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Article

On the Conditional Value at Risk Based on the Laplace Distribution with Application in GARCH Model

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Abstract: In this article, the Laplace distribution is employed in lieu of the well-known normal distribution for finding better scalar values of risk. Explicit formulas for value-at-risk (VaR) and conditional value-at-risk (CVaR) are studied and used to manage the risk involved in a stock movement by using the GARCH model. Numerical simulations are given for a variety of stocks in equity markets to uphold the findings.

Keywords: risk measure; stock market; Laplace distribution; non-normality; fat-tail

MSC: 91G70; 91B30

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1. Introduction and Preliminaries

Unlike the well-known and fundamental value-at-risk (VaR) measure, there is a coherent measure for various investing scenarios named the conditional value-at-risk (CVaR) measure, which assesses risk [1]. It is useful to have better bounds when we are faced with high oscillations of stock movements in a market and to be able to control and predict the involved risk in the investing process [2] (Chapters 14–15).

The best-known measure for controlling the risk of a stock is the VaR. It was originally provided in [3,4] in a different manner to estimate the VaR of a portfolio and in fact to optimize the profit at different levels of risk. By considering α as the pre-determined level of confidence, the VaR is given in what follows [5]:

$$\mathrm{VaR}_{\alpha}(X) := \min\{z \in \mathbb{R} | \alpha \leq F_X(z)\}, \tag{1}$$

where $F_X(\cdot)$ is the cumulative distribution function (CDF), X is a random variable, and $\alpha \in (0,1)$. The sub-additivity property does not hold for the VaR and thus it is not coherent [6,7].

As a matter of fact, for a given distribution, CVaR is the average loss at the extreme tail area. Hence it has enough superiority to be taken into account as an improvement on (1) [2] (Chapter 15). It is given by

$$\mathrm{CVaR}_{\alpha}(X) := \mathbb{E}[X | \mathrm{VaR}_{\alpha}(X) \leq X]. \tag{2}$$

On the contrary, the distribution of Laplace has been shown to provide more accurate results on the economical data once it is in a comparison to the well-known normal distribution. Hence, it is taken into further consideration here to study explicit formulas for the important measures of risk, viz., CVaR and VaR, in order to have reliable tools for managing risk [8,9].

Here we focus on the left tail rather than right tail of the return. This means that we care about what the "lowest" return is in an adverse situation. There is a similar formulation for VaR and CVaR, at which the focus is on the left tail by choosing the confidence level as $1-\alpha$ in lieu of α in (1) or (2), so we can see what implications might occur in an adverse situation.

This article studies explicit formulas for the CVaR and VaR measures under the Laplace distribution. Furthermore, we contribute by examining the risk measures in computing the risks of the daily performance of several well-known stocks by using time series models for forecasting. To fit the financial and economical data, such as stock returns, we here employ the effective process of generalized autoregressive conditional heteroskedasticity which is also known as the GARCH model [10,11]. It is now requisite to recall that the authors in [12] studied the measure of CVaR on several well-known continuous distributions and more particularly obtained a general formulation for the GEV distribution. Our work is distinguished from their work and focuses on the GARCH application by using these risk measures based on the Laplace distribution as well. This work studies risk modeling along the research line of [13,14].

Here we are motivated by selecting the Laplace distribution in forecasting the risk because it leads to wider tails that match better in practice with the observations of unstable equity markets (see also [13]).

The remaining parts of this paper are as follows. In Section 2, the continuous Laplace distribution and its VaR are given. This distribution has wider tails than the normal distribution so it is more applicable to the financial data and provides further understanding of the likelihood of extreme events. Next, the risk measures of CVaR for this heavy-tail distribution is constructed in Section 3. Section 4 is provided to give the existing famous risk measures under the specified distribution in forecasting the risk for several stocks from various markets via the GARCH(1,1) model (see [15] for more background). Several comparisons are worked out along with numerical simulations. At last, a summary of the work along with some comments for forthcoming works are provided in Section 5.

2. Computation of VaR Employing the Laplace Distribution

The double exponential distribution, which is also named as the Laplace distribution, is the distribution of differences with identical exponential distributions between two independent variates. As a matter of fact, for two identically distributed exponential random variables which are independent, their difference is expressed under a Laplace distribution, as is a Brownian motion [16] calculated at an exponentially distributed random time.

Over the set of real numbers, the Laplace distribution can be defined with a σ as the scale parameter and a μ as its mean. More precisely, if

$$X \sim \text{Laplace}(\mu, \sigma), \tag{3}$$

then its probability density function (PDF) can be given by

$$f(x) = \begin{cases} \frac{e^{\frac{x-\mu}{\sigma}}}{2\sigma}, & x < \mu, \\ \frac{e^{\frac{\mu-x}{\sigma}}}{2\sigma}, & x \geq \mu. \end{cases} \tag{4}$$

Its CDF is presented as follows:

$$F(x) = \begin{cases} 1 - \frac{1}{2} e^{\frac{\mu-x}{\sigma}}, & x \geq \mu, \\ \frac{1}{2} e^{\frac{x-\mu}{\sigma}}, & x < \mu. \end{cases} \tag{5}$$

The PDF (4) and CDF (5) of this distribution are given in Figure 1 with zero mean, which shows that its PDF has wider tails in comparison to the famous normal distributions or those of Gumbel. The parameters σ and μ illustrate the overall height and steepness and the horizontal location, respectively. Additionally, the required moments in this case could be provided as follows:

$$\text{Variance }(X) = 2\sigma^2, \text{ Skewness }(X) = 0, \tag{6}$$

$$\text{Kurtosis }(X) = 6, \text{ Median }(X) = \mu, \text{ Mean }(X) = \mu. \tag{7}$$

Here the target is not to replace the normal distribution with the Laplace distribution (3). In fact, the risk measures are defined under any distributions. On the other hand, the need for the other distributions come from the fact that prices/returns from stock indices or similar financial derivatives may not follow the standard normal distribution in general, and it is requisite to generalize/investigate the other heavy-tailed distributions for various purposes in financial engineering. The Laplace distribution is useful in predicting when a fatter tail on the underlying prices exists in market and to express extremely different events [17].

We note that this distribution is proper for stochastic modeling because it is stable under geometric rather than ordinary summation. In order to model the regression, when the errors follow this distribution, then the least absolute deviation (LAD) estimate is the maximum likelihood estimate as well, equivalent to the least squared deviation estimate when the errors possess a distribution of normal. This fact could be observed from (4), which distinguishes mostly from the normal distribution by including a term for the mean absolute rather than a squared deviation of a random variable [17].

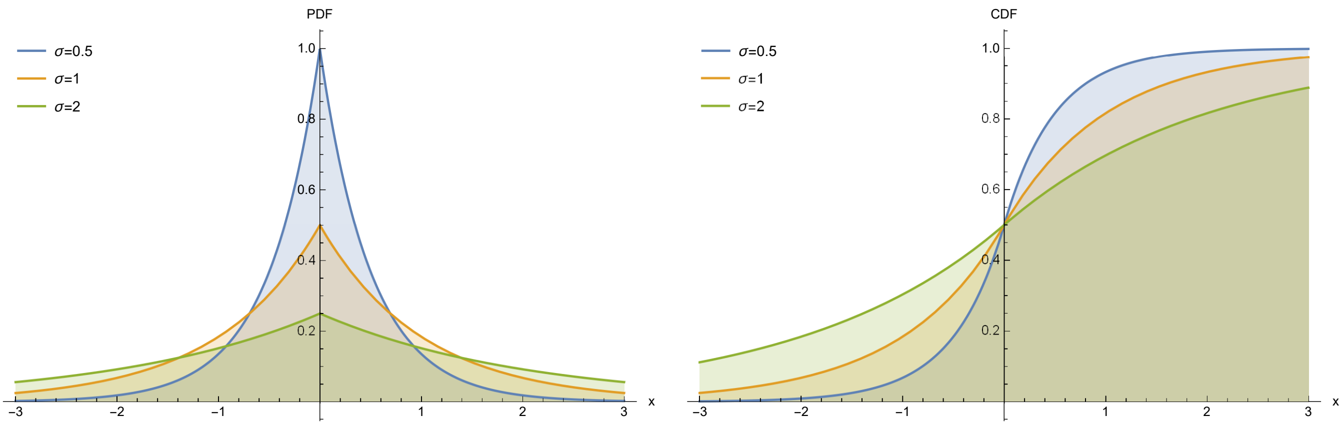

Figure 1. (**Left**) The PDFs of the Laplace distribution by varying the scale parameter. (**Right**) The CDFs of the Laplace distribution by varying the scale parameter.

Theorem 1. *Let $X \in L^p$ be a random variate indicating the loss for the Laplace distribution(μ, σ). The risk measure of VaR is obtained in an explicit form.*

Proof. Let $p \geq 1$, and the random variate X be in L^p space to ensure the existence of the expectation. Now by considering (1), one obtains

$$\begin{aligned} \text{VaR}_\alpha(X) &= \min\{t \in \mathbb{R} \mid p(X \leq t) \geq \alpha\}, &(8)\\ &= \min\{t \in \mathbb{R} \mid \alpha \leq F_X(t)\}, &(9)\\ &= \min\left\{t \in \mathbb{R} \mid \frac{e^{\frac{\mu - x}{\sigma}}}{2\sigma} \geq \alpha\right\}, &(10)\\ &= \mu - \sigma \log(2 - 2\alpha), \qquad 1/2 < \alpha < 1, &(11) \end{aligned}$$

noting that log stands for the natural logarithm. The proof is complete. □

Here the two main parameters are the time horizon and the confidence level α, which should be selected with attention and care for the aim of the risk allocation (corporate risk management, regulatory reporting, and so on). In addition, the asymmetric Laplace distribution (ALD) is a continuous probability distribution and is considered as an extension over (3). It includes two exponential distributions of unequal scale back-to-back around the real location, adjusted to assure normalization and continuity.

The VaR measure under this generalized distribution has been discussed in terms of applicability and usefulness in the works of [18,19]. Consequently, our work focuses on the CVaR case in the next section, and its application to time series fitting on financial data is our main contribution.

3. CVaR

We start by noting that the sensitivity of CVaR is higher than VaR for errors of approximation [20]. The challenge of our aim now is how to aggregate various types of risk and distributions. Such an investigation is outlined in the theorem below.

Theorem 2. *Considering the assumptions of Theorem 1, the measure of CVaR employing the Laplace distribution, is obtained via (14).*

Proof. Employing a similar methodology as in the Theorem 1's proof and having (2), we obtain

$$\begin{aligned} \mathrm{CVaR}_\alpha(X) &= \mathbb{E}[X|X \geq \mathrm{VaR}_\alpha(X)], && (12)\\ &= \mathbb{E}[X|X \geq \mu - \sigma\log(-\log(\alpha))], && (13)\\ &= -\sigma\log(2-2\alpha) + \mu + \sigma. && (14) \end{aligned}$$

The proof is ended here. □

A comparison between (1) and (2) under (3) is shown in Figure 2 revealing that CVaR takes much higher risk values than the VaR.

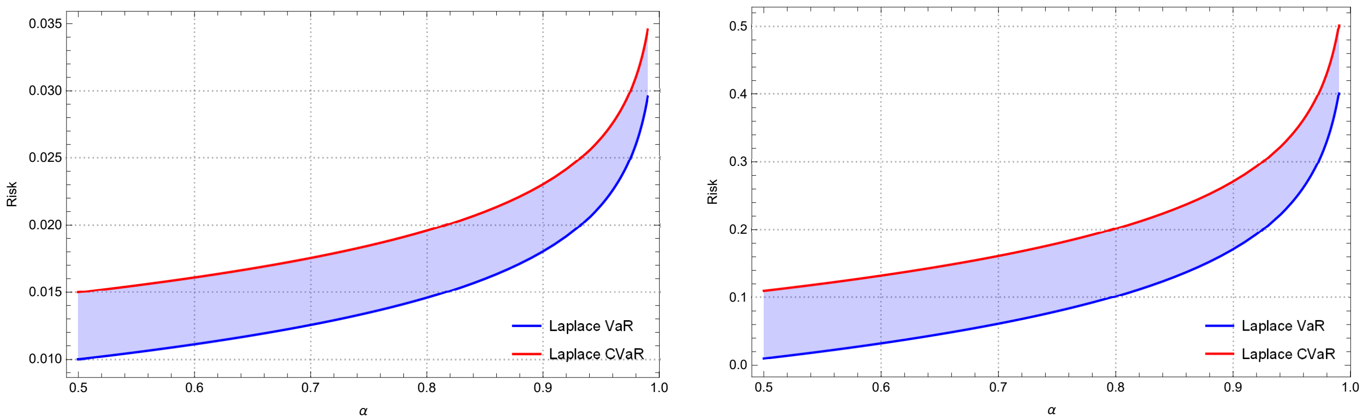

Figure 2. Laplace VaR and CVaR by varying the predetermined confidence level for $\mu = 0.01$ and $\sigma = 0.005$ and $\sigma = 0.1$ on the left and right, respectively.

Risk Modeling

Several works have shown that future variance forecasting via state-of-the-art GARCH processes is necessary to handle the risk of portfolio risk effectively [21], due to the presence of the heteroskedasticity's effects. As such, herein we employ the GARCH(1,1) process by defining ϱ as the expected return and $w > 0$ as follows [22,23]:

$$\begin{aligned} r_t &= \varrho + \varepsilon_t = \varrho + \sigma_t z_t,\\ \sigma_t^2 &= w + \lambda\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \end{aligned} \quad (15)$$

where z_t is a stochastic piece. Herein, r_t is the actual return, σ_t is the volatility of the returns on day t (see the discussed models in [24,25] for further understanding about the concept of volatility), and eventually

$$\lambda + \beta < 1,\ \beta \geq 0,\ \lambda \geq 0. \tag{16}$$

Noting that

$$\beta + \lambda < 1 \tag{17}$$

in (16) confirms that we have obtained a stationary solution for the GARCH model [26,27].

It is notable that the GARCH model is not based on normal distribution. Managing the risk based on the GARCH model does not mean that one can replace the normal distribution with fat-tail distributions such as the Laplace distribution. Actually, we have built the VaR and CVaR risk measures under (3) and found some theoretical findings. Then the new relations will be used together with the GARCH model without replacing the normal distribution with the Laplace distribution in some applications in order to show and illustrate how the findings can work in practice.

Now we intend to examine both risk measures of CVaR and VaR to calculate the risk under the Laplace distribution and show their applications in time series forecasting. This is pursued in the next section.

Note that the influence of other distributions can be tracked down. In fact, the use of some distributions such as the Gumbel distribution, due to its unsymmetrical PDF behavior, might be limited to special cases, but that both Gamma and Beta distributions might be helpful to construct better results. Theoretically speaking, one difficulty which occurs here is that for more general distributions, obtaining the closed formulas for the VaR/CVaR as well as closed formulas for their application by the GARCH approach is not that possible. In fact, the involved integrals cannot be computed theoretically, and for other general distributions, one might compute the risk measures numerically. In addition, the fatter the tail, the higher the risk measure values that can be obtained.

4. Numerical Results

4.1. Simulations

The purpose of this subsection is to simulate the theoretical findings of (11) and (14). In fact, it is necessary to check the following relations having the same settings:

$$\text{VaR (Laplace)} \geq \text{VaR (normal)}, \qquad \text{CVaR (Laplace)} \geq \text{CVaR (normal)}. \tag{18}$$

Toward this objective, we have simply compared these risk measures in Table 1 confirming the upper values for risk measures when the Laplace distribution is employed. This assists the traders to obtain more reliable confidence in investing in different stocks on various trading days and avoiding severe loss.

The computational evidence also reveals that the increased risk budget is enough (viz., one does not overestimate risk, by raising the risk budget, thus reulting in overcautiousness).

The results in Table 1 are based on only three parameters, because the only parameters which affect the results of the risk measures are μ, σ, and α based on (11) and (14). The results also confirm the inequalities given in (18) and also show that the VaR and CVaR values tend to each other when the confidence level α tends to 100%.

Table 1. Comparisons of the normal and Laplace distributions in terms of the observed risk values.

μ	σ	α	**VaR (Normal)**	**VaR (Laplace)**	**CVaR (Normal)**	**CVaR (Laplace)**
0.02	0.004	0.90	0.025	0.026	0.027	0.030
0.02	0.004	0.91	0.025	0.026	0.027	0.030
0.02	0.004	0.92	0.025	0.027	0.027	0.031
0.02	0.004	0.93	0.025	0.027	0.027	0.031
0.02	0.004	0.94	0.026	0.028	0.027	0.032
0.02	0.004	0.95	0.026	0.029	0.028	0.033
0.02	0.004	0.96	0.027	0.030	0.028	0.034
0.02	0.004	0.97	0.027	0.031	0.029	0.035
0.02	0.004	0.98	0.028	0.032	0.029	0.036
0.02	0.004	0.99	0.029	0.035	0.030	0.039
0.02	0.006	0.90	0.027	0.029	0.030	0.035
0.02	0.006	0.91	0.028	0.030	0.030	0.036
0.02	0.006	0.92	0.028	0.030	0.031	0.036
0.02	0.006	0.93	0.028	0.031	0.031	0.037
0.02	0.006	0.94	0.029	0.032	0.031	0.038
0.02	0.006	0.95	0.029	0.033	0.032	0.039
0.02	0.006	0.96	0.030	0.035	0.032	0.041
0.02	0.006	0.97	0.031	0.036	0.033	0.042
0.02	0.006	0.98	0.032	0.039	0.034	0.045
0.02	0.006	0.99	0.033	0.043	0.035	0.049
0.04	0.004	0.90	0.045	0.046	0.047	0.050
0.04	0.004	0.91	0.045	0.046	0.047	0.050
0.04	0.004	0.92	0.045	0.047	0.047	0.051
0.04	0.004	0.93	0.045	0.047	0.047	0.051
0.04	0.004	0.94	0.046	0.048	0.047	0.052
0.04	0.004	0.95	0.046	0.049	0.048	0.053
0.04	0.004	0.96	0.047	0.050	0.048	0.054
0.04	0.004	0.97	0.047	0.051	0.049	0.055
0.04	0.004	0.98	0.048	0.052	0.049	0.056
0.04	0.004	0.99	0.049	0.055	0.050	0.059
0.04	0.006	0.90	0.047	0.049	0.050	0.055
0.04	0.006	0.91	0.048	0.050	0.050	0.056
0.04	0.006	0.92	0.048	0.050	0.051	0.056
0.04	0.006	0.93	0.048	0.051	0.051	0.057
0.04	0.006	0.94	0.049	0.052	0.051	0.058
0.04	0.006	0.95	0.049	0.053	0.052	0.059
0.04	0.006	0.96	0.050	0.055	0.052	0.061
0.04	0.006	0.97	0.051	0.056	0.053	0.062
0.04	0.006	0.98	0.052	0.059	0.054	0.065
0.04	0.006	0.99	0.053	0.063	0.055	0.069

4.2. Application

To calculate the risk measures of CVaR and VaR, the volatility forecast for one day in advance has been considered. The aim now is to analyze CVaR and VaR forecasting by the GARCH model to control the risk that occurs on the trading days of an open stock.

Noting that the simulation tests in this article are performed with Mathematica 12.0 [28].

4.2.1. Test on "NASDAQ:GOOG"

Here we use a ticker from NASDAQ as follows: "NASDAQ:GOOG". It is an Alphabet Inc. company in Mountain View, California. Alphabet is an American multinational conglomerate. It was constructed via a re-structuring of Google on 2 October 2015, and became the parent company of Google and several former Google subsidiaries.

The specified period of time to check the usefulness of the discussed risk measures is 1 June 2019 through 9 July 2020, which states having 277 data points based on the daily fractional changes. The name and sectors of this ticker are "Alphabet Class C Shares" and "Internet Content And Information", respectively.

The floating share is also 693,398,350 (at the time of gathering the data for simulations). Note that the floating share is the number of shares available for trading of the stock. The floating share is computed via subtracting closely held shares and restricted stock from a firm's total outstanding shares.

To evaluate the risk measure on real data, we need to first get the stock's returns. This is illustrated in Figure 3. The returns are the difference of two subsequent logarithms of prices in two trading days. Please see Appendix A.1.

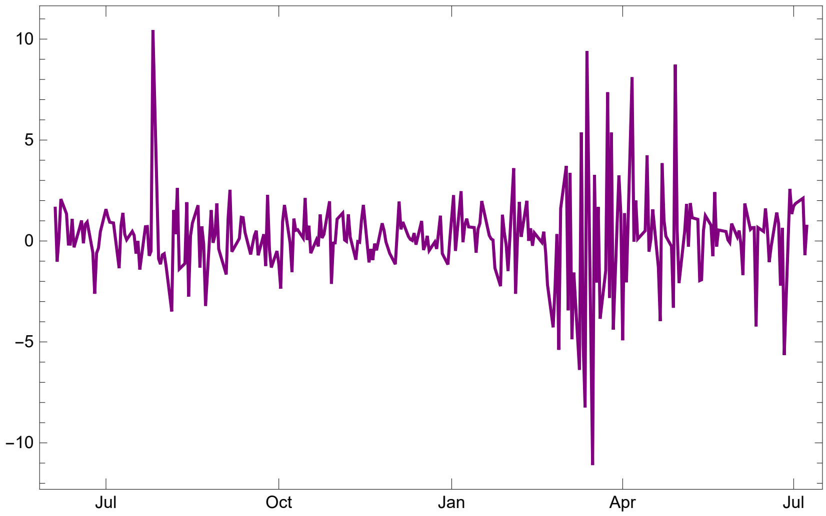

Figure 3. The daily returns of the NASDAQ:GOOG on the considered time period.

We employ the process (15) on the stock's returns via a time series fit methodology. This part is further illustrated in Appendix A.2.

Now it in fact yields the characteristics in Table 2 and the following matrices for covariance and information, respectively:

$$A = \begin{pmatrix} 1.37041 & -1.46373 \\ -1.46373 & 2.55142 \end{pmatrix}, \tag{19}$$

$$B = \begin{pmatrix} 1.88435 & 1.08104 \\ 1.08104 & 1.01212 \end{pmatrix}. \tag{20}$$

The correlation function is also drawn for this test in Figure 4 based on the `modelfit` given above. Here, after some calculations, the normal VaR and Laplace CVaR under the GARCH(1,1) are respectively contributed by

$$\text{Normal VaR} = \text{erfc}^{-1}(2\alpha)\left(-\sqrt{0.677528x^2 + 0.366733y^2 + 2.275}\right), \tag{21}$$

$$\text{Normal CVaR} = -\frac{e^{-\text{erfc}^{-1}(2\alpha)^2}\sqrt{0.053916x^2 + 0.0291837y^2 + 0.181038}}{\alpha - 1}, \tag{22}$$

where x, y are filled based on the times series thread (its two components). Moreover, the Laplace VaR and CVaR under the GARCH(1,1) would be obtained ultimately, respectively, as follows:

$$\text{Laplace VaR} = \log(2 - 2\alpha)\left(-\sqrt{0.575643x^2 + 0.109422y^2 + 1.61859}\right), \tag{23}$$

$$\text{Laplace CVaR} = (\log(2-2\alpha)-1)\left(-\sqrt{0.575643x^2+0.109422y^2+1.61859}\right). \quad (24)$$

To illustrate further, (23) is obtained after imposing $\mu = 0$ and $\sigma = \sqrt{\text{par}[[1]]+\text{par}[[2,1]]x^2+\text{par}[[3,1]]y^2}$ in (11). Here "par" stands for the best time-series model fit on the returns via the GARCH process and the maximum likelihood as the process estimator. This is illustrated in the following piece of code:

```
gVaRfn =  gdVaR /. {\mu -> 0,
\sigma -> Sqrt[par[[1]] + par[[2, 1]] x^2 + par[[3, 1]] y^2]};
gVaRfn // FullSimplify
```

where `gdVaR` is (11).

This is in a similar fashion as (24). It is a fact that GARCH(1,1) has been found by several works of literature to perform best in modeling the volatility of stock returns.

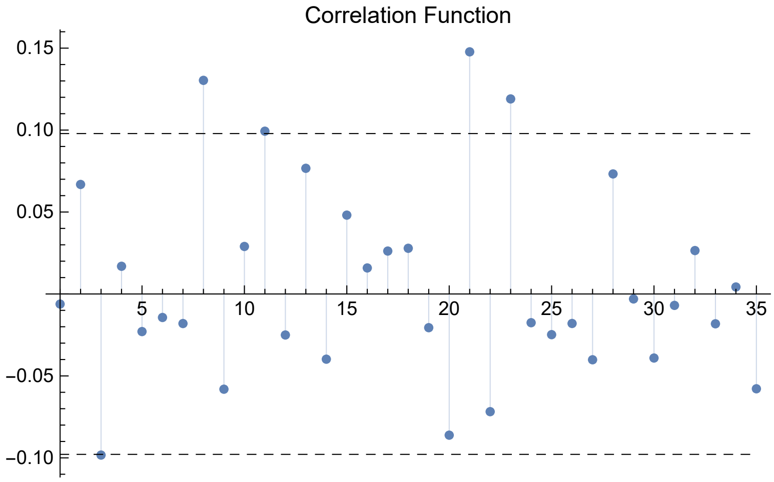

Figure 4. The correlation function obtained by the GARCH(1,1) process on NASDAQ:GOOG.

Remark 1. *It is also necessary to point out that one may ask why GARCH(1,1) is an interesting example but why it is a limited one and why other GARCH models must be employed for the sake of comparisons. To respond this, we indicate that we let the programming package choose the best process for fitting on the input of financial data according to its own criteria such as AIC. The point is that for all such cases, the best-fit model given by the programming package is GARCH(1,1). Due to this, providing a comparison to other ones for such a circumstance is not requisite and has not been included.*

The simulation evidence given in Figures 5–7 for this experiment also reveal that

- both risk measures tend to one another by increasing the pre-determined confidence level, and
- selecting $\alpha = 95\%$ seems to be a reliable selection for a highly volatile stock until we employ measures based on the Laplace distribution.

Computational evidence confirms the analytic provided at Section 3, by showing that the measure of CVaR when using the Laplace distribution is a reliable risk measure for risk allocation in the market during various time frames.

Table 2. Fitted parameters of the GARCH(1,1) process of the NASDAQ:GOOG.

w	λ	β	**Variance of Error**
1.61859	0.575643	0.109422	148.687

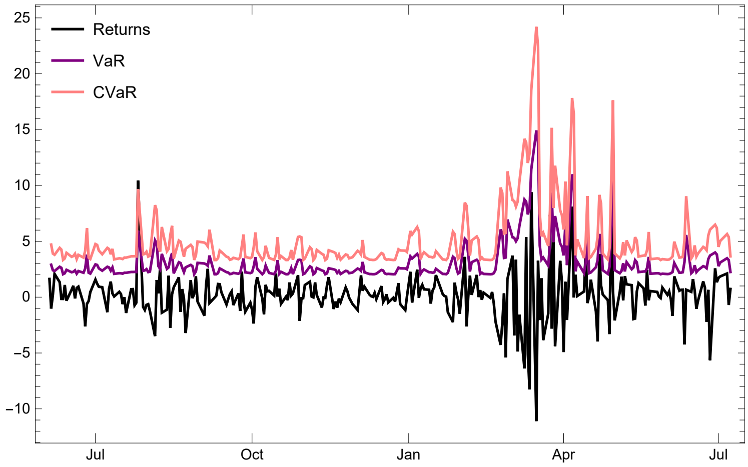

Figure 5. Results of risk comparisons (based on Laplace distribution) by the pre-determined confidence level $\alpha = 90\%$ for the NASDAQ:GOOG. The horizontal and vertical axes stand for the time and risk values and returns, respectively.

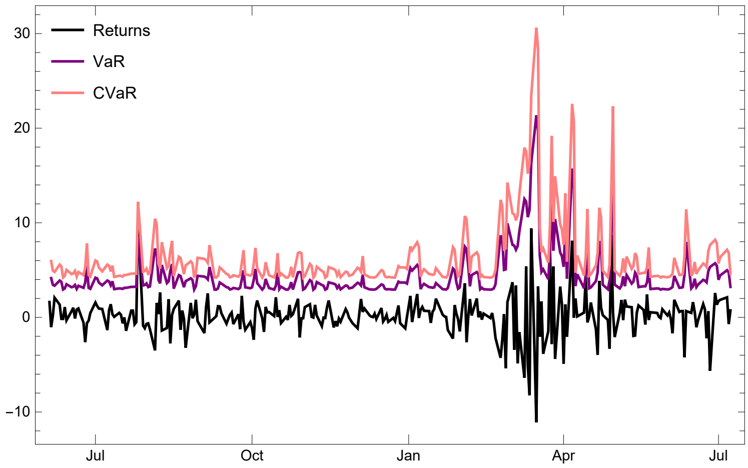

Figure 6. Results of risk comparisons (based on Laplace distribution) by the pre-determined confidence level $\alpha = 95\%$ for the NASDAQ:GOOG. The horizontal and vertical axes stand for the time and risk values and returns, respectively.

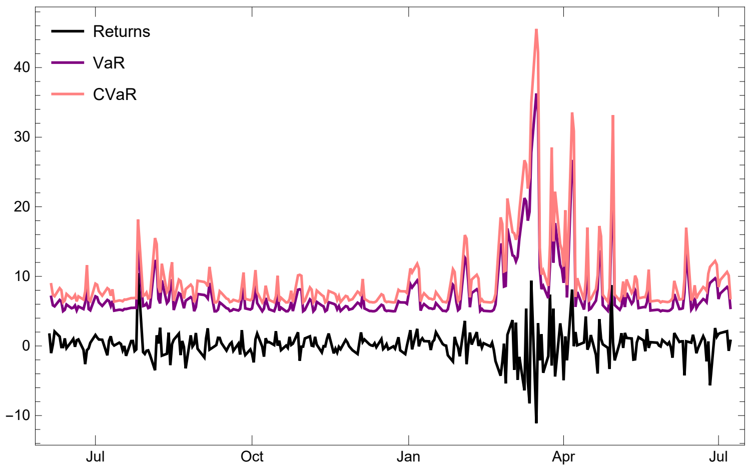

Figure 7. Results of risk comparisons (based on the Laplace distribution) by the pre-determined confidence level $\alpha = 99\%$ for the NASDAQ:GOOG. The horizontal and vertical axes stand for the time and risk values and returns, respectively.

We end this subsection by comparing the normal CVaR and Laplace CVaR for risk management under the GARCH process and the given experiment features. The computational evidence in this case is portrayed in Figure 8, which again manifests that CVaR under the Laplace distribution is a good choice for risk management because it owns fatter tails, and thus more reliable values for risk can be provided for trading in investment strategies.

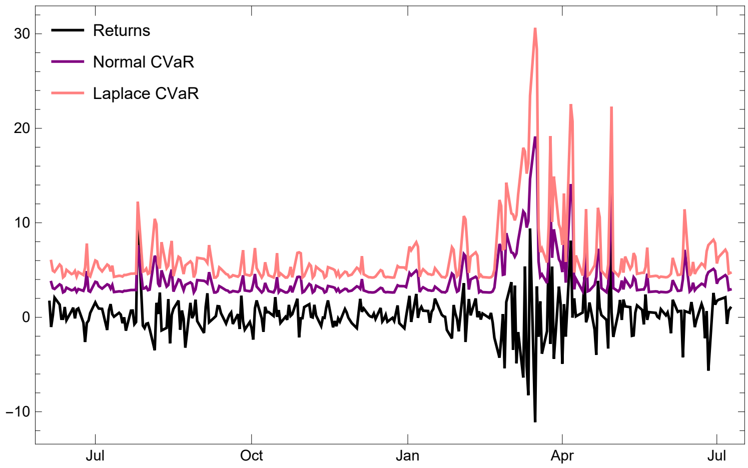

Figure 8. Computational evidence of the NASDAQ:GOOG for risk comparisons with $\alpha = 95\%$ showing the superiority of the fatter-tailed distribution. The horizontal and vertical axes stand for the time and risk values and returns, respectively.

4.2.2. Test on "NYSE:MGM"

Now we must ask if this proposed method can handle bigger data, as the previous test has smaller elements. Hence, the goal of this test is just to assess the applicability and usefulness of the discussed risk measures for a stock on a large time scale. This means that the number of observations is quite high enough to cover two different financial crises, i.e., the 2008 financial crises and the Covid-19 crisis of 2020.

We use a ticker from NYSE as follows: "NYSE:MGM" with the name "MGM Mirage", which is under the Resorts and Casinos sector; its floating share is 493,281,168 (at the time of gathering the data for writing up this work).

The working framework to evaluate the fruitfulness of the presented risk measures is 1 January 2007 through 3 November 2020, which states having 3484 data observations based on the daily fractional changes.

Here we do not include all the details, as in the previous test, and just illustrate the comparisons of the risk measure in Figure 9. Results re-confirm the theoretical discussions. In fact, even for cases in which there are so many observations, the proposed VaR and CVaR risk measures under the Laplace distribution can yield reliable results over time. CVaR risk values are always higher than VaR values as expected as well.

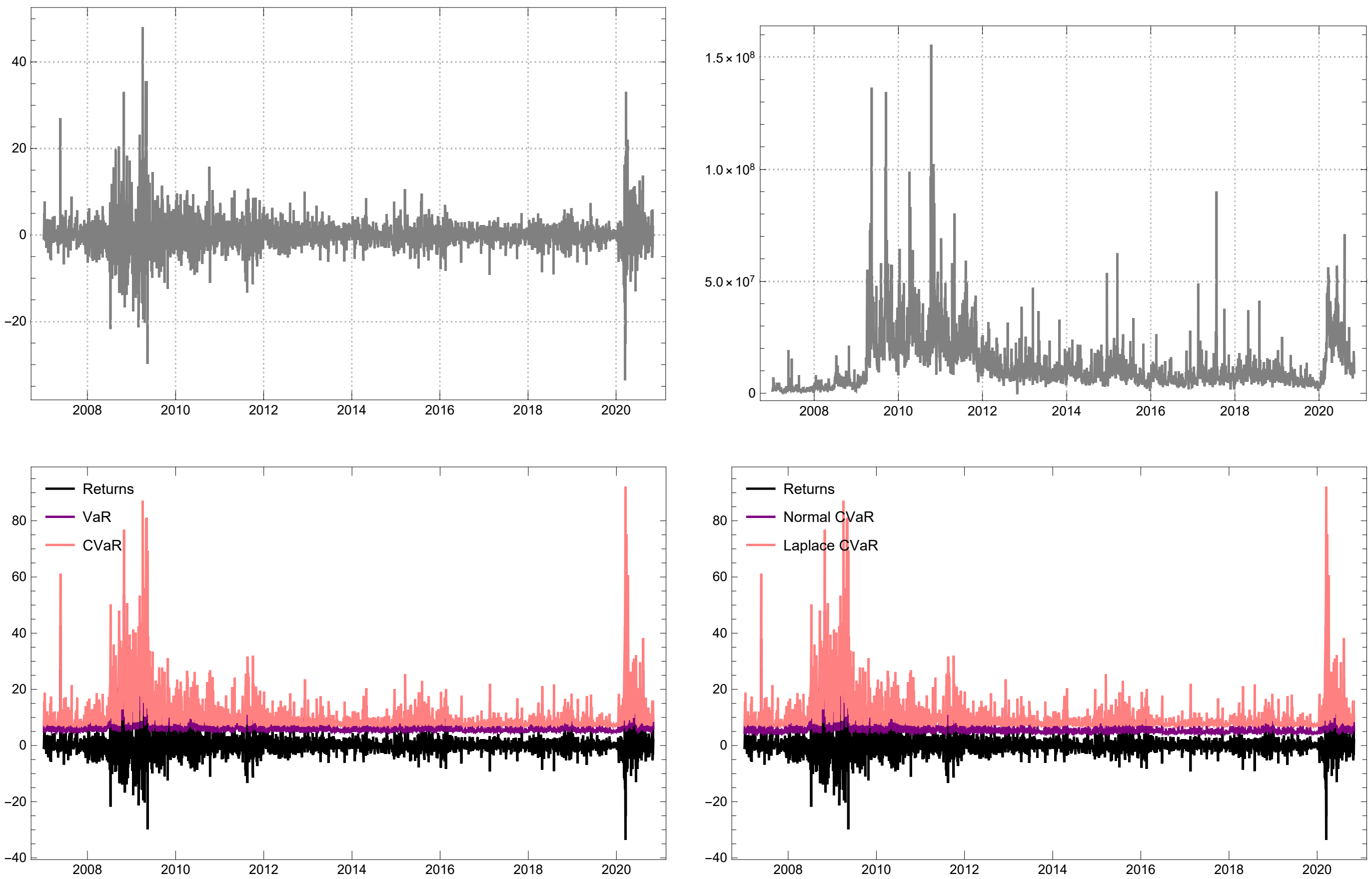

Figure 9. Computational evidence of the "NYSE:MGM" with $\alpha = 95\%$. (**Top left**) the fractional change over the time. (**Top right**) Its volume over the time. (**Bottom left**) Laplace VaR and CVaR comparisons. (**Bottom right**) Laplace and Normal CVaR comparisons. The horizontal axis stand for the time of evaluation.

5. Conclusions

A useful characteristic of GARCH-type processes is that they capture the fat-tailedness along with clustering of the volatility. In this paper, we have investigated the closed forms of the VaR and CVaR under the Laplace fat-tailed distribution and applied these risk measures for controlling the risk of stock movements. In fact, the paper has sought to extend the frontiers of modeling volatility by forecasting via GARCH with VaR and CVaR measures under the Laplace distribution. The results confirmed the applicability of the Laplace distribution when compared to the normal or Gumbel distributions for risk managements. The extension of our discussions based on other well-known fat-tailed distributions and with application of some financial stochastic models [29] are under investigation by our team for future studies.

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Appendix A

Appendix A.1. Program 1

To make the understanding of the procedure easier, the following piece of Mathematica code is used:

```
return = FinancialData["NASDAQ:GOOG",
"FractionalChange", {{2019, 06, 01}, {2020, 07, 09}, "Daily"}]
DateListPlot[return, ImageSize -> 450, PlotStyle -> Purple,
PlotRange -> All, FrameLabel -> {"Time", "Fractional Change"}]
```

In this case, the output for the returns would be

```
TimeSeries[ Time: 04 Jun 2019 to 09 Jul 2020
            Data points: 279
            Regular: False          Output dimension: 1
            Metadata: 1 property    Minimum increment: {1, Day}
                                    Resampling: HoldValueFromLeft
```

Appendix A.2. Program 2

This is simplified in our programming language via the following piece of code:

```
modelfit = TimeSeriesModelFit[return, "GARCH",
ProcessEstimator -> "MaximumLikelihood"]
par = modelfit["Properties"]
```

Which results in:

```
TimeSeriesModel[ Family: GARCH
                 Order: {1, 1}
                 Time: 3 768 595 200 to 3 803 241 600
                 Data points: 402
                 Candidates: 2
                 Criterion:
```

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