectively, and find the value domain (upper and lower bounds) of the two objective functions.

Objective A.

$$\begin{aligned} &\min \sum_{j \in Z} \sum_{p \in P} (t_j - t_0) c_{1j} x_{ijk}^{p} + c_{2j} d_{ipk} \theta t_j,\ while\ t_j \leq t_0,\ t_j - t_0 = 0 \\ &s.t.\ constraint(1) - constraint(13) \end{aligned} \tag{20}$$

Objective B.

$$\begin{aligned} &\min\left\{\sum_{p \in P} 1 - d_{ipk}(1 - \widetilde{\theta} t_j) / \widetilde{D}_{ip},\ i \in Z,\ k \in K\right\} \\ &s.t.\ constraint(1) - constraint(13) \end{aligned} \tag{21}$$

2. Step 1 finds the minimum value of objective A as m, and then adds $Z_A \leq a$ as a constraint to get the result of objective B. Construct a single-objective optimization problem for objective B. If the problem has a feasible solution, find the optimal solution for objective B as Z_B^*, and go to Step 3; if there is no feasible solution, go to Step 4.

3. Then, add $Z_B \leq Z_B^*$ as a constraint to objective A to construct a single objective optimization problem for objective A. Similarly, if there is a feasible solution to the problem, the optimal solution to objective A is found at Z_A^*, at which point the solution obtained in the above step is counted in the Pareto solution set; if there is no feasible solution, then go to step 4.

4. Make $a = a + \epsilon$. ϵ is a fixed step; go to step 2 to continue solving.

5. Stop when a is greater than the maximum value of target A.

3.5. The Basic Process of DE-WOA

The improved differential evolutionary whale algorithm is computed as Figure 1.

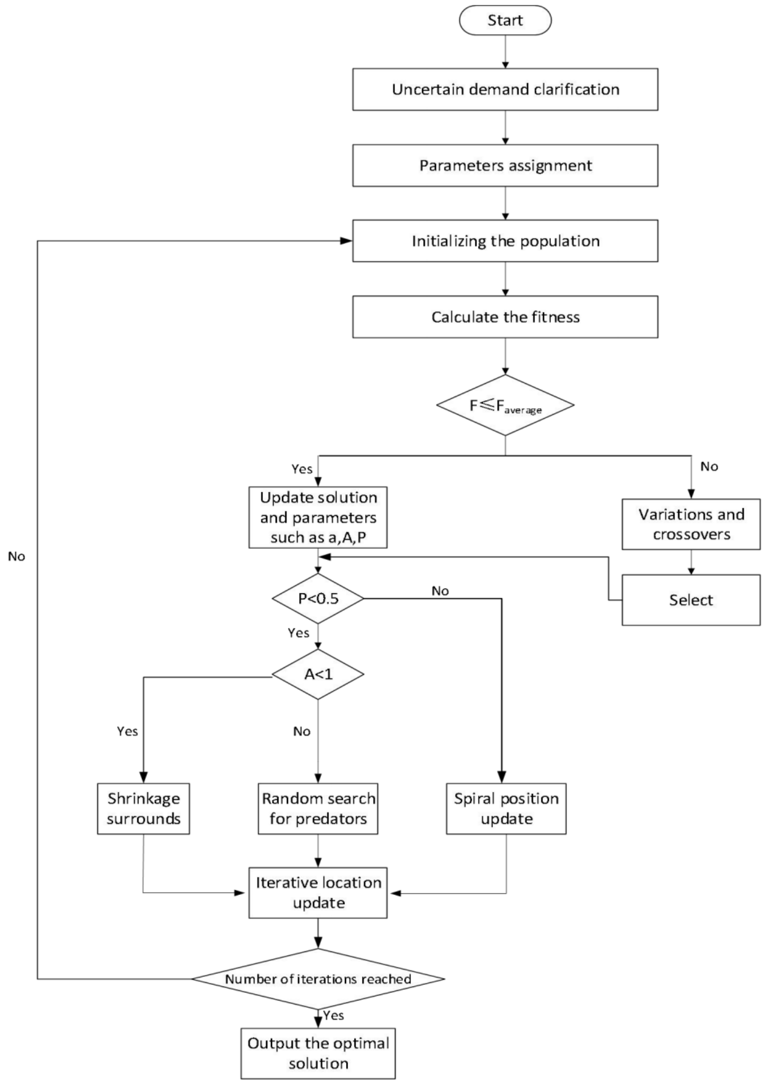

Figure 1. Flowchart of the DE-WOA algorithm.

Step 1: Uncertain demand clarification. The demand parameters in the model are the interval boundary. The most probable weight method is used to convert the interval boundary into definite values and replace them in the model.

Step 2: Initialize parameters. Assign values to parameters, such as population size pop, the maximum number of iterations M, the logarithmic spiral shape constant b, the scaling factor F and the crossover probability CR.

Step 3: Calculate the individual fitness function at F' and the population average fitness function at $F'_{average}$. Based on the obtained fitness function values, record the location of the global optimal solution in the initial population x_{best}, where the global optimal value is F'_{best}.

Step 4: When $F' \leq F'_{average}$, iteratively update the solution and calculate the values of parameters such as a, A, p, C and l; otherwise adapt it for global exploitation using $X_i(t+1) = X_{best}(t) - A * D, A = 2a * r - a, C = 2 * r$ to expand the population diversity.

Step 5: When $P < 0.5$ and $|A| < 1$, the whale position is updated using $D = |C * X_{best}(t) - x_i(t)|$; when $P \geq 0.5$ and $|A| < 1$, the whale position is updated using $D' = |X_{best}(t) - x_i(t)|$; when $P < 0.5$ and $|A| \geq 1$, the whale position is updated using $D = |C * X_{rand}(t) - x_i(t)|$.

Step 6: Update the global optimal solution x_{best} and the global optimal value F'_{best}.

Step 7: Stop the iteration if the algorithm stopping condition is met; otherwise, repeat step 4–step 7.

4. Analysis of Numerical Examples and Computational Results

In this section, we report the results of numerical experiments that were applied to verify the feasibility and effectiveness of the constructed model and proposed algorithm. All experiments were tested on a PC equipped with an Intel(R) Core(TM) i7-9750H CPU @ 2.60 GHz 2.59 and 8 GB of RAM. The model programming was solved by Python 3.8.1.

4.1. Parameter Setting

There are one distribution center and ten disaster locations labeled in order from 0 to 10. Related information, including coordinate values, is shown in Table 3. The network topology between the distribution center and the affected points is shown in Figure 2.

Table 3. Coordinates of the distribution center and locations of affected points.

	No.	**X**	**Y**
Distribution Center	0	30	70
Affected sites	1	35	55
	2	38	73
	3	25	70
	4	30	55
	5	32	85
	6	38	62
	7	43	79
	8	40	60
	9	38	85
	10	24	65

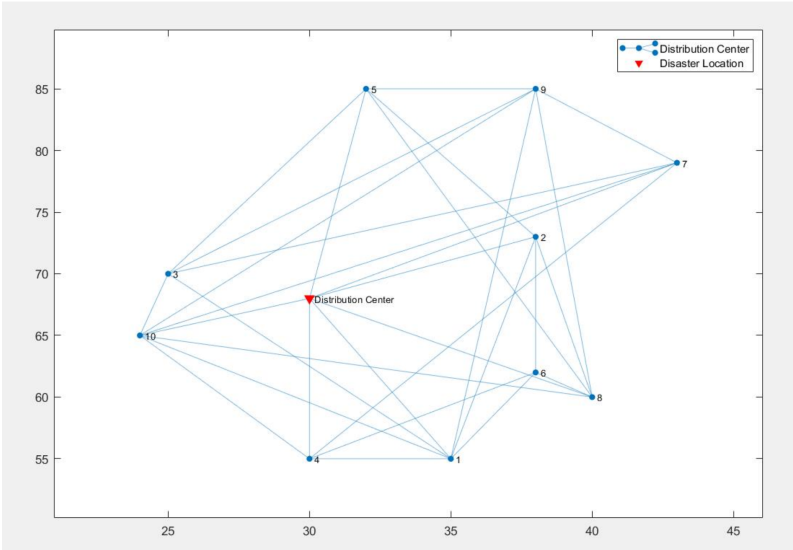

Figure 2. Network topology in the affected area.

Due to the lack of information on data from the affected areas, the demand for emergency perishable goods and the speed of vehicle movements at each affected location need to be estimated based on published information, such as local casualties and the probability of secondary disasters. The specific demand parameters q_1 (pessimistic value), q_2 (most likely value) and q_3 (optimistic value) are shown in Table 4.

Table 4. Demand parameters.

Point of Need	0	1	2	3	4	5	6	7	8	9	10
q_1	0	103	52	78	210	53	41	43	80	65	52
q_2	0	125	70	86	226	70	50	56	91	74	61
q_3	0	140	82	100	242	81	56	64	100	81	70
Demand	0	123.833	69	87	226	69	49.5	55.163	90.667	73.667	61

The distribution center has three small trucks of the same type. In order to obtain a more accurate distribution time, the actual distance between any two points is measured according to the latitude and longitude of the map, and the maximum speed of the vehicles traveling on each road is estimated according to the road damage. The transport network parameters (a, b) and vehicle parameters are shown in Tables 5 and 6.

Table 5. Transport network parameters.

	1	2	3	4	5	6	7	8	9	10
0	(30,30)	(22.4,38)	(-,-)	(6.5,45)	(18.6,60)	(-,-)	(-,-)	(32.5,35)	(15.2,41)	(7.7,46)
1	0,0	(40.6,37)	(31.5,40)	(-,-)	(15.7,42)	(20.4,33)	(-,-)	(43.7,37)	(22.1,41)	(-,-)
2	(40.6,37)	0,0	(51.2,40)	(36.2,38)	(25.6,42)	(-,-)	(17.5,39)	(-,-)	(-,-)	(27.1,45)
3	(-,-)	(51.2,40)	0,0	(-,-)	(-,-)	(34.6,47)	(15.3,39)	(-,-)	(22.2,30)	(-,-)
4	(6.5,45)	(36.2,38)	(-,-)	0,0	(-,-)	(17.8,33)	(-,-)	(33.6,42)	(-,-)	(29.5,40)
5	(18.6,60)	(25.6,42)	(-,-)	(-,-)	0,0	(-,-)	(29.3,36)	(19.3,42)	(-,-)	(-,-)
6	(-,-)	(-,-)	(34.6,47)	(17.8,33)	(-,-)	0,0	(-,-)	(-,-)	(42.1,36)	(14.2,45)
7	(-,-)	(17.5,39)	(15.3,39)	(-,-)	(29.3,36)	(-,-)	0,0	(-,-)	(23.4,35)	(-,-)
8	(32.5,35)	(-,-)	(-,-)	(33.6,42)	(19.3,42)	(-,-)	(-,-)	0,0	(-,-)	(47.1,60)
9	(15.2,41)	(-,-)	(-,-)	(-,-)	(-,-)	(42.1,36)	(23.4,35)	(-,-)	0,0	(38.9,39)
10	(7.7,46)	(27.1,45)	(29.5,40)	(29.5,40)	(-,-)	(14.2,45)	(-,-)	(47.1,60)	(38.9,39)	0,0

Table 6. Vehicle parameters.

Vehicle Type	Quantity (Volume)	Max. Loading Capacity (kg)	Average Travel Speed (km/h)
Small trucks	3	500	60

In Table 5, a denotes the distance between two points (km), b denotes the actual speed of the vehicle traveling between this path v_k (km/h) and "-" denotes that this section is impassable, resulting in the delivery time parameters shown in Table 7.

Table 7. Distribution time parameters.

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.5	0.22	-	0.04	0	-	-	0.39	0.12	0.02
1	0.5	0	0.42	0.26	-	0.11	0.28	-	0.45	0.17	-
2	0.22	0.42	0	0.43	0.35	0.18	-	0.16	-	-	0.15
3	-	0.26	0.43	0	-	-	0.16	0.14	-	0.37	-
4	0.04	-	0.35	-	0	-	0.24	-	0.24	-	0.25
5	0	0.11	0.18	-	-	0	-	0.33	0.14	-	-
6	-	0.28	-	0.16	0.24	-	0	-	-	0.47	0.08
7	-	-	0.16	0.14	-	0.33	-	0	-	0.28	-
8	0.39	0.45	-	-	0.24	0.14	-	-	0	-	0
9	0.12	0.17	-	0.37	-	-	0.47	0.28	-	0	0.35
10	0.22	-	0.15	-	0.25	-	0.08	-	0	0.35	0

4.2. Results

After several trials, the parameters of the improved whale algorithm (DE-WOA) based on the difference algorithm were set as shown in Table 8.

Table 8. DE-WOA parameter settings.

Parameter	Description	Value
pop_num	Initial population size	80
Max_iteration	Maximum number of iterations	300
R	Maximum vehicle loading capacity	500 kg
θ	Corruption rate	0.02 kg/h
σ	Minimum permissible rate of spoilage of materials during vehicle transport	0.90
k	Number of vehicles	3
β	Minimum allowable loading rate during vehicle transport	0.5

After five trials, a Pareto frontier solution set for the problem was obtained and is shown in Figure 3. The horizontal coordinates and vertical coordinates, respectively, represent the value of objective A (the distribution delay penalty and corruption cost) and the value of objective B (total amount of demand that is not met). Each point represents a distribution solution that satisfies the Pareto optimum. The decision maker can choose a relative compromise by weighing the relationship between multiple objectives according to the situation in practice.

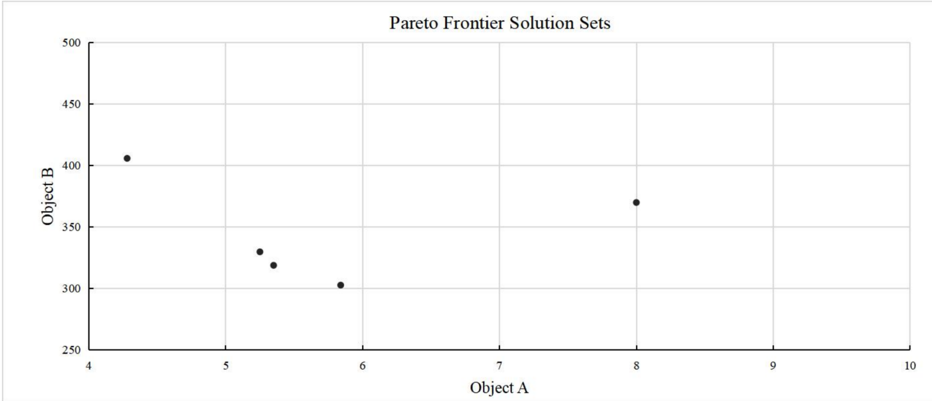

Figure 3. Pareto frontier solution set.

The relationship between the transport volume and the optimal solution at each affected point is obtained, and the optimal path is output as shown in Tables 9 and 10.

Table 9. Relationship between transport volumes at affected sites and optimal path.

First Period	Affected Sites	Transport Volume	Second Period	Affected Sites	Transport Volume
	0	0		0	0
Vehicle 1	5	69	Vehicle 1	2	73.2
	1	123.83		0	0
	8	90.67	Vehicle 2	10	63.66
	0	0		8	86
Vehicle 2	2	69		5	74.37
	7	55.17		7	50
	3	87		3	92.8
	6	49.5		9	74.33
	9	73.67		0	0
	0	0	Vehicle 3	4	232.83
Vehicle 3	10	61		6	53.5
	4	226		1	156
	0	0		0	0

Table 10. Distribution vehicle paths and objective function values.

Periodicity	Vehicles	Transport Routes	Objective A	Objective B
1	1 2 3	0-5-1-8-0 0-2-7-3-6-9-0 0-10-4-0	318.760	5.358
2	1 2 3	0-2-0 0-10-8-5-7-3-9-0 0-4-6-1-0	490.679	7.790

4.3. Algorithm Comparison

Two algorithms, the standard whale algorithm (WOA) and the improved differential evolutionary whale algorithm (DE-WOA), were used to solve the algorithms, resulting in the set of Pareto front solutions under both algorithms shown in Figure 4. It can be seen from Figure 4 that the Pareto ranks of the solutions of the improved differential evolutionary whale algorithm are lower than the Pareto ranks of the solutions of the standard whale algorithm, thereby indicating that the improved differential evolutionary whale algorithm can effectively improve the local search capability and increase the diversity of solutions in the population.

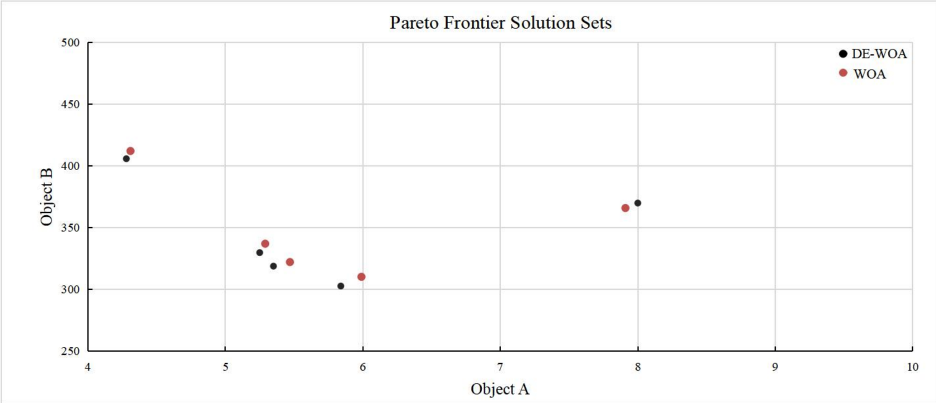

Figure 4. Set of Pareto front solutions under both algorithms.

After 100 runs of both algorithms, the optimal objective values were obtained as shown in Table 8, and the convergence of the algorithms under the two objectives was obtained as shown in Figures 5 and 6.

1. Comparison of target results.

Algorithms	Minimal Distribution Delay Penalty and Corruption Costs	Minimize Total Amount of Demands That Are Not Met
WOA	332.120	6.015
DE-WOA	318.760	5.358

2. Analysis of convergence effects.

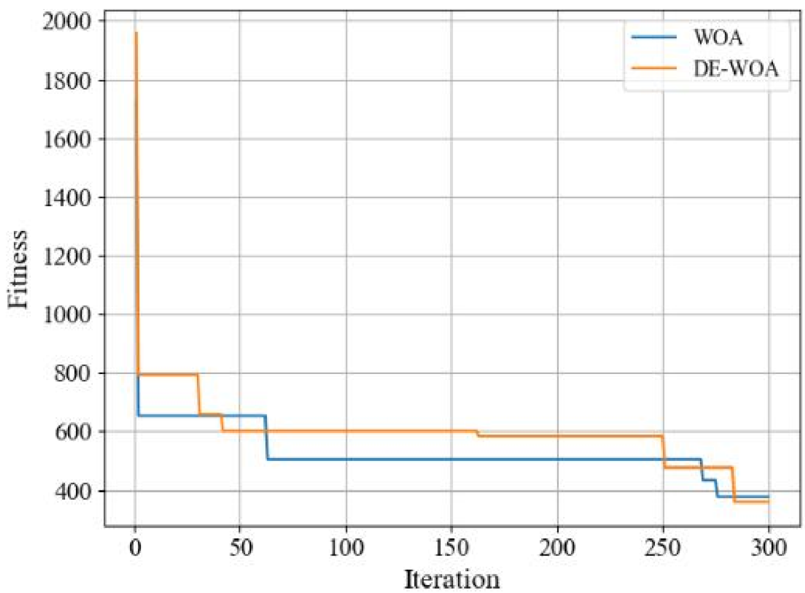

Figure 5. Convergence diagram of distribution delay penalty and corruption costs.

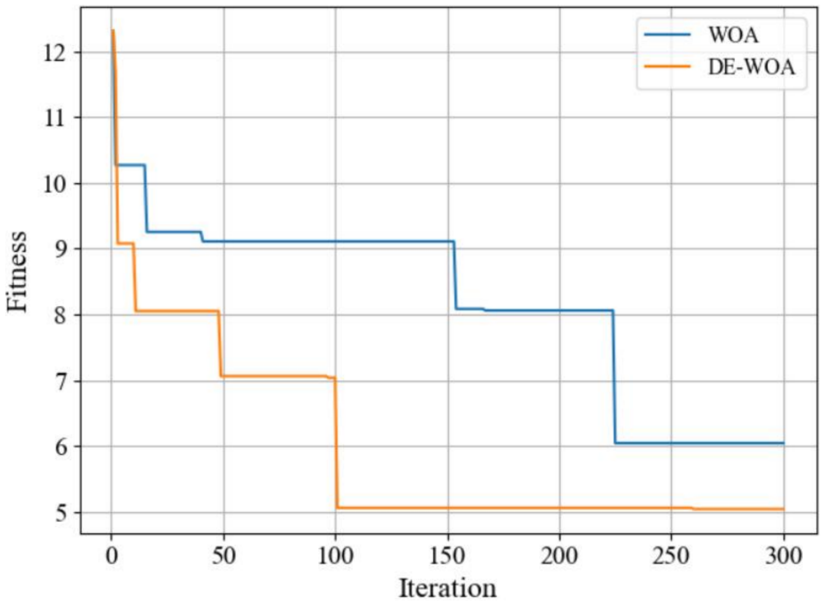

Figure 6. Convergence diagram of total amount of demand that is not met.

By comparison, it can be seen that the improved differential evolutionary whale algorithm outperformed the standard whale algorithm in terms of solution results and converged faster when solving. This shows that the improved differential evolutionary whale algorithm outperforms the standard whale algorithm in solving the dual objective model of this paper, thereby also verifying the validity of the model and algorithm.

5. Discussion and Conclusions

For the optimization problem of the multi-cycle distribution of emergency perishable materials under a uncertain demand, this paper draws the following conclusions.

(1) In this paper, we studied the multi-cycle distribution problem for emergency perishable materials under the situation of sufficient materials in distribution centers after disasters, and considered characteristics such as the degree of road destruction in real situations. We established a dual-objective PVRP distribution optimization model minimizing the cost penalty of distribution delay and the total corruption during delivery, and minimizing the total amount of demand that is not met. Additionally, the uncertain demand in the model is processed using the interval number and the most probable weight method, and the dual objective is processed with the idea of a constraint method. It was verified by an example that the solution is more accurate and faster after the model is processed.

(2) Combined with the real application conditions and scenarios, the whale optimization algorithm was chosen due to the characteristics of the model for optimization. To solve the shortcomings of small population diversity and falling into a local optimum of the standard whale optimization algorithm, the idea of combination with the differential evolution algorithm was proposed. It was improved by adding the characteristics of easy operation and strong global search ability of the differential evolution algorithm. Finally, the analysis of the numerical calculation results of the earthquake in Jiuzhaigou County, Sichuan, showed that the improved differential evolutionary whale algorithm can find a better distribution solution than the standard whale optimization algorithm with less distribution time and the less material corruption. Additionally, it improves the demand satisfaction, and converges faster, which further verifies the feasibility and applicability of the algorithm in practical applications.

The main purpose of this paper was to provide a set of scientific distribution scheme for emergency rescue, through the reasonable distribution of emergency perishable materials and reasonable arrangement of vehicles, so as to effectively reduce the damage caused by an earthquake, reduce casualties, improve the rescue work efficiency, etc. The research model and algorithm proposed in this paper can be applied not only to the disaster scenario, but also to the logistics distribution in urban and rural areas in practical daily life, which can effectively improve the operation efficiency among supply chains.

The model we proposed in this paper also has a few shortcomings. This case operates under the assumption that the distribution center has sufficient supplies and only distributes a single variety of perishable materials, though in practice the distribution center is often short of supplies and the demand for emergency perishable materials at the disaster site is often multi-species. In future work, we will take this shortcoming into account and consider how to combine and distribute multiple species of emergency perishable materials and improve the model to take more factors into account and build a more realistic emergency material distribution model. At the same time, as the complexity of the model increases, more efficient algorithms should be designed to correspondingly solve the model.

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Abbreviations

VRP	Vehicle Routing Problem
PVRP	Period Vehicle Routing Problem
PVRPTW	Period Vehicle Routing Problem with Time Window
MAPVRP	Multidepot and Periodic Vehicle Routing Problems
DPVRPD	The Dynamic Multi-period Vehicle Routing Problem
ERS	Emergency Resource Scheduling
MOAE/D	Multi-objective Evolutionary Algorithm
VRPFPG	The vehicle routing problem for perishable goods
MOGE	Multi-objective Gradient Evolution Algorithm
MILP	Mixed-integer Linear Programming
GA	Genetic Algorithm
PSO	Particle Swarm Optimization
WOA	Whale Optimization Algorithm
DE	Differential Evolutionary Algorithm
DE-WOA	Differential Evolutionary-Whale Optimization Algorithm

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