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Article

Lump Collision Phenomena to a Nonlinear Physical Model in Coastal Engineering

Tukur Abdulkadir Sulaiman [1], Abdullahi Yusuf [1,*], Ali Saleh Alshomrani [2] and Dumitru Baleanu [3,4,5]

[1] Department of Computer Engineering, Biruni University, Istanbul 34010, Turkey
[2] Department of Mathematics, King Abdul Aziz University, Jeddah 21589, Saudi Arabia
[3] Department of Mathematics, Cankaya University, Ankara 06530, Turkey
[4] Institute of Space Sciences, Magurele, 077125 Bucharest, Romania
[5] Department of Natural Sciences, School of Arts and Sciences, Lebanese American University, Beirut 11022801, Lebanon
* Correspondence: ayusuf@biruni.edu.tr

Abstract: In this study, a dimensionally nonlinear evolution equation, which is the integrable shallow water wave-like equation, is investigated utilizing the Hirota bilinear approach. Lump solutions are achieved by its bilinear form and are essential solutions to various kind of nonlinear equations. It has not yet been explored due to its vital physical significant in various field of nonlinear science. In order to establish some more interaction solutions with some novel physical features, we establish collision aspects between lumps and other solutions by using trigonometric, hyperbolic, and exponential functions. The obtained novel types of results for the governing equation includes lump-periodic, two wave, and breather wave solutions. Meanwhile, the figures for these results are graphed. The propagation features of the derived results are depicted. The results reveal that the appropriate physical quantities and attributes of nonlinear waves are related to the parameter values.

Keywords: shallow water wave-like scalar equation; Hirota bilenear method; breather wave solution; lump-periodic solution; two-wave solution

MSC: 35A08; 35A09; 35A25

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1. Introduction

Nonlinear models have a pleasant features to comprehend so many physical problems, and researchers consider research in nonlinear fields as one of the most significant constraints for comprehending the universe. The study of a variety of nonlinear partial differential equations is essential for the mathematical modeling of complicated time-varying phenomena. As a result, during the past few decades, one of the most delightful and exciting fields of research has been the examination of results to the aforementioned aspects, as well as the associated problem of constructing closed form wave solutions to a wider group of nonlinear equations. Solitary wave solutions with a closed form provide more report about those instances. As a result, a large number of mathematicians and physical researchers have worked hard to find closed form wave solutions for nonlinear PDEs, as well as a kind of powerful and adapted approaches [1–14].

Nonlinear PDEs can generate a large variety of solutions. Lump solutions are rational function solutions that have been empirically investigated in all directions [15–22]. Lump solutions are among the most important results for nonlinear PDEs [22–26]. Lump solutions occur in several non-integrable equations. Moreover, several studies have shown that collision aspects between lumps and other forms of solutions to nonlinear equations exist [25–32].

Moreover, lump solutions to mathematical equations are required for understanding the qualitative features of many occurrences and processes in several disciplines of natural science. Lump solutions of nonlinear differential equations graphically depict and explain a variety of sophisticated nonlinear phenomena, such as the spatial localization of transfer processes, the existence of peaking regimes, and the multiplicity or absence of steady states under various situations. Furthermore, simple solutions are frequently utilized as particular examples highlighting key notions of a theory that allow for mathematical exposition in many courses. Many equations in physics, chemistry, and biology have empirical parameters or empirical functions, which should be noted. Exact solutions enable researchers to develop and carry out studies to identify these parameters or functions by setting adequate natural conditions [33–37].

However, as far as we are aware, breather and lump-periodic wave solutions have not been investigated for the shallow water wave-like scalar equation [38]. The most significant processes in the world are described using nonlinear equations. It continues to be a basic issue in applied mathematics and physics to find innovative approximate or precise solutions to nonlinear equations. To achieve this, many approaches must be used. One of the most prominent analytical methods for resolving nonlinear equations is the Hirota transformation technique. We are motivated to construct a unique lump-like solution for the shallow water wave-like scalar equation supplied by [38]:

$$\phi_t + \phi_x + \frac{3}{2}\phi\phi_{xt} - \frac{3}{2}\phi_x\phi_t + \frac{1}{2}\phi^2\phi_t = 0, \tag{1}$$

because of the lasting character of lump solutions and their power to grasp a wide spectrum of nonlinear events in the cross-field.

Shallow-water wave equations are a collection of hyperbolic partial differential equations that describe fluid flow beneath a pressure surface. When the horizontal length scale is substantially larger than the vertical length scale, the Navier–Stokes equations are depth-integrated to create the water wave equations. As a result, the fluid's vertical velocity scale is thought to be less significant than its horizontal velocity scale according to the principle of conservation of mass. The momentum equation demonstrates that vertical pressure gradients are virtually hydrostatic and that horizontal pressure gradients are caused by the displacement of the pressure surface, indicating that the horizontal velocity field is constant throughout the fluid's depth. The vertical velocity can be taken out of the equations using vertical integration. This leads to the derivation of the shallow-water equations [39].

The rest of the paper is arranged as follows: The next part concentrates on the breather wave solution. The Lump-periodic solutions of the governing equation are constructed in Section 3. In Section 4, the two-wave solutions have been established in Section 5, the physical interpretation of the obtained results has been given in Section. In Section 6, concluding remarks are provided.

2. Breather Wave Solution

Here, a class of breather wave solutions is provided.

Assume that

$$\phi(x,t) = 3(\ln\psi)_x. \tag{2}$$

Inserting Equation (2) into (1), provides [38]:

$$2\psi(\psi_{xt} + \psi_{xx}) - 9\psi_{xt}\psi_{xx} + \psi_x(9\psi_{xxt} - 2(\psi_t + \psi_x)) = 0. \tag{3}$$

Consider:

$$\psi(x,t) = \gamma_1\cos(\vartheta_0(t\varpi_0 + x)) + \gamma_2 e^{(\vartheta_1(\varepsilon_0 t + x))} + e^{-\vartheta_1(\varepsilon_0 t + x)}. \tag{4}$$

Inserting Equation (4) into (3) yields the following set of equations:

$$
\begin{aligned}
&-36\gamma_2\varepsilon_0\vartheta_1^4+8\gamma_2\varepsilon_0\vartheta_1^2-9\gamma_1^2\vartheta_0^4\varpi_0-2\gamma_1^2\vartheta_0^2\varpi_0-2\gamma_1^2\vartheta_0^2+8\gamma_2\vartheta_1^2=0,\\
&9\gamma_1\varepsilon_0\vartheta_1^3\vartheta_0-2\gamma_1\varepsilon_0\vartheta_1\vartheta_0-9\gamma_1\vartheta_1\vartheta_0^3\varpi_0-2\gamma_1\vartheta_1\vartheta_0\varpi_0-4\gamma_1\vartheta_1\vartheta_0=0,\\
&9\gamma_1\varepsilon_0\vartheta_1^2\vartheta_0^2+2\gamma_1\varepsilon_0\vartheta_1^2+9\gamma_1\vartheta_1^2\vartheta_0^2\varpi_0-2\gamma_1\vartheta_0^2\varpi_0-2\gamma_1\vartheta_0^2+2\gamma_1\vartheta_1^2=0,\\
&-9\gamma_1\gamma_2\varepsilon_0\vartheta_1^3\vartheta_0+2\gamma_1\gamma_2\varepsilon_0\vartheta_1\vartheta_0+9\gamma_1\gamma_2\vartheta_1\vartheta_0^3\varpi_0+2\gamma_1\gamma_2\vartheta_1\vartheta_0\varpi_0+4\gamma_1\gamma_2\vartheta_1\vartheta_0=0,\\
&9\gamma_1\gamma_2\varepsilon_0\vartheta_1^2\vartheta_0^2+2\gamma_1\gamma_2\varepsilon_0\vartheta_1^2+9\gamma_1\gamma_2\vartheta_1^2\vartheta_0^2\varpi_0-2\gamma_1\gamma_2\vartheta_0^2\varpi_0-2\gamma_1\gamma_2\vartheta_0^2+2\gamma_1\gamma_2\vartheta_1^2=0.
\end{aligned} \tag{5}
$$

Simplifying Equation (5), provides the following solutions:
(I): As

$$\varepsilon_0=\frac{2\left(9\vartheta_0^2-2\right)}{81\vartheta_0^2\vartheta_1^2+4},\ \varpi_0=-\frac{2\left(9\vartheta_1^2+2\right)}{81\vartheta_0^2\vartheta_1^2+4},\ \gamma_2=-\frac{\gamma_1^2\vartheta_0^2}{4\vartheta_1^2},$$

we get

$$\psi_1(x,t)=-\frac{\gamma_1^2\vartheta_0^2e^{\left(\vartheta_1\left(\frac{2t\left(9\vartheta_0^2-2\right)}{81\vartheta_0^2\vartheta_1^2+4}+x\right)\right)}}{4\vartheta_1^2}+e^{\left(-\vartheta_1\left(\frac{2t\left(9\vartheta_0^2-2\right)}{81\vartheta_0^2\vartheta_1^2+4}+x\right)\right)}+\gamma_1\cos\left(\vartheta_0\left(x-\frac{2t\left(9\vartheta_1^2+2\right)}{81\vartheta_0^2\vartheta_1^2+4}\right)\right).$$

Consequently,

$$\phi_1(x,t)=\frac{3\left(-\frac{\gamma_1^2\vartheta_0^2e^{\left(\vartheta_1\left(\frac{2t\left(9\vartheta_0^2-2\right)}{81\vartheta_0^2\vartheta_1^2+4}+x\right)\right)}}{4\vartheta_1}-\vartheta_1e^{\left(-\vartheta_1\left(\frac{2t\left(9\vartheta_0^2-2\right)}{81\vartheta_0^2\vartheta_1^2+4}+x\right)\right)}-\gamma_1\vartheta_0\sin\left(\vartheta_0\left(x-\frac{2t\left(9\vartheta_1^2+2\right)}{81\vartheta_0^2\vartheta_1^2+4}\right)\right)\right)}{-\frac{\gamma_1^2\vartheta_0^2e^{\left(\vartheta_1\left(\frac{2t\left(9\vartheta_0^2-2\right)}{81\vartheta_0^2\vartheta_1^2+4}+x\right)\right)}}{4\vartheta_1^2}+e^{\left(-\vartheta_1\left(\frac{2t\left(9\vartheta_0^2-2\right)}{81\vartheta_0^2\vartheta_1^2+4}+x\right)\right)}+\gamma_1\cos\left(\vartheta_0\left(x-\frac{2t\left(9\vartheta_1^2+2\right)}{81\vartheta_0^2\vartheta_1^2+4}\right)\right)}. \tag{6}$$

(II): As

$$\vartheta_1=\frac{\sqrt{2}}{3},\ \varepsilon_0=\frac{9\vartheta_0^2-2}{9\vartheta_0^2+2},\ \varpi_0=-\frac{4}{9\vartheta_0^2+2},\ \gamma_2=\frac{1}{8}(-9)\gamma_1^2\vartheta_0^2,$$

we obtain

$$\psi_2(x,t)=e^{\left(-\frac{1}{3}\sqrt{2}\left(\frac{t\left(9\vartheta_0^2-2\right)}{9\vartheta_0^2+2}+x\right)\right)}-\frac{1}{8}9\gamma_1^2\vartheta_0^2e^{\frac{1}{3}\sqrt{2}\left(\frac{t\left(9\vartheta_0^2-2\right)}{9\vartheta_0^2+2}+x\right)}+\gamma_1\cos\left(\vartheta_0\left(x-\frac{4t}{9\vartheta_0^2+2}\right)\right).$$

Consequently,

$$\phi_2(x,t)=\frac{3\left(-\frac{1}{3}\sqrt{2}e^{\left(-\frac{1}{3}\sqrt{2}\left(\frac{t\left(9\vartheta_0^2-2\right)}{9\vartheta_0^2+2}+x\right)\right)}-\frac{3\gamma_1^2\vartheta_0^2e^{\frac{1}{3}\sqrt{2}\left(\frac{t\left(9\vartheta_0^2-2\right)}{9\vartheta_0^2+2}+x\right)}}{4\sqrt{2}}-\gamma_1\vartheta_0\sin\left(\vartheta_0\left(x-\frac{4t}{9\vartheta_0^2+2}\right)\right)\right)}{e^{\left(-\frac{1}{3}\sqrt{2}\left(\frac{t\left(9\vartheta_0^2-2\right)}{9\vartheta_0^2+2}+x\right)\right)}-\frac{1}{8}9\gamma_1^2\vartheta_0^2e^{\frac{1}{3}\sqrt{2}\left(\frac{t\left(9\vartheta_0^2-2\right)}{9\vartheta_0^2+2}+x\right)}+\gamma_1\cos\left(\vartheta_0\left(x-\frac{4t}{9\vartheta_0^2+2}\right)\right)}. \tag{7}$$

3. Lump-Periodic Solution

Here, a class of lump-periodic wave solutions is provided.
Consider:

$$\psi(x,t)=\tau_2\cos(t\vartheta_4+x\vartheta_3)+\tau_1\cosh(t\vartheta_2+x\vartheta_1)+\tau_3\cosh(t\vartheta_6+x\vartheta_5). \tag{8}$$

Inserting Equation (8) into (3) yields the following set of equations:

$$\begin{aligned}
&-9\tau_1^2\vartheta_2\vartheta_1^3+2\tau_1^2\vartheta_1^2+2\tau_1^2\vartheta_2\vartheta_1-2\tau_2^2\vartheta_3^2-9\tau_2^2\vartheta_3^3\vartheta_4-2\tau_2^2\vartheta_3\vartheta_4+2\tau_3^2\vartheta_5^2-9\tau_3^2\vartheta_5^3\vartheta_6+2\tau_3^2\vartheta_5\vartheta_6=0,\\
&9\tau_1\tau_2\vartheta_3\vartheta_4\vartheta_1^2+2\tau_1\tau_2\vartheta_1^2+9\tau_1\tau_2\vartheta_2\vartheta_3^2\vartheta_1+2\tau_1\tau_2\vartheta_2\vartheta_1-2\tau_1\tau_2\vartheta_3^2-2\tau_1\tau_2\vartheta_3\vartheta_4=0,\\
&-9\tau_1\tau_2\vartheta_2\vartheta_3\vartheta_1^2+4\tau_1\tau_2\vartheta_3\vartheta_1+9\tau_1\tau_2\vartheta_3^2\vartheta_4\vartheta_1+2\tau_1\tau_2\vartheta_4\vartheta_1+2\tau_1\tau_2\vartheta_2\vartheta_3=0.\\
&-9\tau_1\tau_3\vartheta_5\vartheta_6\vartheta_1^2+2\tau_1\tau_3\vartheta_1^2-9\tau_1\tau_3\vartheta_2\vartheta_5^2\vartheta_1+2\tau_1\tau_3\vartheta_2\vartheta_1+2\tau_1\tau_3\vartheta_5^2+2\tau_1\tau_3\vartheta_5\vartheta_6=0.\\
&9\tau_1\tau_3\vartheta_2\vartheta_5\vartheta_1^2-4\tau_1\tau_3\vartheta_5\vartheta_1+9\tau_1\tau_3\vartheta_5^2\vartheta_6\vartheta_1-2\tau_1\tau_3\vartheta_6\vartheta_1-2\tau_1\tau_3\vartheta_2\vartheta_5=0,\\
&9\tau_2\tau_3\vartheta_5\vartheta_6\vartheta_3^2-2\tau_2\tau_3\vartheta_3^2+9\tau_2\tau_3\vartheta_4\vartheta_5^2\vartheta_3-2\tau_2\tau_3\vartheta_4\vartheta_3+2\tau_2\tau_3\vartheta_5^2+2\tau_2\tau_3\vartheta_5\vartheta_6=0,\\
&9\tau_2\tau_3\vartheta_4\vartheta_5\vartheta_3^2+4\tau_2\tau_3\vartheta_5\vartheta_3-9\tau_2\tau_3\vartheta_5^2\vartheta_6\vartheta_3+2\tau_2\tau_3\vartheta_6\vartheta_3+2\tau_2\tau_3\vartheta_4\vartheta_5=0.
\end{aligned}\tag{9}$$

Simplifying Equation (9), provides the following solutions:
(I): As

$$\vartheta_1=\frac{\sqrt{2}}{3},\ \vartheta_2=\frac{3\sqrt{2}\vartheta_5^2+\frac{2\sqrt{2}}{3}}{9\vartheta_5^2-2},\ \vartheta_6=\frac{4\vartheta_5}{9\vartheta_5^2-2},\ \tau_1=-\frac{3\tau_3\vartheta_5}{\sqrt{2}},\ \tau_2=0,$$

we obtain

$$\psi_1(x,t)=\tau_3\cosh\left(\frac{4t\vartheta_5}{9\vartheta_5^2-2}+x\vartheta_5\right)-\frac{3\tau_3\vartheta_5\cosh\left(\frac{t\left(3\sqrt{2}\vartheta_5^2+\frac{2\sqrt{2}}{3}\right)}{9\vartheta_5^2-2}+\frac{\sqrt{2}x}{3}\right)}{\sqrt{2}}.$$

Consequently,

$$\phi_1(x,t)=\frac{3\left(\tau_3\vartheta_5\sinh\left(\frac{4t\vartheta_5}{9\vartheta_5^2-2}+x\vartheta_5\right)-\tau_3\vartheta_5\sinh\left(\frac{t\left(3\sqrt{2}\vartheta_5^2+\frac{2\sqrt{2}}{3}\right)}{9\vartheta_5^2-2}+\frac{\sqrt{2}x}{3}\right)\right)}{\tau_3\cosh\left(\frac{4t\vartheta_5}{9\vartheta_5^2-2}+x\vartheta_5\right)-\frac{3\tau_3\vartheta_5\cosh\left(\frac{t\left(3\sqrt{2}\vartheta_5^2+\frac{2\sqrt{2}}{3}\right)}{9\vartheta_5^2-2}+\frac{\sqrt{2}x}{3}\right)}{\sqrt{2}}}.\tag{10}$$

(II): As

$$\vartheta_1=-\frac{\sqrt{2}}{3},\ \vartheta_2=\frac{2\sqrt{2}-9\sqrt{2}\vartheta_3^2}{3(9\vartheta_3^2+2)},\ \vartheta_4=-\frac{4\vartheta_3}{9\vartheta_3^2+2},\ \tau_1=-\frac{3\tau_2\vartheta_3}{\sqrt{2}}.\ \tau_3=0,$$

we obtain

$$\psi_2(x,t)=\tau_2\cos\left(x\vartheta_3-\frac{4t\vartheta_3}{9\vartheta_3^2+2}\right)-\frac{3\tau_2\vartheta_3\cosh\left(\frac{\sqrt{2}x}{3}-\frac{t\left(2\sqrt{2}-9\sqrt{2}\vartheta_3^2\right)}{3\left(9\vartheta_3^2+2\right)}\right)}{\sqrt{2}}.$$

Consequently,

$$\phi_2(x,t)=\frac{3\left(\tau_2\vartheta_3\left(-\sin\left(x\vartheta_3-\frac{4t\vartheta_3}{9\vartheta_3^2+2}\right)\right)-\tau_2\vartheta_3\sinh\left(\frac{\sqrt{2}x}{3}-\frac{t\left(2\sqrt{2}-9\sqrt{2}\vartheta_3^2\right)}{3\left(9\vartheta_3^2+2\right)}\right)\right)}{\tau_2\cos\left(x\vartheta_3-\frac{4t\vartheta_3}{9\vartheta_3^2+2}\right)-\frac{3\tau_2\vartheta_3\cosh\left(\frac{\sqrt{2}x}{3}-\frac{t\left(2\sqrt{2}-9\sqrt{2}\vartheta_3^2\right)}{3\left(9\vartheta_3^2+2\right)}\right)}{\sqrt{2}}}.\tag{11}$$

4. Two-Wave Solution

Here, a class of two-wave solutions are presented.
Consider:

$$\psi(x,t) = c_1 e^{(\delta_2 t+\delta_1 x)} + c_2 e^{(-(\delta_2 t+\delta_1 x))} + c_3 \sin(\delta_4 t+\delta_3 x) + c_4 \sinh(\delta_6 t+\delta_5 x). \tag{12}$$

Putting Equation (12) into (3) provides:

$$\begin{aligned}
&-36c_1c_2\delta_2\delta_1^3 + 8c_1c_2\delta_1^2 + 8c_1c_2\delta_2\delta_1 - 2c_3^2\delta_3^2 - 2c_4^2\delta_5^2 - 9c_3^2\delta_3^3\delta_4 - 2c_3^2\delta_3\delta_4 + 9c_4^2\delta_5^3\delta_6 - 2c_4^2\delta_5\delta_6 = 0,\\
&2c_1c_3\delta_1^2 + 9c_1c_3\delta_3\delta_4\delta_1^2 + 9c_1c_3\delta_2\delta_3^2\delta_1 + 2c_1c_3\delta_2\delta_1 - 2c_1c_3\delta_3^2 - 2c_1c_3\delta_3\delta_4 = 0,\\
&2c_2c_3\delta_1^2 + 9c_2c_3\delta_3\delta_4\delta_1^2 + 9c_2c_3\delta_2\delta_3^2\delta_1 + 2c_2c_3\delta_2\delta_1 - 2c_2c_3\delta_3^2 - 2c_2c_3\delta_3\delta_4 = 0,\\
&9c_1c_3\delta_2\delta_3\delta_1^2 - 4c_1c_3\delta_3\delta_1 - 9c_1c_3\delta_3^2\delta_4\delta_1 - 2c_1c_3\delta_4\delta_1 - 2c_1c_3\delta_2\delta_3 = 0,\\
&-9c_2c_3\delta_2\delta_3\delta_1^2 + 4c_2c_3\delta_3\delta_1 + 9c_2c_3\delta_3^2\delta_4\delta_1 + 2c_2c_3\delta_4\delta_1 + 2c_2c_3\delta_2\delta_3 = 0,\\
&2c_1c_4\delta_1^2 - 9c_1c_4\delta_5\delta_6\delta_1^2 - 9c_1c_4\delta_2\delta_5^2\delta_1 + 2c_1c_4\delta_2\delta_1 + 2c_1c_4\delta_5^2 + 2c_1c_4\delta_5\delta_6 = 0,\\
&2c_2c_4\delta_1^2 - 9c_2c_4\delta_5\delta_6\delta_1^2 - 9c_2c_4\delta_2\delta_5^2\delta_1 + 2c_2c_4\delta_2\delta_1 + 2c_2c_4\delta_5^2 + 2c_2c_4\delta_5\delta_6 = 0,\\
&-2c_3c_4\delta_3^2 + 9c_3c_4\delta_5\delta_6\delta_3^2 + 9c_3c_4\delta_4\delta_5^2\delta_3 - 2c_3c_4\delta_4\delta_3 + 2c_3c_4\delta_5^2 + 2c_3c_4\delta_5\delta_6 = 0,\\
&9c_1c_4\delta_2\delta_5\delta_1^2 - 4c_1c_4\delta_5\delta_1 + 9c_1c_4\delta_5^2\delta_6\delta_1 - 2c_1c_4\delta_6\delta_1 - 2c_1c_4\delta_2\delta_5 = 0,\\
&-9c_2c_4\delta_2\delta_5\delta_1^2 + 4c_2c_4\delta_5\delta_1 - 9c_2c_4\delta_5^2\delta_6\delta_1 + 2c_2c_4\delta_6\delta_1 + 2c_2c_4\delta_2\delta_5 = 0,\\
&-9c_3c_4\delta_4\delta_5\delta_3^2 - 4c_3c_4\delta_5\delta_3 + 9c_3c_4\delta_5^2\delta_6\delta_3 - 2c_3c_4\delta_6\delta_3 - 2c_3c_4\delta_4\delta_5 = 0.
\end{aligned} \tag{13}$$

Simplifying Equation (13) provides:
(I): As

$$\delta_5 = -\delta_1,\ \delta_6 = \frac{9\delta_2\delta_1^2 - 4\delta_1 - 2\delta_2}{9\delta_1^2 - 2},\ c_2 = -\frac{c_4^2}{4c_1},\ c_3 = 0,$$

we obtain

$$\psi_1(x,t) = -\frac{c_4^2 e^{\delta_1(-x)-\delta_2 t}}{4c_1} + c_1 e^{\delta_2 t+\delta_1 x} - c_4 \sinh\left(\delta_1 x - \frac{(9\delta_2\delta_1^2 - 4\delta_1 - 2\delta_2)t}{9\delta_1^2 - 2}\right).$$

Consequently,

$$\phi_1(x,t) = \frac{3\left(\frac{c_4^2\delta_1 e^{\delta_1(-x)-\delta_2 t}}{4c_1} + c_1\delta_1 e^{\delta_2 t+\delta_1 x} - c_4\delta_1 \cosh\left(\delta_1 x - \frac{(9\delta_2\delta_1^2-4\delta_1-2\delta_2)t}{9\delta_1^2-2}\right)\right)}{-\frac{c_4^2 e^{\delta_1(-x)-\delta_2 t}}{4c_1} + c_1 e^{\delta_2 t+\delta_1 x} - c_4 \sinh\left(\delta_1 x - \frac{(9\delta_2\delta_1^2-4\delta_1-2\delta_2)t}{9\delta_1^2-2}\right)}. \tag{14}$$

(II): When

$$\delta_2 = \frac{2\delta_1}{9\delta_1^2 - 2},\ \delta_3 = -\delta_1,\ \delta_4 = -\frac{2\delta_1}{9\delta_1^2 - 2},\ c_4 = 0,$$

we obtain

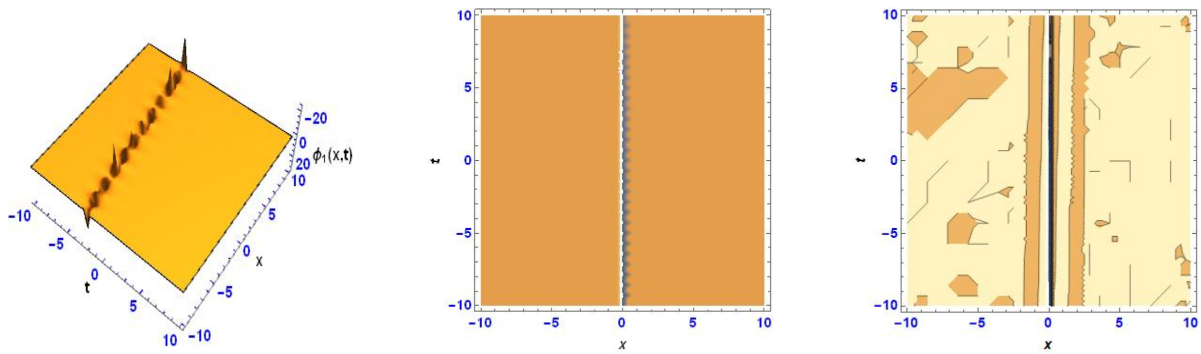
$$\psi_2(x,t) = c_1 e^{\frac{2\delta_1 t}{9\delta_1^2-2}+\delta_1 x} + c_2 e^{\delta_1(-x)-\frac{2\delta_1 t}{9\delta_1^2-2}} - c_3 \sin\left(\frac{2\delta_1 t}{9\delta_1^2 - 2} + \delta_1 x\right).$$

Consequently,

$$\phi_2(x,t) = \frac{3\left(c_1\delta_1 e^{\frac{2\delta_1 t}{9\delta_1^2-2}+\delta_1 x} - c_2\delta_1 e^{\delta_1(-x)-\frac{2\delta_1 t}{9\delta_1^2-2}} - c_3\delta_1 \cos\left(\frac{2\delta_1 t}{9\delta_1^2-2} + \delta_1 x\right)\right)}{c_1 e^{\frac{2\delta_1 t}{9\delta_1^2-2}+\delta_1 x} + c_2 e^{\delta_1(-x)-\frac{2\delta_1 t}{9\delta_1^2-2}} - c_3 \sin\left(\frac{2\delta_1 t}{9\delta_1^2-2} + \delta_1 x\right)}. \tag{15}$$

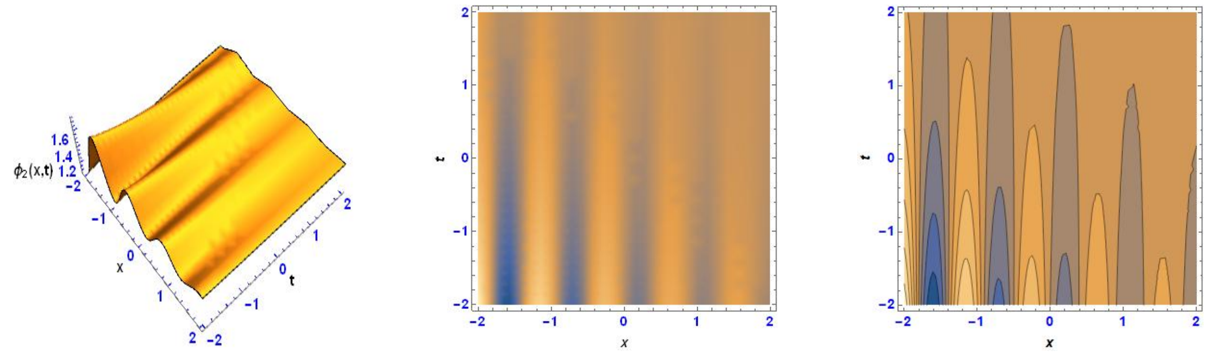
5. Physical Interpretation

This study investigates the lump interaction aspects to a shallow water wave-like equation using the Hirota bilinear approach. One of the top solutions for nonlinear evolution equations has been demonstrated to be the lump solutions. We successfully reported some breather wave, lump-periodic, and two-wave solutions. A breather is a nonlinear wave in physics that has energy concentrated in a focused, oscillating manner. The expectations drawn from the analogous linear system for infinitesimal amplitudes, which lean toward an even distribution of originally localized energy, are in conflict with this. The word "breather" comes from the fact that the majority of breathers oscillate (breathe) in time and are confined in location. Alternatively, oscillations that are localized in time and place are referred to as a break [40]. An expanding dynamic disturbance of one or more values is known as a wave in physics, mathematics, and related subjects. When a wave is periodic, its constituent parts repeatedly oscillate at a given frequency around an equilibrium value. A traveling wave is one where the entire waveform is moving in one direction; in contrast, a standing wave is one where two superimposed periodic waves are moving in opposite directions. A standing wave has nulls in the vibrational amplitude at some locations where the wave amplitude seems reduced or even zero. The standing wave field of two opposing waves known as a wave equation or a one-way wave equation for the dynamics of a single wave in a particular direction are two common ways to explain waves [41]. Under the choice of the good values of the parameters, three-dimensional, density, and contour figures are plotted. Figures 1 and 2 display the collision aspects between lump, exponential function, and singular periodic wave for the breather solutions (6) and (7). Figures 3 and 4 display the collision aspects between lump, exponential function, periodic, and singular periodic waves for the lump-periodic wave solutions (10) and (11). Figures 5 and 6 display the collision aspects between lump, exponential function, periodic, and singular periodic waves for the two-wave solutions (14) and (15).



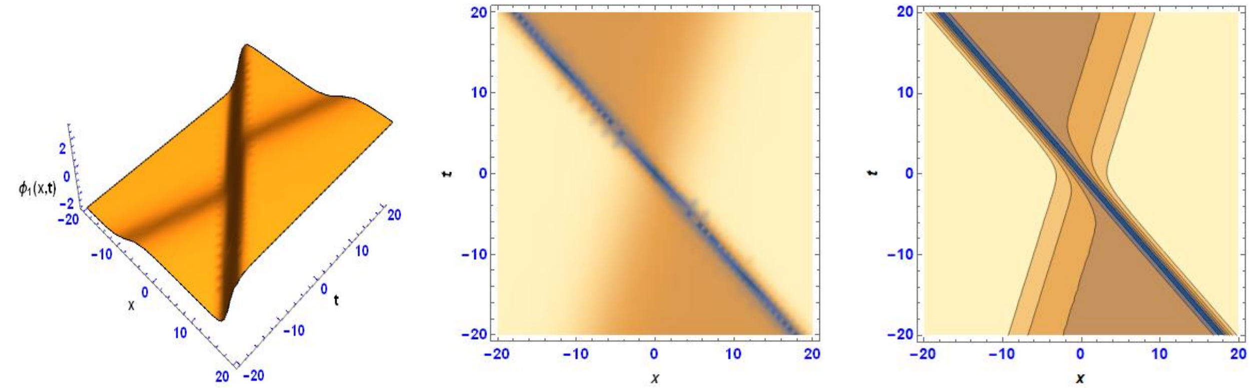
(**a**) (**b**) (**c**)

Figure 1. (**a**) Three-dimensional, (**b**) density, and (**c**) contour images of (6) under the values $\vartheta_0 = 6$, $\vartheta_1 = 0.75$, $\gamma_1 = -3.5$.



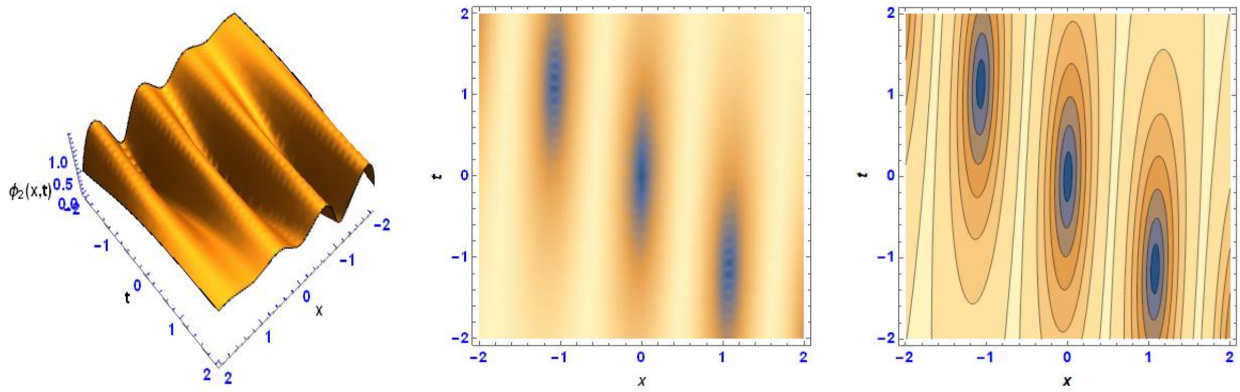
(**a**) (**b**) (**c**)

Figure 2. (**a**) Three-dimensional, (**b**) density, and (**c**) contour images of (7) under the values $\vartheta_0 = -7$, $\gamma_1 = -9.55$, $\vartheta_1 = 0.471$.



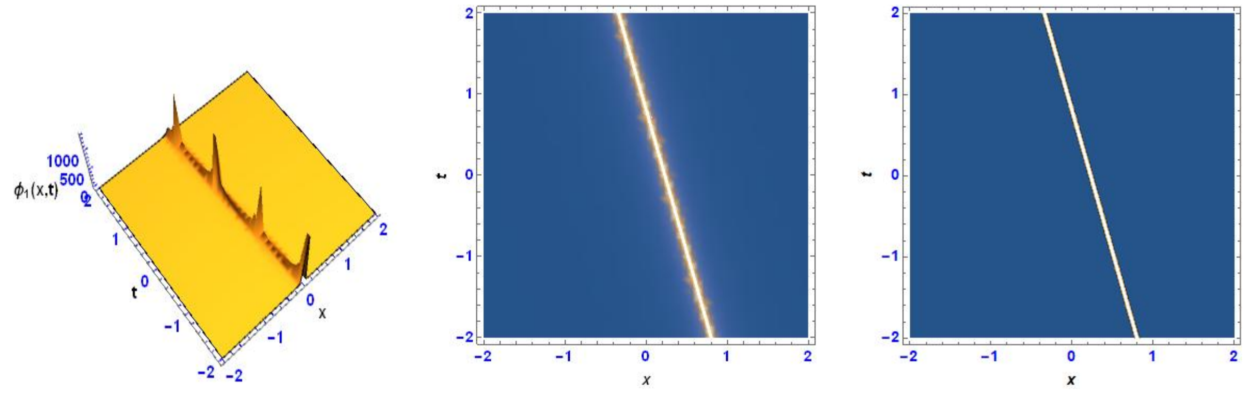
(**a**) (**b**) (**c**)

Figure 3. (**a**) Three-dimensional, (**b**) density, and (**c**) contour images of (10) under the values $\vartheta_5 = -1$, $\tau_3 = 18.5$.



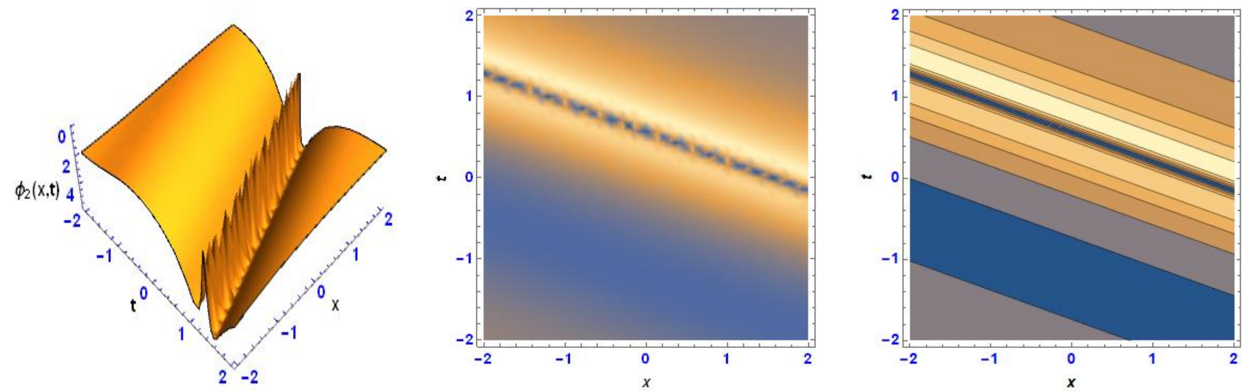
(**a**) (**b**) (**c**)

Figure 4. (**a**) Three-dimensional, (**b**) density, and (**c**) contour images of (11) under the values $\vartheta_3 = 3.14$, $\tau_2 = 18.28$.



(**a**) (**b**) (**c**)

Figure 5. (**a**) Three-dimensional, (**b**) density, and (**c**) contour images of (14) under the values $\delta_1 = 1,\ \delta_2 = -2,\ c_1 = 1.1, c_4 = -2$.



(**a**) (**b**) (**c**)

Figure 6. (**a**) Three-dimensional, (**b**) density, and (**c**) contour images of (15) under the values $\delta_1 = -0.5,\ c_2 = 5.65, c_1 = -1.1,\ c_3 = 17.2$.

6. Conclusions

The shallow water wave-like equation has been investigated. The well-known and efficient Hirota bilinear approach was employed to construct several novel solutions to the equation under consideration. Lump-periodic, two-wave, and breather wave solutions were produced as novel forms of results for the governing equation. In the meantime, the figures for these results have been graphed. The propagation properties of the generated solutions are illustrated in the plotted figures using the contour and three-dimensional plots. The results reveal that the appropriate physical quantities and attributes of nonlinear waves are related to the parameter values. The findings may be applied to a wide range of areas to assist readers in better comprehending difficult physical elements. The equation under consideration agreed with the attained solutions.

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