*mathematics*

*Article*

# Multi-Objective Robust Optimization for the Sustainable Location-Inventory-Routing Problem of Auto Parts Supply Logistics

**Ao Lv and Baofeng Sun ***

College of Transportation, Jilin University, Changchun 130022, China
* Correspondence: sunbf@jlu.edu.cn

**Abstract:** A great loss of transportation capacity has been caused in auto parts supply logistics due to the independent transportation from auto parts suppliers (APSs) to the automobile production line (APL). It is believed that establishing distribution centers (DCs) for centralized collection and unified distribution is one effective way to address this problem. This paper proposes a unified framework simultaneously considering the location-inventory-routing problem (LIRP) in auto parts supply logistics. Integrating the idea of sustainable development, a multi-objective MIP model is developed to determine the location and inventory capacity of DCs and routing decisions to minimize the total system cost and carbon emissions while concerning multi-period production demand. In addition, a robust optimization model is developed further in the context of uncertain demand. Numerical experiments and sensitivity analyses are conducted to verify the effectiveness of our proposed deterministic and robust models. The results show that synergistically optimizing the location and capacity of DCs and routing decisions are beneficial in reducing total system cost and carbon emissions. The analysis can provide guidelines to decision-makers for the effective management of auto parts supply logistics.

**Keywords:** auto parts supply chain; sustainable logistics; robust optimization; location-inventory-routing optimization; multi-period demand

**MSC:** 90B05; 90B06; 90B10

check for updates

**Citation:** Lv, A.; Sun, B. Multi-Objective Robust Optimization for the Sustainable Location-Inventory-Routing Problem of Auto Parts Supply Logistics. *Mathematics* **2022**, *10*, 2942. https://doi.org/10.3390/math10162942

Academic Editor: Yaping Ren

Received: 16 July 2022
Accepted: 12 August 2022
Published: 15 August 2022



## 1. Introduction

With the increase and diversification of end-customer demands in the commercial vehicle market, the automobile logistics service size and system complexity will unavoidably continue growing. As a core concept of the automobile supply chain, auto parts supply logistics has caught the abundant attention of academic and auto manufacturers [1]. The existing literature has indicated that if the auto parts are delivered separately by the auto part suppliers (APSs), several problems will be caused such as the great loss of transportation capacity and higher transport cost [2]. Hence, to address the practical issues of the auto parts supply logistics, our research aims to establish distribution centers (DCs) integrated with the routing problem [3] for centralized pickup and unified delivery. In order to better respond to the idea of emission reduction [4], it is critical to determine the location-inventory-routing problem from a sustainable perspective while reducing the total transport cost.

Our problem is a typical location-inventory-routing problem (LIRP) involving strategic, tactical, and operational decisions. Specifically, the strategic decisions are to determine the number and location of DCs, which undertake the tasks of the centralized pick-up of auto parts from APSs, storage, and their unified delivery to the automobile production line (APL). Unreasonable locations of the DCs are likely to fail in meeting production demands and reduce the operating efficiency of the auto parts supply logistics. The tactical decisions are to determine the inventory capacity of the opened DCs. Subject to the construction budget, it is not ideal to establish DCs with a large inventory capacity. On the other hand, when the inventory capacity is insufficient, there is a risk of a shortage of auto parts in DCs. In that case, the required auto parts of the APL cannot be satisfied by DCs, requiring APSs to deliver auto parts directly to the APL, which will significantly increase the transport cost. Moreover, operational decisions refer to the assignment between DCs and APSs [5] routing decisions for centralized pickup and unified delivery. The location selection of DCs, inventory capacity, and routing decisions are interrelated. With respect to the periodicity and time-sensitive characteristics of auto parts, this paper considers multi-period production demand to describe the LIRP of auto parts supply logistics.

Another essential factor that has to be considered is the uncertain demand. In automobile production activities, the production demand in the APL depends on the market orders, which are impacted deeply by indeterministic factors. In order to make up for the production uncertainty, it is essential to expand the inventory capacity of DCs or to order more auto parts in advance. However, these solutions have the potential to increase the delivery cost significantly. At the same time, it is difficult to obtain an exact probability distribution of uncertain demand. Therefore, a robust optimization method is needed to deal with the uncertainty of production demand.

Overall, this paper proposes a unified framework to describe the multi-objective LIRP while considering the multi-period uncertain demand of auto parts supply logistics. The main contributions of this paper are as follows:

(1) Concerning the multi-period deterministic demand, this paper first proposes a multi-objective mixed-integer programming model (MIP) to investigate the LIRP in auto parts supply logistics from a sustainable perspective. Specifically, this model determines the location and capacity of DCs and routing decisions to minimize the total system cost and carbon emissions.
(2) Further, a robust optimization model is developed to capture the multi-period uncertain production demand in the APL. To the authors' knowledge, this is the first time the LIRP in auto parts supply logistics is simultaneously optimized while considering multi-period uncertain demand.
(3) Numerical experiments are conducted to demonstrate the usefulness of the proposed models. The sensitivity analysis results show that the location, inventory capacity, and delivery routing decisions are highly affected by various cost parameters.

The remainder of this paper is structured as follows. Section 2 briefly reviews the relevant literature, and the LIRP and the MIP model are formulated in Section 3. Numerical experiments, results analysis, and managerial insights are presented in Section 4. Finally, Section 5 summarizes the conclusions and future direction of this research.

## 2. Literature Review

As a typical topic, numerous studies focus on the LIRP in various fields. To comprehensively review the problem in auto parts supply logistics, we summarize the literature review from three aspects: the LIRP in the auto parts supply network, the environmentally sustainable LIRP, and the LIRP with uncertain factors.

### *2.1. LIRP in Auto Parts Supply Logistics*

The complexities of the LIRP in auto parts supply logistics are closely related to location, inventory, and routing decisions. The location models have been widely investigated to determine the site of DCs. Ref. [6] developed a location model for an auto parts warehouse to minimize construction and transport costs. An improved particle swarm optimization algorithm was proposed and benchmark experiments were conducted to prove the effectiveness of the location model. In addition, the inventory capacity is another essential component in the LIRP, which is tightly associated with supply efficiency and inventory cost. In addition, route planning at the regional level has significant implications for regional transportation planning [7]. Compared to the zero inventory strategy that several automobile companies adopt, Ref. [8] proposed a new logistics strategy integrating progress-lane and vehicle routing problems. A mixed-integer model was established to minimize the total cost of inbound logistics, which is demonstrated to be more effective and economical than the zero inventory strategy. Further, Ref. [9] developed an adaptive Visual Basic Application (VBA) program to largely enhance the utilization rate of DCs and reduce the inventory cost. A multi-level location-inventory model was proposed by [10] and solved with the Lagrangian relaxation method. Concerning the auto parts demand, Ref. [11] provided the second weighted moving average method to forecast the future demand in advance, which is beneficial in reducing the inventory cost.

As auto parts transportation is the main component of the total system cost and the primary source of carbon emission [12,13], it is of great significance to optimize the routing problem in the auto parts supply logistics. Previous research has mainly concentrated on the location-inventory and routing problems separately. Therefore, it is essential to investigate the LIRP in auto parts supply logistics comprehensively.

### 2.2. Environmentally Sustainable LIRP

Traditional supply chain optimization problems mainly focus on minimizing the economic costs, which ignores the goal of reducing carbon emissions. With the popularity of the low-carbon concept, recent literature [14] has paid more attention to the LIRP from a sustainable view in various fields. For instance, to address the fresh food delivery problem, Ref. [15] proposed a model to discuss the vehicle routing problem with time windows (VRPTW) while decreasing carbon emissions. To deal with urban waste, Ref. [16] developed a multi-warehouse location-routing model and proposed a hybrid genetic algorithm and a simulated annealing algorithm. Numerical experiments demonstrated that the method and algorithm are effective in determining the location of parking lots and the vehicles routing to collect garbage. A two-layer planning model based on a carbon emission trading policy was developed in [17] to optimize the location problem of cold chain logistics. Similarly, Ref. [18] proposed an LIRP model that considers the carbon trading mechanism in the cold chain logistics network. Through simulation results, it was demonstrated that the improved NSGA-II can effectively reduce the carbon emissions of enterprises. Combining pollution-related routing, Ref. [19] developed a mathematical model to discuss the inventory-routing problem and proposed a hybrid adaptive particle swarm optimization algorithm. Considering the green location inventory problem, Ref. [20] developed a two-stage stochastic mathematical model and proposed a hierarchical heuristic algorithm. This study further proved the impact of the carbon trading scheme on strategic decision-making. To achieve a sustainable supply chain, Ref. [21] proposed a two-stage approach and built a multi-objective mixed-integer model. Based on the environmental consideration, the location-path-inventory system in a three-level supply chain network was studied, and Ref. [22] formulated a bi-objective mixed-integer programming model for the above system and developed a multi-objective particle swarm optimization algorithm.

Only a few studies discussed energy saving and emission reduction in auto parts supply logistics. To minimize the transport cost and carbon emissions, Ref. [23] established a routing planning model and verified the effectiveness through a real case. Based on emission reduction and resource sharing, a decision-making optimization model of the auto parts supply chain was established by [24] to minimize the total system cost and carbon emission. Variational inequality was utilized to analyze the optimal conditions.

In order to simultaneously minimize economic cost and carbon emissions, this study described a multi-objective model for the LIRP in the auto parts supply network.

### 2.3. LIRP with Uncertain Factors

Another key challenge for the LIRP is the demand uncertainty [25]. Reviewing the literature in the LIRP, researchers have made various assumptions when describing the demand. For instance, assuming that the demand is satisfied by a normal distribution, Ref. [26] formulated a dual-objective stochastic model to minimize the total cost and maximize the service time while considering various multi-period products, and a heuristic algorithm was adopted to obtain the Pareto set. Concerning the uncertain topology of the hub location, Ref. [27] proposed an interactive method to delineate the design problem of the dangerous goods transportation network. Two heuristics based on the lower bound and rolling horizon were proposed to solve the model in a large-scale case.

In the presence of demand uncertainty, robust optimization and stochastic optimization are two main streams. Ref. [28] formulated a stochastic programming mixed-integer model to determine the location and inventory strategy at the same time. Considering carbon emissions and energy consumption, Ref. [29] applied stochastic programming to the LIRP and proposed a sustainable closed-loop model to achieve economic, environmental, and social trade-offs. Robust optimization concentrates on min-max risk control. With respect to the uncertain demand in the medical supplier network, a robust LIRP model was proposed by [30] to reduce the total system cost effectively. Integrating with big data technology, Ref. [31] developed a multi-center location-routing optimization model of medical logistics considering several uncertain factors.

To give a clear representation of the innovation of this paper, we summarize the related literature review in Table 1. Up until now, there has been a lack of studies on a robust LIRP in auto parts supply logistics from a sustainable perspective while considering the multi-type, multi-period characteristic. To address the existing issues, this paper investigates the deterministic LIRP first and proposes a robust model combined with the multi-period uncertain demand to optimize the total transport cost and carbon emissions.

**Table 1.** List of referenced articles.

| Article | Problem | Multi-Objective | Sustainable | Uncertainty | Optimization Method | Multi-Period |
|---|---|---|---|---|---|---|
| Ghasemi et al., 2022 [32] | LIRP | | | ✓ | Two-stage | |
| Yang et al., 2021 [33] | LIRP | | ✓ | | Integrated | |
| Biuki et al., 2020 [21] | LIRP | ✓ | ✓ | ✓ | Two-stage | ✓ |
| Chao et al., 2019 [34] | LIRP | | | | Two-stage | ✓ |
| Li et al., 2022 [18] | LIRP | ✓ | ✓ | | Integrated | |
| Ji et al., 2022 [22] | LIRP | ✓ | ✓ | | Integrated | |
| Shang et al., 2022 [30] | LIRP | | | ✓ | Integrated | ✓ |
| Aydemir-Karadag, 2022 [35] | LIRP | ✓ | ✓ | | Two-stage | ✓ |
| Liu et al., 2021 [36] | LIRP | ✓ | ✓ | | Integrated | |
| Yavari et al., 2020 [37] | LIRP | | | ✓ | Integrated | ✓ |
| Song Wu, 2022 [38] | LIRP | ✓ | | | Integrated | ✓ |
| This Paper | LIRP | ✓ | ✓ | ✓ | Integrated | ✓ |

## 3. Problem Description and Model Formulation

Motivated by a real-world problem, we aim to present a novel multi-objective optimization model for the LIRP in the auto parts supply network. First, problem formulation and modeling assumptions are described. Then, we develop an integrated deterministic MIP model. Finally, integrating multi-period uncertain demand, a robust optimization model is further developed, and the complex modeling process in this paper is delineated in Figure 1.

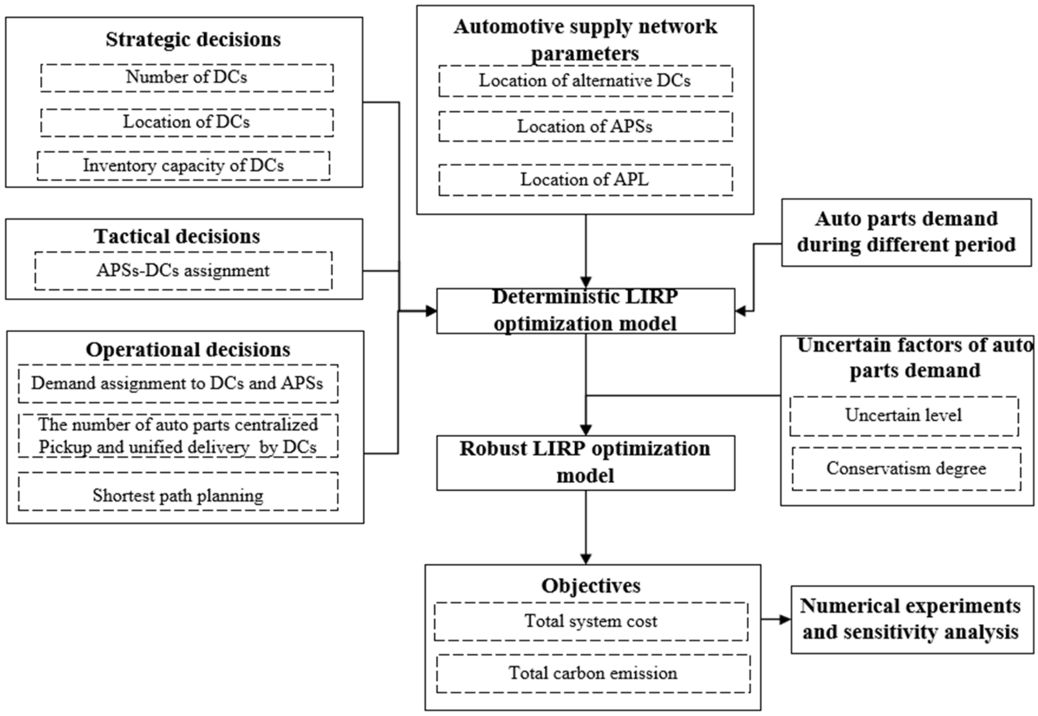



**Figure 1.** The research framework for auto parts supply logistics.

### *3.1. Problem Description*

This paper considers an auto parts supply network, as shown in Figure 1. The strategical decisions in this paper are to determine the location and inventory capacity of DCs. Let $J = \{1, 2, \ldots \overline{J}\}$ represent the alternative points set of DCs, which is indexed by $j$. Denote $y_j$ as a binary variable to describe if the alternative point $j$ is selected as DCs. That is, if $y_j = 1$, the DC is opened. In addition, it is essential to determine a reasonable inventory level for the opened DCs. Let $L = \{1, 2, \ldots \overline{L}\}$ represent the inventory levels set. Denote $y_{jl} = 1$ to describe the opened DC, where $j$ is equipped with the inventory level l. In addition, we have to decide the assignment between DCs and APSs, and we assume each APS has to be assigned with one DC. Suppose the set for APSs is $I = \{1, 2, \ldots \overline{I}\}$, which is indexed by $i$. If the APS i is assigned to DC $j$,we define $x_{ij} = 1$, otherwise $x_{ij} = 0$. To investigate the multi-periods production demand of the APL, the time is discretized into several equal periods $T = \{1, 2, \ldots, \overline{T}\}$, which is indexed by t. Once the APS $i$ is assigned to DC $j$ initially, the assignment will not change. As a side note, the auto parts demand in the APL is satisfied by DCs and APSs. We assume the auto parts demand of APS $i$ is $D_i^t$. One part of the demand of the APS $i$ is accommodated by the responding DC $j$, and another part of the demand is provided by APS $i$ directly. Afterward, there is a need to pick-up auto parts centrally to the DCs so that the demand in the last period can be satisfied. Namely, the DCs will dispatch vehicles to the corresponding assigned APS $i$ to pick-up auto parts according to the order quantity.

This will involve a vehicle routing problem to seek the shortest path. Three kinds of routes are included in Figure 2. The red lines are the direct delivery routing from APSs to the APL, and the black lines are the unified delivery routing from APSs to the APL. In addition, the blue lines are the centralized pickup routing from DCs to APSs and end at the same DCs. Specifically, in order to reduce the system operating cost and carbon emissions, this paper solves a joint decision-making LIRP in auto parts supply logistics.

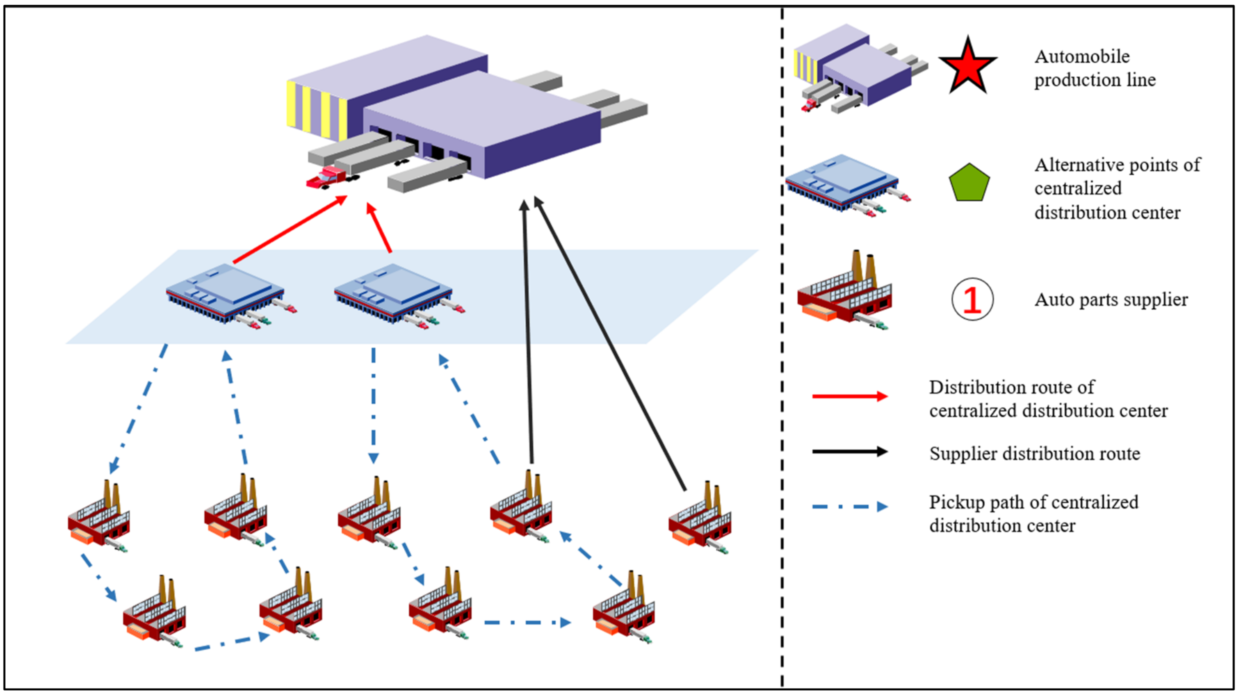



**Figure 2.** LIRP in the auto parts supply network.

### *3.2. Problem Assumption and Notations*

To formulate the complex auto part supply network into a mathematical model, the following assumptions are proposed in this paper.

(1) We assume the APSs have the ability to provide sufficient auto parts and the shortage is out of our investigation scope.
(2) The production demand for the APL can be obtained from historical data and must be satisfied.
(3) It is essential to translate the auto parts into the standard unit so that the various auto parts can be classified as a unified specification.
(4) We suppose that each APSs only provide one kind of auto part.
(5) The carbon emission factor considered in this paper is the carbon emission from vehicles during transportation, and the calculation method refers to the literature [39].
(6) We assume sufficient vehicles to serve the pickup and delivery, and there is no difference in vehicle performance.

Specifications on variables and parameters used in the LIRP are shown in Table 2.

**Table 2.** Variables and parameters used in the LIRP.

| Notations | Detailed Definition |
|---|---|
| **Set** | |
| I | The set for APSs, indexed by $i$ |
| $T$ | The set for time periods, indexed by $t$ |
| $J$ | The candidate point set for DCs, indexed by $j$ |
| L | The set for capacity level, indexed by $l$ |
| K | The set for APL, indexed by $k$ |

**Table 2.** *Cont.*

| Notations | Detailed Definition |
|---|---|
| Parameters | Detailed definition |
| $D_i^t$ | The auto parts demand from APS $i$ during period $t$ |
| UH | Unit inventory holding cost for auto parts |
| $FW_l$ | Fixed cost for establishing one DC with inventory level $l$ |
| $FN$ | Unit transport cost from DCs to the APL |
| FZ | Unit transport cost from APSs to the APL directly |
| $FG$ | Unit transport cost for picking up auto parts from DCs to corresponding APSs |
| $WC_l$ | The inventory capacity corresponding to the inventory level $l$ of the DC |
| $VC$ | Vehicle capacity for picking up auto parts |
| $M_0$ | A large positive number |
| CE | The factor for carbon emission |
| $d_{ik}$ | Distance between APS and the APL |
| $d_{jk}$ | Distance between DC and the APL |
| $d_{ii\prime}$ | Distance between two APSs |
| $d_{ji}$ | Distance between DC and the APS |
| Auxiliary variables | |
| $SY_{ji}^t$ | The remaining quantity of auto part from APS $i$ at DC $j$ during period $t$ |
| $LC_{ji}^t$ | The quantity loaded in the vehicle that picks up auto parts from DC $j$ after it finishes loading at APS $i$ during the period $t$ |
| $or_{ji}^t$ | A positive integer, the quantity ordered from APS $i$ for DC $j$ during period t |
| $q_{ji}^t$ | A binary, if there is a need for DC $j$ to order auto parts from APS $i$ during period t |
| Decision variables | |
| $x_{ij}$ | A binary, equal to 1 if the APS $i$ is assigned to DC $j$, 0 otherwise |
| $y_j$ | A binary, equal to 1 if the candidate point $j$ is selected as the DC, 0 otherwise |
| $y_{jl}$ | A binary, equal to 1 if the capacity of DC $j$ is level l, 0 otherwise |
| $de_{ji}^t$ | A positive integer, the number of auto parts delivered from APS $i$ to APL during period $t$ when APS $i$ is assigned to DC j |
| $gy_{ji}^t$ | A positive integer, the number of auto parts of APS $i$ delivered from DC $j$ to APL during period t |
| $ro_{ii'}^t$ | A binary, equal to 1 if the pickup routs from $i$ to $i'$ during period $t$, 0 otherwise |
| $ro_{ji}^t$ | A binary, equal to 1 if the pickup routs from DC $j$ to APS $i$ during period t, 0 otherwise |
| $ro_{ij}^t$ | A binary, equal to 1 if the pickup routs from APS $i$ to DC $j$ during period $t$, 0 otherwise |

*3.3. Deterministic Model*

Here, we first formulate a deterministic model to describe the LIRP integrating with the prior known multi-period demand of the APL. Our objective is to find the most efficient $x_{ij}$, $y_j$, and $y_{jl}$ and other routing decision variables under proper constraints. As a result, the proposed sustainable LIRP in auto parts supply logistics is formulated as follows:

Objective function:

$$\begin{aligned} &\sum_{j\in J}\sum_{l\in L} FW_l\cdot y_{jl} + \sum_{j\in J}\sum_{i\in I}\sum_{t\in T} UH\cdot \sum_{t=1}^{T} SY_{ji}^t + \sum_{j\in J}\sum_{i\in I}\sum_{t\in T} FZ\left(de_{ji}^t/VC\right)\cdot d_{ik} \\ &+ \sum_{j\in J}\sum_{t\in T} FN\left(\sum_{i\in I} gy_{ji}^t/VC\right)\cdot d_{jk} + \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} FG\cdot ro_{ii'}^t\cdot d_{ii'} \\ &+ \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} FG\cdot ro_{ji}^t\cdot d_{ji} + \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} FG\cdot ro_{ij}^t\cdot d_{ij} \end{aligned} \tag{1}$$

$$\sum_{j\in J}\sum_{i\in I}\sum_{t\in T} CE\left(de_{ji}^{t}/VC\right)\cdot d_{ik} + \sum_{j\in J}\sum_{t\in T} CE\left(\sum_{i\in I} gy_{ji}^{t}/VC\right)\cdot d_{jk} + \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} CE\cdot ro_{ii'}^{t}\cdot d_{ii'} + \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} CE\cdot ro_{ji}^{t}\cdot d_{ji} + \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} CE\cdot ro_{ij}^{t}\cdot d_{ij} \tag{2}$$

Objective function (1) minimizes the total system cost, where the first term is the construction cost for DCs, the second term is the inventory holding cost, the third term is the total transport cost from APSs to the APL, the fourth term is the total transport cost from DCs to the APL, and the last term is the total transport cost for centralized pickup from DCs to APSs.

In additn, objective function (2) is the definition of carbon emissions released by transport vehicles, including the carbon emission released by transport vehicles that route from APSs to the APL, from DCs to the APL, and from DCs to APSs.

Constraints:

$$\sum_{j\in J} x_{ij} = 1\forall i \in I \tag{3}$$

Constraint (3) limits that each APS can only be assigned to one DC.

$$x_{ij} \le y_j \forall i \in I, \forall j \in J \tag{4}$$

Constraint (4) ensures that the APSs can only be assigned to the opened DC.

$$\sum_{l\in L} y_{jl} = y_j, \forall j \in J \tag{5}$$

Constraint (5) determines the capacity level of the opened DC.

$$\sum_{j\in J} gy_{ji}^{t} + \sum_{j\in J} de_{ji}^{t} = D^t \times CP_i, \forall i \in \mathrm{I}, \forall t \in \mathrm{T} \tag{6}$$

Constraint (6) represents that all the demands of the APL are satisfied by APS and corresponding DC. We denote $gy_{ji}^{t}$ to indicate the number of auto parts of APS $i$ delivered from DC $j$ to the APL during period $t$, and $de_{ji}^{t}$ is the quantity of auto parts delivered from APS $i$ to the APL during period $t$ when APS $i$ is assigned to DC $j$.

$$gy_{ji}^{t} \le M_0\cdot x_{ij}, \quad \forall j \in J, t \in T, i \in I \tag{7}$$

$$de_{ji}^{t} \le M_0\cdot x_{ij}, \quad \forall j \in J, t \in T, i \in I \tag{8}$$

Constraints (7) and (8) ensure that auto parts supplies from DCs and APSs need to satisfy the assignment relationship between DCs and APSs.

$$SY_{ji}^{t} = SY_{ji}^{t-1} + or_{ji}^{t-1} - gy_{ji}^{t-1}, \quad \forall i \in I, \forall j \in J, \forall t \in T \tag{9}$$

Constraint (9) indicates the inventory quantity conservation of auto parts from APS $i$ stored in the corresponding assigned DC $j$. That is, the inventory quantity of auto parts from APS $i$ during period $t$ at DC $j$ equals the inventory quantity during period $t-1$, plus the order quantity from APS $i$ to DC $j$, minus the quantity delivered to the APL from DC $j$.

$$gy_{ji}^{t} \le SY_{ji}^{t}, \quad \forall i \in I, \forall j \in J, \forall t \in T \tag{10}$$

Constraint (10) ensures that the quantity of auto parts of APS $i$ delivered from DC $j$ to the APL during period t does not exceed the inventory quantity at DC $j$ during period $t$.

$$\sum_{i\in I} SY_{ji}^{t} \le \sum_{l\in L} WC_l\cdot y_{jl}, \quad \forall j \in J, \forall t \in T \tag{11}$$

Constraint (11) guarantees that the total inventory quantity of all auto parts in the DC $j$ at each period is less than the inventory capacity of DC $j$.

$$SY_{ji}^{t} \leq x_{ij} \cdot M_0, \ \forall i \in I, \forall j \in J, \forall t \in T \tag{12}$$

Constraint (12) indicates that only if the APS $i$ is assigned to the DC $j$, the DC $j$ will store the auto parts of the APS $i$.

$$or_{ji}^{t} \leq M_0 \cdot x_{ij}, \quad \forall j \in J, t \in T, i \in I \tag{13}$$

Constraint (13) points out that only if the APS $i$ is assigned to the DC $j$, the DC $j$ will order auto parts from the APS $i$.

$$or_{ji}^{t} \leq M_0 \cdot q_{ji}^{t}, \quad \forall j \in J, t \in T, i \in I \tag{14}$$

$$q_{ji}^{t} \leq or_{ji}^{t} \quad \forall j \in J, t \in T, i \in I \tag{15}$$

Constraints (14) and (15) ensure that when the DC $j$ has a clear ordering demand for the auto parts of APS $i$, the DC $j$ will dispatch vehicles to APS $i$ to pick-up the auto parts.

$$\sum_{i' \in I} ro_{ji'}^{t} = \sum_{i \in I} ro_{ij}^{t}, \quad \forall t \in T \ \forall j \in J \tag{16}$$

$$ro_{ji'}^{t} \leq q_{ji'}^{t} \quad \forall i' \in I \ \forall t \in T \ \forall j \in J \tag{17}$$

$$ro_{ij}^{t} \leq q_{ji}^{t} \quad \forall i \in I \ \forall t \in T \ \forall j \in J \tag{18}$$

Constraint (16) indicates that the vehicles dispatched by DC $j$ will depart from DC j and finally arrive at DC $j$. Constraints (17) and (18) ensure that the first and final APSs picked up by the vehicles have a clear order need.

$$\sum_{i' \in N_j^t, i \neq i'} ro_{ii'}^{t} + ro_{ji'}^{t} = q_{ji'}^{t} \quad \forall i' \in N_j^t, \forall t \in T, \forall j \in J \tag{19}$$

$$\sum_{i' \in N_j^t, i \neq i'} ro_{ii'}^{t} + ro_{ij}^{t} = q_{ji}^{t} \quad \forall i \in N_j^t, \forall t \in T, \forall j \in J \tag{20}$$

Constraints (19) and (20) describe the path planning for the vehicles to pick-up auto parts. $N_j^t$ is the set for APSs that are assigned to the same DC $j$ and has order needs during period $t$.

$$LC_{ji}^{t} \leq q_{ji}^{t} \cdot M_0 \quad \forall i \in I, \forall t \in T, \forall j \in J \tag{21}$$

$$LC_{ji}^{t} \geq q_{ji}^{t} \quad \forall i \in I, \ \forall t \in T, \ \forall j \in J \tag{22}$$

Constraints (21) and (22) ensure that the auto parts of APS $i$ loaded in the vehicles only occur at the APS, which has a clear order need.

$$LC_{ji}^{t} + \left(1 - ro_{ji}^{t}\right) \cdot M_0 \geq or_{ji}^{t} \forall i \in I, \ \forall t \in T, \forall j \in J \tag{23}$$

$$LC_{ji'}^{t} + \left(1 - ro_{ii'}^{t}\right) \cdot M_0 \geq LC_{ji}^{t} + or_{ji'}^{t} \ \forall i, i' \in I \ (i \neq i'), \forall t \in T, \ \forall j \in J \tag{24}$$

Constraints (23) and (24) calculate the number of auto parts loaded in the vehicle after it finishes loading at APS $i$ during the period $t$.

$$LC_{ji}^{t} \leq VC + \left(1 - ro_{ij}^{t}\right) \cdot M_0 \quad \forall i \in I, \forall t \in T, \ \forall j \in J \tag{25}$$

$$LC_{ji}^{t} \leq VC, \quad \forall i \in I, \forall t \in T, \forall j \in J \tag{26}$$

Constraints (25) and (26) require that the quantity of auto parts loaded in the vehicle is no more than the vehicle capacity.

$$or_{ji}^{t} \leq LC_{ji}^{t} \quad \forall i \in I,\ \forall t \in T\ ,\forall j \in J. \tag{27}$$

Constraint (27) requires that the quantity loaded in the vehicle after it finishes loading at APS $i$ is greater than the order quantity needed.

$$x_{ij}, y_j, y_{jl}, ro_{ii'}^{t}, ro_{ji}^{t},\ ro_{ij}^{t} \in \{0,1\} \forall i \in I,\ \forall t \in T\ ,\forall j \in J \tag{28}$$

$$de_{ji}^{t}\ ,\ gy_{ji}^{t} \geq 0 \forall i \in I,\ \forall t \in T\ ,\forall j \in J \tag{29}$$

Constraints (28) and (29) are the definitional domain of the decision variables.

Based on the above analysis, the comprehensive LIRP is formulated as the following MIP model.

Deterministic model:

$$\begin{gathered}\min \text{Objective } (1)\ (2) \\ \text{s.t. Constraints } (3) - (29)\end{gathered}$$

*3.4. Robust Model*

In daily production activities, automobile production depends on the market demand, which is easily influenced by the preference of consumers and unexpected events. Various uncertain factors may lead to a sharp increase in auto parts demand. To avoid facing a shortage of auto parts, we try to develop a robust model to formulate the uncertain demand. To address the uncertain issue, this paper adopts the robust optimization method of [40] and proposes a robust LIRP optimization model that is able to describe the degree of conservation and uncertainty level.

We assume that the production demand for the APL during each period is unknown and belongs to the symmetric range $\left[D^t - \hat{D}^t, D^t + \hat{D}^t\right]$, where $D^t$ is the nominal values and $\hat{D}^t$ is the maximum deviation value. For the sake of clarifying the uncertain $\hat{D}^t$, we introduce the concept of uncertainty level $\beta \in [0,1]$ to represent the proportion of deviation. Therefore, the automobile production demand for the APL falls in the range of $\left[D^t - \beta\cdot\hat{D}^t, D^t + \beta\cdot\hat{D}^t\right]$. Subsequently, we let $\rho$ describe the number of periods at which the production demand is uncertain, and the value of $\rho$ falls in the range of $\rho \in \left[0, \overline{T}\right]$, in which $\overline{T}$ is the total number of planning periods. Specifically, if $\rho = 0$, there is no uncertainty protection, and the model is deterministic. On the other hand, if $\rho = \ \mathrm{T}$ , there exists uncertain demand during each period, indicating that the production scheme of the APL is fairly conservative. Then, we employ a set $U = \left\{\left(t \middle| \hat{D}^t > 0\right)\right\}$ to describe the period set at which the production demand is uncertain. Based on the above analysis, a robust model is developed. Compared to the deterministic model, the difference is that constraint (6) is substituted by robustness constraints, which are formulated in (30) and (31).

Robust model:

$$\min (1), (2)$$

Subject to: (3)–(5), (7)–(29)

$$\sigma^t = \begin{cases} 1 & t \in \left\{u^t \middle| max_{\{u^t | u^t \in U, |u^t| = \rho\}} \left\{\sum_{t \in u^t} \hat{D}^t\right\}\right\} \\ 0 & others \end{cases} \tag{30}$$

$$\sum_{j \in J} gy_{ji}^{t} + \sum_{j \in J} de_{ji}^{t} = D_i^t + CP_i \cdot \sigma^t \cdot \beta \cdot \hat{D}^t, \quad \forall i \in I, \forall t \in T \tag{31}$$

In constraint (30), a compensation coefficient is introduced and represented as $\sigma^t$, which describes the protection function against the worst case. Equation (31) describes the demand satisfaction constraints under uncertain scenarios.

### 3.5. Solution Approach

Regarding the multi-objective deterministic or robust model, the main challenge is deciding how to manage the trade-offs. We adopt the linear programming solver Gurobi to obtain the optimal solution. Gurobi allows the multi-objectives function to be treated hierarchically. In the hierarchical approach, the priority is set for each objective. Concerning the realistic problem, this paper first sets the same priority for the two objectives in the following sensitivity analysis. Afterward, this paper generates a series of coupled priorities to obtain the Pareto solution. The priority determination method is subjective to a certain extent. The data envelopment analysis (DEA) method in [41] will be discussed in a future study.

## 4. Numerical Experiments

To illustrate the practical application of the proposed models, we conduct numerical experiments based on actual data provided by an automobile manufacturer located in Changchun, China. The following experiments are run by Gurobi 9.5.1 software on a personal computer equipped with an AMD Ryzen 7,5700 G with Radeon Graphics 3.80 GHz and 32.0 GB RAM, using the Microsoft Windows 10 operating system.

The auto parts supply network consists of 30 APSs, 12 DCs, and 1 APL. Figure 3 graphically describes the location distribution of APSs, DCs, and the APL. In addition, the units of travel cost, carbon emissions, and distance are measured with RMB (yuan), kilogram (KG), and kilometers (KM), respectively. Next, we analyze in detail the sensitivity of the deterministic model, the advantages of the robust optimization model, and the usefulness of our proposed models.

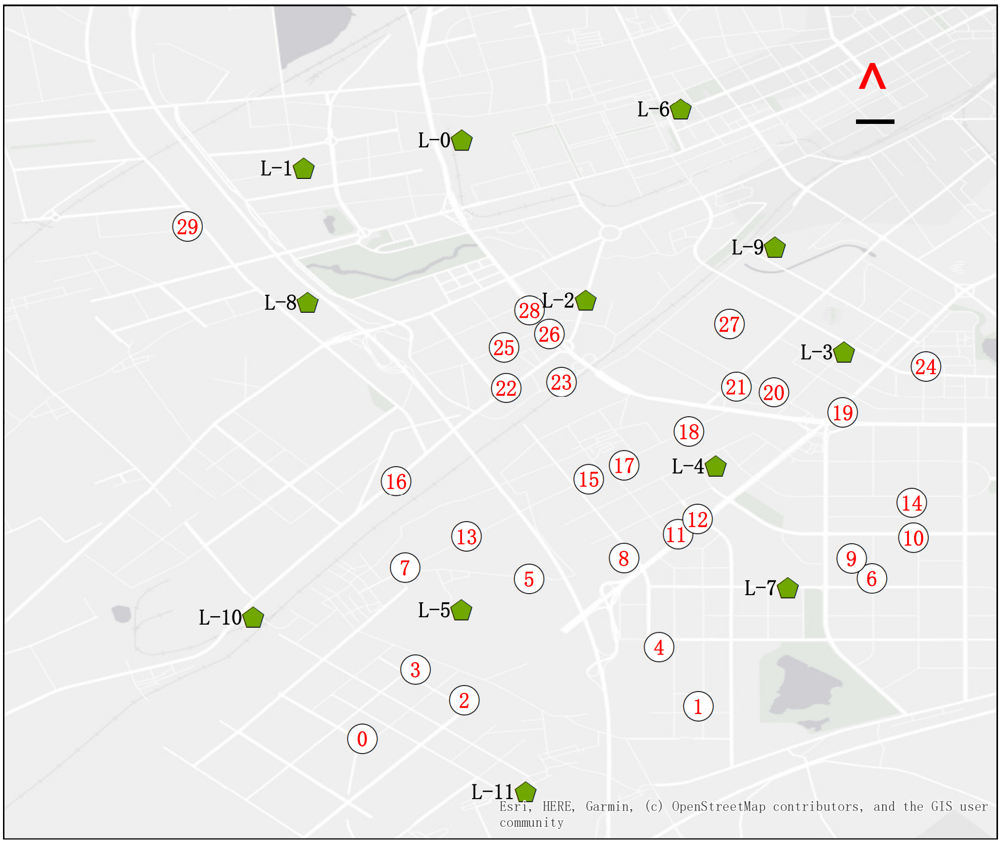



**Figure 3.** Description of the study area.

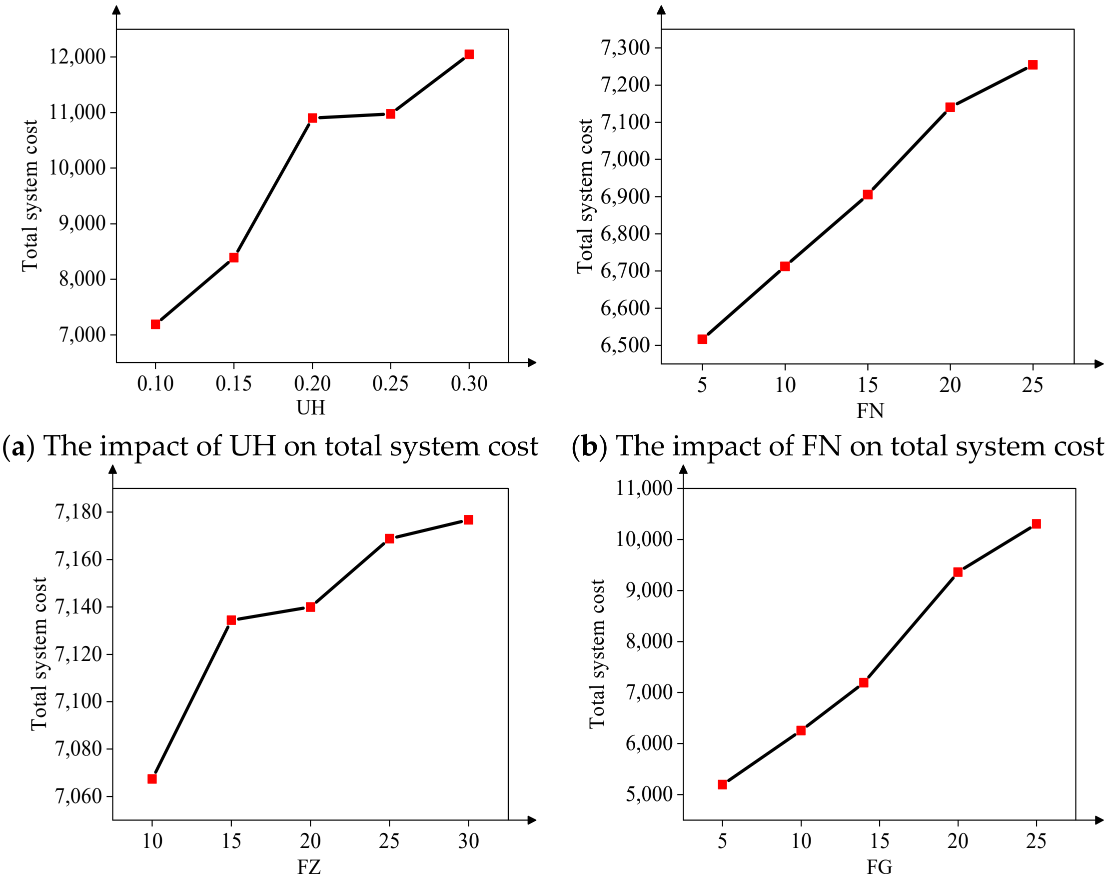
We obtain experimental data from surveying and collecting the material and data in Changchun. Through a series of data processing and simplification processes, the parameters involved in these experiments are listed in Table 3. The automobile demand of the APL is derived from the actual statistical data. According to the corresponding relationship between the automobile and the auto parts, we multiply the coefficients to acquire the auto parts demand of each APS. In this paper, we consider four production periods of the automobile APL, and the vehicle capacity is set as 1500. In addition, the inventory capacity can be chosen from the set (8000, 9000, 10,000, and 11,000). Suppose the unit transport cost from DCs to the APL and from APSs to the APL are the same and set as 20. However, the unit transport cost for picking up auto parts from DCs to the corresponding APSs is less and assumed as 14. Define the unit inventory holding cost for auto parts as 0.1. Finally, the factor of carbon emission is defined as 0.3.

**Table 3.** Key input parameters.

| $T$ | $VC$ | $WC_l$ | $UH$ | $FN/FZ$ | $FG$ | $CE$ |
|---|---|---|---|---|---|---|
| 4 | 1500 | (8000, 9000, 10,000, 11,000) | 0.1 | 20 | 14 | 0.3 |

### 4.1. Sensitivity Analysis of Deterministic Model

This section conducts sensitivity experiments to investigate the impact of essential parameters on the LIRP under deterministic demand. Namely, we discuss how the unit inventory holding cost (UH), unit transport cost from DCs to the APL (FN), unit transport cost from APSs to the APL directly (FZ), and unit transport cost for picking up auto parts from DCs to corresponding APSs (FG) influence the total system cost and the carbon emission. The results are shown in Figures 4 and 5, respectively.

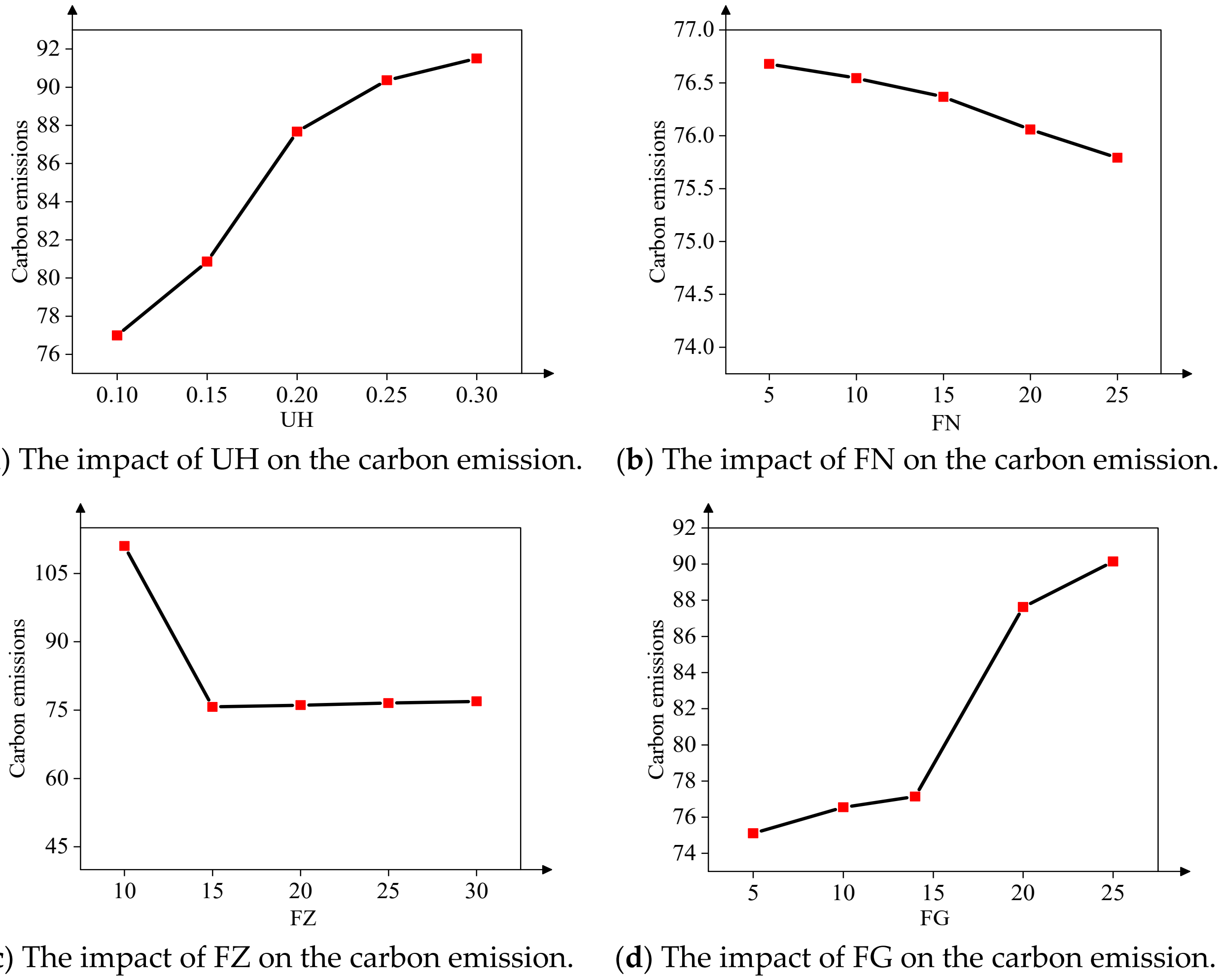



(**a**) The impact of UH on total system cost (**b**) The impact of FN on total system cost

(**c**) The impact of FZ on total system cost (**d**) The impact of FG on total system cost

**Figure 4.** The impact of various parameters on the total system cost.

As can be seen from Figure 4, there is an evident increase in the total system cost with the growth of UH, FN, FZ, and FG, which is in accordance with the realistic situation. For example, Figure 4a describes the relationship between total system cost and the unit inventory holding cost (UH). When the UH changes from 10 to 15, the total system cost increases sharply. As UH continues to increase, the growth rate will slow down. Hence, we can draw the conclusion that the total system cost is more sensitive to the UH when it is lower than 15.




(**a**) The impact of UH on the carbon emission. (**b**) The impact of FN on the carbon emission. (**c**) The impact of FZ on the carbon emission. (**d**) The impact of FG on the carbon emission.

**Figure 5.** The impact of various parameters on the carbon emission.

Further, we analyze how these parameters influence the carbon emissions in Figure 5. Figure 5b,c show that the carbon emission slightly decreases with the growth of the FN and FZ. On the contrary, Figure 5a,d indicate that carbon emission rises significantly with the increase in UH and FG.

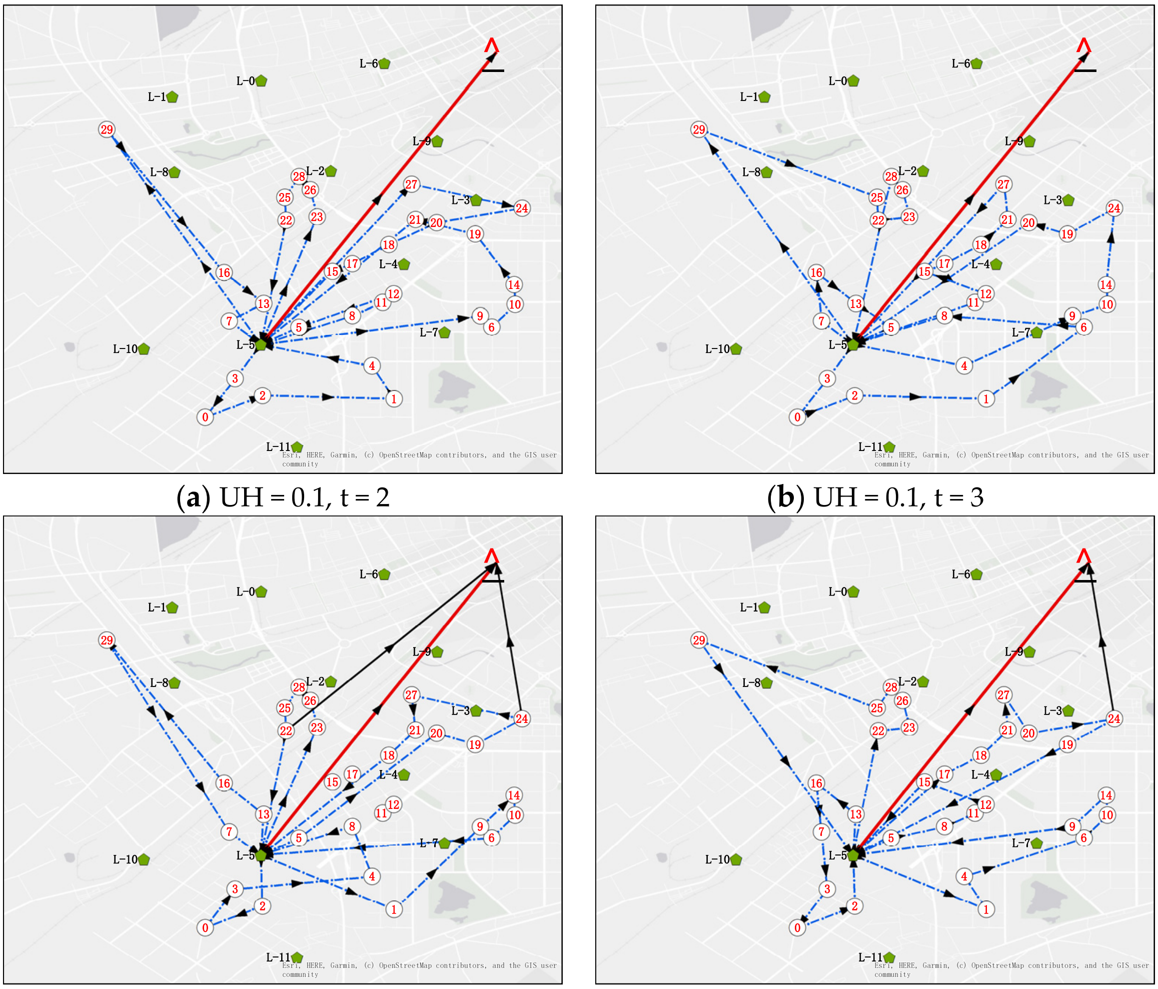
According to Figures 4 and 5, it can be observed that both objectives are closely associated with the UH and FG. The main reason is that the variations in UH and FG greatly influence the delivery routing of auto parts. We propose the indicator Supplier Delivery Amount (SDA) to represent the number of auto parts delivered from APSs to the APL directly. Analogously, Distribution Center Delivery Amount (DCDA) is proposed to describe the number of auto parts supplied from DCs to the APL. In addition, Supplier Delivery Amount Percentage (SDAP) is proposed to capture the proportion of SDA in the total number of auto parts transported.

Therefore, we summarize these indicators under different values of UH and FG in Table 4. It is easy to observe that the SDAP increases from 0% to 14.89% with the growth of UH, which means that if the unit inventory cost is high, more auto parts will be delivered from APSs to the APL directly. Therefore, there is a need for more vehicles to undertake auto parts transport activities. Consequently, the total system cost and the carbon emission will arise. Compared to the UH, FG has a similar effect on the system performance of the auto parts supply logistics network. A larger value for FG indicates that the centralized pickup cost from DCs to corresponding APSs is high, resulting in more auto parts being delivered from APSs to DCs directly, and the SDAP increases from 0% to 21.88%.

**Table 4.** Statistical data for the delivery routing of auto parts under various UH and FG.

| SA | UH | | | | | FG | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **0.1** | **0.15** | **0.2** | **0.25** | **0.3** | **5** | **10** | **14** | **20** | **25** |
| SDA | 0 | 874 | 886 | 2566 | 3561 | 0 | 0 | 0 | 4377 | 5233 |
| DCDA | 23,912 | 23,038 | 23,026 | 21,379 | 20,351 | 23,912 | 23,912 | 23,912 | 19,535 | 18,679 |
| SDAP | 0.00% | 3.66% | 3.71% | 10.59% | 14.89% | 0.00% | 0.00% | 0.00% | 18.30% | 21.88% |

To graphically depict the delivery routing decisions under different values of UH, we portray the delivery routing distribution during periods $t = 2$ and $t = 3$ in Figure 6, respectively. Figure 6a,b describe the routing decisions under the circumstance of UH = 0.1, and Figure 6c,d assume UH = 1.5.




(**a**) UH = 0.1, t = 2 (**b**) UH = 0.1, t = 3

(**c**) UH = 0.15, t = 2 (**d**) UH = 0.15, t = 3

**Figure 6.** The impact of UH on the routing decisions.

As can be seen in Figure 6, candidate L_5 is chosen as DCs and the delivery routing decisions are described. The blue dotted line is the delivery routing from DCs to the corresponding APSs to pick up auto parts, and each circle represents an assigned vehicle. In addition, the solid black lines depict the routing from APSs to the APL, while the solid red line describes the delivery routing from DCs to the APL. Through the comparison between Figure 6a,c, it is clear that there are two extra direct delivery routes from APSs to the APL, and the result again verifies the idea that a higher UH will increase the number of auto parts delivered directly from APSs to the APL. We can obtain the same conclusion by comparing Figure 6b,d. The impact of FG on the delivery decisions is similar to UH, and it is not described in this article, to avoid repetition.

However, compared to UH and FG, FZ has a distinct influence on auto parts delivery routing decisions. In the following, we analyze the delivery routing of auto parts under various FZ in Table 4. Higher FZ indicates that the unit transport cost from APSs to the APL is high. With the FZ upward, more auto parts are assigned to DCs to reduce the transport cost by centralized pickup. Table 5 shows that as the FZ increases from 10 to 15, the SDAP will decrease from 37.09% to 1.09%.

**Table 5.** Statistical data on the delivery routing of auto parts under various FZ.

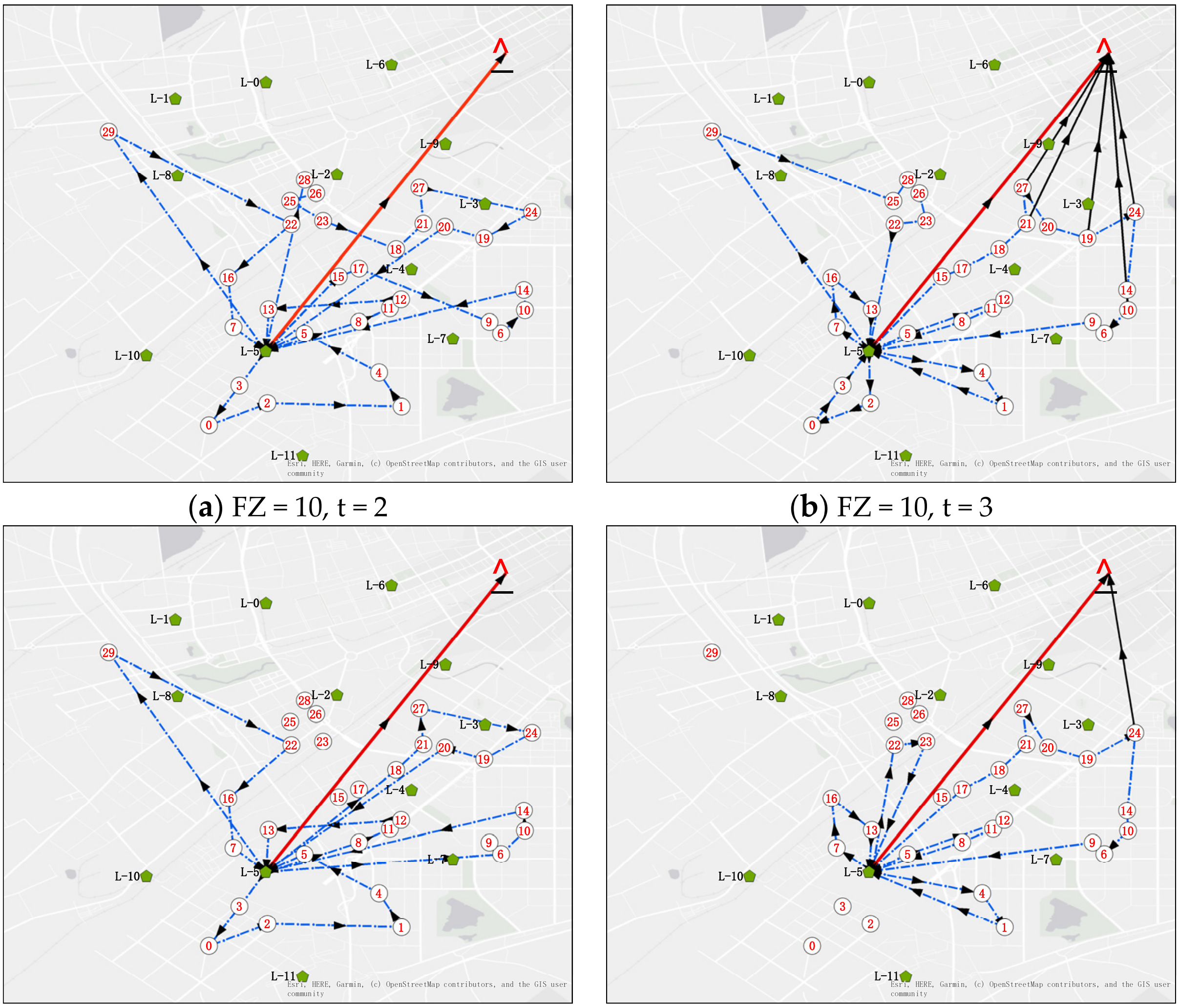
| SA | FZ | | | | |
|---|---|---|---|---|---|
| | 10 | 15 | 20 | 25 | 30 |
| SDA | 8772 | 260 | 0 | 0 | 0 |
| DCDA | 15,140 | 23,652 | 23,912 | 23,912 | 23,912 |
| SDAP | 37.09% | 1.09% | 0.00% | 0.00% | 0.00% |

Similar to Figure 6, to graphically depict the delivery routing decisions under different values of FZ, we portray the routing distribution during periods $t = 2$ and $t = 3$ in Figure 7. Figure 7a,b delineate the routing decisions under the condition of FZ = 10, and Figure 7c,d suppose FZ = 15.

Analogous to Figure 6, candidate point L_5 is selected as DCs. Figure 6a,b show that there are a total of five routes from APSs to the APL during periods t = 2 and t = 3 when FZ = 10. However, when FZ = 15 in Figure 6c,d, the total number of routes from APSs to the APL is reduced to one. This is in accordance with Table 6 that higher FZ leads to more auto parts being delivered through DCs, which perform centralized pickup and unified delivery.

**Table 6.** Variations in LIRP decisions under different uncertainty levels ($\rho = 1$).

| Uncertainty Level | DC | WC | SDA | DCDA | SDAP |
|---|---|---|---|---|---|
| 30% | L_5 | 9000 | 2313 | 24,161 | 8.74% |
| 60% | L_5 | 9000 | 4827 | 24,209 | 16.62% |
| 90% | L_5 | 10,000 | 7410 | 24,188 | 23.45% |




(**a**) FZ = 10, t = 2 (**b**) FZ = 10, t = 3

(**c**) FZ = 15, t = 2 (**d**) FZ = 15, t = 3

**Figure 7.** The impact of FZ on the routing decisions.

The above parameters influence the system performance by affecting the location of DCs, inventory capacity, and delivery routing decisions. Overall, we can conclude that the total system cost and carbon emission are sensitive to the above four parameters.

Concerning the relationship between the total system cost and carbon emission, the Pareto-set output from the deterministic model under different weights is depicted in Figure 8, where one point in the figure represents a particular LIRP solution. The Pareto set has a total system cost ranging from 7062 RMB to 7166 RMB and the carbon emission ranges from 74 kg to 77 kg. It can also be seen in Figure 8 that carbon emission and total system cost have the same tendency, which proves the essence of considering the sustainable LIRP in auto parts supply logistics.

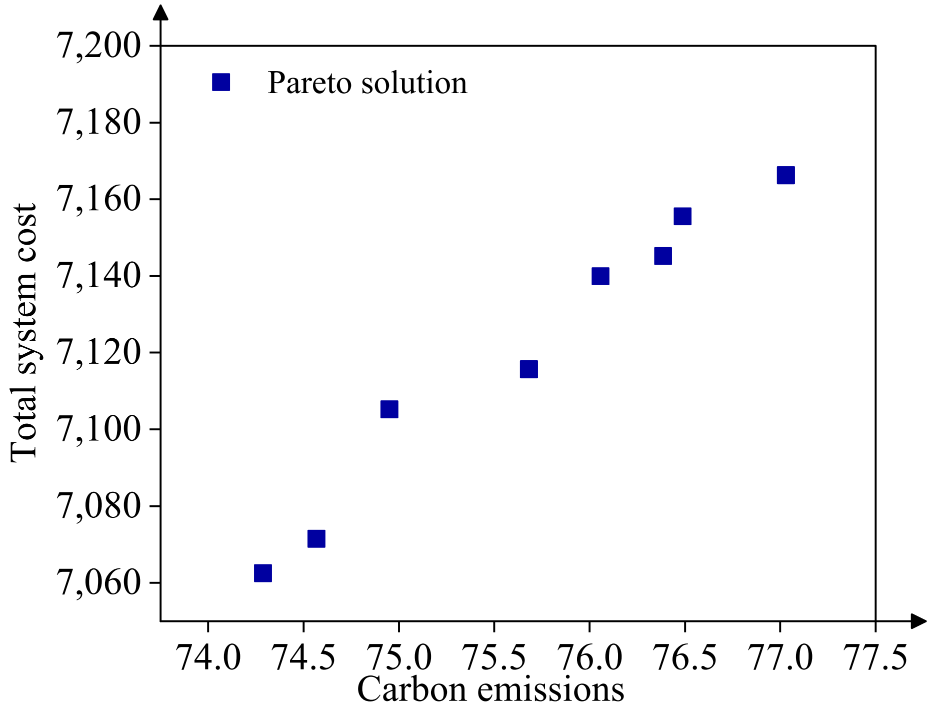
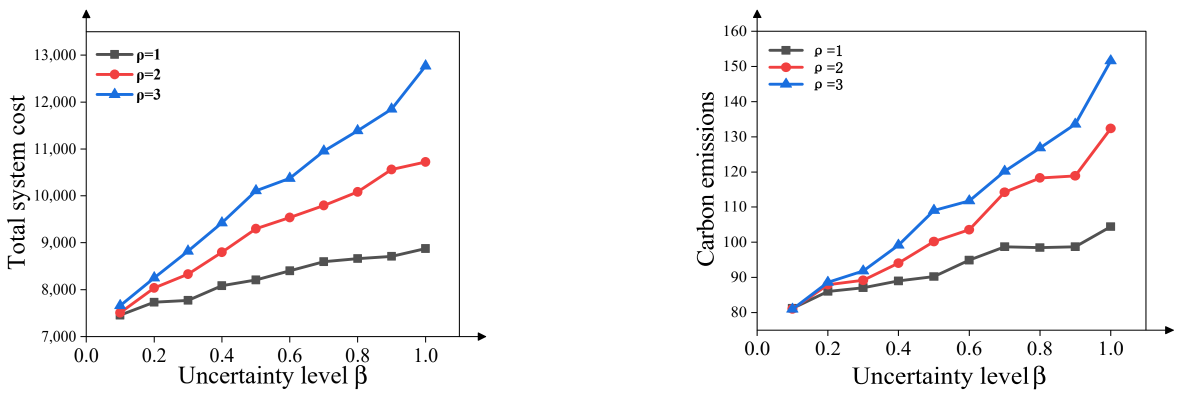



**Figure 8.** Pareto set of LIRP.

### 4.2. *Sensitivity Analysis of Robust Model*

The above parameters sensitivity analysis in the deterministic model are suitable for the robust model. Except for these analyses, this section further discusses how the uncertainty level and the degree of conservatism influence the LIRP. Here, we first define that the deviation value $\hat{D}^t$ equals the nominal values $D^t$. According to the same input parameters as the deterministic model, it is assumed that the uncertain demand is likely to occur at any period, and, as introduced in 3.4, we use $\beta \in [0, 1]$ to represent the uncertainty level. This paper adopts $\rho$ to describe the degree of conservatism.

First, we discuss the effect of uncertainty level on the system performance in Figure 9. Three kinds of degrees of conservatism are considered. Figure 9a delineates the variation in total system cost with increased uncertainty level, and Figure 9b shows the variation in carbon emissions. It can be seen from Figure 8 that with the increase in the uncertainty level, the total system cost and carbon emissions increase significantly under different degrees of conservatism.

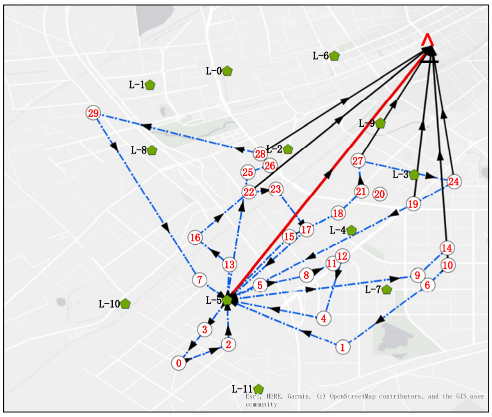



(**a**) Effect of uncertainty level on the total system cost (**b**) Effect of uncertainty level on the carbon emission

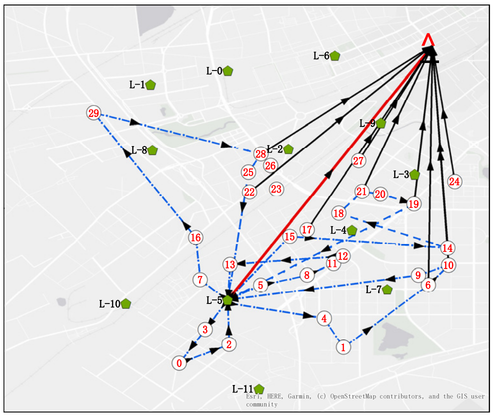
**Figure 9.** Effect of uncertainty level on the system performance.

To further depict the impact of uncertainty level on the location of DCs, inventory capacity, and delivery routing decisions, we take the degree of conservatism $\rho = 1$ as an example and analyze how the decision variables change when the uncertainty level ranges from 30% to 90%. The results are shown in Table 6, where the value of DC is the selected candidate point for DCs and WC is the inventory capacity for DCs.

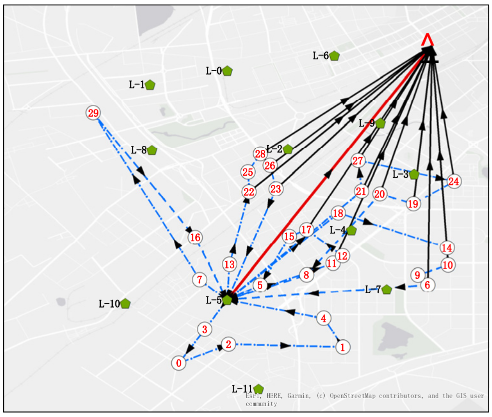
As shown in Table 6, although the location of DC does not change, the inventory capacity of DC increases from 9000 to 10,000 when the uncertainty level grows. Another interesting finding is that the proportion of auto parts delivered from APSs to the APL directly increases from 8.74% to 23.45%. The reason may be that when the uncertainty level grows, the production demand in the APL for the worst case is extremely high, which is likely to exceed the inventory capacity of DCs. Therefore, the auto parts in APSs close to the APL will be transported directly without needing transfers in DCs. The detailed delivery routing decisions are graphically shown in Figure 10. It is clear to see that more solid black lines are depicted with the increase in the uncertainty level, verifying that more auto parts close to the APL are delivered directly.



(**a**) $\beta = 30\%, \rho = 1$



(**b**) $\beta = 60\%, \rho = 1$



(**c**) $\beta = 90\%, \rho = 1$

**Figure 10.** The impact of uncertainty level on the delivery routing decisions.

Subsequently, the effects of the degree of conservatism on the location, inventory, and delivery routing decisions are summarized in Table 7. We take the uncertainty level $\beta = 100\%$ as an example and analyze how the decision variables change when the degree of conservatism changes from 1 to 3. Similar to the uncertainty level, the inventory capacity increases from 10,000 to 11,000 with the growth of the degree of conservatism, and the proportion of auto parts delivered from APSs to the APL directly increases from 29.48% to 55.10%. This is because more production demand is needed under a more conservative environment. Limited to the inventory capacity, more auto parts close to the APL will be delivered directly, and the detailed delivery routing distribution is similar to Figure 9, which we do not display again.

**Table 7.** Variations in LIRP decisions under different conservatism degrees ($\beta = 100\%$).

| Degree of Protection | DC | WC | SDA | DCDA | SDAP |
|---|---|---|---|---|---|
| 1 | L_5 | 10,000 | 9566 | 22,886 | 29.48% |
| 2 | L_5 | 10,000 | 18,439 | 22,126 | 45.46% |
| 3 | L_5 | 11,000 | 26,353 | 21,471 | 55.10% |

### *4.3. Model Comparisons*

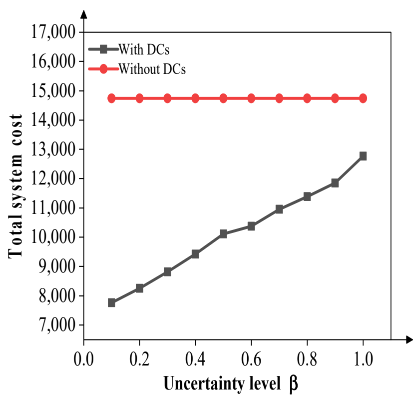
At present, the auto parts are delivered directly from APSs to the APL, resulting in a large waste in vehicle capacity and a high transport cost. This paper proposes a robust LIRP in auto parts supply logistics to address the real problem of establishing DCs to minimize the total system cost. We define the two scenarios as "Without DCs" and "With DCs". In the following, we conduct comparative analyses on the utilization rate of vehicle capacity, total system cost, and carbon emission. The results are summarized in Table 8.

**Table 8.** Comparative analyses between without and with DCs.

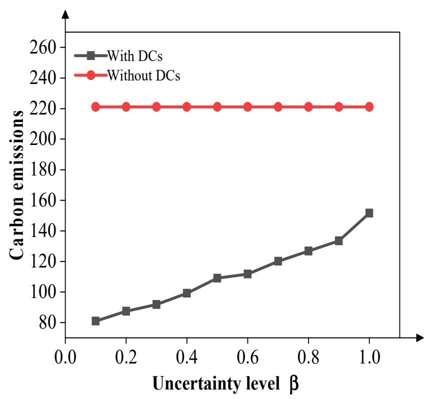
| Scenarios | The Utilization Rate of Vehicle Capacity | Carbon Emission | Total System Cost |
|---|---|---|---|
| Without DCs | 17.71% | 221.1108 | 14,740.72 |
| With DCs | 96.29% | 76.05862 | 7139.92 |

Obviously, the total system cost and carbon emissions will decrease when establishing DCs using the proposed LIRP model, and the utilization rate of vehicle capacity will be dramatically improved. Consequently, we can draw the conclusion that the proposed LIRP model is largely effective in the auto parts supply logistics network.

Moreover, to demonstrate the significance of considering uncertain demand, we compared the system performance between the scenarios "Without DCs" and "With DCs" under uncertain demand. Assuming the degree of conservatism $\rho = 3$, we further discuss the variation in total system cost and carbon emission while the uncertainty level changes from 0.1 to 1. As shown in Figure 11, the total system cost and the carbon emission obtained by our robust LIRP model are significantly lower than the scenario "without DCs".




(**a**) Comparison in total system cost




(**b**) Comparison in Carbon emissions

**Figure 11.** Comparative analyses between without and with DCs considering uncertain demand.

## 5. Discussion

In this paper, a multi-objective MIP model is proposed to deal with the LIRP of the automotive parts supply chain with deterministic and uncertain demands. The objective of the model is to minimize the cost and carbon emissions in the whole system.

Based on the above numerical analysis, we can have the following practical implications and insights on the LIRP system design problem. First, the model proposed in this paper provides an integrated optimization scheme for automotive parts supply chain optimization and gives a basis for micro-analysis of operation strategies. Compared with the two-stage model [35], the integrated optimization model we choose can more adequately consider the relationship between the three-level decisions. Second, compared to the LIRP model of [42], we fully consider the idea of sustainability, which largely reduces carbon emissions released from transport vehicles, and the results reveal that the total system cost optimization direction is consistent with sustainable optimization. Finally, considering the uncertainty in production demand, the robust optimization approach adopted in this paper significantly outperforms the emergency order direct delivery by APSs in terms of both total system cost reduction and carbon emission reduction.

The environment is becoming an increasingly important criterion in planning automotive parts supply networks. The model presented in this paper has the potential to assist decision-makers and managers solve the LIRP in the supply network configuration. It also provides constructive suggestions for auto parts supply chain planners to select the reasonable DC location and determine the cost-optimal routing decisions for centralized collection and unified distribution. The results demonstrate that the method proposed in this paper would contribute to significant savings in total system cost and reduce the environmental impact.

Although this study proves that the LIRP model we proposed is effective in auto parts supply logistics, there is still a few limitations that we would like to emphasize for future research. First, for the sake of simplification, only one APL is considered. It is more realistic to investigate multiple APLs, which will increase the complexity of this problem. Secondly, as the number of periods and APSs increases, the size of the problem could become very large, making it difficult to address by the solver. Therefore, it is necessary to explore an efficient algorithm. With various metaheuristics available and many possibilities for customization, future work might explore the best options for realistic networks.

## 6. Conclusions

This paper proposes a unified framework simultaneously considering the location-inventory-routing problem in auto parts supply logistics, which are rarely considered from a sustainable perspective. Within this framework, a novel multi-objective MIP model is proposed to estimate the system performance. Specifically, this model determines the location and capacity of DCs and routing decisions to minimize the total system cost and carbon emissions while considering multi-period production demand. Concerning uncertain factors in production activities, a robust optimization method is developed further in the context of uncertain demand in the APL. A numerical example is investigated to illustrate the effectiveness of the proposed framework in the LIRP. Sensitivity analyses of essential parameters yield several managerial insights. The results show that the location, inventory capacity, and delivery routing decisions are highly affected by various cost parameters. Finally, we observe that the utilization of vehicle capacity will be dramatically improved by our LIRP model, indicating that studying the LIRP of auto parts supply logistics is extremely meaningful.

The current research can be extended in various directions to optimize the automotive parts supply network LIRP. First, it is idealistic to assume the auto parts are classified as a unified specification in the context of actual situations. It is of great significance to discuss the diverse specifications of auto parts. Secondly, it seems that uncertain incidents can occur in random stages. Hence, it is not enough to consider the uncertain demand in the APL, but the uncertain circumstance during the supply and transportation should also be taken into account. Finally, the urban road network structure deeply affects transportation routing decisions, which will be included in our future study.

**Author Contributions:** Conceptualization, B.S.; methodology, A.L.; software, A.L.; validation, B.S.; formal analysis, A.L.; investigation, A.L.; resources, A.L.; data curation, A.L.; writing—original draft preparation, A.L.; writing—review and editing, A.L.; visualization, A.L.; supervision, A.L.; project administration, A.L.; funding acquisition, B.S. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the National Natural Science Foundation of China (grant number 61873109); the Natural Science Foundation of Jilin Province (grant number 20210101055JC); the FAW Technology Innovation Project (grant number KF2020-20006).

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Acknowledgments:** The authors appreciate the support from Jilin University.

**Conflicts of Interest:** The authors declare no conflict of interest.

## References

1. Ranjbaran, F.; Kashan, A.H.; Kazemi, A. Mathematical formulation and heuristic algorithms for optimisation of auto-part milk-run logistics network considering forward and reverse flow of pallets. *Int. J. Prod. Res.* **2020**, *58*, 1741–1775. [CrossRef]
2. La Rosa, L.M.V.-D.; Villarreal-Villarreal, L.A.; Alarcón-Martínez, G. Quality and innovation as drivers for manufacturing competitiveness of automotive parts suppliers. *TQM J.* **2019**, *33*, 966–986. [CrossRef]
3. Wang, Y.; Chen, F. Packed parts delivery problem of automotive inbound logistics with a supplier park. *Comput. Oper. Res.* **2019**, *101*, 116–129. [CrossRef]
4. Yu, W.; Wang, T.; Xiao, Y.; Chen, J.; Yan, X. A Carbon Emission Measurement Method for Individual Travel Based on Transportation Big Data: The Case of Nanjing Metro. *Int. J. Environ. Res. Public Health* **2020**, *17*, 5957. [CrossRef]
5. Wu, W.; Zhou, W.; Lin, Y.; Xie, Y.; Jin, W. A hybrid metaheuristic algorithm for location inventory routing problem with time windows and fuel consumption. *Expert Syst. Appl.* **2021**, *166*, 114034. [CrossRef]
6. Yaobao, Z.; Ping, H.; Shu, Y. An Improved Particle Swarm Optimization for the Automobile Spare Part Warehouse Location Problem. *Math. Probl. Eng.* **2013**, *2013*, 726194. [CrossRef]
7. Russo, F.; Rindone, C. Regional Transport Plans: From Direction Role Denied to Common Rules Identified. *Sustainability* **2021**, *13*, 9052. [CrossRef]
8. Mao, Z.; Huang, D.; Fang, K.; Wang, C.; Lu, D. Milk-run routing problem with progress-lane in the collection of automobile parts. *Ann. Oper. Res.* **2020**, *291*, 657–684. [CrossRef]
9. Khongkaew, P. Random Location under Fixed Zone Storage Strategy A Case Study of Automobile and Electronic Parts Manufacturing Factory. In Proceedings of the International Conference on Industrial Engineering and Operations Management, Bangkok, Thailand, 5–7 March 2019.
10. Diabat, A.; Richard, J.-P.; Codrington, C.W. A Lagrangian relaxation approach to simultaneous strategic and tactical planning in supply chain design. *Ann. Oper. Res.* **2013**, *203*, 55–80. [CrossRef]
11. Zeng, R. Research on the Demand Prediction of Parts Inventory for Auto Customer Service. *J. Phys. Conf. Ser.* **2020**, *1654*, 012115. [CrossRef]
12. Li, J.; Wang, F.; He, Y. Electric Vehicle Routing Problem with Battery Swapping Considering Energy Consumption and Carbon Emissions. *Sustainability* **2020**, *12*, 10537. [CrossRef]
13. Zou, Y.; Wu, H.; Yin, Y.; Dhamotharan, L.; Chen, D.; Tiwari, A.K. An improved transformer model with multi-head attention and attention to attention for low-carbon multi-depot vehicle routing problem. *Ann. Oper. Res.* **2022**, 1–20. [CrossRef]
14. Qiu, Y.; Qiao, J.; Pardalos, P.M. A branch-and-price algorithm for production routing problems with carbon cap-and-trade. *Omega* **2017**, *68*, 49–61. [CrossRef]
15. Chen, J.; Dan, B.; Shi, J. A variable neighborhood search approach for the multi-compartment vehicle routing problem with time windows considering carbon emission. *J. Clean. Prod.* **2020**, *277*, 123932. [CrossRef]
16. Yu, V.F.; Aloina, G.; Susanto, H.; Effendi, M.K.; Lin, S.-W. Regional Location Routing Problem for Waste Collection Using Hybrid Genetic Algorithm-Simulated Annealing. *Mathematics* **2022**, *10*, 2131. [CrossRef]
17. Zhang, S.; Chen, N.; She, N.; Li, K. Location optimization of a competitive distribution center for urban cold chain logistics in terms of low-carbon emissions. *Comput. Ind. Eng.* **2021**, *154*, 107120. [CrossRef]
18. Li, K.; Li, D.; Wu, D. Carbon Transaction-Based Location-Routing- Inventory Optimization for Cold Chain Logistics. *Alex. Eng. J.* **2022**, *61*, 7979–7986. [CrossRef]

19. Kumar, R.S.; Kondapaneni, K.; Dixit, V.; Goswami, A.; Thakur, L.; Tiwari, M. Multi-objective modeling of production and pollution routing problem with time window: A self-learning particle swarm optimization approach. *Comput. Ind. Eng.* **2016**, *99*, 29–40. [CrossRef]
20. Wang, M.; Wu, J.; Kafa, N.; Klibi, W. Carbon emission-compliance green location-inventory problem with demand and carbon price uncertainties. *Transp. Res. Part E Logist. Transp. Rev.* **2020**, *142*, 102038. [CrossRef]
21. Biuki, M.; Kazemi, A.; Alinezhad, A. An integrated location-routing-inventory model for sustainable design of a perishable products supply chain network. *J. Clean. Prod.* **2020**, *260*, 120842. [CrossRef]
22. Ji, S.; Tang, J.; Sun, M.; Luo, R. Multi-objective optimization for a combined location-routing-inventory system considering carbon-capped differences. *J. Ind. Manag. Optim.* **2022**, *18*, 1949. [CrossRef]
23. Quan, C.; He, Q.; Ye, X.; Cheng, X. Optimization of the Milk-run route for inbound logistics of auto parts under low-carbon economy. *J. Algorithms Comput. Technol.* **2021**, *15*, 17483026211065387. [CrossRef]
24. Liu, Z.; Hu, B.; Huang, B.; Lang, L.; Guo, H.; Zhao, Y. Decision Optimization of Low-Carbon Dual-Channel Supply Chain of Auto Parts Based on Smart City Architecture. *Complexity* **2020**, *2020*, 2145951. [CrossRef]
25. Govindan, K.; Mina, H.; Esmaeili, A.; Gholami-Zanjani, S.M. An Integrated Hybrid Approach for Circular supplier selection and Closed loop Supply Chain Network Design under Uncertainty. *J. Clean. Prod.* **2019**, *242*, 118317. [CrossRef]
26. Nekooghadirli, N.; Tavakkoli-Moghaddam, R.; Ghezavati, V.; Javanmard, S. Solving a new bi-objective location-routing-inventory problem in a distribution network by meta-heuristics. *Comput. Ind. Eng.* **2014**, *76*, 204–221. [CrossRef]
27. Zahiri, B.; Suresh, N.C. Hub network design for hazardous-materials transportation under uncertainty. *Transp. Res. Part E: Logist. Transp. Rev.* **2021**, *152*, 102424. [CrossRef]
28. Zahiri, B.; Suresh, N.C.; de Jong, J. Resilient hazardous-materials network design under uncertainty and perishability. *Comput. Ind. Eng.* **2020**, *143*, 106401. [CrossRef]
29. Zhalechian, M.; Tavakkoli-Moghaddam, R.; Zahiri, B.; Mohammadi, M. Sustainable design of a closed-loop location-routing-inventory supply chain network under mixed uncertainty. *Transp. Res. Part E Logist. Transp. Rev.* **2016**, *89*, 182–214. [CrossRef]
30. Shang, X.; Zhang, G.; Jia, B.; Almanaseer, M. The healthcare supply location-inventory-routing problem: A robust approach. *Transp. Res. Part E Logist. Transp. Rev.* **2022**, *158*, 102588. [CrossRef]
31. Yuan, Z.; Gao, J. Dynamic Uncertainty Study of Multi-Center Location and Route Optimization for Medicine Logistics Company. *Mathematics* **2022**, *10*, 953. [CrossRef]
32. Ghasemi, P.; Goodarzian, F.; Muñuzuri, J.; Abraham, A. A cooperative game theory approach for location-routing-inventory decisions in humanitarian relief chain incorporating stochastic planning. *Appl. Math. Model.* **2022**, *104*, 750–781. [CrossRef]
33. Yang, Y.; Zhang, J.; Sun, W.; Pu, Y. Research on NSGA-III in Location-routing-inventory problem of pharmaceutical logistics intermodal network. *J. Intell. Fuzzy Syst.* **2021**, *41*, 699–713. [CrossRef]
34. Chao, C.; Zhihui, T.; Baozhen, Y. Optimization of two-stage location–routing–inventory problem with time-windows in food distribution network. *Ann. Oper. Res.* **2019**, *273*, 111–134. [CrossRef]
35. Aydemir-Karadag, A. Bi-Objective Adaptive Large Neighborhood Search Algorithm for the Healthcare Waste Periodic Location Inventory Routing Problem. *Arab. J. Sci. Eng.* **2022**, *47*, 3861–3876. [CrossRef]
36. Liu, A.; Zhu, Q.; Xu, L.; Lu, Q.; Fan, Y. Sustainable supply chain management for perishable products in emerging markets: An integrated location-inventory-routing model. *Transp. Res. Part E Logist. Transp. Rev.* **2021**, *150*, 102319. [CrossRef]
37. Yavari, M.; Enjavi, H.; Geraeli, M. Demand management to cope with routes disruptions in location-inventory-routing problem for perishable products. *Res. Transp. Bus. Manag.* **2020**, *37*, 100552. [CrossRef]
38. Song, L.; Wu, Z. An integrated approach for optimizing location-inventory and location-inventory-routing problem for perishable products. *Int. J. Transp. Sci. Technol.* **2022**. [CrossRef]
39. Qin, G.; Tao, F.; Li, L. A Vehicle Routing Optimization Problem for Cold Chain Logistics Considering Customer Satisfaction and Carbon Emissions. *Int. J. Environ. Res. Public Health* **2019**, *16*, 576. [CrossRef]
40. Bertsimas, D.; Sim, M. The Price of Robustness. *Oper. Res.* **2004**, *52*, 35–53. [CrossRef]
41. Musolino, G.; Rindone, C.; Vitetta, A. Evaluation in Transport Planning: A Comparison between Data Envelopment Analysis and Multi Criteria Decision Making Methods. In Proceedings of the 31st Annual European Simulation and Modelling Conference, Lisbon, Portugal, 25–27 October 2017; pp. 233–237.
42. Saragih, N.; Bahagia, S.; Suprayogi, S.; Syabri, I. Location-inventory-routing model with considering urban road networks. *J. Ind. Eng. Manag.* **2021**, *14*, 830–849. [CrossRef]