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# Numerical Solutions of Fractional-Order Electrical RLC Circuit Equations via Three Numerical Techniques

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**Abstract:** In this article, three different techniques, the Fractional Perturbation Iteration Method (FPIA), Fractional Successive Differentiation Method (FSDM), and Fractional Novel Analytical Method (FNAM), have been introduced. These three iterative methods are applied on different types of Electrical RLC-Circuit Equations of fractional-order. The fractional series approximation of the derived solutions can be established by using the obtained coefficients. These three algorithms handle the problems in a direct manner without any need for restrictive assumptions. The comparison displays an agreement between the obtained results. The beauty of this paper lies in the error analysis between the exact solution and approximate solutions obtained by these three methods which prove that the Approximate Solution obtained by FNAM converge very rapidly to the exact solution.

**Keywords:** Fractional Novel Analytical Method; Fractional Perturbation Iteration Method; Fractional Successive Differentiation Method; caputo fractional operator; fractional-order electrical RLC-circuit equations

**MSC:** 65Nxx; 35R11

## 1. Introduction

This last decade has seen significant advances in research on fractional calculus. Many new studies on the use of fractional calculus to the real-world issues have appeared in the literature [1–7]. When compared to the traditional derivative, fractional derivatives offer several advantages. The first benefit is that fractional derivatives account for memory. A memory effect is a basic characteristic of differential equation of non-integer type. This explains why fractional derivatives are used to describe physical systems better than the classical derivative [8–11]. Another advantage was that the fractional derivatives created a large number of diffusion processes. For further information on super-diffusion, hyper-diffusion, ballistic diffusion, and other diffusion processes, see [12].

Many new mathematical models emerge as a result of fractional derivatives. Researchers have lately begun to use fractional-order derivatives in electrical circuits. Many forms of fractional electrical circuits have lately been presented in the literature; see [8,13–15] for more information. In [16–20], we learned about fractional RL circuit modeling. In [16–20], we have a fractional RC circuit model. The fractional LC circuit was first introduced in [18]. The numerical and analytical solutions are at the heart of many fractional electrical circuits studies [19,20].

Authors in [18] use the AB derivative to investigate the numerical solutions for fractional RL and RC circuits. The solutions of the electrical RC, LC, and RL circuits of non-integer order given by Mittag–Leffler fractional derivative are proposed by Aguilar et al. in [12]. Rawdan et al. investigate fractional-order RL and LC circuits in [19]. This paper suggests that fractional-order electrical circuits can be compared to normal electrical circuits. Aguilar et al. and Sene et al. gives the description of electrical RC and LC circuits in terms of fractional derivatives in [21–23]. Aguilar et al. discuss research on fractional electrical circuits characterized by a non-integer derivative with regular Kernel in their paper [22].

Here, we cover the basic equations of electric circuits involving capacitors, resistors and inductors. We analyze the following FDEs for three different types of circuits [18]:

$$D_{\beth}^{2\alpha} I(\beth) + \frac{1}{LC} I(\beth) = \frac{E(\beth)}{L}, \tag{1}$$

$$D_{\beth}^{\alpha} V(\beth) + \frac{1}{CR} V(\beth) = \frac{E(\beth)}{R}, \tag{2}$$

$$D_{\beth}^{\alpha} I(\beth) + \frac{R}{L} I(\beth) = \frac{E(\beth)}{L}. \tag{3}$$

In the circuit, $I(\beth)$ is the Current and $V(\beth)$ is a charge at time $\beth$, $E(\beth)$ is the supplied source (volt), $C$ is the Capacitance (farad), $L$ is the Inductance (henry), and $R$ is the Resistance (ohms). In the above equations, Equation (1) represents the inductor–capacitor ($LC$) circuit, Equation (2) represents the resistor–capacitor ($RC$) circuit and Equation (3) represents the inductor–resistor ($LR$) circuit of fractional order. Some preliminary numerical results for RLC–circuits extensions to electric circuits and other applications of differential equations are presented in [24–42].

This manuscript is organized as follows. In Section 2, we start with the description of Fractional Perturbation Iteration Algorithm (FPIA). In Section 3, Fractional Successive Differentiation Method (FSDM) is presented with theoretical background. Fractional Novel Analytical Method (FNAM) is discussed in Section 4. Section 5 looks into the application of the FPIA, FSDM, and FNAM to some electrical RLC circuits equations of fractional-order and a comparison is made with the existing classical solutions which were reported in other published literature. Finally, a brief conclusion is presented in the last section.

The operator which we used for the fractional derivative is of Caputo-type [43], which is defined as:

$$ {}^{C}_{0}D_{\beth}{}^{\alpha}[f(\beth)] = \begin{cases} \dfrac{1}{\Gamma[m-\alpha]} \int_0^{\beth} \dfrac{f^{(m)}(\tau)}{(\beth-\tau)^{\alpha-m+1}} d\tau, & m-1<\alpha<m \\[2ex] \dfrac{d^m}{dt^m} f(\beth), & \alpha = m. \end{cases} $$

A fundamental feature of the Caputo fractional derivative [43] is that,

$$J_{\beth}^{\alpha}\left[{}^{C}_{0}D_{\beth}{}^{\alpha} f(\beth)\right] = f(\beth) - \sum_{k=0}^{+\infty} f^{(k)}(0^+)\frac{\beth^k}{k!}.$$

## 2. Fractional Perturbation Iteration Algorithm (FPIA)

Consider the following initial value problem,

$$D_{\beth_0}^{\alpha} S(\beth_0) + M[S(\beth_0)] + N[S(\beth_0)] = u(\beth_0), \;\; 0 < \alpha \leq 1, \;\; \beth_0 \in R, \tag{4}$$

with initial condition $S^{(k)}(0) = S_k$, $k = 0, 1, 2, \ldots, m-1$ and $m-1 < \alpha \leq m$. Here $M$ is a linear operator, $N$ is a non-linear operator and $u(\beth_0)$ is known analytical function, respectively. Introducing $\varepsilon$ with non-linear term, yield,

$$D^{\alpha}_{\beth_0} S(\beth_0) + M[S(\beth_0)] + \varepsilon N[S(\beth_0)] - u(\beth_0) = 0.$$

In the present paper, the simplest Perturbation Iteration Algorithm $PIA(1,1)$ is used by taking one correction term in the perturbation expansion and correction terms of only fractional order derivatives in the Taylor series expansion, that is, $n = 1$, $m = 1$. The correction term, as discussed in [44], is given with the help of following equation:

$$\begin{aligned} S_{n+1} &= S_n + \varepsilon (S_c)_n, \\ S'_{n+1} &= S'_n + \varepsilon \left(S'_c\right)_n. \end{aligned} \tag{5}$$

Substituting Equation (5) in to Equation (4) and writing in the Taylors Series expansion for only 1st order derivative yields,

$$\begin{aligned} &G\left(S_n^{(\alpha)}, S_n^{(m)}, S_n^{(m-1)}, \ldots, S'_n, S_n, 0\right) + G_{S^{(\alpha)}}\left(S_n^{(\alpha)}, S_n^{(m)}, S_n^{(m-1)}, \ldots, S'_n, S_n, 0\right)\varepsilon\left(S_c^{(\alpha)}\right)_n \\ &+ G_{S^{(m)}}\left(S_n^{(\alpha)}, S_n^{(m)}, S_n^{(m-1)}, \ldots, S'_n, S_n, 0\right)\varepsilon\left(S_c^{(m)}\right)_n + G_{S^{(m-1)}}\left(S_n^{(\alpha)}, S_n^{(m)}, S_n^{(m-1)}, \ldots, S'_n, S_n, 0\right)\varepsilon\left(S_c^{(m-1)}\right)_n + \ldots \\ &+ G_{S'}\left(S_n^{(\alpha)}, S_n^{(m)}, S_n^{(m-1)}, \ldots, S'_n, S_n, 0\right)\varepsilon\left(S'_c\right)_n + G_S\left(S_n^{(\alpha)}, S_n^{(m)}, S_n^{(m-1)}, \ldots, S'_n, S_n, 0\right)\varepsilon(S_c)_n + \\ &G_\varepsilon\left(S_n^{(\alpha)}, S_n^{(m)}, S_n^{(m-1)}, \ldots, S'_n, S_n, 0\right)\varepsilon = 0, \end{aligned} \tag{6}$$

or

$$\left(S_c^{(\alpha)}\right)_n \frac{\partial G}{\partial S^{(\alpha)}} + \left(S_c^{(m)}\right)_n \frac{\partial G}{\partial S^{(m)}} + \left(S_c^{(m-1)}\right)_n \frac{\partial G}{\partial S^{(m-1)}} + \ldots + \left(S'_c\right)_n \frac{\partial G}{\partial S'} + (S_c)_n \frac{\partial G}{\partial S} + \frac{\partial G}{\partial \varepsilon} + \frac{G}{\varepsilon} = 0, \tag{7}$$

here $(.)'$ represents the derivative with respect to independent variable and

$$G_\varepsilon = \frac{\partial G}{\partial \varepsilon}, G_S = \frac{\partial G}{\partial S}, G_{S'} = \frac{\partial G}{\partial S'}, \ldots \tag{8}$$

where all derivatives are evaluated at $\varepsilon = 0$. Starting with the initial condition $S_0$ first corrected term $[S_c]_0$ has been calculated by the help of Equation (7). Then, we substitute $[S_c]_0$ into Equation (5) to find $S_1$ iteration process is repeated using Equation (7) and Equation (5) until we obtain a satisfactory result.

## 3. Fractional Successive Differentiation Method (FSDM)

We consider the generalize $n$th order FODEs as

$$D^{n\alpha}_{\beth} S(\beth) = N\left(S(\beth), D^{\alpha}_{\beth} S(\beth), D^{2\alpha}_{\beth} S(\beth), \ldots, D^{(n-1)\alpha}_{\beth} S(\beth)\right) + h(\beth), \tag{9}$$

with initial conditions are given as,

$$\begin{aligned} S(0) &= \beta_0, \\ D^{\alpha}_0 S(0) &= \beta_1, \\ D^{2\alpha}_0 S(0) &= \beta_2, \\ &\vdots \\ D^{(n-1)\alpha}_0 S(0) &= \beta_{n-1}. \end{aligned} \tag{10}$$

In general, the initial condition is written as $D_0^{j\alpha}S(0) = \beta_j, 0 \leq j \leq n-1$. Apply many times fractional differentiations on both sides of Equation (9), say $m$ times, we obtain,

$$(D_\daleth^{n\alpha}S)^{(m\alpha)}(\daleth) = N^{(m\alpha)}\Big(S(\daleth), D_\daleth^{\alpha}S(\daleth), D_\daleth^{2\alpha}S(\daleth), \ldots, D_\daleth^{(n-1)\alpha}S(\daleth)\Big) + h^{(m\alpha)}(\daleth), \quad m \geq 1. \tag{11}$$

By substituting $\daleth = 0$ at each differentiating step of Equation (11) we calculate the values of the functions $\left(S^{(n\alpha)}\right)^{\alpha}(0), \left(S^{(n\alpha)}\right)^{2\alpha}(0), \left(S^{(n\alpha)}\right)^{3\alpha}(0), \ldots$. Having estimated these values by using the initial conditions and the fractional series approximation of the solution $S(\daleth)$ follows instantly.

$$S(\daleth) = \sum_{n=0}^{+\infty} \frac{D_0^{n\alpha}S(0)}{\Gamma[n\alpha+1]}\daleth^{n\alpha}, \ \alpha > 0, \ \daleth \geq 0. \tag{12}$$

For detail on successive differentiation method, please see [45]. We will employ the FSDM to the three different types of electrical RLC circuit equations of fractional order.

## 4. Fractional Novel Analytical Method (FNAM)

We will discuss the elementary concepts of constructing a FNAM [46,47] for the FDE in this section. Consider the following general FDE:

$$D_\daleth^{2\beta}S(\daleth) = \mathcal{F}\Big(\daleth, S, D_\daleth^{\beta}S, \ldots\Big), \tag{13}$$

with initial condition

$$S(0) = \varphi_0, \quad and \ \ D_\daleth S(0) = \varphi_1. \tag{14}$$

Taking Fractional Integral (FI) both sides of Equation (13) from 0 to $\daleth$, we get,

$$D_\daleth^{\beta}S(\daleth) - D_\daleth^{\beta}S(0) = I_\daleth^{\beta}\mathcal{F}[S],$$

$$D_\daleth^{\beta}S(\daleth) - \varphi_1 = I_\daleth^{\beta}\mathcal{F}[S].$$

Then,

$$D_\daleth^{\beta}S(\daleth) = \varphi_1 + I_\daleth^{\beta}\mathcal{F}[S], \tag{15}$$

where $\mathcal{F}[S] = \mathcal{F}\Big(\daleth, S, D_\daleth^{\beta}S, \ldots\Big)$. Then, again taking the FI from 0 to $\daleth$, on both sides of Equation (15). We obtain,

$$S(\daleth) - S(0) = \varphi_1\frac{\daleth^{\beta}}{\Gamma(\beta+1)} + I_\daleth^{2\beta}\mathcal{F}[S],$$

$$S(\daleth) - \varphi_0 = \varphi_1\frac{\daleth^{\beta}}{\Gamma(\beta+1)} + I_\daleth^{2\beta}\mathcal{F}[S].$$

Thus,

$$S(\daleth) = \varphi_0 + \varphi_1\frac{\daleth^{\beta}}{\Gamma(\beta+1)} + I_\daleth^{2\beta}\mathcal{F}[S]. \tag{16}$$

For $\mathcal{F}[S]$ the Fractional Taylor Series (FTS) is extended about $\daleth = 0$,

$$\mathcal{F}[S] = \sum_{k=0}^{+\infty} \frac{D_\daleth^{k\beta}\mathcal{F}[S_0]}{\Gamma(k\beta+1)}\daleth^{k\beta}, \quad \beta > 0$$

$$\mathcal{F}[S] = \mathcal{F}[S_0] + \frac{D_\daleth^{\beta}\mathcal{F}[S_0]}{\Gamma(\beta+1)}\daleth^{\beta} + \frac{D_\daleth^{2\beta}\mathcal{F}[S_0]}{\Gamma(2\beta+1)}\daleth^{2\beta} + \frac{D_\daleth^{3\beta}\mathcal{F}[S_0]}{\Gamma(3\beta+1)}\daleth^{3\beta} + \ldots + \frac{D_\daleth^{k\beta}\mathcal{F}[S_0]}{\Gamma(k\beta+1)}\daleth^{k\beta} + \ldots. \tag{17}$$

Substituting Equation (17) by Equation (16), we obtain

$$S(\beth) = \varphi_0 + \varphi_1 \frac{\beth^{\beta}}{\Gamma(\beta+1)} + I_{\beth}^{2\beta}\left[\mathcal{F}[S_0] + \frac{D_{\beth}^{\beta}\mathcal{F}[S_0]}{\Gamma(\beta+1)}\beth^{\beta} + \frac{D_{\beth}^{2\beta}\mathcal{F}[S_0]}{\Gamma(2\beta+1)}\beth^{2\beta} + \frac{D_{\beth}^{3\beta}\mathcal{F}[S_0]}{\Gamma(3\beta+1)}\beth^{3\beta} + \ldots + \frac{D_{\beth}^{k\beta}\mathcal{F}[S_0]}{\Gamma(k\beta+1)}\beth^{k\beta} + \ldots\right],$$

$$S(\beth) = \varphi_0 + \varphi_1 \frac{1}{\Gamma(\beta+1)}\beth^{\beta} + \frac{\mathcal{F}[S_0]}{\Gamma(2\beta+1)}\beth^{2\beta} + \frac{D_{\beth}^{\beta}\mathcal{F}[S_0]}{\Gamma(3\beta+1)}\beth^{3\beta} + \frac{D_{\beth}^{2\beta}\mathcal{F}[S_0]}{\Gamma(4\beta+1)}\beth^{4\beta} + \ldots + \frac{D_{\beth}^{k\beta}\mathcal{F}[S_0]}{\Gamma((k+2)\beta+1)}\beth^{(k+2)\beta} + \ldots,$$

$$S(\beth) = \mathfrak{a}_o + \mathfrak{a}_1 \frac{\beth^{\beta}}{\Gamma(\beta+1)} + \mathfrak{a}_2 \frac{\beth^{2\beta}}{\Gamma(2\beta+1)} + \mathfrak{a}_3 \frac{\beth^{3\beta}}{\Gamma(3\beta+1)} + \mathfrak{a}_4 \frac{\beth^{4\beta}}{\Gamma(4\beta+1)} + \ldots + \mathfrak{a}_k \frac{\beth^{k\beta}}{\Gamma(k\beta+1)} + \ldots, \tag{18}$$

where,

$$\begin{aligned}
\mathfrak{a}_0 &= \varphi_0, \\
\mathfrak{a}_1 &= \varphi_1, \\
\mathfrak{a}_2 &= \mathcal{F}[S_0], \\
\mathfrak{a}_3 &= D_{\beth}^{\beta}\mathcal{F}[S_0], \\
\mathfrak{a}_4 &= D_{\beth}^{2\beta}\mathcal{F}[S_0], \\
&\vdots \\
\mathfrak{a}_k &= D_{\beth}^{(k-2)\beta}\mathcal{F}[S_0],
\end{aligned}$$

such that highest derivative of $S$ is $k$. The endorsement of Equation (18) is to extend FTS for $S$ about $\beth = 0$. It means that,

$$\begin{aligned}
\mathfrak{a}_0 &= S(0), \\
\mathfrak{a}_1 &= D_{\beth}^{\beta}S(0), \\
\mathfrak{a}_2 &= D_{\beth}^{2\beta}S(0), \\
\mathfrak{a}_3 &= D_{\beth}^{3\beta}S(0), \\
\mathfrak{a}_4 &= D_{\beth}^{4\beta}S(0), \\
&\vdots \\
\mathfrak{a}_k &= D_{\beth}^{k\beta}S(0).
\end{aligned}$$

So, we can obtain our desired numerical solution easily.

## 5. Application for RLC Circuits

In this portion, we apply FPIA, FSDM, and FNAM techniques to Equations (1), (2), and (3), while assuming the time-invariant source ($E(\beth) = E_0$) in the underlying series circuit models and obtain results by comparing their approximate solutions with the corresponding classical solutions.

### *5.1. Inductor–Capacitor (LC) Circuit of Fractional Order*

Only charged inductor and capacitor are present in the circuit and its FDE [17,18,48] is given as:

$$D_{\beth}^{2\alpha}I(\beth) + \frac{1}{LC}I(\beth) = \frac{E_0}{L}, \quad 0 < \alpha \leq 1,$$

with initial conditions $I(0) = i_0$ and $DI(0) = 0$. In this work fractional order derivatives are understood to be in the Caputo sense. The classical solution of LC Circuit equation of fractional-order Equation (1) at $\alpha = 1$ is $I(\beth) = CE_0 - CE_0 \cos\left(\frac{\beth}{\sqrt{CL}}\right) + i_0 \cos\left(\frac{\beth}{\sqrt{CL}}\right)$.

FPIA: Introducing $\varepsilon$ with non-linear term yield,

$$D_{\beth}^{2\alpha} I(\beth) + \frac{\varepsilon}{LC} I(\beth) = \varepsilon \frac{E_0}{L}.$$

Applying Fractional Perturbation Iteration Algorithm $FPIA(1,1)$ yields,

$$\left[\left(D_{\beth}^{2\alpha} I(\beth)\right)_n\right]_c = -\frac{D_{\beth}^{2\alpha} I(\beth)}{\varepsilon} - \frac{1}{LC} I(\beth) + \frac{E_0}{L}.$$

By taking fractional integral on both sides, yields

$$[(I(\beth))_n]_c = J_{\beth}^{2\alpha}\left[-\frac{D_{\beth}^{2\alpha} I(\beth)}{\varepsilon} - \frac{1}{LC} I(\beth) + \frac{E_0}{L}\right]. \tag{19}$$

An initial guess satisfying the initial condition should be selected. Using the algorithm of FPIA with Equation (19) and initial guess, the approximate solution at each step is:

$$\begin{aligned}
I_0(\beth) =& i_0, \\
I_1(\beth) =& \frac{E_0}{L\Gamma(1+2\alpha)}\beth^{2\alpha} + i_0 - \frac{i_0}{CL\Gamma(1+2\alpha)}\beth^{2\alpha}, \\
I_2(\beth) =& \frac{E_0}{L\Gamma(1+2\alpha)}\beth^{2\alpha} - \frac{E_0}{CL^2\Gamma(1+4\alpha)}\beth^{4\alpha} + i_0 - \frac{i_0}{CL\Gamma(1+2\alpha)}\beth^{2\alpha} + \frac{i_0}{C^2L^2\Gamma(1+4\alpha)}\beth^{4\alpha}, \\
I_3(\beth) =& \frac{E_0}{L\Gamma(1+2\alpha)}\beth^{2\alpha} - \frac{E_0}{CL^2\Gamma(1+4\alpha)}\beth^{4\alpha} + \frac{E_0}{C^2L^3\Gamma(1+6\alpha)}\beth^{6\alpha} + i_0 - \frac{i_0}{CL\Gamma(1+2\alpha)}\beth^{2\alpha} + \frac{i_0}{C^2L^2\Gamma(1+4\alpha)}\beth^{4\alpha} - \\
& \frac{i_0}{C^3L^3\Gamma(1+6\alpha)}\beth^{6\alpha}, \\
I_4(\beth) =& \frac{E_0}{L\Gamma(1+2\alpha)}\beth^{2\alpha} - \frac{E_0}{CL^2\Gamma(1+4\alpha)}\beth^{4\alpha} + \frac{E_0}{C^2L^3\Gamma(1+6\alpha)}\beth^{6\alpha} - \frac{E_0}{C^3L^4\Gamma(1+8\alpha)}\beth^{8\alpha} + i_0 - \frac{i_0}{CL\Gamma(1+2\alpha)}\beth^{2\alpha} + \\
& \frac{i_0}{C^2L^2\Gamma(1+4\alpha)}\beth^{4\alpha} - \frac{i_0}{C^3L^3\Gamma(1+6\alpha)}\beth^{6\alpha} + \frac{i_0}{C^4L^4\Gamma(1+8\alpha)}\beth^{8\alpha}, \\
& \vdots
\end{aligned}$$

and so on. We calculated 10th iterations of FPIA.

FSDM: By using FSDM, we get,

$$\begin{aligned}
I(\beth) =& \beta, \\
D_{\beth}^{\alpha} I(\beth) =& \gamma, \\
D_{\beth}^{2\alpha} I(\beth) =& -\frac{1}{LC} I(\beth) + \frac{E_0}{L}, \\
D_{\beth}^{3\alpha} I(\beth) =& -\frac{1}{LC} D_{\beth}^{\alpha} I(\beth) + D_{\beth}^{\alpha}\frac{E_0}{L}, \\
& \vdots
\end{aligned} \tag{20}$$

and so on. Substituting $\beth = 0$ in each derivative result of Equation (20) gives the values of the fractional derivatives. We observe that $\beta = i_0$, $\gamma = 0$ and $\alpha, 3\alpha, 5\alpha, \ldots$ order derivatives terms are zero, i.e., $D_0^{\alpha}[I(0)] = D_0^{3\alpha}[I(0)] = D_0^{5\alpha}[I(0)] = \ldots = 0$. In view of this, we obtain the Fractional series approximation,

$$I(\beth) = \frac{E_0}{L\Gamma(1+2\alpha)}\beth^{2\alpha} - \frac{E_0}{CL^2\Gamma(1+4\alpha)}\beth^{4\alpha} + \frac{E_0}{C^2L^3\Gamma(1+6\alpha)}\beth^{6\alpha} - \frac{E_0}{C^3L^4\Gamma(1+8\alpha)}\beth^{8\alpha} + \ldots + i_0 - \frac{i_0}{CL\Gamma(1+2\alpha)}\beth^{2\alpha} + \frac{i_0}{C^2L^2\Gamma(1+4\alpha)}\beth^{4\alpha} - \frac{i_0}{C^3L^3\Gamma(1+6\alpha)}\beth^{6\alpha} + \frac{i_0}{C^4L^4\Gamma(1+8\alpha)}\beth^{8\alpha} + \ldots$$

FNAM: By following the steps carefully elaborated in the FNAM, we attain the following series of solution,

$$I(\beth) = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}E_0\beth^{2n\alpha}}{C^{n-1}L^n\Gamma(1+2n\alpha)} + \sum_{n=0}^{+\infty} \frac{(-1)^n i_0\beth^{2n\alpha}}{C^nL^n\Gamma(1+2n\alpha)}.$$

Figure 1 represents the Time v/s current graph and graphical comparison of classical solution and numerical solutions obtained by FPIA, FSDM, and FNAM of the electrical inductor–capacitor circuit equation. The graphs of classical solution and numerical solutions represents a simple harmonic oscillation. Graph of classical solution and FNAM solution overlap each other representing the harmonic oscillation in the values of current as time progresses. Figure 2 represents the graphical comparisons of absolute errors of obtained solutions by FPIA, FSDM, and FNAM of the electrical inductor–capacitor circuit equation and the graph clearly shows that FNAM contains less error as compared to FPIA and FSDM. Obtained solutions by FPIA, FSDM, and FNAM of the fractional-order electrical inductor–capacitor circuit equation at different values of $\alpha$ are represented in Figure 3.

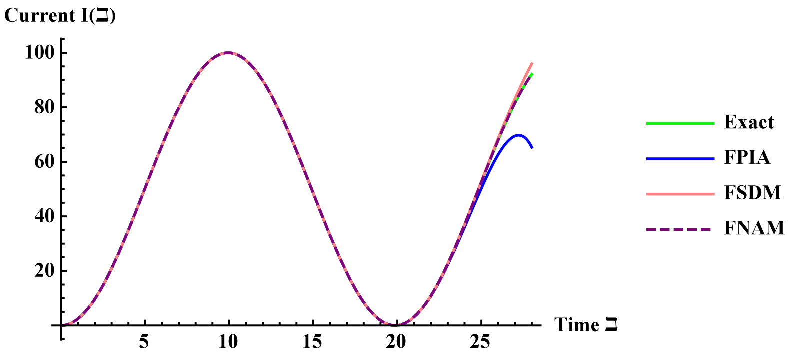



**Figure 1.** Graphical comparison of classical solution and approximate solutions of inductor–capacitor circuit at $\alpha = 1$, $L = 2$, $C = 5$, $E_0 = 10$ and $i_0 = 0$.

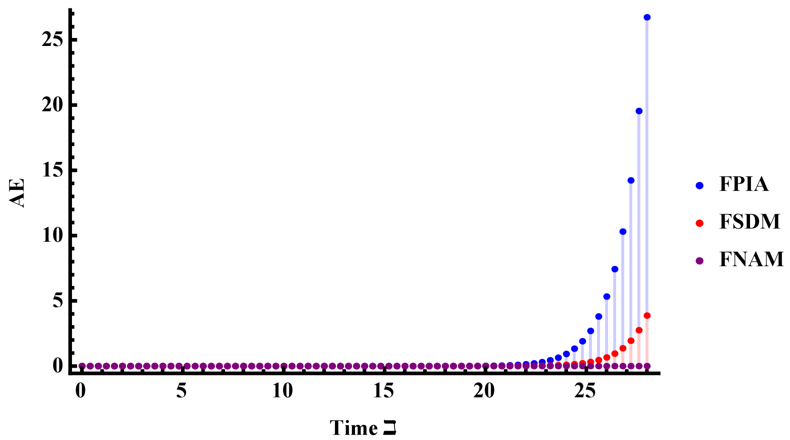
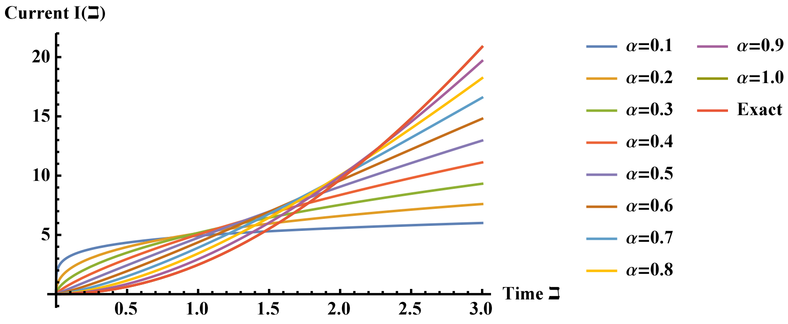
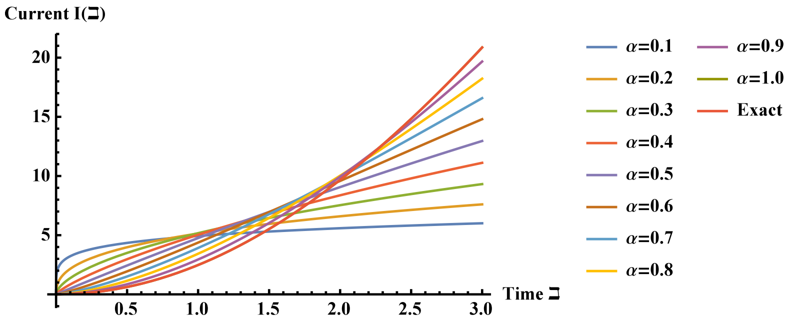


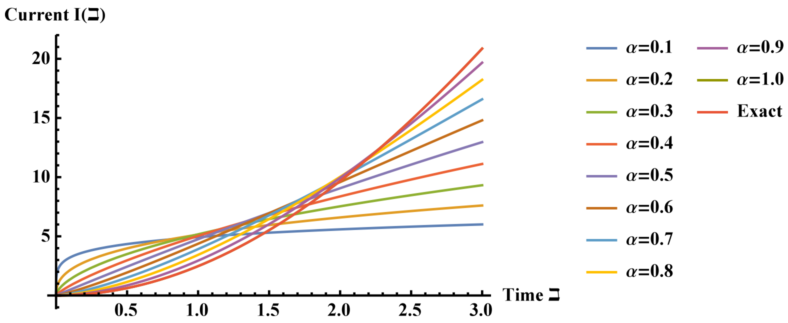

**Figure 2.** Graphical comparisons of Absolute Errors (AE) of obtained solutions by FPIA, FSDM, and FNAM.




(**a**) Solutions via FPIA




(**b**) Solutions via FSDM




(**c**) Solutions via FNAM

**Figure 3.** Numerical solutions obtained by (**a**) FPIA, (**b**) FSDM, and (**c**) FNAM of the electrical inductor–capacitor circuit equation of fractional-order at different $\alpha$ values.

### *5.2. Resistor–Capacitor (RC) Circuit of Fractional Order*

Only charged resistor and capacitor are present in the circuit and its FDE [18,48] is given as:

$$D^{\alpha}_{\beth}V(\beth) + \frac{1}{CR}V(\beth) = \frac{E_0}{R}, \quad 0 < \alpha \leq 1,$$

with initial condition $V(0) = v_0$. The classical solution of RC circuit equation is $V(\beth) = E_0C + (v_0 - E_0C)\exp\left(-\frac{1}{RC\alpha}\beth^{\alpha}\right)$.

FPIA: Introducing $\varepsilon$ with non-linear term yield,

$$D^{\alpha}_{\beth}V(\beth) + \varepsilon\frac{1}{CR}V(\beth) = \varepsilon\frac{E_0}{R}.$$

Applying Fractional Perturbation Iteration Algorithm PIA$(1,1)$ yields,

$$\left[(D^{\alpha}_{\beth}V(\beth))_n\right]_c = -\frac{D^{\alpha}_{\beth}V(\beth)}{\varepsilon} - \frac{1}{CR}V(\beth) + \frac{E_0}{R}.$$

By taking fractional integral on both sides yields,

$$[(V(\daleth))_n]_c = J_{\daleth}^{\alpha}\left[-\frac{D_{\daleth}^{\alpha}V(\daleth)}{\varepsilon} - \frac{1}{CR}V(\daleth) + \frac{E_0}{R}\right]. \tag{21}$$

An initial guess satisfying the initial condition should be selected. Using the algorithm of FPIA with Equation (21) and initial guess, the approximate solution at each step is:

$$\begin{aligned}
V_0(\daleth) =& v_0,\\
V_1(\daleth) =& \frac{E_0}{R\Gamma(1+\alpha)}\daleth^{\alpha} + v_0 - \frac{v_0}{CR\Gamma(1+\alpha)}\daleth^{\alpha},\\
V_2(\daleth) =& \frac{E_0}{R\Gamma(1+\alpha)}\daleth^{\alpha} - \frac{E_0}{CR^2\Gamma(1+2\alpha)}\daleth^{2\alpha} + v_0 - \frac{v_0}{CR\Gamma(1+\alpha)}\daleth^{\alpha} + \frac{v_0}{C^2R^2\Gamma(1+2\alpha)}\daleth^{2\alpha},\\
V_3(\daleth) =& \frac{E_0}{R\Gamma(1+\alpha)}\daleth^{\alpha} - \frac{E_0}{CR^2\Gamma(1+2\alpha)}\daleth^{2\alpha} + \frac{E_0}{C^2R^3\Gamma(1+3\alpha)}\daleth^{3\alpha} + v_0 - \frac{v_0}{CR\Gamma(1+\alpha)}\daleth^{\alpha} + \frac{v_0}{C^2R^2\Gamma(1+2\alpha)}\daleth^{2\alpha} -\\
& \frac{v_0}{C^3R^3\Gamma(1+3\alpha)}\daleth^{3\alpha},\\
V_4(\daleth) =& \frac{E_0}{R\Gamma(1+\alpha)}\daleth^{\alpha} - \frac{E_0}{CR^2\Gamma(1+2\alpha)}\daleth^{2\alpha} + \frac{E_0}{C^2R^3\Gamma(1+3\alpha)}\daleth^{3\alpha} - \frac{E_0}{C^3R^4\Gamma(1+4\alpha)}\daleth^{4\alpha} + v_0 - \frac{v_0}{CR\Gamma(1+\alpha)}\daleth^{\alpha} +\\
& \frac{v_0}{C^2R^2\Gamma(1+2\alpha)}\daleth^{2\alpha} - \frac{v_0}{C^3R^3\Gamma(1+3\alpha)}\daleth^{3\alpha} + \frac{v_0}{C^4R^4\Gamma(1+4\alpha)}\daleth^{4\alpha},\\
& \vdots
\end{aligned}$$

and so on.

FSDM: By using FSDM, we get,

$$\begin{aligned}
V(\daleth) =& \beta,\\
D_{\daleth}^{\alpha}V(\daleth) =& -\frac{1}{CR}V(\daleth) + \frac{E_0}{R},\\
D_{\daleth}^{2\alpha}V(\daleth) =& -\frac{1}{CR}D_{\daleth}^{\alpha}V(\daleth) + D_{\daleth}^{\alpha}\frac{E_0}{L},\\
D_{\daleth}^{3\alpha}V(\daleth) =& -\frac{1}{CR}D_{\daleth}^{2\alpha}V(\daleth) + D_{\daleth}^{2\alpha}\frac{E_0}{L},\\
D_{\daleth}^{4\alpha}V(\daleth) =& -\frac{1}{CR}D_{\daleth}^{3\alpha}V(\daleth) + D_{\daleth}^{3\alpha}\frac{E_0}{L},\\
& \vdots
\end{aligned} \tag{22}$$

and so on. Substituting $\daleth = 0$ in each derivative result of Equation (22) gives the values of the fractional derivatives. In view of this, we obtain the Fractional series approximation,

$$\begin{aligned}
V(\daleth) =& \frac{E_0\daleth^{\alpha}}{R\Gamma(1+\alpha)} + \frac{E_0\daleth^{2\alpha}}{R^2C\Gamma(1+2\alpha)} - \frac{E_0\daleth^{3\alpha}}{R^3C^2\Gamma(1+3\alpha)} + \frac{E_0\daleth^{4\alpha}}{R^4C^3\Gamma(1+4\alpha)} - \frac{E_0\daleth^{5\alpha}}{R^5C^4\Gamma(1+5\alpha)} + \ldots + v_0 -\\
& \frac{v_0\daleth^{\alpha}}{RC\Gamma(1+\alpha)} + \frac{v_0\daleth^{2\alpha}}{R^2C^2\Gamma(1+2\alpha)} - \frac{v_0\daleth^{3\alpha}}{R^3C^3\Gamma(1+3\alpha)} + \frac{v_0\daleth^{4\alpha}}{R^4C^4\Gamma(1+4\alpha)} - \frac{v_0\daleth^{5\alpha}}{R^5C^5\Gamma(1+5\alpha)} + \ldots.
\end{aligned}$$

FNAM: By following the steps carefully elaborated in the FNAM, we attain the following series of solution,

$$V(\daleth) = \sum_{n=1}^{+\infty}\frac{(-1)^{n+1}E_0\daleth^{n\alpha}}{C^{n-1}R^n\Gamma(1+n\alpha)} + \sum_{n=0}^{+\infty}\frac{(-1)^n v_0\daleth^{n\alpha}}{C^nR^n\Gamma(1+n\alpha)}.$$

Figure 4 represents the Time v/s Voltage graph and graphical comparison of classical solution and numerical solutions obtained by FPIA, FSDM, and FNAM of the electrical resistor–capacitor circuit equation of fractional-order. Classical and FNAM solution graph overlaps each other at $\alpha = 1$ representing the charge in the circuit as time progresses.

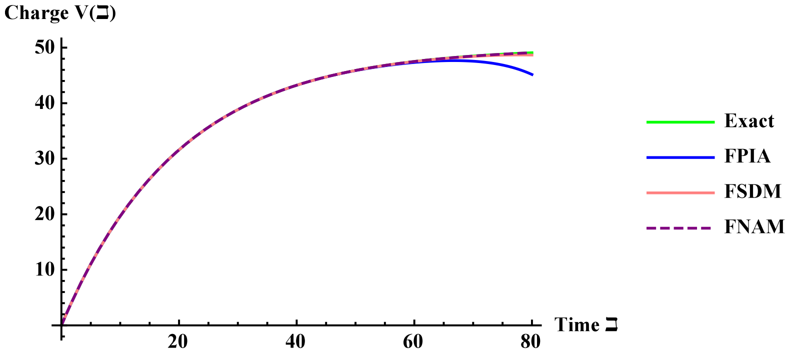



**Figure 4.** Graphical comparisons of classical solution and approximate solutions of resistor–capacitor circuit at $\alpha = 1$, $R = 4$, $C = 5$, $E_0 = 10$ and $v_0 = 0$.

Graphical comparisons of absolute errors of obtained solutions by FPIA, FSDM, and FNAM of the electrical resistor–capacitor circuit equation is represented in Figure 5 and the graph clearly shows that FNAM contains less error as compared to FPIA and FSDM. Figure 6 represents the graphs of obtained solutions by FPIA, FSDM, and FNAM of the fractional-order electrical resistor–capacitor circuit equation at different values of $\alpha$.

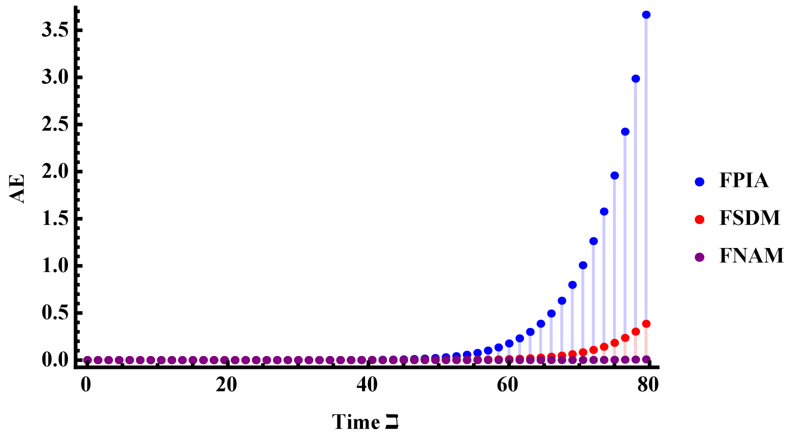
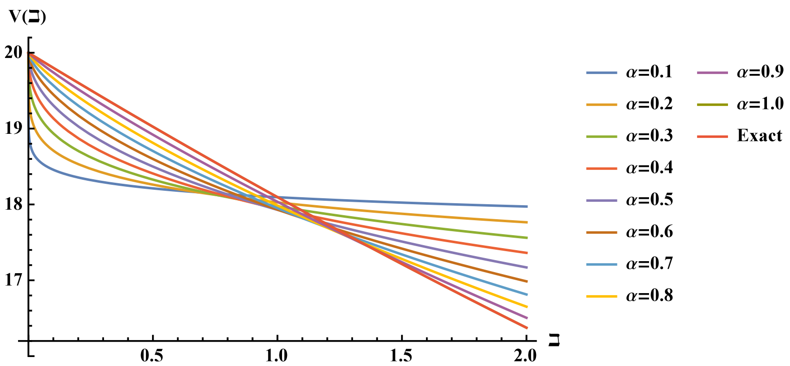
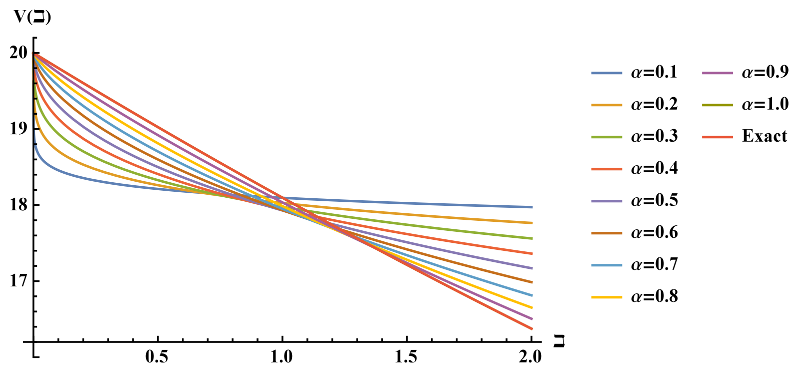



**Figure 5.** Graphical comparisons of Absolute Errors (AE) of obtained solutions by FPIA, FSDM, and FNAM.

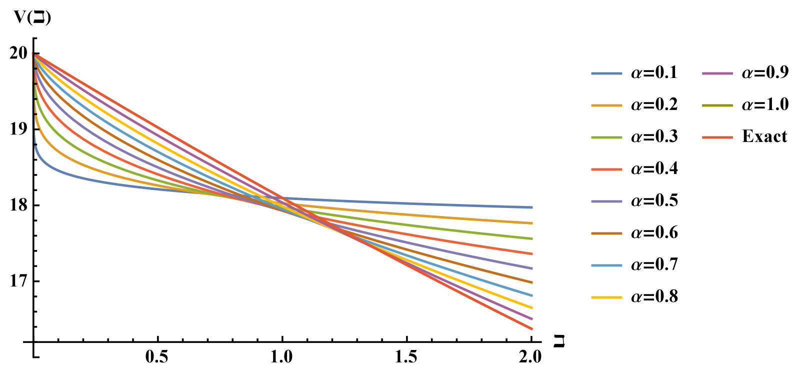



(**a**) Solutions via FPIA

**Figure 6.** *Cont.*




(**b**) Solutions via FSDM




(**c**) Solutions via FNAM

**Figure 6.** Numerical solutions obtained by (**a**) FPIA, (**b**) FSDM, and (**c**) FNAM of the electrical resistor–capacitor circuit equation of fractional-order at different $\alpha$ values.

### *5.3. Inductor–Resistor (RL) Circuit of Fractional Order*

Just resistor, a non-variant voltage source and inductor are present in the circuit and its FDE [17,18,48] is given as,

$$D^{\alpha}_{\beth} I(\beth) + \frac{R}{L} I(\beth) = \frac{E_0}{L}, \quad 0 < \alpha \le 1,$$

with initial condition $I(0) = i_0$. The classical solution of RL Circuit is $I(\beth) = \frac{E_0}{R} + \left(i_0 - \frac{E_0}{R}\right) \exp\left(-\frac{R}{L\alpha}\beth^{\alpha}\right)$.

FPIA: Introducing $\varepsilon$ with non-linear term yield,

$$D^{\alpha}_{\beth} I(\beth) + \varepsilon \frac{R}{L} I(\beth) = \varepsilon \frac{E_0}{L}.$$

Applying the Fractional Perturbation Iteration Algorithm PIA$(1,1)$ yields,

$$\left[(D^{\alpha}_{\beth} I(\beth))_n\right]_c = -\frac{D^{\alpha}_{\beth} I(\beth)}{\varepsilon} - \frac{R}{L} I(\beth) + \frac{E_0}{L}.$$

By taking fractional integral on both sides, we get,

$$[(I(\beth))_n]_c = J^{\alpha}_{\beth}\left[-\frac{D^{\alpha}_{\beth} I(\beth)}{\varepsilon} - \frac{R}{L} I(\beth) + \frac{E_0}{L}\right]. \tag{23}$$

An initial guess satisfying the initial condition should be selected. Using the algorithm of FPIA with Equation (23) and initial guess, the approximate solutions at each step are:

$$
\begin{aligned}
I_0(\daleth) =&i_0,\\
I_1(\daleth) =&\frac{E_0}{L\Gamma(1+\alpha)}\daleth^{\alpha}+i_0-\frac{i_0R}{L\Gamma(1+\alpha)}\daleth^{\alpha},\\
I_2(\daleth) =&\frac{E_0}{L\Gamma(1+\alpha)}\daleth^{\alpha}-\frac{E_0R}{L^2\Gamma(1+2\alpha)}\daleth^{2\alpha}+i_0-\frac{i_0R}{L\Gamma(1+\alpha)}\daleth^{\alpha}+\frac{i_0R^2}{L^2\Gamma(1+2\alpha)}\daleth^{2\alpha},\\
I_3(\daleth) =&\frac{E_0}{L\Gamma(1+\alpha)}\daleth^{\alpha}-\frac{E_0R}{L^2\Gamma(1+2\alpha)}\daleth^{2\alpha}+\frac{E_0R^2}{L^3\Gamma(1+3\alpha)}\daleth^{3\alpha}+i_0-\frac{i_0R}{L\Gamma(1+\alpha)}\daleth^{\alpha}+\frac{i_0R^2}{L^2\Gamma(1+2\alpha)}\daleth^{2\alpha}-\\
&\frac{i_0R^3}{L^3\Gamma(1+3\alpha)}\daleth^{3\alpha},\\
I_4(\daleth) =&\frac{E_0}{L\Gamma(1+\alpha)}\daleth^{\alpha}-\frac{E_0R}{L^2\Gamma(1+2\alpha)}\daleth^{2\alpha}+\frac{E_0R^2}{L^3\Gamma(1+3\alpha)}\daleth^{3\alpha}-\frac{E_0R^3}{L^4\Gamma(1+4\alpha)}\daleth^{4\alpha}+i_0-\frac{i_0R}{L\Gamma(1+\alpha)}\daleth^{\alpha}+\\
&\frac{i_0R^2}{L^2\Gamma(1+2\alpha)}\daleth^{2\alpha}-\frac{i_0R^3}{L^3\Gamma(1+3\alpha)}\daleth^{3\alpha}+\frac{i_0R^4}{L^4\Gamma(1+4\alpha)}\daleth^{4\alpha},\\
&\vdots
\end{aligned}
$$

and so on.

SFDM: By using SFDM, we get,

$$
\begin{aligned}
I(\daleth) =&\beta,\\
D^{\alpha}_{\daleth}I(\daleth) =&-\frac{R}{L}I(\daleth)+\frac{E_0}{L},\\
D^{2\alpha}_{\daleth}I(\daleth) =&-\frac{R}{L}D^{\alpha}_{\daleth}I(\daleth),\\
D^{3\alpha}_{\daleth}I(\daleth) =&-\frac{R}{L}D^{2\alpha}_{\daleth}I(\daleth),\\
D^{4\alpha}_{\daleth}I(\daleth) =&-\frac{R}{L}D^{3\alpha}_{\daleth}I(\daleth),\\
&\vdots
\end{aligned}
\tag{24}
$$

and so on. Substituting $\daleth = 0$ in each derivative result of Equation (24) gives the values of the fractional derivatives. In view of this, we obtain the fractional series approximation,

$$
\begin{aligned}
I(\daleth) =&i_0+\left(\frac{E_0}{L}-\frac{R}{L}i_0\right)\frac{\daleth^{\alpha}}{\Gamma(1+\alpha)}+\left(-\frac{E_0R}{L^2}+\frac{i_0R^2}{L^2}\right)\frac{\daleth^{2\alpha}}{\Gamma(1+2\alpha)}+\left(\frac{E_0R^2}{L^3}-\frac{i_0R^3}{L^3}\right)\frac{\daleth^{3\alpha}}{\Gamma(1+3\alpha)}+\\
&\left(-\frac{E_0R^3}{L^4}+\frac{i_0R^4}{L^4}\right)\frac{\daleth^{4\alpha}}{\Gamma(1+4\alpha)}+\left(\frac{E_0R^4}{L^5}-\frac{i_0R^5}{L^5}\right)\frac{\daleth^{5\alpha}}{\Gamma(1+5\alpha)}+\dots.
\end{aligned}
$$

FNAM: By following the steps carefully elaborated in the FNAM, we attain the following series of solution,

$$
I(\daleth) = \sum_{n=1}^{+\infty}\frac{(-1)^{n+1}E_0R^{n-1}\daleth^{n\alpha}}{L^n\Gamma(1+n\alpha)}+\sum_{n=0}^{+\infty}\frac{(-1)^{n}i_0R^{n}\daleth^{n\alpha}}{L^n\Gamma(1+n\alpha)}.
$$

Figure 7 represents the Time v/s Current graph and comparison of classical solution and numerical solutions obtained by FPIA, FSDM, and FNAM of the electrical inductor–resistor circuit equation of fractional-order, whereas the graph of classical solution and FNAM solution behave similarly at $\alpha = 1$ having a non-trivial overlap over each other. Graphical comparisons of absolute errors of obtained solutions by FPIA, FSDM, and FNAM of the electrical inductor–resistor circuit equation are represented in Figure 8 and the graph clearly shows that FNAM contains less error as compared to FPIA and FSDM. Figure 9 represents the graphs of obtained solutions by FPIA, FSDM, and FNAM of the fractional-order electrical inductor–resistor circuit equation at different values of $\alpha = 1$.

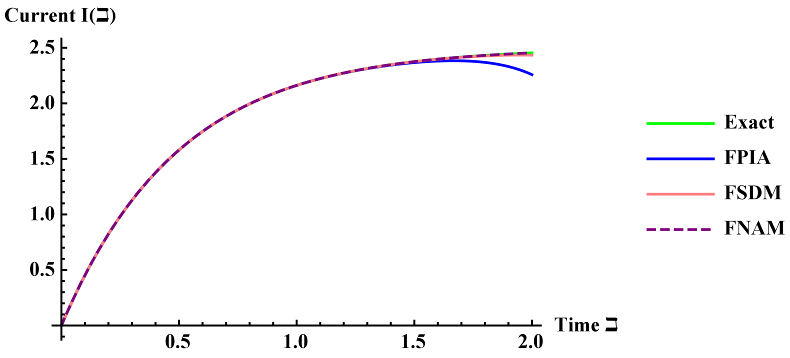



**Figure 7.** Graphical comparisons of classical solution and approximate solutions of the inductor–resistor circuit at $\alpha = 1$, $R = 4$, $L = 2$, $E_0 = 10$, and $i_0 = 0$.

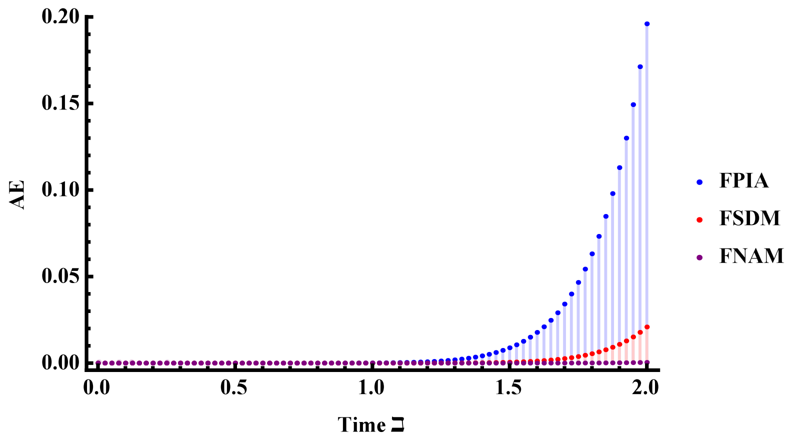
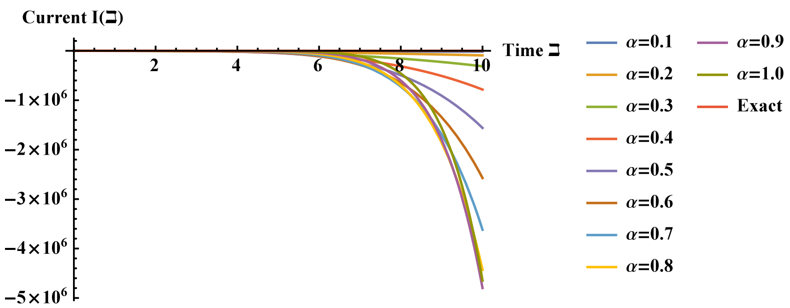
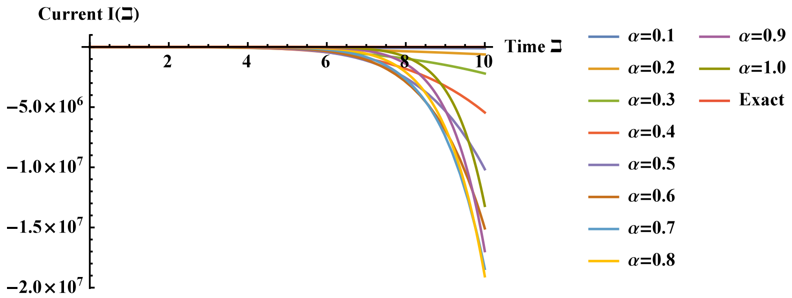



**Figure 8.** Graphical comparisons of Absolute Errors (AE) of obtained solutions by FPIA, FSDM, and FNAM.

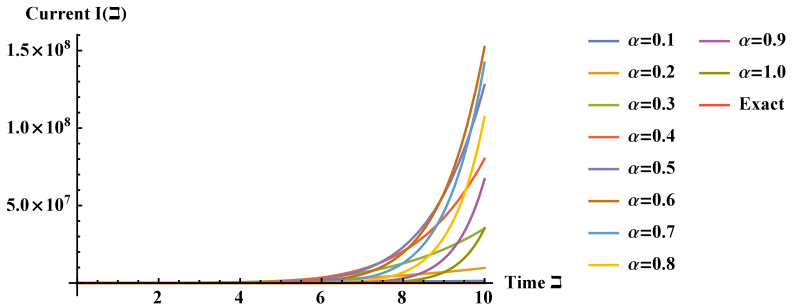



(**a**) Solutions via FPIA

**Figure 9.** *Cont.*




(**b**) Solutions via FSDM




(**c**) Solutions via FNAM

**Figure 9.** Numerical solutions obtained by (**a**) FPIA, (**b**) FSDM, and (**c**) FNAM of the fractional-order electrical inductor–resistor circuit equation at different $\alpha$ values.

## 6. Concluding Remarks

We have applied the Fractional Perturbation Iteration Method (FPIA), Fractional Successive Differentiation Method (FSDM), and Fractional Novel Analytical Method (FNAM) to three different types of electrical RLC circuit equations of fractional-order and compared them with their classical solutions. As observed after studying the numerical comparisons of graphs, we can state that FNAM acts as a generalization to the classical solutions. We calculated the 10th iteration of FPIA and FSDM and the 9th coefficients of FNAM, after which we compared the obtained results of FPIA, FSDM, and FNAM. The graphs clearly show that FNAM gives the best accuracy as compared to FPIA and FSDM and contains minimum Absolute Error. FNAM takes less computational time. This method works effectively to the Fractional Partial Differential Equations and system of Fractional Differential Equations and other physical models to develop a better understanding of use of FNAM in real-life problems.

**Author Contributions:** Conceptualization, M.S.; Data curation, U.A. and O.B.; Formal analysis, A.A.A.-m.; Investigation, M.S. and O.B.; Methodology, U.A.; Project administration, K.N.; Resources, U.A. and O.B.; Software, U.A. and A.H.A.; Supervision, K.N.; Validation, A.H.A.; Visualization, A.H.A.; Writing—original draft, A.A.A.-m.; Writing—review & editing, K.N. All authors have read and agreed to the published version of the manuscript.

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