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The City as a Tool for STEAM Education: Problem-Posing in the Context of Math Trails

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Abstract: This study presents an experience that combines problem-posing and Math Trails in the context of future teachers' instruction. Pre-service teachers in the third year of their studies were faced with the design of tasks to be included in Math Trails for primary school students. The study analyzes, from a quantitative approach, 117 tasks contained in 11 Math Trails. The analysis was performed on the basis of classification variables (grade, mathematical content and object or real element involved in every task) and research variables which provide information about the nature of the tasks (procedural vs. problem-solving, level of cognitive demand, degree of contextualization, openness and creativity). Additionally, relationships between the different categories of analysis have been studied. The results reveal certain biases in the tasks in relation to the contents addressed (an abundance of tasks with a geometric component and a scarcity of tasks involving algebra or probability concepts). Most of the tasks are presented in a real context. However, a higher percentage of procedural tasks than problem-solving tasks is observed, with a predominance of low openness, creativity and cognitive demand. These results provide useful lines of work to address difficulties faced by future teachers in the STEAM field.

Keywords: Math Trails; problem-posing; initial teacher training; task-design

MSC: 97B50

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1. Introduction

The abstract nature of mathematics and the specificity of its symbolic language constitute a learning barrier in the initial stages of education that eventually leads to a reduction in general interest in the discipline and other related areas, the so-called STEM (science, technology, engineering and mathematics). The growing demand for qualified professionals in these fields is compromised, with the prospect of employment difficulties in the coming decades. The situation is not encouraging even after adding the STEAM perspective, derived from STEM and Arts, which allows for a more motivating and personal approach for students [1–3].

This perception of mathematics is maintained over time by the perpetuation of myths, such as "Mathematics is difficult", "There are people who are naturally gifted in mathematics" or "School mathematics and real-life mathematics are two separate worlds with little relation to each other" [4–6]. Combating this deep-rooted view in our societies goes beyond the limits of the area of Mathematics Education and should be considered a problem of social interest in which gender, class and racial exclusion also converge [7,8].

The perpetuation of these myths is fueled by work in the mathematics classroom, or in any other STEM discipline, based on teaching–learning processes with a behavioral approach: reduction to simple rules and resolution of standard cases that can be memorized through repetition. These traditional educational models lead to the decontextualization of knowledge because the space and time of the classroom are far from the place and time where the knowledge acquired will be needed. The problems derived from this situation are the loss of motivation of the students, the lack of significance of the subject matter and the difficulty of transferring what has been learned to its application [9,10].

1.1. Math Trails: Situated Learning in Mathematics Education

Situated learning (SL) is based on the central idea that the teaching–learning process is directly influenced by the environment (spatial and social) in which it takes place and, as a result, there will be environments that are more appropriate than others for achieving certain types of learning [11].

Working in out-of-class environments allows students to contextualize abstract knowledge, facilitates a cross-curricular vision of knowledge and increases interaction between participants. Furthermore, these experiences favor the linking of students with their immediate environment, thereby favoring the development of critical thinking and social involvement [12,13].

Among the situated practices, a distinction is made between those that take the classroom to real environments (field trips and educational visits, field projects or meetings with experts) and those that, in the opposite direction, bring the real context to schools (such as the creation of workshops, laboratories or school orchards). The first group includes Mathematical Trails.

Math Trails can be defined as a route with marked stops where interesting mathematical problems are formulated, discussed and solved. They are based on using the urban or natural elements present in the environment as a source of data for investigations or problems; revealing the mathematics that is present in our everyday world [14,15].

Designing a Math Trail is a creative challenge that starts from the consideration of the city, or any other environment, as an educational space in itself. This vision of the role of cities in the education of their inhabitants is the focal point of movements such as the one promoted by the international association Educating Cities. Its theoretical proposal [16] considers that education should not be confined to places with a specific functionality, such as the school or the museum, but that all spaces in a city can contribute to the education of its inhabitants to varying degrees. This holistic approach is the starting point for the educational proposal presented in this paper.

The creation of a mathematical walk, from the teaching point of view, requires, on the one hand, the design of the educational experience that its implementation will entail: duration, distance to be covered, materials needed, level of autonomy of the participants, etc. and, on the other hand, it requires the creation of the tasks that will be proposed in the walk.

This second process, associated with the problem-posing concept, implies, firstly, the development of an adequate mathematical outlook capable of recognizing the educational possibilities present in each element of the city, and secondly, the implementation of all the mathematical contents and processes that will be necessary to apply in the process of solving each problem, taking as a basis the level of knowledge that will be required of the future problem-solvers [15,17].

The design of contextualized tasks, within the framework of a Math Trail, allows a shift from academic mathematical situations to real-life mathematical situations. In addition, the teaching objective is shifted towards the development of problem-solving skills rather than the development of purely procedural skills [18–20].

Finally, Math Trails support the STEAM educational approach by addressing mathematics in a contextualized way on the basis of real objects, sometimes drawn directly from the fields of architecture, engineering or the arts; by promoting the use of technology: mobiles, GPS or measuring instruments among others; and through group work and specialized roles [21].

1.2. Problem-Solving and Problem-Posing: Fostering Mathematical Creativity in Initial Teacher Training

Despite the reality of the classroom, problem-solving has gained popularity in the last decades as having a leading role in the organization of mathematics curricula all over the world, as opposed to traditional teaching based on rote learning of rules and definitions and the acquisition of computational skills [22].

Problem-posing has been incorporated to Mathematics teaching in the recent years as a complement to problem-solving. Invention, modification or the search for new problems are considered precursor activities to creativity in all artistic fields and can easily be linked to key moments in the advancement of mathematical science and thinking [23,24].

Problem-posing connects directly with the STEAM educative approach's interest in fostering creativity and motivation of learners [25,26]. In both cases, the aim is to shift the focus of educational interest from the application of convergent thinking, which in principle is typical of problem-solving or STEM disciplines and projects, to divergent thinking, related to problem-posing and the STEAM approach, capable of generating innovative and varied products and solutions [3] (p. 549).

The development of problem-posing skills is a relevant aspect of teacher training, especially in the initial stages of education where teachers are not specialists in the field. It is not only a matter of proposing problems that connect with the appropriate level of knowledge and that provide meaningful and motivating contexts for the solvers, but also that the choice of problems made by the teacher will shape the students' future experience with mathematics and their conception of what mathematical practice is [27]. The lack of training in this field in primary education teacher training, which is the focus of this study, results in an excessive dependence on the textbook and a total lack of knowledge of other design options such as the possibility of posing problems with open solutions [18,28] or proposing creative alternatives to the known reality [29].

1.3. Didactic Proposal: Sevilla Math City

The "Sevilla Math City" project was carried out with 37 students in the third year of the Bilingual Primary Education Degree at Loyola Andalucía University, who were enrolled in their second subject of Didactics of Mathematics.

The students, organized into groups of 3 and 4 individuals, were asked to design a mathematical itinerary connecting public spaces in the city of Seville. Each group was free to choose the spaces in which to work. It was explicitly pointed out that it was not only a question of choosing open-air spaces, such as squares, parks or streets, but also closed spaces, such as courtyards or building halls, which are accessible to the public.

The public spaces chosen were to be considered as stops on the itinerary in which mathematical problems were to be solved, working on various contents of the primary school stage. The itineraries should be adjusted in time and distance to an external outing for a class session of a specific primary school in the city, although they are also available to any family or person interested in following them completely or partially.

The didactic objectives of the proposal, on which the evaluation of it was based, were the following:

1. Adequacy of the total itinerary (duration and distance) to its real use as a Math Trail.
2. Identification of the resolution of the task with the chosen space or physical element (monuments, buildings, pavement, views, nature, etc.) in such a way that the presence of the solver is compulsory.
3. Correct definition of the objectives pursued, and the contents worked on in the area and adaptation of the statement and materials provided to the proposed objectives.
4. Adequacy of the complexity of the task to the target primary education cycle.
5. Quality of the oral presentation, design, materials and texts.

It is worth mentioning that the students did not receive specific training on the creation of tasks in the context of Math Trails. Furthermore, most of them had not experienced this teaching tool in the pre-university stages of their education. They only received a comprehensive bibliography on Math Trails from which they were able to extract examples of interesting activities and possible urban elements to work with.

A total of 11 walks and 117 mathematical tasks were carried out. The results of the project were materialized in work dossiers, student notebooks, necessary for the Math Trail, and summary posters that were exhibited by each group in a final class session.

Additionally, all the trails created by the students were collected on a freely accessible Googlemaps map (Figure 1). Students learned how to use this technological tool in the context of the project with the support of a selection of online tutorials and peer learning supervised by the teacher during the two class work sessions.

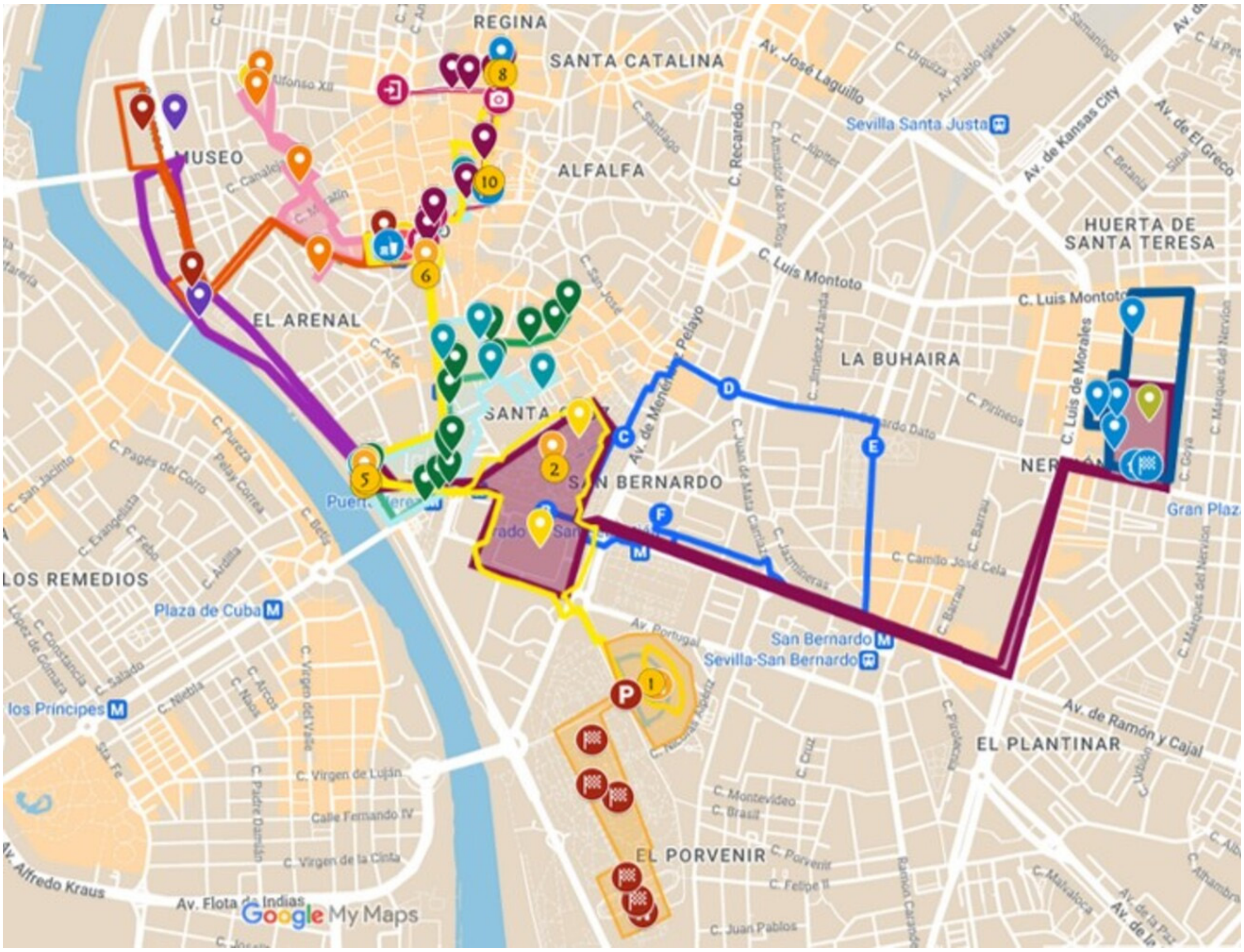

Figure 1. Interactive map of Seville with all the Math Trails developed by the students. The lines with different colors represent the itineraries and the markers are the stops of the Math Trails. Available at: https://bit.ly/3IT25ZT (accessed on 19 July 2022).

This paper explores the production of problems for Math Trails by pre-service teachers in the third year of the Primary Education Degree. The research questions are:

1. How are the tasks and the Math Trails created by the students characterized according to different classification variables and research variables?
2. What shortcomings or biases can be detected in the students' productions after having worked freely on the design of the tasks?
3. Are there any relationships between the different variables in the tasks produced by the students?

2. Materials and Methods

This article presents an exploratory and descriptive study which analyzes, from a quantitative approach, 117 tasks contained in 11 Math Trails designed by pre-service teachers in the context of the project "Sevilla Math City". We consider three classification variables: *Grade*, mathematical *Content* and *Object* or real element involved in every task, and five investigation variables:

(i) *Proc-PS*: a distinction between *procedural* and *problem-solving* tasks.
(ii) *Demand*: the classification of the level of cognitive demand proposed in Stein, Grover, and Henningsen [30] and in Schwan and Stein [31] related to the processes and strategies required to complete the task.
(iii) *Context*: a distinction between academic, semi-real and real-life tasks.
(iv) *Openness:* a distinction between open, open-ended and closed tasks in relation to the type of answer of the task.
(v) *Creativity:* a distinction between the problem-posing categories *accepting data* and *what-if-not* proposed in Brown and Walter [29], considering task design as a creative activity.

The data was collected from the student's dossiers presented at the end of the project and the results are presented in Tables 2–9. The process of classifying the tasks was performed by two researchers (R1 and R2), who independently classified all the tasks. In cases of discordance, researchers R1 and R2 met with researcher R3 and discussed one by one, completing the consensus on the analysis of the types of tasks.

A study of correlations between the variables considered in the study is presented in Table 10. A matrix of correlation coefficients and the matrix of p-values for testing the hypothesis that there is no relationship between the observed phenomena was calculated with MatLab Toolbox Stats (Matlab R2021a, Mathworks, Inc., Natick, Massachusetts, USA). If an off-diagonal element of p-values matrix is smaller than the significance level (0.05), then the corresponding correlation is considered significant.

2.1. Classification Variables: Grade, Mathematical Content and Objetc

The tasks were geared to grades 1st to 6th (years 6 to 12) in primary school. Every task in the same trail was designed for the same grade so we could assign a unique grade to each of the trails. See Table 2 for the results of the study.

For the mathematical content, tasks were classified into the areas of Numbers, Geometry, Measure and Probability and Statistics, each of them subdivided in several sub-areas (see Table 3). We include in the study cases where a single task contains subtasks involving more than one mathematical area.

The different types of real elements or objects in which the tasks are based are listed in Table 1. The category *2D or 3D space* considers the location where the task is taking place as a whole, as an empty place. In addition, the category of *Other* includes tasks containing elements or activities which are not directly related with an object of the surrounding.

Table 1. Object or element type in Math Trails tasks.

Object	Examples
Urban elements	Bench, streetlight, flowerpot, fountain.
Architectural elements	Facade, courtyard, window, door.
Buildings	University main building, tower.
Interior elements	Ceiling, furniture, well.
Natural elements	Trunk, tree, flower, bush.
2D or 3D space	Square, street.
Other	Food, people, cars.

2.2. Investigation Variables

We introduce in what follows the four investigation variables used to further classify the type of task in more depth.

2.2.1. Procedural vs. Problem-Solving (Proc-PS) and Cognitive Demand of Tasks

We say that a task is a *problem-solving* activity when it involves problem-solving strategies to be completed (such as guess and check, use a model, find a pattern and solve a simple problem), whereas *procedural tasks* are the ones involving practicing on procedures [18]. Note that this distinction depends on the student since it happens that some problem-solving tasks may become procedural with time and practice.

In the context of our Math Trails, an example of a procedural task may be *"Look for a square in the floor and calculate its perimeter"*, and an example of a problem-solving task would be *"Estimate the number of persons that can fit in the patio"*.

Problem-solving tasks propose challenges for students, and they are not a direct application of a procedure, thus they require some high-level thinking. In this sense one can relate this classification with the *cognitive demand* required to complete a task. Stein, Grover and Henningsen [30] proposed a categorification of tasks into four types: the two low level demand categories of:

- *Memorization:* tasks which need to reproduce previous learnings and to memorize facts, formulas or definitions.
- *Procedures without connections* (to concepts or meanings): tasks which are algorithmic, reproducing procedures that are explicitly specified or previously known from prior instruction or experience;
- And the two high level demand categories of:
- *Procedures with connections* (to concepts or meanings): tasks where the use of procedures are closely connected to the underlying mathematical concepts and ideas. Tasks are usually represented in multiple ways (visual, manipulatives, symbols and problem situations), making connections among multiple representations.
- *Doing mathematics:* tasks which require complex thinking which is not algorithmic, and the solving approach is not known nor explicit in the statement of the task. These tasks create the need for students to impose their own structure and procedure to solve the task.

See also [31] for a task-analysis guide with a list of characteristics of each cognitive demand.

In the context of Math Trails, an example of a task with the cognitive demand level of procedures without connections (low level) may be *"Draw on paper one of the circles you can see in the facade and identify its elements (radius, center, . . .)"*, and an example of a cognitive demand level task of procedures with connections (high level) would be *"Classify all the types of windows in the facade, organize the information in a table and draw a bar chart on your sheet of paper"*.

2.2.2. Context: Academic, Semi-Real and Real-Life Tasks

A task is considered to be *academic* if it refers only to mathematics, having no context at all. Classical examples are exercises appearing in math textbooks like *"Calculate the greatest common divisor of 24 and 42"*. The inclusion of real context into mathematical tasks can be conducted in two different ways: *semi-real tasks* and *real-life tasks*.

Semi-real tasks include a storyline or a daily-life situation, but the reality has been constructed, the data provided is artificial [20]. For example, in a math textbook one can find exercises such as: *"Shopkeeper A sells dates for 85p per kg., and shopkeeper B sells them at 1.2kg for 1 pound. Which shop is cheaper? What is the difference between the prices charged by the two shopkeepers for 15kg of dates?"* [20] (p. 126). In this case the shops and the prices are not real, but they are used to bring some sense of reality to the tasks.

In a Math Trail a semi-real task may involve a real object and the student can be present in a real setting, but the task creates a new situation or adds artificial data to the context.

Real-life tasks use real data (transport schedules, sports statistics, unemployment graphs, etc.), refer to an applied situation (workplace, architecture or engineering problem) or they take place in an actual real-life context such as a Math Trail in a street or a field trip to a park [18]. Nevertheless, these situations may not be real to the students, whereas for example playing mathematical-rich games, such as Nim or Tower of Hanoi where winning or losing matters, are closer to their experience and interest [32]. In this approach, real-life tasks are related to meaningfulness rather than realism or usefulness [33].

A Math Trail is a suitable context to involve students in real-life tasks. There are plenty of examples of this type of task: *"Estimate the volume of the pond"*, *"Create a replica of the tower of 1/5 of its size"* or in general any task that involve an exploration and in-depth understanding of a real object (bench, flowerpot, etc.). In fact, Math Trails are capable of fulfilling the four principles for problems in real-life contexts [19]: the realistic principle (use authentic real-world contexts to engage in real-world problems-solving); the mathematical principle (tasks should engage students to think and work with mathematical concepts); the activity principle (it should engage students in mathematical thinking processes); and the documentation principle (make visible students' thinking as much as possible).

2.2.3. Openness: Closed, Open-Ended and Open Tasks

In *closed tasks* the goal and the answer are closed. The goal is specified in the statement and there is only one correct answer [18]. Typical examples of closed tasks can be found in textbooks when they require student to practice some formula or procedure, i.e., *"Calculate the area of a circle of radius 2"*.

There are several ways to increase the *openness* of a task. One can consider the *open-ended tasks* proposed by Becker and Shimada [28] to be the tasks that have multiple correct answers. These authors give the following example of an open-ended task: *"A transparent flask in the shape of a right rectangular prism is partially filled with water. When the flask is placed on a table and tilted, with one edge of its base being fixed, several geometric shapes of various sizes are formed by the cuboid's faces and the surface of the water. The shapes and sizes may vary according to the degree of tilt or inclination. Try to discover as many invariant relations (rules) concerning these shapes and sizes as possible. Write down all your findings."* [28] (p. 10). In this example the answer is ill-defined since there is no way to specify all correct answers.

One can also consider a task to be open-ended if the method of solution is open, that is, problems where the focus is not on the answer of the problem but rather on developing different methods to solve it. For example, asking students to *"find the total number of handshakes among 14 participants of a workshop where they shake only once with each participant"* [18] (p. 14) where several problem-solving strategies can be used to obtain the solution.

Even though open-ended tasks facilitate mathematics understanding more than standard questions, they are not widely used in classrooms [34]. Authors such as Lowrie [35] have focused on these types of questions in problem-posing activities, concluding that with teacher guidance the students can increasingly start to propose open-ended tasks.

Finally, one can consider tasks to be *open* if there are multiple valid answers, and there is no such thing as a correct answer. For example, the task "*Design a playground for the school*" is open since the correctness depends on an ill-defined interpretation and one has to consider validness instead. They usually involve a certain amount of creativity and may be related to investigation tasks.

In the context of Math Trails, an example of a closed task is "Students have to look for a bike and study its wheel, stating the name and calculating the length of its parts (radius, perimeter, arc, … "; an example of an open-ended task is "Draw a shape with the same number of symmetries as a figure on a ceiling"; and an example of an open task would be "Find a pattern of polygonal shapes that can fill out the floor of the room".

2.2.4. Creativity: Accepting Data vs. What-If-Not

Brown and Walter [29] proposed two categories of problem-posing activities. In the first category, called *accepting data*, the problem is formulated from a static situation (a sentence, a condition, a picture, a diagram, etc.), which is not modified, or the data presented do not change. The second category is called *what-if-not*, and it is based on the negation of an attribute of the given situation. It extends a given task by changing what is given.

For example, "How many squares can you form in a 5×5 geoboard?" is an accepting-data task, while the problem "What if the geoboard it is not a square? Imagine that you have a circular geoboard of radius 10 cm, how many nails (pins) in a grid of 2×2 cm can you place in it?" presents a different situation to the student, so it is a what-if-not task.

The *what-if-not* approach requires knowing in depth the information contained in a problem, identifying its conditions and limitations, in order to modify one or more of these aspects to generate new questions. Barbosa and Vale [17] conducted a study with pre-service teachers working from photographs, from which they posed questions or formulated problems. They observed that they mainly used *accepting data* as a problem-posing strategy.

3. Results

In this section we present the tables containing the data collected from the 11 Math Trails designed by the pre-service teachers, starting from the classification variables of *Grade*, mathematical *Content* and *Object*.

Table 2 shows that the majority of the trails were geared to the 6th grade, indicating a preference for designing tasks in the course where there is a wider range of mathematical concepts. For instance, any task involving the calculation of volume, one of the most recurrent tasks as Table 3 shows, must be placed in this grade.

Table 2. Relation of trails and their grade in primary education.

Grade	Trails (% of Total)	Grade	Trails
1st	0 (0%)	4th	1 (9%)
2nd	1 (9%)	5th	1 (9%)
3rd	2 (18%)	6th	6 (55%)

Table 3 shows the mathematical content of the tasks contained in the trails. Some of the tasks involved more than one area so the total number in this table is 183, which is greater than 117. One can see the predominance of tasks related to Geometry (54.1%), primarily related to 2D and 3D geometric shapes, rather than spatial orientation and transformations. It is worth noting the very low percentage of tasks related to Algebra (0%) and Probability and Statistics (2.2%), as well as with other concepts such as divisibility, sequences, series and some measurement magnitudes.

One can see from Table 4, that in relation to the type of object almost 60% of the tasks involved urban and architectural elements (benches, flowerpots, fountains and facade elements such as doors, windows or geometric ornaments, which were the most common), since most of the trails were designed in streets, parks or other open-air public spaces. The category of *Other* included several interesting objects and related tasks. For instance, there were tasks counting people, tasks asking to do activities with their own bodies or a task related to the food ordered in a restaurant.

Tables 5 and 6 contain the data related to procedural vs. problem-solving tasks (variable *Proc-PS*) and the cognitive demand (variable *Demand*). The left of both tables shows the percentage of each category for the total of tasks, while on the right is the breakdown for each of the 11 trails. There were no tasks with the highest level of cognitive demand (*Doing mathematics*), concentrating mainly at Levels 2 and 3 (Table 6), while procedural and problem-solving tasks appear to be more balanced within the trails (Table 5).

Table 3. Mathematical content of the tasks.

Area	Content	Tasks	Area %	Total %
Numbers	Elementary operations	24	53.3%	24.6%
Measure	Ordinals	0	0%	19.1%
Geometry	Fractions and decimals	1	2.2%	54.1%
Probability and Statistics	Percentages, proportionality	4	8.9%	2.2%
	Divisibility	0	0%	
	Counting	14	31.1%	
	Estimation	1	2.2%	
	Sequences and series	1	2.2%	
Algebra	Equations, patterns, relations, functions	0	0%	0%
Measurement	Geometrical magnitudes: length, area, volume	24	68.6%	19.1%
	Weight	0	0%	
	Capacity	1	2.9%	
	Currency	1	2.9%	
	Time	1	2.9%	
	Angular units	2	5.7%	
	Estimation	6	17.1%	
Geometry	2D figures	49	49.5%	54.1%
	3D figures	17	17.2%	
	Orientation	5	5.1%	
	Transformations, symmetry	9	9.1%	
	Perimeter, area, volume	16	16.2%	
	Coordinates	3	3%	
Probability and Statistics	Tables and graphs	3	75%	2.2%
	Statistical measures: mean, mode, range	1	25%	
	Random experiments	0	0%	
	Probability of events	0	0%	

Table 4. Type of real element or object present in the tasks.

Object	Percentage
Urban elements	28.2%
Architectural elements	27.2%
Buildings	12.8%
Interior elements	7.7%
Natural elements	4.3%
2D or 3D space	6%
Other	13.7%

Table 5. Procedural vs. problem-solving tasks (*Proc-PS*) and breakdown for each trail.

Proc-PS	%	Trail	Procedural	Problem-Solving
Procedural	56.4%	1	30.8%	69.2%
Problem-solving	43.6%	2	62.5%	37.5%
		3	55.6%	44.4%
		4	38.5%	61.5%
		5	72.7%	27.3%
		6	70%	30%
		7	80%	20%
		8	92.3%	7.7%
		9	50%	50%
		10	22.2%	77.8%
		11	44.4%	55.6%

Table 6. Cognitive demand (*Demand*) of the tasks and breakdown for each trail.

Demand	%	Trail	Level 1	Level 2	Level 3	Level 4
Low Level 1. Memorization	17.9%	1	7.7%	23.1%	53.8%	15.4%
Low Level 2. Procedures without connections	38.5%	2	37.5%	25%	37.5%	0%
High Level 3. Procedures with connections	40.2%	3	33.3%	22.2%	44.4%	0%
High Level 4. Doing mathematics	3.4%	4	15.4%	23.1%	46.2%	15.4%
		5	0%	72.7%	27.3%	0%
		6	40%	30%	30%	0%
		7	30%	50%	20%	0%
		8	38.5%	53.8%	0%	0%
		9	0%	50%	50%	0%
		10	0%	22.2%	77.8%	0%
		11	0%	44.4%	55.6%	0%

From the data in Tables 5 and 6 one can characterize trails according to this variable. For instance, Trail 10 can be considered as problem-solving oriented as it contains more than 75% of problem-solving tasks (or oriented to high levels of cognitive demands), Trail 8 is procedural oriented (or require low levels of cognitive demand), while Trails 3, 9 and 11 have a balance among the type of tasks and cognitive levels of demand.

For the results on the variable *Context*, the distinction of tasks as academic, semi-real or real is shown in Table 7. The real tasks are highly predominant (76.1%) due to the contextualized nature of Math Trails, and one can see that few tasks were proposed as academic (6.8%).

Table 7. Academic, semi-real and real tasks (*Context*) and breakdown for each trail.

Context	%	Trail	Academic	Semi-Real	Real
Academic	6.8%	1	7.7%	30.8%	61.5%
Semi-real	17.1%	2	25%	12.5%	62.5%
Real	76.1%	3	11.1%	33.3%	55.6%
		4	7.7%	23.1%	69.2%
		5	0%	9.1%	90.9%
		6	10%	20%	70%
		7	10%	0%	90%
		8	0%	0%	100%
		9	8.3%	25%	66.7%
		10	0%	11.1%	88.9%
		11	0%	22.2%	77.8%

One example of a real task in our study is shown in Figure 2, where the ant and its speed are made up, and the purpose of the task is to use the dimensions of the real bucket.

Considering the type of answer that the task requires, the openness of tasks in the Math Trails analyzed is shown in Table 8. The pre-service teachers chose mainly closed tasks (75.2%) and there were very few open tasks (3.4%). In the breakdown of trails one can see that only Trail 5 has all closed tasks, so most of the trails included some amount of openness to a greater or lesser extent.

Table 8. *Openness* of tasks and breakdown for each trail.

Openness	%	Trail	Closed	Open-Ended	Open
Closed	75.2%	1	61.5%	30.8%	7.7%
Open-ended	21.4%	2	87.5%	0%	12.5%
Open	3.4%	3	88.9%	0%	11.1%
		4	84.6%	15.4%	0%
		5	100%	0%	0%
		6	80%	20%	0%
		7	50%	50%	0%
		8	84.6%	15.4%	0%
		9	66.7%	25%	8.3%
		10	55.6%	44.4%	0%
		11	66.7%	33.3%	0%

Figure 2. "The well of the garden has a bucket hanging on it. Imagine that there is an ant alone inside the bucket and wants to get out by climbing up the wall. If it walks 10 cm in 1 min, how much time will it take to walk the whole bucket?".

An example of an open task created by a pre-service teacher participating in the study is shown in Figure 3.

With regard to *Creativity,* the characterization of the tasks is shown in Table 9. Only 4.3% of the tasks are included in the *what-if-not* category, so students tended to use the surroundings as given to create mathematical tasks.

An example of a *what-if-not* task designed by the participants of the study is shown in Figure 4.

Table 10 shows the matrix of correlations between the investigation variables.

The correlation analysis confirms the natural relation of the variables *Proc-PS* and *Demand*, with the highest value of positive correlation: procedural tasks are related to low levels of cognitive demand and problem-solving tasks are related to high levels of cognitive demand.

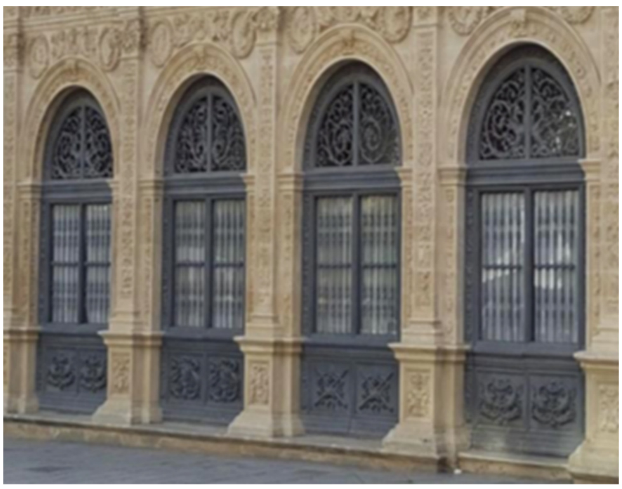

Figure 3. "Look at the door of the facade of the building and recognize the geometrical figures that appear on it. Can you think of a different way of dividing the door with different geometrical figures?".

Table 9. Problem-posing type (*Creativity* variable) and breakdown for each trail.

Creativity	%	Trail	Accepting Data	What-if-Not
Accepting data	95.7%	1	92.3%	7.7%
What-if-not	4.3%	2	87.5%	12.5%
		3	88.9%	11.1%
		4	100%	0%
		5	100%	0%
		6	90%	10%
		7	100%	0%
		8	100%	0%
		9	91.7%	8.3%
		10	100%	0%
		11	100%	0%

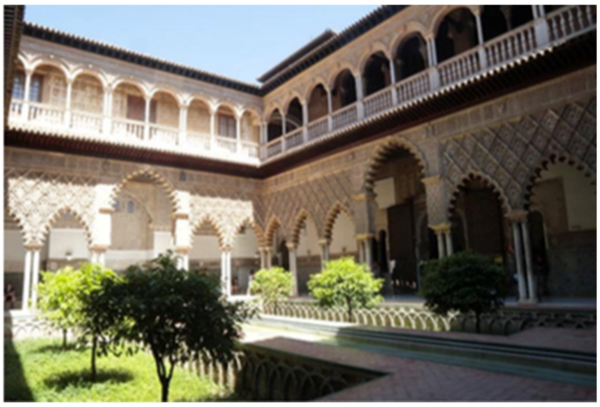

Figure 4. "Consider the patio as a Cartesian plane, as if the X-axis were the short side and Y-axis the long side (each column is a point of the axis). Represent the coordinates (3,3) (0,0) (3,6) (0,4) and (2,7) locating yourselves in the correct point".

The variable *Openness* has a positive correlation with *Proc-PS*, *Demand* and *Creativity* (although it is only significant in the case of *Creativity*). The *Creativity* variable is positively correlated with *Proc-PS*, *Demand* and *Openness*, all being significant.

Table 10. Matrix of correlations between variables.

	Proc-PS	Demand	Context	Openness	Creativity
Proc-PS	1.00	0.87 *	−0.21 *	0.15	0.24 *
Demand	0.87 *	1.00	−0.16	0.08	0.19 *
Context	−0.21 *	−0.16	1.00	0.03	−0.18
Openness	0.15	0.13	0.03	1.00	0.21 *
Creativity	0.24 *	0.19 *	−0.18	0.21 *	1.00

* Significant correlation (*p*-value 0.05).

Against expectations, *Context* presents a negative correlation with *Proc-PS*, *Demand* and *Creativity* (although it is only significant in the case of *Proc-PS*). This indicates that the tasks having more context (as they get closer to real tasks) are more procedural, with less cognitive demand and creativity.

With respect to the classification variables, the study shows that *Proc-PS* and *Demand* are dependent on the mathematical *Content*. In fact, there is more cognitive demand and more presence of problem-solving tasks in Numbers and Probability and Statistics, although the latter is not representative since there are only three tasks of this content. The investigation variables of *Context*, *Openness* and *Creativity* do not present a dependence on *Content*.

The classification variable of *Grade* does not present a dependence on any of the investigation variables studied. The classification criterion of *Object* was not considered to be relevant in this analysis.

4. Discussion

In this study, we present the analysis of tasks designed by pre-service teachers in the framework of Math Trails. Challenging future teachers to design tasks built on the potential of problem-posing as a tool for developing problem-solving skills and training good problem-solvers [36]. In addition, problem-posing is enhanced by the experience of contextualized outdoor mathematics, which has been identified as being of great value, in the context of teacher education, for developing cooperative work, critical thinking and establishing mathematical connections [37].

For the analysis of the tasks, different variables were considered and resulted useful in exploring how prospective teachers face the challenge of problem-posing. The analysis of the tasks has also led to some findings in terms of teacher education which are detailed below.

Concerning the content, Geometry was the one with the widest presence in the tasks designed. The outdoor experience seems to favor the proposal of tasks with a spatial or geometric component, which should be taken into account in teacher education, as there are studies that reveal certain weaknesses of future teachers in relation to the sense of space and measurement [38,39].

The course chosen by most students was 6th grade. This recurrence may be due to the fact that students feel more confident in creating tasks closer to their own level of knowledge. Additionally, since Geometry was the most used mathematical area by students, it reveals a superficial perception of what a geometry task is, mostly being tasks of calculation of perimeters, areas and volumes.

With regard to investigation variables, results show that there was a higher percentage of tasks classified as procedural with low level or high level of cognitive demand (Level 2 and Level 3 of *Demand*). This result can be explained by the fact that prospective teachers show a poorer performance in devising problem-solving strategies than in activating declarative and procedural knowledge, as reported by [40,41]. Additionally, these results support the idea that prospective teachers are aware of the characteristics of interesting mathematical problems but their ability to propose them is limited, in line with the findings of [27].

The Math Trails approach also seems to encourage the design of real-life tasks (variable *Context*). This may result in being useful in teacher education as a tool to overcome the difficulties in contextual knowledge shown by pre-service teachers, which lead to limitations in working with mathematical models and solving real problems [42], which would be in line with the work of [43], where it is reported how mathematical city walks can promote competences in mathematising.

In relation to variables such as *Openness* and *Creativity* of tasks, the participants were mostly inclined towards *closed* and *accepting data* type tasks, which corroborates the works of [18,28] and the need to include certain degree of openness (*open-ended* or *open*) and *what-if-not* tasks in teacher education.

5. Conclusions

In the same way that there are studies that show that students have acquired mathematical experiences and their performance in mathematics improves with Math Trails activities [44], in view of the results obtained, the creation of Math Trails emerges as a didactic tool that can be very useful to address mathematical needs of future teachers.

In relation to the characterization of tasks according to the variables considered in this study, the majority were related with concepts of Geometry involving calculations with 2D and 3D figures. The objects considered were mainly urban and architectural elements: benches, flowerpots, fountains and facades were among the most used by students to create their mathematical tasks. The percentage of procedural or lower cognitive demand tasks (56.4%) were slightly higher than problem-solving or higher demand tasks (43.6%). This difference is also present when looking at the trails, where the 55% were procedural oriented (that is, it contains majority of procedural tasks), the 36% were problem-solving oriented and 9% have a balance between procedural and problem-solving tasks. Students designed mostly real tasks (just the 6.8% were academic), closed tasks (over 75% of the total) and very few of them were classified as type *what-if-not.*

Some of the findings of the study show shortcomings or biases in students' views and approaches when facing a problem-posing activity in the context of Math Trails. As noted at the end of the Discussion, students mostly created closed tasks which did not include variations of the situation. In addition, most of the trails (and tasks) were aimed for the last grade of primary school, which may show less confidence in pre-service teachers to create interesting or meaningful mathematical tasks for younger students. Similarly, students picked Numbers and Geometry as the most preferred contents, while there were no tasks related to Algebra and very few to Probability and Statistics. Teacher training should give them enough knowledge and confidence in these areas to see their potential in the context of Math Trails.

About the relationships between the different variables, the study confirms the natural relation between *Proc-PS* (procedural vs. problem-solving) and the cognitive *Demand* of the tasks. It also relates positively the amount of openness of a task with the cognitive demand and the creativity as a problem-posing activity. The study shows a negative correlation between the *Context* (academic, semi-real and real tasks) and *Proc-PS.* It would be interesting to look further into this fact, since it points out the difficulty of proposing contextualized problems that require problem-solving strategies.

The didactic proposal developed in this work, based on situated learning in an urban environment and on the creation of mathematical tasks, allows for an integrated approach to the STEAM educational model. Mathematics has been applied in processes of element recognition, data collection and problem-solving working with real elements of the urban context belonging to fields such as architecture, engineering or the arts. The importance of not forgetting mathematical work based on more creative problem types (open and what-if-not) has been stressed. The future teachers have learned to use technological tools (Googlemaps, Google, Mountain View, California, USA) to improve the understanding of their project, their Math Trail in the urban context of Seville, and to communicate it to other people and users. Finally, design and illustration have also played a prominent role in some of the elements in which the work of the groups was materialized: the student notebooks, addressed to the children who will solve the Math Trail and the summary posters that were exhibited and explained by each group in the final session of the project.

Among the limitations of the present study, the difficulty of sorting out mathematical tasks into a classification must be mentioned. Furthermore, the experience and previous instruction of the students for which the task is aimed plays a subtle role in any classification (a problem-solving task may be procedural for an advance or talented student). Despite the methodology followed in the classification, incorporating the judgement of two researchers and confronting it with that of a third, there are some mathematical tasks that can be associated with more than one category. Smith and Stein [31] mention similar difficulties when testing their cognitive demand categories with teachers and pre-service teachers, finding out that their classification does not always agree with the researchers' criteria.

This work is an initial exploratory study whose results are limited to the context of the participants who created the tasks: third-year students of the Degree in Primary Education, who had already passed a subject in Didactics of Mathematics but without specific training in problem-posing. The results should be compared with those obtained in other contexts in order to gain a more complete overview of the competences, shortcomings and biases of trainee teachers when proposing mathematical tasks for primary education.

In this regard, the improvements of the didactic proposal which are being implemented in an ongoing project include the following:

- An initial stage where students experience a pre-designed math trail. In one hand it would serve as a model for students of different types of tasks and a variety of urban elements on which one can create and do mathematical problems. On the other hand, it is important that pre-service teachers first take the role of users of a Math Trail since most of them did not experience this activity before.
- The use of mobile technological tools, such as MathCityMap, which give students the opportunity to practice with the advantages of their inclusion into Math Trails: GPS localization of the trail and tasks, immediate feedback or gamification capabilities among others [14].
- A selection of the Math Trails created by the pre-service teachers are proposed to be tested by primary school students. This last step seeks to provide students with a self-assessment, which will help them to redesign their mathematical tasks and as a final reflection for their future professional practice.

The analysis of Math Trails as complete elements has only been approached in a descriptive way in this work. An interesting prospect for future work would be the cluster analysis of the walks in search of emerging patterns. In this field, the work of Haas, Kreis and Lavicza on the analysis of Math Trails created by pre-service teachers according to the STEAM approach is worth highlighting [21].

The need to provide specific training in problem-posing to future teachers in the initial stages, who are not specialists in STEAM areas, is fundamental to the emergence of a rich and varied awareness of what a good mathematical problem is [17,27]. This study has pointed out some shortcomings in terms of the content and nature of problems proposed in the context of Math Trails that need to be taken into consideration so that future teachers, and the students who will be trained by them, can develop a deeper understanding of what it means to do mathematics.

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