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Global Prescribed-Time Stabilization of High-Order Nonlinear Systems with Asymmetric Actuator Dead-Zone

Xin Guo [1], Hejun Yao [1] and Fangzheng Gao [1,2,*]

[1] School of Mathematics and Statistics, Anyang Normal University, Anyang 455000, China; 01321@aynu.edu.cn (X.G.); yaohj@aynu.edu.cn (H.Y.)
[2] School of Automation, Nanjing Institute of Technology, Nanjing 211167, China
* Correspondence: gaofz@njit.edu.cn

Abstract: This paper is concerned with the global prescribed-time stabilization problem for a class of uncertain high-order nonlinear systems (HONSs) with an asymmetric actuator dead-zone. Firstly, a new state-scaling transformation (SST) is developed for high-order nonlinear systems to change the original prescribed-time stabilization into the finite-time stabilization of the transformed one. The defects of the conventional one introduced in Song et al. (2017), which is unable to ensure the closed-loop stability behind a prespecified convergence time and a closed-loop system, which is only driven to the neighborhood of destination, is successfully overcome by introducing a switching mechanism in our proposed SST. Then, by using the adding a power integrator (API) technique, a state feedback controller is explicitly constructed to achieve the requirements of the closed-loop prescribed time convergence. Lastly, a liquid-level system is utilized to validate the theoretical results.

Keywords: high-order nonlinear systems (HONSs); asymmetric actuator dead-zone; state-scaling transformation (SST); prescribed-time stabilization

MSC: 93D15; 93D40

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Citation: Guo, X.; Yao, H.; Gao, F. Global Prescribed-Time Stabilization of High-Order Nonlinear Systems with Asymmetric Actuator Dead-Zone. *Mathematics* **2022**, *10*, 2147. https://doi.org/10.3390/math10122147

Academic Editors: Dimplekumar N. Chalishajar and Ivanka Stamova

Received: 21 April 2022
Accepted: 17 June 2022
Published: 20 June 2022

1. Introduction

During the past few years, high-order nonlinear systems (HONSs) have received a great deal of attention due to their significant value both in theory and practice [1,2]. However, the Jacobian linearization of HONSs is neither controllable nor feedback linearized, and feedback stabilization of such systems has been recognized as meaningful and challenging work. Fortunately, with the help of an added power integrator (API) [3] technique, many significant results have been produced regarding the asymptotic stabilizing/tracking control of HONSs; for examples, see [4–7] and references therein.

On the other hand, in recent years, the research on finite-time control has become a hot research area due to the finite-time stable system performing superior properties, such as both fast response and good disturbance rejection. Since the groundbreaking work in which the Lyapunov finite-time stability of nonlinear systems was introduced in [8], lots of important results have been established [9–17]. However, it should be pointed out that the above-mentioned results are subject to the issue that settling time functions is seriously dependent on initial system conditions, which causes their convergence time to experience unacceptably large increases as the magnitude of the initial conditions grows. To address this faultiness, the idea of fixed-time stability, which requires the upper boundedness of the involved settling time function be independent of initial system conditions, is put forward in [18]. So, under this new framework, many works have appeared to address fixed-time control of linear/nonlinear systems. Generally speaking, there are two kinds of existing design methods of fixed-time control: the bi-limit homogeneous-based method [18,19] and the Lyapunov-based method [20–28]. It should be mentioned that both methods suffer from some inherent shortcomings. In the former, the upper bound of the settling time

(UBST) function exists, but it is usually unknown, and in the latter, the UBST is bounded and adjustable, but it is difficult or even impossible to prespecify discretionarily in the light of requirements because the derived settling time function relies on design parameters, whose choice is laborious when satisfying a pregiven settling time.

In fact, a prespecifiable settling time is desirable in many practical applications, e.g., missile guidance [29]. As a result, prescribed/predefined-time control has become an active research topic [30–35]. In particular, research has explored scaling the states using a function that increases unboundedly, trending towards the terminal time, a state-scaling design method to address the prescribed-time stabilization (PTS) of Brunovsky systems in [30]. By employing a switching mechanism to address the computationally singular aspect of the proposed controller, this technique was further extended to study the PTS of planar linear systems in [35]. However, only the linear case is studied in [35], which implies the proposed technique has difficulty handling strongly nonlinear systems, e.g., high-order nonlinear systems (HONSs) which are intrinsically nonlinear systems. This describes many practical systems [1]. Moreover, another common drawback of the aforementioned results is that the effect of the actuator dead-zone is ignored. Nevertheless, owing to physical limitations of device, many actual systems are usually inevitably subjected to input dead-zone nonlinearity during operation. Such undesirable property may significantly degrade the system's performance or even system damage [36–42]. Therefore, an interesting question naturally arises: *For HONSs with actuator dead-zone, is it possible to devise a controller to achieve the PTS? If possible, how can one design it?* This paper focuses on addressing the problem of global PTS for a kind of HONSs with asymmetric actuator dead-zone and giving an affirmative answer to the above question. The significant contributions are underlined as follows.

(i) Different from finite-time convergence in [41] or fixed-time convergence in [42], fully taking into consideration of practical system requirements, both asymmetric actuator dead-zone and prescribed-time convergence are included to study the stabilization problem of HONSs.
(ii) To effectively overcome the computationally singular problem of the conventional scaling function-based design in [30], a novel switched scaling function with the switching rule involving both state and time is introduced.
(iii) Under some weaker restricted conditions on characterizing system nonlinearities, a systematic design method is proposed by delicately utilizing the API technique to ensure the achievement of the performance requirements.

Notations. In this paper, the used notations are standard. Specifically, for a vector $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, denote $\bar{x}_s = (x_1, \ldots, x_s)^T \in \mathbb{R}^s$, $s = 1, \ldots, n$, and the function $\lceil x \rceil^c$ is defined as $\lceil x \rceil^c = \mathrm{sign}(x)|x|^c$, with $\mathrm{sign}(\cdot)$ being the signum function.

2. Problem Formulation and Preliminaries

2.1. Problem Formulation

Consider a kind of HONSs given by

$$\begin{cases} \dot{z}_1 = d_1(t)\lceil z_2 \rceil^{p_1} + f_1(z_1), \\ \dot{z}_2 = d_2(t)\lceil z_3 \rceil^{p_2} + f_2(\bar{z}_2), \\ \quad\vdots \\ \dot{z}_{n-1} = d_{n-1}(t)\lceil z_n \rceil^{p_{n-1}} + f_{n-1}(\bar{z}_{n-1}), \\ \dot{z}_n = d_n(t)D(u) + f_n(\bar{z}_n), \end{cases} \tag{1}$$

where $\bar{z}_i = (z_1, \ldots, z_i)^T \in \mathbb{R}^i$ is the system state (vector), and $d_i \in \mathbb{R}$, $p_i \in \mathbb{R}^+$ (with $p_n = 1$), $i = 1, \ldots, n$ are the control coefficients and the power orders of the system, respectively. $f_i : \mathbb{R}^i \to \mathbb{R}$, $i = 1, \ldots, n$ are uncertain continuous functions satisfying $f_i(0) = 0$. $u \in \mathbb{R}$ is the real control input, and $D \in \mathbb{R}$ denotes the asymmetric actuator dead-zone which can be modeled as

$$D(u) = \begin{cases} m_r(t)(u - b_r(t)), & u \geq b_r(t), \\ 0, & -b_l(t) < u < b_r(t), \\ m_l(t)(u + b_l(t)), & u \leq -b_l(t), \end{cases} \tag{2}$$

where $m_r(t)$, $m_l(t)$ and $b_r(t)$, $b_l(t)$ are the corresponding the slopes and the breakpoints of the dead-zone characteristic, respectively.

The aim of this paper is to find a state feedback control mechanism which stabilizes system (1) within prescribed finite time under the following wild assumptions.

Assumption 1. *For $i = 1, \ldots, n$, there are smooth functions $\varphi_i \geq 0$ and a constant $\tau > 0$ such that*

$$|f_i(\bar{z}_i)| \leq \varphi_i(\bar{z}_i) \sum_{j=1}^{i} |z_j|^{\frac{\lambda_i - \tau}{\lambda_j}}, \tag{3}$$

where λ_is are recursively defined by

$$\lambda_{n+1} = \tau, \;\; p_i \lambda_{i+1} = \lambda_i - \tau \geq 0, \;\; i = 1, \ldots, n-1. \tag{4}$$

Assumption 2. *There are positive constants $\underline{d}_i$ and $\overline{d}_i$, $i = 1, \ldots, n$ such that $\underline{d}_i \leq d_i(t) \leq \overline{d}_i$.*

Assumption 3. *There are positive constants $\underline{m}_r$, $\underline{m}_l$, $\overline{b}_r$ and $\overline{b}_l$, such that*

$$\underline{m}_r \leq m_r(t), \;\; \underline{m}_l \leq m_l(t), \;\; b_r(t) \leq \overline{b}_r, \;\; b_l(t) \leq \overline{b}_l. \tag{5}$$

Remark 1. *Assumption 1 can be regarded as a new growth-like condition where λ_is are different from the traditional ones [4–7] where they are recursively given by $\lambda_1 = 1$, $p_i \lambda_{i+1} = \lambda_i - \tau \geq 0$, $i = 1, \ldots, n$. In addition, it should be mentioned that it is reasonable in engineering practice to impose the control coefficients and the unknown dead-zone parameters (i.e., the slopes and breakpoints) bounded in Assumptions 2 and 3. Similar requirements can be found in the existing literature [40,41].*

2.2. Preliminaries

Consider the nonlinear system

$$\dot{z} = f(t, z), \;\; z(0) = z_0 \in \mathbb{R}^n, \tag{6}$$

where $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous with respect to x and satisfies $f(t, 0) = 0$.

Definition 1 ([8])**.** *The origin of system (6) is said to be globally finite-time stable if it is globally asymptotically stable and for any $z_0 \in \mathbb{R}^n$, a settling time function $T : \mathbb{R}^n \setminus \{0\} \to (0, +\infty)$ exists to ensure all solutions $z(t, z_0)$ of (6) satisfying $z(t, z_0) = 0$, $\forall t \geq T(z_0)$.*

Definition 2 ([20])**.** *The origin of system (6) is said to be globally fixed-time stable if it is globally finite-time stable and its settling-time function $T(z_0)$ is bounded by a positive constant; that is to say, $\exists\, T_{\max} > 0$, s.t. $T(z_0) \leq T_{\max}$, $\forall z_0 \in \mathbb{R}^n$.*

Definition 3 ([32])**.** *The origin of system (6) is said to be globally prescribed-time stable if it is globally fixed-time stable and for any prescribed time $T_p > 0$, a tunable designing parameter $\theta \in \mathbb{R}$ exists to ensure $T(z_0) \leq T_p$, $\forall z_0 \in \mathbb{R}$.*

Lemma 1 ([8])**.** *For system (6), if there is a C^1 and positive definite function $V(z)$ and some real numbers $c > 0$ and $0 < \alpha < 1$ such that*

$$\dot{V}(z) \leq -cV^{\alpha}(z), \;\; \forall z \in \mathbb{R}^n.$$

Then, the origin of system (6) *is finite-time stable with*

$$T(z_0) \leq \frac{V^{1-\alpha}(0)}{c(1-\alpha)}, \ \forall z \in \mathbb{R}^n$$

Lemma 2 ([3])**.** *For any $u \in \mathbb{R}$, $v \in \mathbb{R}$, and a constant $m \geq 1$, one has (i)$|u+v|^m \leq 2^{m-1}|u^m + v^m|$; (ii)$(|u|+|v|)^{1/m} \leq |u|^{1/m} + |v|^{1/m} \leq 2^{(m-1)/m}(|u|+|v|)^{1/m}$.*

Lemma 3 ([3])**.** *For any constants $c, d > 0$ and real-valued function $\delta(u,v) > 0$, one has $|u|^c|v|^d \leq \frac{c}{c+d}\delta(u,v)|u|^{c+d} + \frac{d}{c+d}\delta^{-c/d}(u,v)|v|^{c+d}$.*

Lemma 4 ([4,43])**.** *For any $u, v \in \mathbb{R}$ and constants $0 < m \leq 1$ and $b > 0$, then , one has $|\lceil u \rceil^{bm} - \lceil v \rceil^{bm}| \leq 2^{1-m}|\lceil u \rceil^b - \lceil v \rceil^b|^m$.*

3. Main Results

This section proposes a constructive switching design mechanism of a state feedback controller to stabilize system (1) within any prescribed finite time $T_p > 0$.

3.1. Scaling Function

For the aim of this paper, the following switched scaling function is introduced.

$$F = \begin{cases} \Gamma, & z \in \{\mathbb{R}^n - \Xi\} \ \& \ t < T_s, \\ 1, & \text{otherwise}, \end{cases} \tag{7}$$

where Ξ is a small closed neighborhood of origin and

$$\Gamma = \frac{T_s}{T_s - t}, \tag{8}$$

with the positive design parameter T_s satisfying $0 < T_s < T_p$.

Remark 2. *It is obvious that Γ monotonically increases on $[0, T_s)$ with $\Gamma(0) = 1$ and $\Gamma(T_s) = +\infty$. To address the incapability of ensuring the closed-loop viability and stability behind T_s, a new switched scaling function* (7) *is introduced in this paper. In comparison with the one used in [30], its novelty is that the switching rule is dependent on both state and time, i.e., it uses a small closed neighborhood of origin Ξ to replace the origin, which renders the system trajectory $z(t)$ to the switching set Ξ at some moment before T_s can effectively overcome the computationally singular problem ($\infty \times 0$ type) of the resulting controller as $t \to T_s$.*

3.2. Controller Design

Based on the above switched scaling function, the following novel coordinate transformation is given.

$$\zeta_i = F^{(1+c)\lambda_i} z_i, \quad i = 1, \ldots, n, \quad D(v) = F^{(1+c)\lambda_{n+1}} D(u), \tag{9}$$

where $c \geq (1/\tau) - 1$ is a design constant.

With the aid of (9), system (1) is redescribed as

$$\begin{cases} \dot{\zeta}_1 = F^{(1+c)\tau}\big(d_1\lceil\zeta_2\rceil^{p_1} + F_1(\zeta_1)\big), \\ \dot{\zeta}_2 = F^{(1+c)\tau}\big(d_2\lceil\zeta_3\rceil^{p_2} + F_2(\bar{\zeta}_2)\big), \\ \qquad \vdots \\ \dot{\zeta}_{n-1} = F^{(1+c)\tau}\big(d_{n-1}\lceil\zeta_n\rceil^{p_{n-1}} + F_{n-1}(\bar{\zeta}_{n-1})\big), \\ \dot{\zeta}_n = F^{(1+c)\tau}\big(d_n D(v) + F_n(\bar{\zeta}_n)\big), \end{cases} \tag{10}$$

where

$$F_i(\bar{\zeta}_i) = \zeta_i \frac{(1+c)\lambda_i \dot{F}}{F^{1+(1+c)\tau}} + F^{(1+c)(\lambda_i - \tau)} f_i(\bar{z}_i), \quad i = 1, \ldots, n, \tag{11}$$

for which there are the smooth functions $\bar{\varphi}_i(\bar{\zeta}_i) \geq 0$ such that

$$|F_i(\bar{\zeta}_i)| \leq \bar{\varphi}_i(\bar{\zeta}_i) \sum_{j=1}^{i} |\zeta_j|^{\frac{\lambda_i - \tau}{\lambda_j}}, \ \ i = 1, \ldots, n. \tag{12}$$

Next, a state feedback stabilizing controller of system (10) is designed by employing the API technique.

Step 1. Let $\rho \geq \max_{1 \leq i \leq n}\{\lambda_i\}$ being a constant and take the Lyapunov function V_1 as

$$V_1 = W_1 = \int_0^{\zeta_1} \left\lceil \lceil s \rceil^{\frac{\rho}{\lambda_1}} - 0 \right\rceil^{\frac{2\rho - \lambda_1}{\rho}} ds. \tag{13}$$

Applying Assumptions 1 and 2 and (12) produces

$$\begin{aligned} \dot{V}_1 &= F^{(1+c)\tau} \lceil \pi_1 \rceil^{\frac{2\rho - \lambda_1}{\rho}} (d_1 \lceil \zeta_2 \rceil^{p_1} + F_1) \\ &\leq F^{(1+c)\tau} \left(\lceil \pi_1 \rceil^{\frac{2\rho - \lambda_1}{\rho}} d_1 (\zeta_2 \rceil^{p_1} - \lceil \zeta_2^* \rceil^{p_1}) + d_1 \lceil \pi_1 \rceil^{\frac{2\rho - \lambda_1}{\rho}} \lceil \zeta_2^* \rceil^{p_1} + |\pi_1|^{\frac{2\rho - \lambda_1}{\rho}} \bar{\varphi}_1 \right), \end{aligned} \tag{14}$$

where $\pi_1 = \lceil \zeta_1 \rceil^{\frac{\rho}{\lambda_1}}$ and ζ_2^* is the virtual controller of ζ_2.

Take the virtual controller

$$\zeta_2^* = -\lceil \pi_1 \rceil^{\frac{\lambda_2}{\rho}} \beta_1^{\frac{\lambda_2}{\rho}}(\zeta_1), \tag{15}$$

where

$$\beta_1(\zeta_1) \geq \left(\frac{n + \bar{\varphi}_1}{\underline{d}_1} \right)^{\frac{\rho}{p_1 \lambda_2}}, \tag{16}$$

is a smooth function. Then, by substituting (15) and (16) into (14), we have

$$\dot{V}_1 \leq -nF^{(1+c)\tau} |\pi_1|^{\frac{2\rho - \tau}{\rho}} + F^{(1+c)\tau} d_1 \lceil \pi_1 \rceil^{\frac{2\rho - \lambda_1}{\rho}} (\lceil \zeta_2 \rceil^{p_1} - \lceil \zeta_2^* \rceil^{p_1}). \tag{17}$$

Step 2. Define $\pi_2 = \lceil \zeta_2 \rceil^{\frac{\rho}{\lambda_2}} - \lceil \zeta_2^* \rceil^{\frac{\rho}{\lambda_2}}$ and take the Lyapunov function $V_2 = V_1 + W_2$ with

$$W_2 = \int_{\zeta_2^*}^{\zeta_2} \left\lceil \lceil s \rceil^{\frac{\rho}{\lambda_2}} - \lceil \zeta_2^* \rceil^{\frac{\rho}{\lambda_2}} \right\rceil^{\frac{2\rho - \lambda_2}{\rho}} ds. \tag{18}$$

From

$$\left\{ \begin{aligned} \frac{\partial W_2}{\partial \zeta_2} &= \lceil \pi_2 \rceil^{\frac{2\rho - \lambda_2}{\rho}}, \\ \frac{\partial W_2}{\partial \zeta_1} &= -\frac{2\rho - \lambda_2}{\rho} \frac{\partial \left(\lceil \zeta_2^* \rceil^{\frac{\rho}{\lambda_2}} \right)}{\partial \zeta_1} \times \int_{\zeta_2^*}^{\zeta_2} \left| \lceil s \rceil^{\frac{\rho}{\lambda_2}} - \lceil \zeta_2^* \rceil^{\frac{\rho}{\lambda_2}} \right|^{\frac{\rho - \lambda_2}{\rho}} ds, \end{aligned} \right. \tag{19}$$

a direct calculation gives

$$
\begin{aligned}
\dot{V}_2 \leq\; & -nF^{(1+c)\tau}|\pi_1|^{\frac{2\rho-\tau}{\rho}} + F^{(1+c)\tau}d_1\lceil\pi_1\rfloor^{\frac{2\rho-\lambda_1}{\rho}}(\lceil\zeta_2\rfloor^{p_1} - \lceil\zeta_2^*\rfloor^{p_1}) \\
& + \frac{\partial W_2}{\partial\zeta_1}F^{(1+c)\tau}(d_1\lceil\zeta_2\rfloor^{p_1} + F_1) + \frac{\partial W_2}{\partial\zeta_2}F^{(1+c)\tau}(d_2\lceil\zeta_3\rfloor^{p_2} + F_2) \\
\leq\; & -nF^{(1+c)\tau}|\pi_1|^{\frac{2\rho-\tau}{\rho}} + F^{(1+c)\tau}d_1\lceil\pi_1\rfloor^{\frac{2\rho-\lambda_1}{\rho}}(\lceil\zeta_2\rfloor^{p_1} - \lceil\zeta_2^*\rfloor^{p_1}) \\
& + F^{(1+c)\tau}\Bigg(\frac{\partial W_2}{\partial\zeta_1}(d_1\lceil\zeta_2\rfloor^{p_1} + F_1) + d_2\lceil\pi_2\rfloor^{\frac{2\rho-\lambda_2}{\rho}}(\lceil\zeta_3\rfloor^{p_2} - \lceil\zeta_3^*\rfloor^{p_2}) \\
& + d_2\lceil\pi_2\rfloor^{\frac{2\rho-\lambda_2}{\rho}}\lceil\zeta_3^*\rfloor^{p_2} + \lceil\pi_2\rfloor^{\frac{2\rho-\lambda_2}{\rho}}F_2\Bigg),
\end{aligned} \tag{20}
$$

where ζ_3^* is the virtual controller to be designed later. To continue, the following upper bound estimates for some terms of (20) are needed.

Firstly, from the definitions of π_j and ζ_j^* $(j = 1,2)$ and Lemma 4, one has

$$
\begin{aligned}
|\lceil\zeta_2\rfloor^{p_1} - \lceil\zeta_2^*\rfloor^{p_1}| &= \left|\left(\lceil\zeta_2\rfloor^{\frac{\rho}{\lambda_2}}\right)^{\frac{\lambda_2 p_1}{\rho}} - \left(\lceil\zeta_2^*\rfloor^{\frac{\rho}{\lambda_2}}\right)^{\frac{\lambda_2 p_1}{\rho}}\right| \\
&\leq 2^{1-\frac{\lambda_2 p_1}{\rho}}\left|\lceil\zeta_2\rfloor^{\frac{\rho}{\lambda_2}} - \lceil\zeta_2^*\rfloor^{\frac{\rho}{\lambda_2}}\right|^{\frac{\lambda_2 p_1}{\rho}} \\
&= 2^{1-\frac{\lambda_2 p_1}{\rho}}|\pi_2|^{\frac{\lambda_2 p_1}{\rho}}.
\end{aligned} \tag{21}
$$

Thus, from (21), Assumption 2 and Lemma 3, it is obtained that

$$
\begin{aligned}
d_1\lceil\pi_1\rfloor^{\frac{2\rho-\lambda_1}{\rho}}(\lceil\zeta_2\rfloor^{p_1} - \lceil\zeta_2^*\rfloor^{p_1}) &\leq 2^{1-\frac{\lambda_2 p_1}{\rho}}\bar{d}_1|\pi_1|^{\frac{2\rho-\lambda_1}{\rho}}|\pi_2|^{\frac{\lambda_2 p_1}{\rho}} \\
&\leq \frac{1}{3}|\pi_1|^{\frac{2\rho-\tau}{\rho}} + |\pi_2|^{\frac{2\rho-\tau}{\rho}}\varrho_{21},
\end{aligned} \tag{22}
$$

where $\varrho_{21} \geq 0$ is a smooth function.

Secondly, from (12) and Lemma 2, one gets

$$
\begin{aligned}
|F_2| &\leq \bar{\varphi}_2\left(|\zeta_1|^{\frac{\lambda_2-\tau}{\lambda_1}} + |\zeta_2|^{\frac{\lambda_2-\tau}{\lambda_2}}\right) \\
&\leq \bar{\varphi}_2\left(|\pi_1|^{\frac{\lambda_2-\tau}{\rho}} + |\pi_2|^{\frac{\lambda_2-\tau}{\rho}} + \beta_1^{\frac{\lambda_2-\tau}{\rho}}|\pi_1|^{\frac{\lambda_2-\tau}{\rho}}\right) \\
&\leq \tilde{\varphi}_2\left(|\pi_1|^{\frac{\lambda_2-\tau}{\rho}} + |\pi_2|^{\frac{\lambda_2-\tau}{\rho}}\right),
\end{aligned} \tag{23}
$$

where $\tilde{\varphi}_2 \geq \left(1 + \beta_1^{\frac{\lambda_2-\tau}{\rho}}\right)\bar{\varphi}_2$ is a smooth function.

Using (23) and Lemma 3 yields

$$
\begin{aligned}
\lceil\pi_2\rfloor^{\frac{2\rho-\lambda_2}{\rho}}F_2 &\leq \lceil\pi_2\rfloor^{\frac{2\rho-\lambda_2}{\rho}}\tilde{\varphi}_2\left(|\pi_1|^{\frac{\lambda_2-\tau}{\rho}} + |\pi_2|^{\frac{\lambda_2-\tau}{\rho}}\right) \\
&\leq \frac{1}{3}|\pi_1|^{\frac{2\rho-\tau}{\rho}} + |\pi_2|^{\frac{2\rho-\tau}{\rho}}\varrho_{22},
\end{aligned} \tag{24}
$$

where $\varrho_{22} \geq 0$ is a smooth function.

Finally, note that

$$
\begin{aligned}
\frac{2\rho-\lambda_2}{\rho}\int_{\zeta_2^*}^{\zeta_2}\left|\lceil s\rfloor^{\frac{\rho}{\lambda_2}} - \lceil\zeta_2^*\rfloor^{\frac{\rho}{\lambda_2}}\right|^{\frac{\rho-\lambda_2}{\rho}}ds &\leq \frac{2\rho-\lambda_2}{\rho}|\pi_2|^{\frac{\rho-\lambda_2}{\rho}}|\zeta_2 - \zeta_2^*| \\
&\leq \frac{2\rho-\lambda_2}{\rho}2^{1-\frac{\lambda_2}{\rho}}|\pi_2|,
\end{aligned} \tag{25}
$$

and

$$\begin{aligned}\left|\frac{\partial\left(\lceil\zeta_2^*\rceil^{\frac{\rho}{\lambda_2}}\right)}{\partial\zeta_1}\right| &= \left|\frac{\partial(\beta_1\lceil\pi_1\rfloor)}{\partial\zeta_1}\right| \\ &\leq \left|\frac{\partial\beta_1}{\partial\zeta_1}\right||\pi_1| + \frac{\rho}{\lambda_1}\beta_1|\pi_1|^{\frac{\rho-\lambda_1}{\rho}} \\ &\leq |\pi_1|^{\frac{\rho-\lambda_1}{\rho}}\gamma_2,\end{aligned} \tag{26}$$

where $\gamma_2 \geq 0$ is a smooth function.

Therefore, in the light of (23), (25), (26) and Lemma 3, one concludes that

$$\begin{aligned}&\frac{\partial W_2}{\partial\zeta_1}(d_1\lceil\zeta_2\rfloor^{p_1} + F_1) \\ \leq\ & \frac{2\rho-\lambda_2}{\rho}\int_{\zeta_2^*}^{\zeta_2}\left|\lceil s\rfloor^{\frac{\rho}{\lambda_2}} - \lceil\zeta_2^*\rfloor^{\frac{\rho}{\lambda_2}}\right|^{\frac{\rho-\lambda_2}{\rho}} ds \times \left|\frac{\partial\left(\lceil\zeta_2^*\rfloor^{\frac{\rho}{\lambda_2}}\right)}{\partial\zeta_1}\right|(d_1\lceil\zeta_2\rfloor^{p_1} + F_1) \\ \leq\ & \frac{1}{3}|\pi_1|^{\frac{2\rho-\tau}{\rho}} + |\pi_2|^{\frac{2\rho-\tau}{\rho}}\varrho_{23},\end{aligned} \tag{27}$$

where $\varrho_{23} \geq 0$ is a smooth function.

Substituting (22), (24) and (27) into (21) yields

$$\begin{aligned}\dot{V}_2 \leq\ & -(n-1)F^{(1+c)\tau}|\pi_1|^{\frac{2\rho-\tau}{\rho}} + F^{(1+c)\tau}d_2\lceil\pi_2\rfloor^{\frac{2\rho-r_2}{\rho}}(\lceil\zeta_3\rfloor^{p_2} - \lceil\zeta_3^*\rfloor^{p_2}) \\ &+F^{(1+c)\tau}\left(d_2\lceil\pi_2\rfloor^{\frac{2\rho-r_2}{\rho}}\lceil\zeta_3^*\rfloor^{p_2} + (\varrho_{21}+\varrho_{22}+\varrho_{23})|\pi_2|^{\frac{2\rho-\tau}{\rho}}\right).\end{aligned} \tag{28}$$

Then, one can design the virtual controller

$$\zeta_3^* = -\lceil\pi_2\rfloor^{\frac{\lambda_3}{\rho}}\beta_2^{\frac{\lambda_3}{\rho}}(\bar{\zeta}_2), \tag{29}$$

where the smooth function β_2 satisfies

$$\beta_2(\bar{\zeta}_2) \geq \left(\frac{n-1+\varrho_{21}+\varrho_{22}+\varrho_{23}}{\underline{d}_2}\right)^{\frac{\rho}{q_2\lambda_3}}, \tag{30}$$

such that

$$\dot{V}_2 \leq -(n-1)F^{(1+c)\tau}\left(|\pi_1|^{\frac{2\rho-\tau}{\rho}} + |\pi_2|^{\frac{2\rho-\tau}{\rho}}\right) + F^{(1+c)\tau}d_2\lceil\pi_2\rfloor^{\frac{2\rho-r_2}{\rho}}(\lceil\zeta_3\rfloor^{p_2} - \lceil\zeta_3^*\rfloor^{p_2}). \tag{31}$$

Following the same arguments of Step 2 for details, for Step i ($i = 2,\ldots,n$), we can find a C^1 and positive definite Lyapunov function $V_i = \sum_{j=1}^{i} W_j$ with

$$W_i = \int_{\zeta_i^*}^{\zeta_i}\left\lceil\lceil s\rfloor^{\frac{\rho}{\lambda_i}} - \lceil\zeta_i^*\rfloor^{\frac{\rho}{\lambda_i}}\right\rfloor^{\frac{2\rho-\lambda_i}{\rho}} ds, \tag{32}$$

and a row of continuous virtual controllers $\zeta_{j+1}^* = -\lceil\pi_j\rfloor^{\frac{\lambda_{j+1}}{\rho}}\beta_j^{\frac{\lambda_{j+1}}{\rho}}(\bar{\zeta}_j)$, $j = 1,\ldots,i$, such that

$$\dot{V}_i \leq -(n-i+1)F^{(1+c)\tau}\sum_{j=1}^{i}|\pi_j|^{\frac{2\rho-\tau}{\rho}} + F^{(1+c)\tau}d_i\lceil\pi_i\rfloor^{\frac{2\rho-r_i}{\rho}}(\lceil\zeta_{i+1}\rfloor^{p_i} - \lceil\zeta_{i+1}^*\rfloor^{p_i}). \tag{33}$$

As a result, the previous inductive step indicates that there is a desired dead-zone output

$$\zeta_{n+1}^* = -\lceil \pi_n \rceil^{\frac{\lambda_{n+1}}{\rho}} \beta_n^{\frac{\lambda_{n+1}}{\rho}}(\bar{\zeta}_n), \tag{34}$$

such that

$$\begin{aligned}\dot{V}_n &\le -F^{(1+c)\tau}\sum_{j=1}^{n}|\pi_j|^{\frac{2\rho-\tau}{\rho}} + F^{(1+c)\tau}\lceil \pi_n \rceil^{\frac{2\rho-\lambda_n}{\rho}}\left(D(v)-\zeta_{n+1}^*\right) \\ &\le -F^{(1+c)\tau}\sum_{j=1}^{n}|\pi_j|^{\frac{2\rho-\tau}{\rho}} + F^{2(1+c)\tau}\lceil \pi_n \rceil^{\frac{2\rho-\lambda_n}{\rho}}\Big(D(u)-F^{-(1+c)\tau}\zeta_{n+1}^*\Big).\end{aligned} \tag{35}$$

where

$$V_n = \sum_{j=1}^{n} W_j = \sum_{j=1}^{n}\int_{\zeta_j^*}^{\zeta_j}\left\lceil \lceil s \rceil^{\frac{\rho}{\lambda_j}} - \lceil \zeta_j^* \rceil^{\frac{\rho}{\lambda_j}}\right\rceil^{\frac{2\rho-\lambda_j}{\rho}} ds. \tag{36}$$

Therefore, Assumption 3 instructs that the state feedback control u designed as

$$u = \begin{cases} \left(\dfrac{F^{-(1+c)\tau}\zeta_{n+1}^*}{\underline{m}_r} + \bar{b}_r\right), & \zeta_{n+1}^* > 0, \\ 0, & \zeta_{n+1}^* = 0, \\ \left(\dfrac{F^{-(1+c)\tau}\zeta_{n+1}^*}{\underline{m}_l} - \bar{b}_l\right), & \zeta_{n+1}^* < 0, \end{cases} \tag{37}$$

renders

$$\begin{aligned}&\Big(D(u) - F^{-(1+c)\tau}\zeta_{n+1}^*\Big) \\ &= \begin{cases} m_r\left(\dfrac{F^{-(1+c)\tau}\zeta_{n+1}^*}{\underline{m}_r} + \bar{b}_r - b_r\right) - F^{-(1+c)\tau}\zeta_{n+1}^* > 0, & \zeta_{n+1}^* > 0, \\ 0, & \zeta_{n+1}^* = 0, \\ m_l\left(\dfrac{F^{-(1+c)\tau}\zeta_{n+1}^*}{\underline{m}_l} - \bar{b}_l + b_l\right) - F^{-(1+c)\tau}\zeta_{n+1}^* < 0, & \zeta_{n+1}^* < 0. \end{cases}\end{aligned} \tag{38}$$

By noting $-\lceil \pi_n \rceil^{\frac{2\rho-\lambda_n}{\rho}}\zeta_{n+1}^* \ge 0$, one gets

$$\dot{V}_n \le -F^{(1+c)\tau}\sum_{j=1}^{n}|\pi_j|^{\frac{2\rho-\tau}{\rho}} \le -\sum_{j=1}^{n}|\pi_j|^{\frac{2\rho-\tau}{\rho}}. \tag{39}$$

Consequently, the following result is obtained.

Theorem 1. *For system* (1) *with Assumptions 1–3, the state feedback controller* (37) *drives the states of the CLS to zero within prescribed finite time* $T_p > 0$.

Proof. Since V_n is positive definite and proper, therefore from (39) and Lemma 4.3 in [44], there are class $\mathcal{K}_\infty$ functions η_1, η_2 and η_3, such that

$$\eta_1(|\zeta|) \le V_n(\zeta) \le \eta_2(|\zeta|), \tag{40}$$

$$\dot{V}_n \le -\eta_3(|\zeta|), \tag{41}$$

which indicates that $\zeta(t)$ is asymptotically convergent and bounded.

On the other hand, the SST (9) gives

$$z_i(t) = F^{-(1+c)\lambda_i}\zeta_i(t) = \left(\frac{T_c - t}{T_c}\right)^{(1+c)\lambda_i}\zeta_i(t), \quad i = 1, \ldots, n. \tag{42}$$

Consequently, it is further obtained that

$$\lim_{t \to T_s} z_i(t) = \lim_{t \to T_s}\left(\frac{T_s - t}{T_s}\right)^{(1+c)\lambda_i}\zeta_i(t) = 0, \quad i = 1, \ldots, n. \tag{43}$$

Therefore, when $F = \Gamma$, (39) indicates that the domain Ξ is prescribed-time attractive and the convergence time satisfies $T_a < T_s$.

When $F = 1$, let $C = \max_{\zeta \in \Xi} V_n(\zeta)$. Then, (39) indicates that the origin of the CLS is locally finite-time stable in the attraction domain Ξ and the convergence time satisfies

$$T_l \leq \frac{2V_n^{\frac{\tau}{2}}(0)}{c\tau} \leq \frac{2C^{\frac{\tau}{2}}}{c\tau}. \tag{44}$$

Therefore, by selecting $c \geq (2C^{\frac{\tau}{2}})/(\tau T_1 - \tau T_s)$, one has $T_l \leq T_p - T_s$.

From the existence and continuation of the solutions properties, we know that the whole system is globally Lyapunov stable. As a result, it is concluded that the origin of the CLS is globally prescribed-time stable within $T_a + T_l < T_p$. Thus, the proof is completed. □

Remark 3. *The frame structure of control system* (1) *is as:*

Step 1: *For the considered system* (1) *and any given $T > 0$, take $T_1 = \vartheta T$ with $\vartheta \in (0, 1)$.*

Step 2: *The designed controller* (37) *with $F = \Gamma$ drives the system state $\zeta(t)$ to a (small) prespecified attraction domain Ξ at some $T_a < T_1$.*

Step 3: *Appropriately choosing parameter c, which, together with designed controller* (37) *with $F = 1$, ensures that once the system state $\zeta(t)$ enters the attraction domain Ξ, it converges to and stays at the origin $\zeta = 0$ for all $t \geq T_a + T_l < T_p$.*

4. An Application Example

To verify the applicability of the proposed control scheme, we consider a liquid-level system exhibited in Figure 1, the dynamics of which are represented by

$$\begin{array}{rcl} C_1\dot{H}_1 & = & Q_1 \\ C_2\dot{H}_2 & = & Q - Q_1 - Q_2 \\ Q_1 & = & \begin{cases} k_1\sqrt{2g|H_2 - H_1|}, & H_2 \geq H_1, \\ -k_1\sqrt{2g|H_2 - H_1|}, & H_2 < H_1, \end{cases} \\ Q_2 & = & k_2\sqrt{2gH_2}, \end{array} \tag{45}$$

where the physical meanings of system parameters are as

H_i	liquid levels of tank i;
H	steady-state liquid levels of two tanks;
C_i	cross sections of tank i;
k_1	cross sections of the inlet manual valves of tanks 1 and 2;
k_2	cross sections of the right outlet manual valves of tank 2;
Q	inflow rate of this system;
Q_1	inflow rate from tank 2 to tank 1;
Q_2	outflow rate of this system;
g	gravitational acceleration.

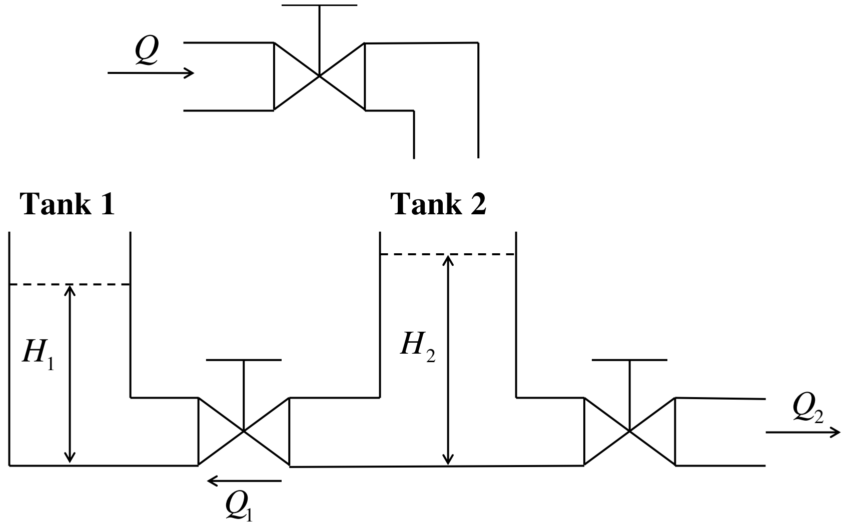

Figure 1. Schematic diagram of the liquid-level system.

By introducing the variable changes

$$z_1 = H_1 - H,\ z_2 = H_2 - H_1,\ u = \frac{Q}{C_2} - \frac{k_2\sqrt{2gH}}{C_2}, \tag{46}$$

and taking the input dead-zone nonlinearity into account, the dynamics of (45) can be further modeled as

$$\begin{aligned} \dot{z}_1 &= d_1\lceil z_2 \rfloor^{\frac{1}{2}}, \\ \dot{z}_2 &= D(u) + f_2(\bar{z}_2), \end{aligned} \tag{47}$$

where $d_1 = \frac{k_1\sqrt{2g}}{C_1}$ and $f_2(\bar{z}_2) = -\frac{C_1}{C_2}d_1\lceil z_2\rfloor^{\frac{1}{2}} - \frac{k_2\sqrt{2g}}{C_2}\lceil z_1 + z_2 + H\rfloor^{\frac{1}{2}} + \frac{k_2\sqrt{2g}}{C_2}\lceil H\rfloor^{\frac{1}{2}}$, D denotes the output of dead-zone input nonlinearity described by (2) with $m_r = m_l = 1 + 0.1\sin t$, $b_r = 0.2 + 0.1\sin t$ and $b_l = 0.4 + 0.1\cos t$, respectively. Based on Lemma 4, it is easily verified that Assumptions 1–3 hold with $\lambda_3 = \tau = 1$, $\lambda_1 = \lambda_2 = 2$, $\underline{m}_r = \underline{m}_l = 0.9$, $\bar{b}_r = 0.3$, $\bar{b}_l = 0.5$, $\varphi_2 = \frac{\sqrt{2g}}{C_2}(k_1 + k_2)$.

Introducing $\zeta_i = F^{(1+c)\lambda_i} z_i$, $i = 1, 2$ with

$$F = \begin{cases} \dfrac{T_s}{T_s - t}, & \zeta \in \{\mathbb{R}^2 - \Xi\}\ \&\ t < T_s, \\ 1, & \text{otherwise}, \end{cases} \tag{48}$$

where Ξ is a small closed neighborhood of origin and taking $\rho = 2$ and $c = 0$, according to Theorem 1, one can design a state feedback controller

$$u = \begin{cases} \left(\dfrac{F^{-1}\zeta_3^*}{\underline{m}_r} + \bar{b}_r\right), & \zeta_3^* > 0, \\ 0, & \zeta_3^* = 0, \\ \left(\dfrac{F^{-1}\zeta_3^*}{\underline{m}_l} - \bar{b}_l\right), & \zeta_3^* < 0, \end{cases} \tag{49}$$

$$\zeta_3^* = -(0.1 + \varrho_{21} + \varrho_{22} + \varrho_{23})\lceil \pi_2 \rfloor^{\frac{1}{2}}, \tag{50}$$

with $\beta_1 = (1.1 + \frac{2}{T_s}(1 + \zeta_1^2)^{\frac{1}{2}})/d_1$ if $\zeta \in \{\mathbb{R}^2 - \Xi\}\ \&\ t < T_s$ and $\beta_1 = 1.1/d_1$ otherwise, $\pi_2 = \zeta_2 - \zeta_2^*$, $\zeta_2^* = -\beta_1\zeta_1^2$, $\tilde{\varphi}_2 = (1 + \beta_1^{\frac{1}{2}})(1 + c)\lambda_2|\zeta_2|^{\tau/\lambda_2}/T_s + \varphi_2$, $\varrho_{21} = 3.7712 d_1^{\frac{3}{2}}$, $\varrho_{22} = 0.6667\tilde{\varphi}_2^{\frac{3}{2}} + \tilde{\varphi}_2$, $\varrho_{23} = |\frac{\partial\zeta_2^*}{\partial\zeta_1}d_1| + 0.6667|\frac{\partial\zeta_2^*}{\partial\zeta_1}|^3(d_1\beta_1^{\frac{1}{2}} + \frac{2}{T_s}(1 + \zeta_1^2)^{\frac{1}{2}})^3$, which can render the system (47) globally prescribed-time stable.

For the simplicity, take the system parameters as $H = 100$ cm, $g = 9.8$ m/s^2, $C_1 = C_2 = \sqrt{2g} = 4.427$ cm^2, $k_1 = 1$ cm^2 and $k_2 = 0.25$ cm^2 and the prescribed time as $T_p = 4$ s, $T_s = 3.8$ s, $\Xi = \{\zeta : \zeta_1^2 + \zeta_2^2 \leq 0.01\}$. For $(z_1(0), z_2(0)) = (0.5, -1)$ and $(z_1(0), z_2(0)) = (5, -10)$, Figures 2–5 are given to exhibit the responses of the CLS via MATLAB2020. It can be clearly observed that the convergence time of the system maintains below the prescribed time 4s, which confirms the effectiveness of the control scheme.

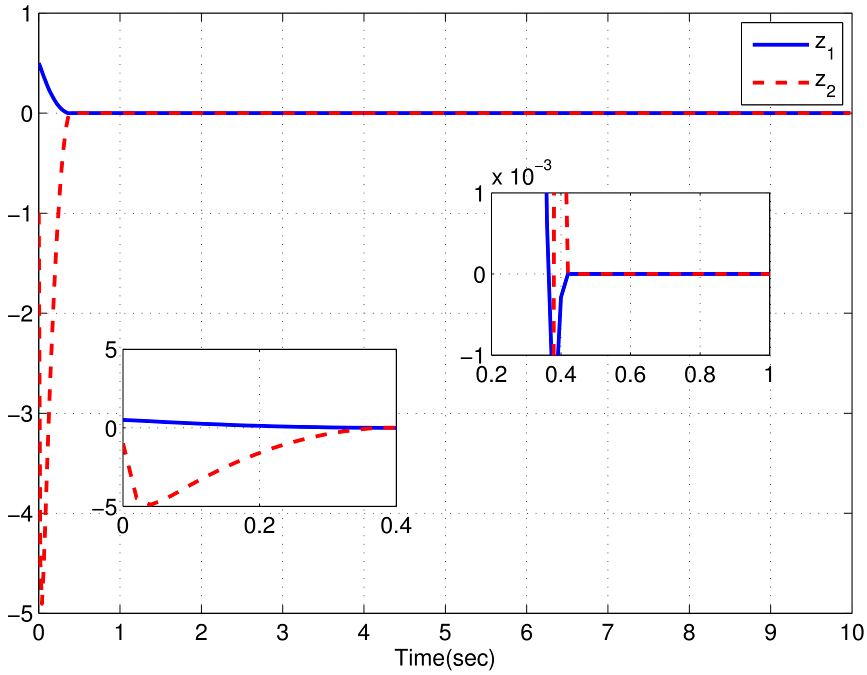

Figure 2. The state responses of the CLS with initial condition $(z_1(0), z_2(0)) = (0.5, -1)$.

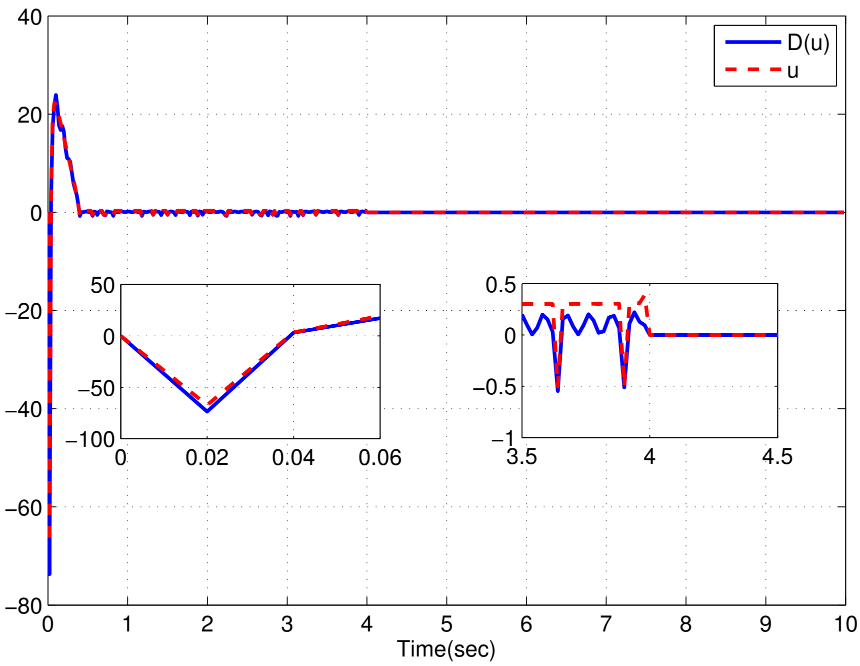

Figure 3. The input responses of the CLS with initial condition $(z_1(0), z_2(0)) = (0.5, -1)$.

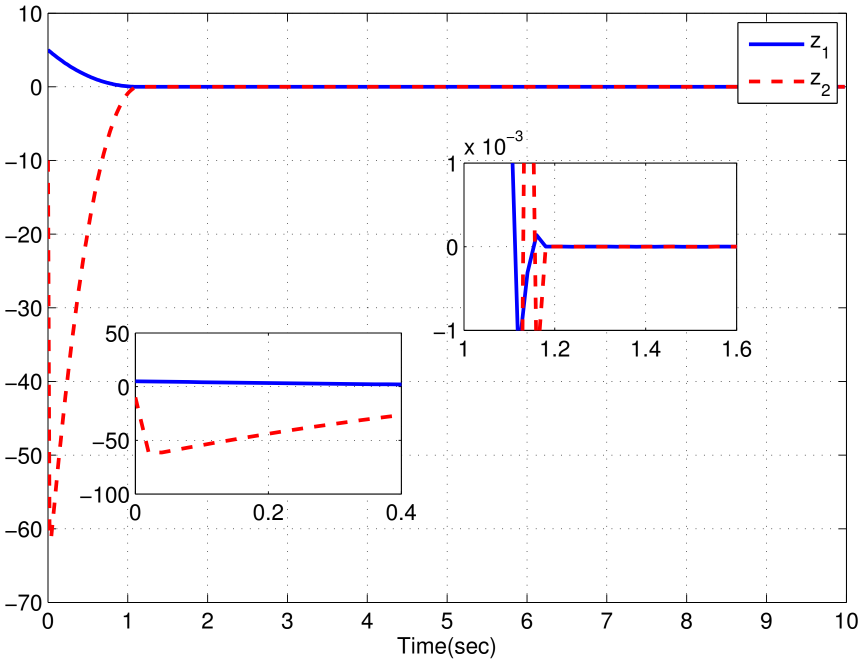

Figure 4. The state responses of the CLS with initial condition $(z_1(0), z_2(0)) = (5, -10)$.

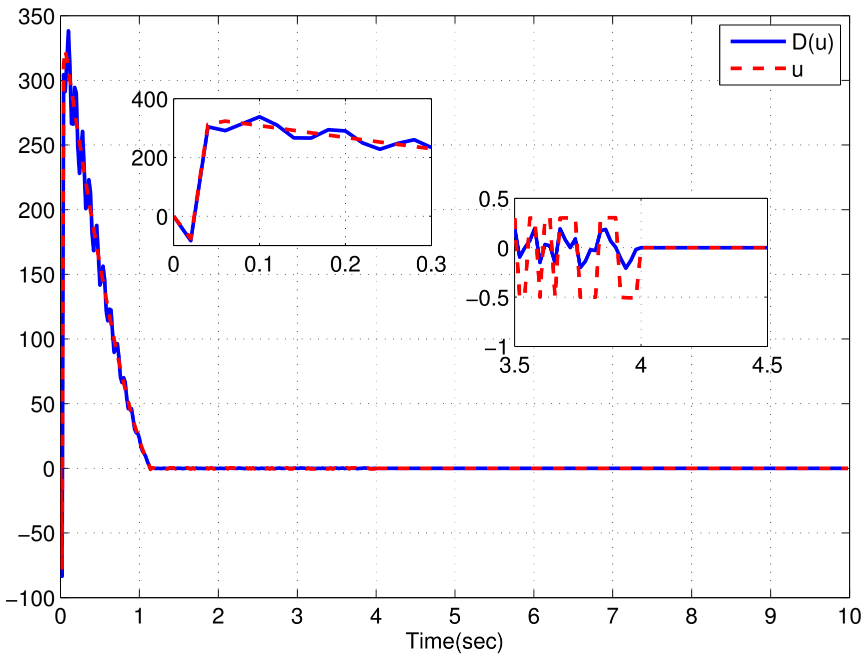

Figure 5. The input responses of the CLS with initial condition $(z_1(0), z_2(0)) = (5, -10)$.

To text the robustness of the proposed controller, the external disturbance $\omega(t) = 0.1 \sin t$ is introduced in the input channel. With the same parameters, the responses of the CLS are given in Figures 6 and 7, from which it is observed that the proposed controller is robust against small external disturbances.

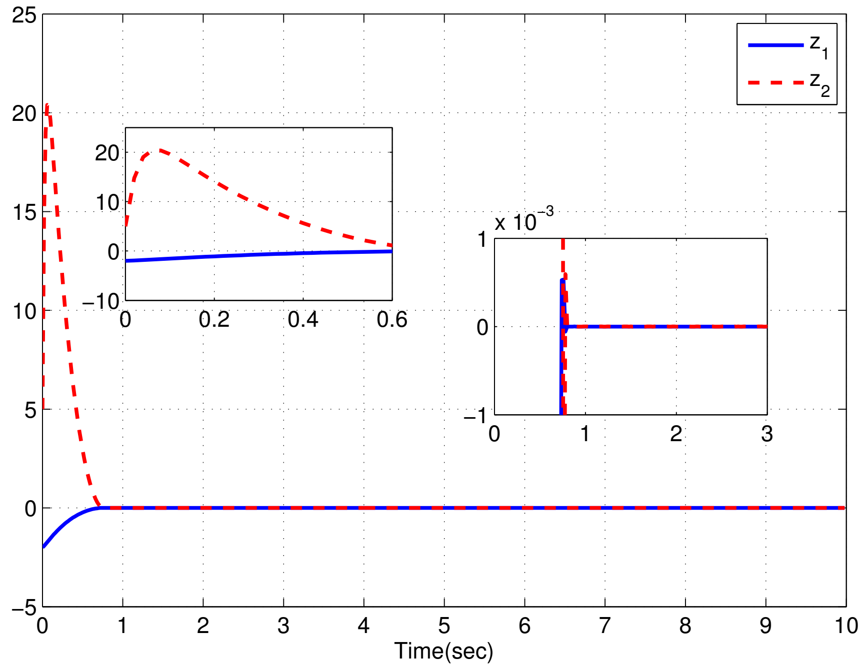

Figure 6. The state responses of the CLS with external disturbance and initial condition $(z_1(0), z_2(0)) = (-1, 5)$.

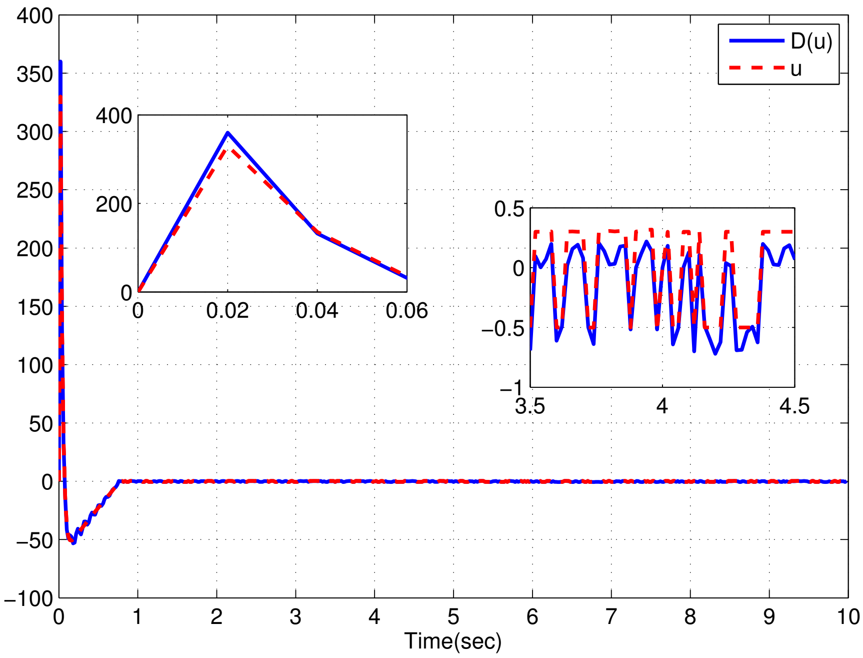

Figure 7. The input responses of the CLS with external disturbance and initial condition $(z_1(0), z_2(0)) = (-1, 5)$.

5. Conclusions

In this paper, a global prescribed-time stabilizing controller has been developed for a kind of HONSs with asymmetric actuator dead-zone. Due to the novel introduced SST, the significant advantage of the presented scheme is that its settling time can be preset and adjusted arbitrarily according to practical requirements by choosing the positive design parameter T_s. How to develop a prescribed-time controller for HONSs with parameter uncertainty and/or disturbances will be a topic for our future works. Applying the proposed method to practical systems such as rigid bodies [45], quadrotor UAVs [46] and induction motors [47] is another of our future topics.

Author Contributions: X.G.: Conceptualization, methodology, software, investigation, writing—original draft. H.Y.: methodology, software, investigation, writing—review and editing. F.G.: methodology, validation, software, resources, writing—review and editing, supervision. All authors have read and agreed to the published version of the manuscript.

Funding: This work was partially supported by the National Natural Science Foundation of China under Grants 61873120 and 62073187, the National Natural Science Foundation of Jiangsu Province under Grants BK20201469 and BE2021016-5 and the Qing Lan project of Jiangsu Province.

Institutional Review Board Statement: The study was conducted according to the guidelines of the Declaration of Helsinki and approved by the Institutional Review Board.

Informed Consent Statement: Not applicable.

Data Availability Statement: No data were used to support this study.

Conflicts of Interest: The authors declare no conflict of interest.

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