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An Adaptive EWMA Control Chart Based on Principal Component Method to Monitor Process Mean Vector

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Abstract: The special causes of variations, which is also known as a shift, can occur in a single or more than one related process characteristics. Statistical process control tools such as control charts are useful to monitor shifts in the process parameters (location and/or dispersion). In real-life situation, the shift is emerging in different sizes, and it is hard to identify it with classical control charts. Moreover, more than one process of characteristics required special attention because they must monitor jointly due to the association among them. This study offers two adaptive control charts to monitor the different sizes of a shift in the process mean vector. The novelty behind this study is to use dimensionally reduction techniques such as principal component analysis (PCA) and an adaptive method such as Huber and Bi-square functions. In brief, the multivariate cumulative sum control chart based on PCA is designed, and its plotting statistic is utilized as an input in the classical exponentially weighted moving average (EWMA) control chart. The run length (RL) properties of the proposed and other control charts are obtained by designing algorithms in MATLAB through a Monte Carlo simulation. For a single shift, the performance of the control charts is assessed through an average of RL, standard deviation of RL, and standard error of RL. Likewise, overall performance measures such as extra quadratic loss, relative ARL, and the performance comparison index are also used. The comparison reveals the superiority over other control charts. Furthermore, to emphasize the application process and benefits of the proposed control charts, a real-life example of the wind turbine process is included.

Keywords: average run length; control charts; Monte Carlo simulation; multivariate CUSUM; principal component

MSC: 62P30; 62H25; 62H25

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1. Introduction

In real-life, most of the manufacturing and/or business processes depend on more than one related characteristic. These process characteristic parameters (location and/or dispersion) contain two types of variations known as natural and un-natural variations. The random (natural) variations are an essential part of every process and are harmless to the outcome because they are small in magnitude. In contrary, special (un-natural) variations appeared due to faults such as the raw material not being in accordance with the specifications, improper adjustments of machines, human errors, etc. The special variations can be identified in the process parameters by implementing the diagnostic techniques such as statistical process control (SPC) tool kit to improve the product's quality. The SPC tool kit consists of a histogram, Pareto chart, fishbone diagram, flowchart, scatter diagram, and control chart tools (see [1]). The control chart has better performance to detect the shift (i.e., special variations) and is easy to implement in real-life as compared to other SPC tools.

Initially, ref. [2] introduced control charts known as Shewhart to identify the large size of shift in the process parameters. Later, to diagnose small-to-moderate shift, refs. [3,4] offered a cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts, respectively. These control charts are efficient enough to detect small-to-moderate shifts in the process location as compared to other control charts. The Shewhart control charts are also called memory-less because they use only current information, while the classical CUSUM and EWMA control charts use current and previous information, which is why they are known as memory control charts as well. Above-mentioned classical control charts are capable of detecting a shift of a characteristic in the process parameter but cannot handle more than one related process characteristic jointly. To distinguish a shift in the process's mean vector, multivariate control charts play a vital role. In this regard, reference [5] designed Hotelling's T^2 control chart. The Hotelling's T^2 control chart detects a large amount of shift quickly in the process mean vector. The Hotelling's T^2 control chart is also *analogous* of the classical Shewhart control chart if there is only one characteristic.

Similarly, to distinguish a small-to-moderate shift in the process mean vector, ref. [6] designed single multivariate CUSUM (MCUSUM) control chart. Later, refs. [7,8] also proposed multivariate CUSUM control charts, denoted as MCUSUM and MC_1, respectively, to identify small-to-moderate shift for the process mean vector. Similarly, ref. [9] suggested a multivariate EWMA (MEWMA) control char to identify small-to-moderate shifts in the process mean vector. Mathematical structures of the MCUSUM, MC_1, and MEWMA control charts depend on the Mahalanobis distance (MD) statistic (see [10]); the MD statistic utilizes the entire set of the interested variable's information; however, in practice, all variables do not contribute significantly. Therefore, the control charts based on an MD statistic face some issues, such as that it is hard to find out which variable became the cause of an out-of-control signal because the MD statistic assumed each variable contributes equally (see [1]). Additionally, as the number of variables increased, the out-of-control average run length denoted as ARL_0 increased. Lastly, if the variables have perfect correlation (i.e., 1 or -1), the MD statistic cannot be calculated because of variance–covariance matrix properties (see [11]).

To handle the above-mentioned issues, an alternative technique such as principal component analysis (PCA) is very helpful to enhance the performance of control charts (see [12]). The PCA is a statistical technique; it uses a common latent structure through the linear grouping of the original correlated variables to explain or present the maximum variation of all variables. It reduces the high-dimensionality problem (see [13]). Moreover, the new latent variables are orthogonal to each other, and their variances are in descending order as well. The PCA method has the ability to integrate with control charts to identify the shift (see [14]) to enhance control charts' performance. For example, ref. [15] mixed PCA with Hotelling's T^2 control chart. Correspondingly, ref. [16] designed an MEWMA control chart by utilizing PCA procedure.

In [17], the authors designed a self-starting control chart which utilized recursive residuals to monitor linear profiles issues for unknown parameters. This chart has the ability to identify shifts in intercept, the slope, or the standard deviation, and it is also easy to design and implement to obtain any desired in-control average run length. Similarly, ref. [18] proposed an EWMA control chart by utilizing a variable sampling interval technique and a parametric diagnostic approach for a general linear profile concept to additionally enhance the performance. Moreover, instructions on how to use PCA in control charts can be found in studies [19–22], and comprehensive details are given in these references. However, the latest development of PCA techniques in control charts can be observed in the studies of [12,23].

The classical control charts (i.e., MCUSUM, MC_1, and MEWMA) and control charts based on PCA techniques are efficient to detect a targeted shift (see [24]) or that for which they are designed for by assuming a specific value of parameters. This is also related to the fact that, in real-life, it is hard to forecast the real magnitude of the shift. Therefore, these control charts may not perform well when future shifts are infrequently unknown or are changed over time. Fortunately, this issued can be tackled or countered through an adaptive technique. Hence, the control charts along an adaptive technique can diagnose a certain range of unknown shifts in the process parameters. For example, ref. [25] proposed an adaptive CUSUM (ACUSUM) control chart to identify a broad range of shifts in the process location parameter. Similarly, ref. [26] recommended an adaptive EWMA (AEWMA) control chart by integrating score functions with the basic structure of the classical EWMA control chart to diagnose a certain range of shifts for the process location. Likewise, ref. [27] designed an ACUSUM based on the classical EWMA statistic and Huber function to distinguish changes in the sizes of a shift in the process location. On similar lines, ref. [28] also proposed an adaptive control chart. Similarly, refs. [29–33] offered AEWMA control charts based on score functions. More enhancement and restructuring h in the classical memory control charts to detect shift effectively can be seen in [34–45] studies.

The utilization of PCA and adaptive techniques to enhance the performance of memory control charts cannot be seen in the SPC literature; therefore, this point is taken as a motivation to propose this study. To accomplish the stated objective, this study offers two AEWMA control charts based on the PCA method, symbolized as $\text{AEWMA}_{\text{PCA}}$, to identify the different sizes of a shift in the mean vector. The proposed $\text{AEWMA}_{\text{PCA}}$ control charts accept the plotting statistic of the MC_1^{PCA} control chart as an input, whereas as the MC_1^{PCA} control chart (see [23]) is the modified form of the MC_1 control chart based on PCA statistics instead of the variable's vector. For adaptive features, two score functions such as Huber and Bi-square are used. In short, if the proposed $\text{AEWMA}_{\text{PCA}}$ control chart uses the Huber function, it is denoted as $\text{AEWMA}^1_{\text{PCA}}$ and $\text{AEWMA}^2_{\text{PCA}}$ for the Bi-square function. Additionally, the performance of the proposed control charts is assessed against some existing control charts. For an evaluation purpose, performance measures such as average run length (ARL), standard deviation of RL (SDRL), standard error of RL (SERL), extra quadratic loss (EQL), relative average run length (RARL), and performance comparison index (PCI) are used. The Monte Carlo simulation method is utilized to obtain the run length (RL) to compute the performance measures. Moreover, to highlight the application mechanism and advantages of the proposed control charts, a real-life example of the wind turbine process is also incorporated for a practical point of view.

The rest of the study is organized as follows: process characteristics, Mahalanobis distance, Hotelling's T^2 control chart, MC_1 control chart, control chart based on PC, MC_1^{PCA} control chart, the relation between PC and MD, and directional invariance property are given in Section 2. Similarly, the proposed $\text{AEWMA}_{\text{PCA}}$ control charts are introduced in Section 3. Likewise, Section 4 explains the performance ARL, EQL, RARL, and PCI measures, choices of parameters, constructions of the proposed $\text{AEWMA}_{\text{PCA}}$ control charts, and Monte Carlo simulation procedure. Comparative analysis based on the performance of measures of the proposed $\text{AEWMA}_{\text{PCA}}$ control charts against other control charts is carried out in Section 5. The implementation of the proposed and other control charts with real-life data is demonstrated in Section 6. Finally, the summary, conclusion, and recommendations are listed in Section 7.

2. Existing Methods

This section presents the background of existing methodologies which comprise the process characteristics, Mahalanobis distance (see [10]), Hotelling's T^2 [5] control chart, MC_1 control chart by [8], control chart based on PC proposed by [21], MC_1^{PCA} control chart designed by [23], the relation between PCM and MD statistics, and directional invariance property.

2.1. Process Characteristic

Assuming that $\text{X}_{p\times 1}$ represents a vector of the process variables or characteristics with p dimensions, it follows that $\text{X}_{p\times 1} \sim \mathcal{N}(\mu_0,\, \Sigma)$ multivariate normal distribution, where μ_0

$(p \times 1)$ is recognized as a mean vector and Σ $(p \times p)$, called variance–covariance matrix. Further, the X_{ji}, i.e., $(X_{1i},\ X_{2i},\ X_{3i}, \ldots, X_{pi}$ or $\mathrm{X}_{p\times 1})$, is i^{th} sample of j^{th} quality characteristic, where $i = 1, 2, 3, \ldots, n$, and n is the total number of observations to be monitored and $j = 1, 2, 3, \ldots, p$.

2.2. Mahalanobis Distance Statistic

The MD, designed by [46], identifies the *irregularities* (shifts) in more than one variable of $\mathrm{X}_{p\times 1} \sim \mathcal{N}(\mu,\ \Sigma)$. It describes the magnitude of standard deviations in term of distance from the μ in simplified way. It is very sensitive; it uses performance parameter's correlation coefficient, which is normalized (see [47]). Mathematically, it can be presented as below

$$MD_i^2 = (X_{ji} - \mu_0)' \Sigma^{-1} (X_{ji} - \mu_0), \tag{1}$$

where Σ^{-1} is the inverse of Σ.

2.3. Hotelling's T^2 Control Chart

Reference [5] offered Hotelling's T^2 control chart to identify large shifts in the process mean vector. The MD (i.e., Equation (1)) is also called the plotting statistic of Hotelling's T^2 control chart, which is designed as follows:

$$T_i^2 = (X_{ji} - \mu_0)' \Sigma^{-1} (X_{ji} - \mu_0), \tag{2}$$

The $UCL_{T^2} = \chi^2_{\alpha,p}$ is the upper control limit (UCL) of the Hotelling's T^2 control chart, where p represents the total number of variables, and α is acknowledged as a false alarm rate and is known as an αth percentile of the Chi-square distribution. The process is determined to be out-of-control if $T_i^2 > UCL_{T^2}$; otherwise, it is in-control.

2.4. MC_1 Control Chart

The MC_1 control chart recommended by [8] is helpful to monitor small-to-moderate shifts in the process mean vector. The MC_1 control chart accumulates information of the X_{ji} vector and is symbolized as $C_{1(i)}$. The $C_{1(i)}$ is defined as $C_{1(i)} = \sum_{l=i-n_i+1}^{i} (X_{ji} - \mu_0)$. The $C_{1(i)}$ vector can be designed as below:

$$\|C_{1(i)}\| = \sqrt{C_{1(i)}{}'\,\Sigma^{-1} C_{1(i)}}, \tag{3}$$

where $\|.\|$ represents the norm of $C_{1(i)}$ vector. The plotting statistic of the MC_1 control chart can be offered below

$$MC_{1i} = \max\left\{0, \|C_{1(i)}\| - Kn_i\right\}, \tag{4}$$

where $n_i = n_{i-1} + 1$ if $MC_{1(i-1)} > 0$, otherwise $n_i = 1$. The n_i is the total number of subgroups since the most recent renewal is zero (i.e., $MC_{1(0)} = 0$). The process is out-of-control for $MC_{1i} > H_{MC_1}$, otherwise it is in-control. The $\mathrm{H}_{\mathrm{MC}_1}$ is the UCL of the MC_1 control chart.

2.5. Control Chart Based on PCA

The control chart based on the PCA technique helps to improve the process parameters' performance [21]. The PCA statistic, according to the [10], can be written as below

$$PCA_i = \sqrt{\frac{1}{\gamma_{(1)}}(y_1)^2 + \frac{1}{\gamma_{(2)}}(y_2)^2 + \ldots + \frac{1}{\gamma_{(p)}}(y_p)^2} \tag{5}$$

where $y_1 = e_1' X_i = e_{11} X_1 + e_{12} X_2 + \ldots + e_{1p} X_p$, $y_2 = e_2' X_i = e_{21} X_1 + e_{22} X_2 + \ldots + e_{2p} X_p$, $\ldots$, $y_p = e_p' X_i = e_{p1} X_1 + e_{p2} X_2 + \ldots + e_{pp} X_p$ are principal components through special linear combination technique and are also uncorrelated because of the orthogonal feature. Further, the e_1', $e_2', \ldots, e_p'$ represent the eigenvectors of Σ. Likewise, the $\gamma_{(i)}$ can be determined as $\mathrm{var}(y_i) = e_i'\Sigma e_i = \gamma_{(i)}$ and called eigenvalues of Σ, while covariance is $\mathrm{cov}(y_i, y_l) = 0$, where $i = 1, 2, \ldots, p$ & $i \neq l = 1, 2, \ldots, p$. Normally, the maximum numbers of PCAs are equal to the total number of characteristics. For example, at $p = 2$, the Equation (5) can be designed as: $PCA_i = \sqrt{\frac{1}{\gamma_{(1)}}(y_1)^2 + \frac{1}{\gamma_{(2)}}(y_2)^2}$, correspondingly; for $p = 3$, it would be defined as: $PCA_i = \sqrt{\frac{1}{\gamma_{(1)}}(y_1)^2 + \frac{1}{\gamma_{(2)}}(y_2)^2 + \frac{1}{\gamma_{(3)}}(y_3)^2}$ and so on. The PCA_i statistics are also treated as the plotting statistic of the control chart based on PC (PC-chart). If $PCA_i^2 > \mathrm{UCL_{PCM}}$, the process is declared to be out-of-control; otherwise, it is in-control. The $\mathrm{UCL_{PM}}$ is the UCL of PC-chart [21].

The aforementioned lines show that only a few PCA tests are needed to explain the maximum variation of the data. Therefore, the question is how someone can find out the optimum number of PCA. In this regard, ref. [48] recommended some valuable guidelines to choose the optimum number of PCA. These guidelines are: (1) select q such that $\sum_{i=1}^{q} \gamma_{(i)} \geq 0.9 \sum_{j=1}^{p} \gamma_{(i)}$, (2) increase q such that $\gamma_{(i)} \geq \gamma_m$, where $\gamma_m = \frac{\sum_{t=1}^{p} \gamma_{(t)}}{p}$ and $i = 1, 2, \ldots, q$, and (3) plot $\gamma_{(i)}$ against i and select q at the 'knee' in the curve. Moreover, the study by [10] may be helpful in this regard.

2.6. $MC_1{}^{PCA}$ Control Chart

Reference [23] recommended the $\mathrm{MC_1^{PCA}}$ control chart to identify the shift in the process mean vector. The $\mathrm{MC_1^{PCA}}$ control chart structure depends on the $\mathrm{MC_1}$ control chart and PCA_i^2 statistic. In more detail, the $\mathrm{MC_1}$ control chart accepts the PCA_i^2 statistic instead of the $\|C_{1(i)}\|$ to formulate the $\mathrm{MC_1^{PCA}}$ control chart. The $\mathrm{MC_1^{PCA}}$ control chart is defined as follows:

$$MC_{1i}^{PCA} = \max\{\varphi, PCA_i - Kn_i\},$$
$$n_i = \begin{cases} n_{i-1} + 1, & if\ MC_{1(i-1)}^{PCA} > \varphi \\ 1, & if \text{ otherwise} \end{cases} \tag{6}$$

where the MC_{1i}^{PCA} is the plotting statistic of the $\mathrm{MC_1^{PCA}}$ control chart and depends on K and n_i parameters, the $MC_{1(i-1)}^{PCA}$ is the one lag value of the MC_{1i}^{PCA} statistic and $K = kd(\mu)$, where $d(\mu)$ is a non-centrality parameter. Similarly, the φ is a constant and is greater than zero (i.e., $\varphi > 0$), and it can be $\varphi = 0.001$ for this study. It helps to design the proposed $\mathrm{AEWMA_{PCA}}$ control chart because, at $\varphi = 0.00$, the proposed control charts cannot define due to mathematical properties (see Section 3). If the MC_{1i}^{PCA} plotting statistic is above the $\mathrm{H_{MC_1^{PCA}}}$ (i.e., $MC_{PCMi} > \mathrm{H_{MC_1^{PCA}}}$), the process is declared as out-of-control; otherwise, it is in-control [12]. The $\mathrm{H_{MC_1^{PCA}}}$ represent the UCL of the $\mathrm{MC_1^{PCA}}$ control chart.

2.7. Relation between PCM and MD

Equation (5) produces the same results as Equation (1) does; however, it contains the maximum number of PCAs (i.e., $y_1, y_2, \ldots, y_p$) (see [10,21]). Therefore, the objective to use PCA is to utilize the maximum data's variability with the minimum number of PCAs.

2.8. Directional Invariance Property

The ARLs of multivariate control charts totally depends on non-centrality parameter $d(\mu)$ [12] magnitude, whereas the $d(\mu)$ is defined as: $d(\mu) = \sqrt{(\mu_1 - \mu_0)'\Sigma^{-1}(\mu_1 - \mu_0)}$. The $d(\mu)$ shows the overall variation in the X vector regardless of the fluctuation in a specific way of μ_0. For instance, if $p = 2$ with a variance –covariance identity matrix of X at $d(\mu) = 1$, the mean vectors of μ are such that $(1, 0)$, $(0, 1)$, $(0.7071, 0.7071)$, $(-0.7071, -0.7071)$, and $(0.7071, -0.7071)$ produce same ARLs for the control chart because of their equidistant $d(\mu)$ from μ_0 (see references [8,21]).

3. Proposed $AEWMA_{PCA}$ Control Charts

This section contains the methodologies of the two proposed multivariate control charts to monitor a certain range of shift in the mean vector. The proposed $\text{AEWMA}_{\text{PCA}}$ control charts are based on the MC_{1i}^{PCA} statistic, the classical EWMA control chart [4], PCA, and score functions. If the proposed $\text{AEWMA}_{\text{PCA}}$ control chart uses the MC_{1i}^{PCA} statistic, the classical EWMA control chart, and the Huber function, it is denoted as an $\text{AEWMA}_{\text{PCA}}^1$ control chart. For the Bi-square function, it is symbolized as an $\text{AEWMA}_{\text{PCA}}^2$ control chart. Special cases of or for the proposed control charts are also given in Section 3.3. More details are provided in the following subsections.

3.1. Proposed $AEWMA_{PCA}^1$ Control Charts

The plotting statistic of the proposed $\text{AEWMA}_{\text{PCA}}^1$ control chart based on MC_{1i}^{PCA} statistic is defined as follows:

$$AEWMA_{PCAi}^1 = (1 - w_h(e_i))AEWMA_{PCA(i-1)}^1 + w_h(e_i)MC_{1i}^{PCA}, \tag{7}$$

where $w_h(e_i)$ is the ratio (i.e., $w_h(e_i) = \frac{\varnothing_h(.)}{e_i}$) of error e_i and Huber function $\varnothing_h(.)$. The error is designed as: $e_i = MC_{1i}^{PCA} - E_{i-1}$, whereas $E_i = ((1-\lambda)E_{i-1} + \lambda MC_{1i}^{PCA}$ and $E_0 = 0$. Similarly, the Huber function $\varnothing_h(e_i)$ is defined as: $\varnothing_h(e_i) = \begin{cases} e_i + (1-\lambda)\gamma, & \text{if } e_i < -\gamma \\ \lambda e_i, & \text{if } |e_i| \le \gamma \\ e_i - (1-\lambda)\gamma, & \text{if } e_i > \gamma \end{cases}$.

Three conditions are given in the above expression based on e_i and its relationship with constant γ; these conditions help to define the size of shift, which will be either small, small-to-moderate, or large in size. The UCL is defined as:

$$UCL_{AEWMA_{PCA}^1 i} = \mu_{MC_{1i}^{PCA}} + h_{AEWMA_{PCA}^1}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}\sigma_{MC_{1i}^{PCA}}, \tag{8}$$

where $h_{AEWMA_{PCA}^1}$ is constant for control limits which help to determine the fixed out-of-control ARL (ARL_0), and the $\mu_{MC_1^{PCA}}$ and $\sigma_{\text{MC}_1^{\text{PCA}}}$ are the empirical time varying mean and standard deviation of the MC_{1i}^{PCA} statistic, respectively. If the $AEWMA_{PCAi}^1 > UCL_{AEWMA_{PCA}^1 i}$, the process is out-of-control; otherwise, it is in-control.

3.2. Proposed $AEWMA_{PCA}^2$ Control Charts

As with the $\text{AEWMA}_{\text{PCA}}^1$ control chart, the proposed $AEWMA_{PCA}^2$ control chart plotting statistic is given below

$$AEWMA_{PCAi}^2 = (1 - w_b(e_i))AEWMA_{PCA(i-1)}^2 + w_b(e_i)MC_{1i}^{PCA}, \tag{9}$$

where $w_b(e_i)$ is the ratio (i.e., $w_b(e_i) = \frac{\varnothing_b(.)}{e_i}$) of error e_i and Bi-square function $\varnothing_b(e_i)$. The Bi-square function is given as follows:

$$\varnothing_b(e_i) = \begin{cases} e_i\left[1-(1-\lambda)\left[1-\left[\frac{e_i}{\gamma}\right]^2\right]^2\right] & \text{if } |e_i| \le \gamma \\ e_i & \text{otherwise.} \end{cases} \tag{10}$$

The UCL of the proposed $AEWMA_{PCA}^2$ control chart is designed as follows:

$$UCL_{AEWMA_{PCA}^2 i} = \mu_{MC_{1i}^{PCA}} + h_{AEWMA_{PCA}^2}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}\sigma_{MC_{1i}^{PCA}}, \tag{11}$$

If $AEWMA^2_{PCAi} > UCL_{\mathrm{AEWMA}^2_{\mathrm{PCA}}i}$, the process in out-of-control; otherwise, it is in-control, whereas the $h_{AEWMA^2_{PCA}}$ is a constant that helps to fix ARL_0.

3.3. Special Cases of Proposed $AEWMA^1_{PCA}$ and $AEWMA^2_{PCA}$ Control Charts

This subsection describes that $MCE^{(2)}$ and $\mathrm{MC}_1^{\mathrm{PCA}}$ control charts are special cases of the proposed $AEWMA^1_{PCA}$ and $AEWMA^2_{PCA}$ control charts, respectively. More mathematical description is given in the following subsections:

3.3.1. $\mathrm{MCE}^{(2)}$ Control Chart Is a Special of Proposed $AEWMA^1_{PCA}$ Control Chart

For $|e_i| \leq \gamma$ in $\varnothing_h(e_i)$, the proposed $\mathrm{AEWMA}^1_{\mathrm{PCA}}$ control chart tends to become a special case of the $\mathrm{MCE}^{(2)}$ control chart. For example, if $|e_i| \leq \gamma$, then $\varnothing_h(e_i)$ becomes $\varnothing_h(e_i)= \lambda e_i$. Next, exchange the resulted $\varnothing_h(e_i)$ in $w_h(e_i) = \frac{\varnothing_h(.)}{e_i}$, that is, $w_h(e_i) = \lambda e_i / e_i = \lambda$, and replace $w_h(e_i)$ of Equation (7) with λ. Thus, the new form of Equation (7) is given as:

$$AEWMA^1_{PCAi} = (1-\lambda)AEWMA^1_{PCA(i-1)} + \lambda MC^{PCA}_{1i} \tag{12}$$

The $AEWMA^{(1)}_{ci}$ in Equation (12) and $MCE^{(2)}_{(i)}$ in Equation (17) of [12] are identical quantities. They serve the same purpose. Therefore, it can be concluded that the $MCE^{(2)}$ control chart is a special case of the proposed $\mathrm{AEWMA}^1_{\mathrm{PCA}}$ control chart.

3.3.2. $MC_1{}^{PCA}$ Control Chart Is a Special of Proposed $AEWMA^2_{PCA}$ Control Chart

Similarly, when $|e_i| > \gamma$ in $\varnothing_b(e_i)$, the proposed $\mathrm{AEWMA}^2_{\mathrm{PCA}}$ control chart converges to the $\mathrm{MC}_1^{\mathrm{PCA}}$ control chart. For instance, if $|e_i| > \gamma$, then $\varnothing_b(e_i)$ becomes $\varnothing_b(e_i) = e_i$. Next, replace the resulted $\varnothing_b(e_i)$ in $w_b(e_i) = \frac{\varnothing_b(.)}{e_i}$, that is, $w_h(e_i) = e_i / e_i = 1$, and replace the $w_h(e_i)$ of Equation (9) with 1. Thus, the new form of Equation (9) is given as:

$$AEWMA^2_{PCAi} = MC^{PCA}_{1i}, \tag{13}$$

The $AEWMA^2_{PCAi}$ statistic in Equation (13) and MC^{PCA}_{1i} statistic in Equation (6) are identical quantities and serve the same objective. Therefore, it can be established that the $\mathrm{MC}_1^{\mathrm{PCA}}$ control chart is a special case of the proposed $AEWMA^2_{PCA}$ control chart.

4. Performance Evaluation

To obtain the run length (RL) of the proposed control charts, the Monte Carlo simulation technique is used by developing algorithms in MATLAB. Based on RL, the performance measure such as average run length (ARL), standard deviation run length (SDRL), and standard error of run length (SERL) are utilized to evaluate the performance of the control chart for a shift, while, for a certain range of shift, extra quadratic loss (EQL), relative average run length (RARL), and the performance comparison index (PCI) are considered by using ARL. In addition, the choice of parameters, construction of proposed control charts, and the Monte Carlo simulation method are also part of this section.

4.1. Average Run Length

Generally, ARL is classified as either in-control and out-of-control, symbolized as ARL_0 and ARL_1, respectively. The ARL_0 represents the ARL when points are plotted against the control limit(s) under an $H_0 : \mu = \mu_0$ (in-control) environment. Similarly, ARL_1 denotes the ARL when points are plotted against the control limit(s) under an $H_1 : \mu = \mu_1$ (out-of-control) situation. For a fair comparison, a control chart should have the same or greater ARL_0, while ARL_1 should be smaller against other control charts for a specific shift. More details about the ARL (ARL_0 and ARL_1) can be studied in recent articles proposed by [33,49–52] and references therein.

4.2. Overall Performance Measures

The EQL, RARL, and PCI are widely used to evaluate the control charts performance for a certain range of shift. Their complete theoretical and mathematical information is provided below.

4.2.1. Extra Quadratic Loss

The EQL assesses the performance of a control chart for a broad range of shift against other control charts. Statistically, the EQL is designed as follows:

$$\mathrm{EQL} = \frac{1}{d(\mu_1)_{\max} - d(\mu_1)_{\min}} \int_{d(\mu_1)_{\min}}^{d(\mu_1)_{\max}} d^2(\mu_1) ARL_{d(\mu_1)} d(d(\mu_1)), \tag{14}$$

where $d(\mu_1)$ represents the shift in the process mean vector. Further, the $d(\mu_1)_{\max}$ and $d(\mu_1)_{\min}$, known as maximum and minimum (i.e., $d(\mu_1)_{\max} < d(\mu_1) < d(\mu_1)_{\min}$) values of shift, respectively, and $\mathrm{ARL}_{\mathrm{d}(\mu_1)}$ is the ARL value of a control chart for a shift. The EQL incorporates ARL by taking the square of $d(\mu_1)$ as weight (see [53]).

4.2.2. Relative Average Run Length

Quality engineers and researchers also use the RARL as a performance measure tool along EQL. It also measures the overall performance of control charts. Mathematically, it is designed as given below

$$\mathrm{RARL} = \frac{1}{d(\mu_1)_{\max} - d(\mu_1)_{\min}} \int_{d(\mu_1)_{\min}}^{d(\mu_1)_{\max}} \frac{ARL_{d(\mu_1)}}{ARL_{\mathrm{bmk}(d(\mu_1))}} d(d(\mu_1)), \tag{15}$$

where the $\mathrm{ARL}_{\mathrm{bmk}(\mathrm{d}(\mu_1))}$ represents the ARL of a benchmark (bmk) control chart for a specific $d(\mu_1)$. A control chart which has minimum EQL is assumed as benchmark control chart. Consequently, benchmark control chart RARL will be equal to 1 and other control charts have a value greater than 1. Greater than one RARL means that the control charts have lower performance (see [45,54]) against the benchmark control chart.

4.2.3. Performance Comparison Index

The PCI is also famous among quality engineers and researchers to evaluate the overall performance of control charts. In fact, it is the ration (i.e., $\mathrm{PCI} = \frac{EQL}{\mathrm{EQL}_{\mathrm{bmk}}}$) of EQL of a control chart and $\mathrm{EQL}_{\mathrm{bmk}}$, whereas the $\mathrm{EQL}_{\mathrm{bmk}}$ represents the benchmark control chart EQL, and the benchmark control chart has minimum EQL among all control charts. Greater than one PCI (i.e., PCI > 1) of a control chart shows inferior performance [55] as compared to the benchmark control chart.

4.3. Choices of Parameters

The performance of the proposed control charts depends on the optimal choices of λ, γ, and k parameters along $\mathrm{h}_{\mathrm{AEWMA}^1_{\mathrm{PCA}}}$ and $\mathrm{h}_{\mathrm{AEWMA}^2_{\mathrm{PCA}}}$ constants. Therefore, the parameters λ, γ, and k optimal choices matter a lot. The ranges of the λ, γ, and k parameters are chosen as $(0,1]$, $(0,4]$, and $(0,2]$, respectively, as recommend [12]. Their values, other than these, can be chosen as well to explore the behavior of the proposed control charts against other control charts. Additionally, the $\mu = (\delta,\ 0,0,\ldots,0)$ shows that there is a shift only in first variable, while other variables have none. The value of shift, other than a first variable, can be chosen, such as $\mu = (0,\ \delta,0,\ldots,0)$ or $\mu = (\delta,\ \delta,0,\ldots,0)$, etc.; however, the objective is to obtain the specific values of $d(\mu)$, such as 0.25, 0.5, 0.75 1.0, 1.25, 1.5, 1.75 2.0, 2.25 2.5, 2.75, 3.0, 4.0, and 5.0.

4.4. Construction of Proposed $AEWMA^1_{PCA}$ and $AEWMA^2_{PCA}$ Control Charts

This subsection gives the general procedures and guidelines to design the proposed $\mathrm{AEWMA}^1_{\mathrm{PCA}}$ and $\mathrm{AEWMA}^2_{\mathrm{PCA}}$ control charts. These common techniques and procedures will be helpful for the audiences to understand the dynamics of the study to implement it for practical purposes. The proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart is developed by adapting the below steps.

(i) Observations are drawn randomly from $X_{p\times 1} \sim \mathcal{N}(\mu, \Sigma)$ $(i = 1, 2, 3, \ldots)$ when $p = 3$;
(ii) The PCA_i statistic is calculated from Equation (5) using X and considering y_1 and y_2 PCs along their respective eigenvectors (i.e., e_1', and e_2') and eigenvalues (i.e., $\gamma_{(1)}$ and $\gamma_{(2)}$);
(iii) Estimate the MC_{1i}^{PCA} statistic from Equation (6) based on the PCA_i statistic, constant $K = kd(\mu)$, and n_i, while φ is 0.001;
(iv) Calculate the $AEWMA^1_{PCAi}$ statistic by using $w_h(e_i)$ time varying and the MC_{1i}^{PCA} statistic;
(v) Let us assume that $\text{h}_{\text{AEWMA}^1_{\text{PCA}}} = 3$;
(vi) Computed the control limit $UCL_{\text{AEWMA}^1_{\text{PCA}}i}$ from Equation (8), whereas the $\mu_{\text{MC}_{1i}^{\text{PCA}}}$ and $\sigma_{\text{MC}_{1i}^{\text{PCA}}}$ are time-varying mean and standard deviation of the MC_{1i}^{PCA} statistic, respectively calculated by empirical method;
(vii) Plot the $AEWMA^1_{PCAi}$ statistic against the control limit $UCL_{\text{AEWMA}^1_{\text{PCA}}i}$; if the $AEWMA^1_{PCAi} > UCL_{\text{AEWMA}^1_{\text{PCA}}i}$ is true, the process is declared out of control, and the index of run length (RL) is recorded;
(viii) Repeat from step (i) to step (vii) for 10^5 times to generate RLs and calculate their average, which is called ARL_0;
(ix) If the ARL_0 is the targeted one, stop here; otherwise, change the value of $\text{h}_{\text{AEWMA}^1_{\text{PCA}}}$ accordingly in step (v) and repeat the process (i.e., step (i) to step (viii)) until the desired ARL_0 is achieved;
(x) After obtaining the desired value of ARL_0, consider $X \sim \mathcal{N}(\mu_1, \Sigma)$ and repeat steps from (i) to (ix) to obtain ARL_1 values.

The proposed $\text{AEWMA}^2_{\text{PCA}}$ control chart can be constructed by a similar process as described in Section 4.4 by utilizing $w_b(e_i)$ instead of $w_h(e_i)$ in step (iv). Figure 1 also shows precision review of Section 4.4.

4.5. Monte Carlo Simulation Procedure

The RL properties of the proposed $\text{AEWMA}^1_{\text{PCA}}$ and $\text{AEWMA}^2_{\text{PCA}}$ control charts are obtained by designing an algorithm in MATLAB through Monte Carlo simulation. For each displacement of $d(\mu)$, 10^5 RLs are generated, and range of $d(\mu)$ between 1 and 5 is chosen. Further, suitable ranges of the parameters λ, γ, and k play a significant role. For instance, ref. [12] suggested that $K = kd(\mu)(k = 0.5)$ for efficient performance. Similarly, ref. [4] provided some guidelines on λ parameter. Likewise, the γ parameter results in better performance for the 1–4 range [30]. Taking these suggestions and recommendations, the proposed $\text{AEWMA}^1_{\text{PCA}}$ and $\text{AEWMA}^2_{\text{PCA}}$ control charts ARLs are given in Tables 1–3. These results can be estimated through other techniques such as the Markov chain approach.

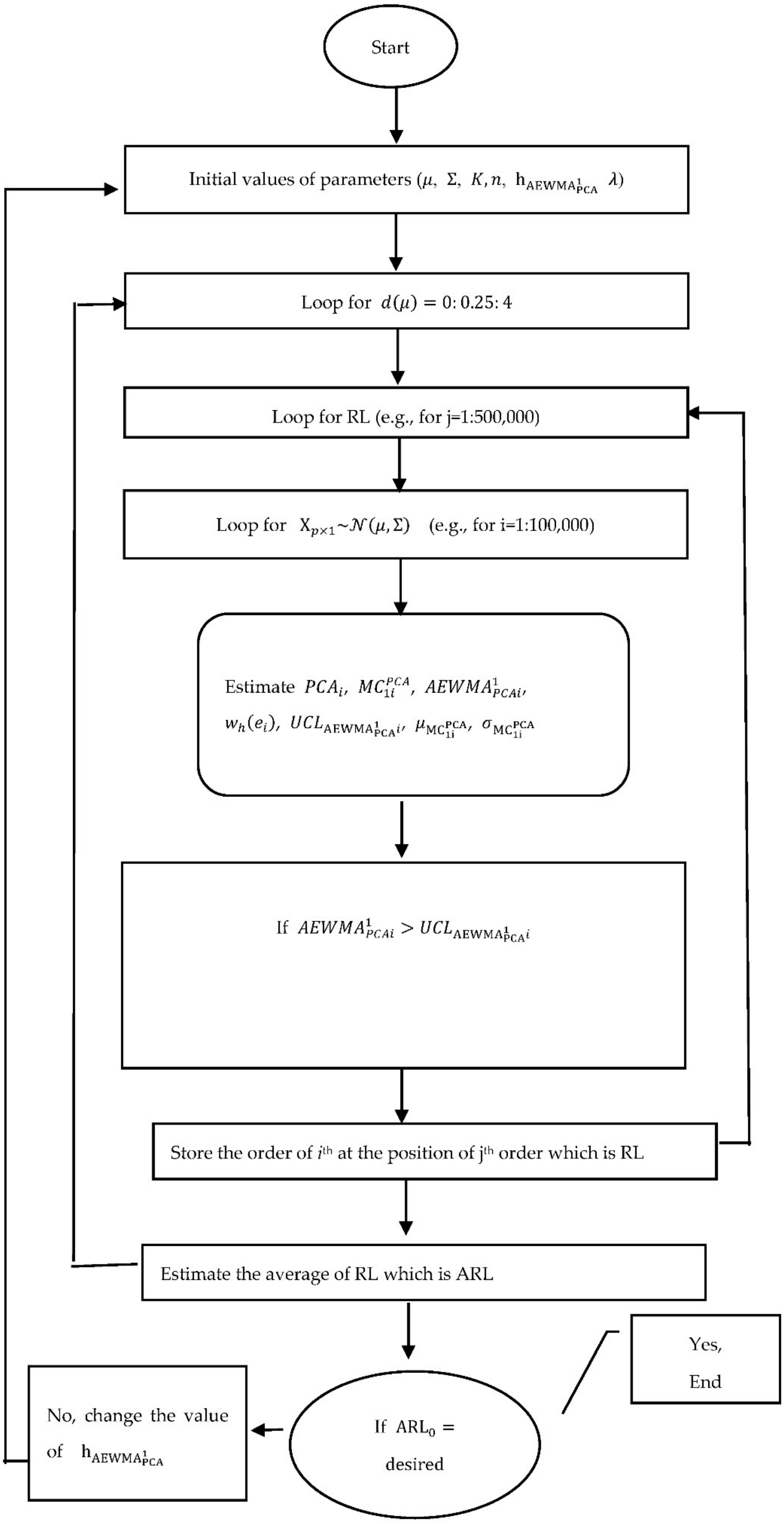


Figure 1. Flow chart of Monte Carlo simulation for proposed $AEWMA^1_{PCA}$ control chart.

Table 1. ARLs', SDRLs', and SERLs' properties of proposed $\mathrm{AEWMA}^1_{\mathrm{PCA}}$ control chart at $\mathrm{ARL}_0 = 200$, $k = 0.50$, $p = 3$, and PCA $= 2$.

$d(\mu)$															
	λ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10	0.10	0.10
	γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	1.00	1.50	2.00	2.50	3.00	3.50	4.00
	$h_{\mathrm{AEWMA}^1_{\mathrm{PCA}}}$	17.52	15.22	12.82	9.02	5.42	4.02	3.58	11.28	10.52	10.18	6.18	4.98	4.48	4.28
0.00	ARL	197	198.71	200	205	200	202	205.38	200	206.368	198	202	202	200	198
	SDRL	225	224	213	214.74	203	195	191	214	212	205.26	201.63	199	193	187
	SERL	1.593	1.585	1.509	1.518	1.433	1.378	1.353	1.516	1.501	1.451	1.426	1.409	1.368	1.324
0.25	ARL	81.79	84.60	87.28	89.07	86.44	86.19	88.40	87.16	90.3022	88.72	88.34	87.26	85.34	84.73
	SDRL	94.24	94.13	90.86	92.37	84.70	78.04	76.23	91.58	91.93	90.77	87.43	82.29	77.78	76.95
	SERL	0.666	0.666	0.642	0.653	0.599	0.552	0.539	0.648	0.650	0.642	0.618	0.582	0.550	0.544
0.50	ARL	24.87	26.42	27.55	28.44	29.00	30.15	31.42	27.89	29.1529	29.67	29.65	30.08	30.20	30.12
	SDRL	26.46	26.27	26.56	26.05	24.74	23.23	23.05	26.88	27.41	26.95	25.45	24.20	23.15	22.61
	SERL	0.187	0.186	0.188	0.184	0.175	0.164	0.163	0.190	0.194	0.191	0.180	0.171	0.164	0.160
0.75	ARL	11.10	12.05	12.73	13.33	14.14	14.91	16.16	12.66	13.1558	13.65	14.21	14.87	15.16	15.49
	SDRL	10.14	9.97	9.91	10.09	10.08	9.69	9.68	10.68	10.44	10.54	10.49	10.09	9.38	9.07
	SERL	0.072	0.071	0.070	0.071	0.071	0.069	0.068	0.076	0.074	0.075	0.074	0.071	0.066	0.064
1.00	ARL	6.37	7.12	7.64	8.09	8.57	9.33	10.17	7.28	7.7447	8.16	8.50	9.06	9.65	10.14
	SDRL	5.13	5.09	5.15	5.02	5.10	5.23	5.36	5.58	5.31	5.36	5.32	5.24	5.04	4.96
	SERL	0.036	0.036	0.037	0.036	0.036	0.037	0.038	0.040	0.038	0.038	0.038	0.037	0.036	0.035
1.25	ARL	4.36	4.94	5.37	5.68	6.07	6.64	7.31	4.92	5.3273	5.65	6.00	6.46	6.99	7.48
	SDRL	3.11	3.10	3.07	3.06	3.14	3.24	3.38	3.35	3.23	3.23	3.24	3.30	3.30	3.24
	SERL	0.022	0.022	0.022	0.022	0.022	0.023	0.024	0.024	0.023	0.023	0.023	0.023	0.023	0.023
1.50	ARL	3.26	3.74	4.14	4.41	4.69	5.12	5.63	3.66	4.0312	4.29	4.57	4.95	5.39	5.82
	SDRL	2.06	2.10	2.10	2.10	2.13	2.24	2.34	2.29	2.24	2.18	2.20	2.27	2.32	2.32
	SERL	0.015	0.015	0.015	0.015	0.015	0.016	0.017	0.016	0.016	0.015	0.016	0.016	0.016	0.016
1.75	ARL	2.63	3.02	3.37	3.60	3.86	4.22	4.58	2.90	3.215	3.46	3.71	4.00	4.38	4.76
	SDRL	1.50	1.52	1.54	1.54	1.58	1.62	1.72	1.66	1.65	1.61	1.63	1.66	1.72	1.76
	SERL	0.011	0.011	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.011	0.012	0.012	0.012	0.012
2.00	ARL	2.20	2.54	2.84	3.06	3.25	3.56	3.88	2.39	2.6923	2.89	3.12	3.36	3.67	3.99
	SDRL	1.15	1.17	1.20	1.21	1.21	1.28	1.32	1.26	1.26	1.25	1.26	1.29	1.34	1.38
	SERL	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.010	0.010
2.25	ARL	1.90	2.19	2.46	2.67	2.85	3.09	3.37	2.03	2.3018	2.52	2.69	2.92	3.18	3.46
	SDRL	0.92	0.97	0.97	0.99	0.99	1.01	1.07	1.00	1.02	1.02	1.02	1.04	1.08	1.12
	SERL	0.007	0.007	0.007	0.007	0.007	0.007	0.008	0.007	0.007	0.007	0.007	0.007	0.008	0.008
2.50	ARL	1.66	1.93	2.18	2.37	2.53	2.75	3.00	1.78	2.0204	2.21	2.39	2.58	2.80	3.05
	SDRL	0.75	0.81	0.82	0.82	0.83	0.85	0.90	0.82	0.86	0.86	0.86	0.86	0.88	0.93
	SERL	0.005	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.007
2.75	ARL	1.49	1.74	1.95	2.12	2.29	2.49	2.70	1.59	1.8058	1.98	2.14	2.33	2.52	2.74
	SDRL	0.65	0.71	0.72	0.71	0.71	0.72	0.76	0.70	0.74	0.75	0.73	0.74	0.75	0.80
	SERL	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.006
3.00	ARL	1.37	1.57	1.77	1.95	2.11	2.28	2.46	1.44	1.6265	1.79	1.96	2.13	2.31	2.49
	SDRL	0.56	0.63	0.65	0.63	0.62	0.63	0.66	0.60	0.66	0.67	0.66	0.66	0.65	0.69
	SERL	0.004	0.004	0.005	0.005	0.004	0.004	0.005	0.004	0.005	0.005	0.005	0.005	0.005	0.005
4.00	ARL	1.08	1.17	1.29	1.43	1.59	1.76	1.92	1.10	1.1942	1.30	1.43	1.58	1.75	1.91
	SDRL	0.27	0.38	0.46	0.51	0.52	0.49	0.44	0.30	0.40	0.47	0.51	0.52	0.49	0.46
	SERL	0.002	0.003	0.003	0.004	0.004	0.004	0.003	0.002	0.003	0.003	0.004	0.004	0.004	0.003
5.00	ARL	1.01	1.02	1.06	1.12	1.20	1.35	1.54	1.01	1.0313	1.06	1.11	1.21	1.35	1.51
	SDRL	0.09	0.15	0.24	0.33	0.40	0.48	0.50	0.10	0.17	0.24	0.32	0.41	0.48	0.50
	SERL	0.001	0.001	0.002	0.002	0.003	0.0034	0.004	0.001	0.001	0.0017	0.002	0.003	0.003	0.004

$d(\mu)$															
	λ	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	1.00	1.50	2.00	2.50	3.00	3.50	4.00
	$h_{\mathrm{AEWMA}^1_{\mathrm{PCA}}}$	7.58	6.68	5.68	5.28	4.70	4.65	4.6	5.93	5.33	4.85	4.59	4.49	4.42	4.42
0.00	ARL	199	200	200	201	197	201	200	199	201	200	201	202	197	197
	SDRL	207	203	203	198.75	195	196	194	203.64	201	201	198.41	199	193.8477	195
	SERL	1.465	1.433	1.434	1.405	1.377	1.388	1.373	1.440	1.423	1.425	1.403	1.409	1.371	1.381
0.25	ARL	88.63	91	89.54	86.77	86	88.47	87.94	89.31	89.30	89.09	88.10	87.92	86.16	87.26
	SDRL	88.68	90.19	88.68	82.19	81.27	83.46	81.51	90.10	86.70	85.80	84.22	83.88	82.4045	83.12
	SERL	0.627	0.638	0.627	0.581	0.575	0.590	0.576	0.637	0.613	0.607	0.596	0.593	0.583	0.588
0.50	ARL	29.51	30.49	30.20	29.97	29.77	30.29	30.05	29.70	29.82	30.31	29.86	29.80	29.54	29.51
	SDRL	26.73	27.41	26.18	25.13	24.61	24.24	23.54	26.73	26.04	25.57	24.70	24.41	24.1859	23.98
	SERL	0.189	0.194	0.185	0.178	0.174	0.171	0.166	0.189	0.184	0.181	0.175	0.173	0.171	0.170
0.75	ARL	13.50	13.85	14.15	14.50	14.62	15.09	14.97	13.69	13.95	14.23	14.56	14.62	14.61	14.66
	SDRL	10.58	10.57	10.40	10.04	9.52	9.58	9.41	10.62	10.44	10.18	10.01	9.88	9.593	9.86
	SERL	0.075	0.075	0.074	0.071	0.067	0.068	0.067	0.075	0.074	0.072	0.071	0.070	0.068	0.070
1.00	ARL	7.99	8.29	8.54	8.96	9.30	9.60	9.66	8.04	8.37	8.69	8.93	9.17	9.16	9.25
	SDRL	5.53	5.56	5.36	5.22	5.00	4.91	4.80	5.46	5.44	5.35	5.16	5.06	4.9861	4.95
	SERL	0.039	0.039	0.038	0.037	0.035	0.035	0.034	0.039	0.039	0.038	0.037	0.036	0.035	0.035
1.25	ARL	5.40	5.64	5.93	6.30	6.67	7.05	7.18	5.58	5.79	6.10	6.39	6.61	6.70	6.79
	SDRL	3.45	3.36	3.34	3.29	3.19	3.10	2.98	3.43	3.35	3.37	3.24	3.10	3.0321	3.01
	SERL	0.024	0.024	0.024	0.023	0.023	0.022	0.021	0.024	0.024	0.024	0.023	0.022	0.021	0.021
1.50	ARL	4.03	4.23	4.53	4.83	5.22	5.53	5.78	4.11	4.34	4.62	4.90	5.16	5.31	5.36
	SDRL	2.37	2.29	2.31	2.30	2.29	2.22	2.12	2.35	2.35	2.33	2.26	2.18	2.1094	2.03
	SERL	0.017	0.016	0.016	0.016	0.016	0.016	0.015	0.017	0.017	0.017	0.016	0.015	0.015	0.014
1.75	ARL	3.15	3.40	3.60	3.89	4.20	4.52	4.79	3.27	3.46	3.71	3.97	4.21	4.39	4.51
	SDRL	1.74	1.72	1.68	1.72	1.71	1.70	1.65	1.75	1.73	1.74	1.71	1.66	1.5854	1.55
	SERL	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.011	0.011
2.00	ARL	2.60	2.80	2.99	3.26	3.54	3.85	4.08	2.68	2.87	3.07	3.32	3.58	3.77	3.87
	SDRL	1.35	1.32	1.34	1.35	1.36	1.40	1.33	1.36	1.35	1.37	1.36	1.35	1.3002	1.23
	SERL	0.010	0.009	0.009	0.010	0.010	0.010	0.009	0.010	0.010	0.010	0.010	0.010	0.009	0.009

Table 1. *Cont.*

$d(\mu)$	λ	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	1.00	1.50	2.00	2.50	3.00	3.50	4.00
	$h_{AEWMA^1_{PCA}}$	7.58	6.68	5.68	5.28	4.70	4.65	4.6	5.93	5.33	4.85	4.59	4.49	4.42	4.42
2.25	ARL	2.20	2.39	2.58	2.78	3.05	3.31	3.54	2.27	2.44	2.62	2.84	3.07	3.26	3.41
	SDRL	1.08	1.07	1.08	1.08	1.11	1.13	1.12	1.10	1.09	1.10	1.11	1.11	1.0782	1.04
	SERL	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.007
2.50	ARL	1.91	2.09	2.25	2.45	2.66	2.91	3.13	1.96	2.10	2.29	2.49	2.70	2.89	3.04
	SDRL	0.89	0.90	0.89	0.90	0.93	0.96	0.96	0.92	0.91	0.92	0.92	0.95	0.9405	0.89
	SERL	0.006	0.006	0.006	0.006	0.007	0.007	0.007	0.007	0.006	0.007	0.007	0.007	0.007	0.006
2.75	ARL	1.69	1.85	2.01	2.19	2.39	2.61	2.81	1.73	1.88	2.03	2.23	2.40	2.60	2.74
	SDRL	0.75	0.78	0.78	0.78	0.79	0.81	0.83	0.78	0.79	0.79	0.81	0.81	0.8078	0.78
	SERL	0.005	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
3.00	ARL	1.52	1.68	1.82	1.97	2.17	2.37	2.53	1.56	1.68	1.83	1.99	2.18	2.34	2.49
	SDRL	0.64	0.68	0.69	0.69	0.70	0.71	0.72	0.67	0.69	0.71	0.72	0.72	0.712	0.70
	SERL	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
4.00	ARL	1.13	1.21	1.30	1.42	1.57	1.75	1.90	1.15	1.21	1.29	1.42	1.56	1.70	1.85
	SDRL	0.35	0.42	0.47	0.51	0.54	0.52	0.49	0.36	0.42	0.47	0.52	0.54	0.5341	0.51
	SERL	0.002	0.003	0.003	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.004	0.004	0.004	0.004
5.00	ARL	1.02	1.04	1.06	1.11	1.20	1.32	1.49	1.02	1.03	1.06	1.11	1.19	1.29	1.42
	SDRL	0.13	0.19	0.25	0.32	0.40	0.47	0.50	0.14	0.18	0.24	0.31	0.39	0.4564	0.50
	SERL	0.001	0.0013	0.0017	0.0022	0.003	0.003	0.004	0.001	0.001	0.002	0.002	0.0028	0.0032	0.004
$d(\mu)$	λ	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	1.00	1.50	2.00	2.50	3.00	3.50	4.00
	$h_{AEWMA^1_{PCA}}$	7.58	6.68	5.68	5.28	4.70	4.65	4.6	5.93	5.33	4.85	4.59	4.49	4.42	4.42
1.75	ARL	3.15	3.40	3.60	3.89	4.20	4.52	4.79	3.27	3.46	3.71	3.97	4.21	4.39	4.51
	SDRL	1.74	1.72	1.68	1.72	1.71	1.70	1.65	1.75	1.73	1.74	1.71	1.66	1.5854	1.55
	SERL	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.011	0.011
2.00	ARL	2.60	2.80	2.99	3.26	3.54	3.85	4.08	2.68	2.87	3.07	3.32	3.58	3.77	3.87
	SDRL	1.35	1.32	1.34	1.35	1.36	1.40	1.33	1.36	1.35	1.37	1.36	1.35	1.3002	1.23
	SERL	0.010	0.009	0.009	0.010	0.010	0.010	0.009	0.010	0.010	0.010	0.010	0.010	0.009	0.009
2.25	ARL	2.20	2.39	2.58	2.78	3.05	3.31	3.54	2.27	2.44	2.62	2.84	2.25	ARL	2.20
	SDRL	1.08	1.07	1.08	1.08	1.11	1.13	1.12	1.10	1.09	1.10	1.11		SDRL	1.08
	SERL	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008		SERL	0.008
2.50	ARL	1.91	2.09	2.25	2.45	2.66	2.91	3.13	1.96	2.10	2.29	2.49	2.50	ARL	1.91
	SDRL	0.89	0.90	0.89	0.90	0.93	0.96	0.96	0.92	0.91	0.92	0.92		SDRL	0.89
	SERL	0.006	0.006	0.006	0.006	0.007	0.007	0.007	0.007	0.006	0.007	0.007		SERL	0.006
2.75	ARL	1.69	1.85	2.01	2.19	2.39	2.61	2.81	1.73	1.88	2.03	2.23	2.40	2.60	2.74
	SDRL	0.75	0.78	0.78	0.78	0.79	0.81	0.83	0.78	0.79	0.79	0.81	0.81	0.8078	0.78
	SERL	0.005	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
3.00	ARL	1.52	1.68	1.82	1.97	2.17	2.37	2.53	1.56	1.68	1.83	1.99	2.18	2.34	2.49
	SDRL	0.64	0.68	0.69	0.69	0.70	0.71	0.72	0.67	0.69	0.71	0.72	0.72	0.712	0.70
	SERL	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
4.00	ARL	1.13	1.21	1.30	1.42	1.57	1.75	1.90	1.15	1.21	1.29	1.42	1.56	1.70	1.85
	SDRL	0.35	0.42	0.47	0.51	0.54	0.52	0.49	0.36	0.42	0.47	0.52	0.54	0.5341	0.51
	SERL	0.002	0.003	0.003	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.004	0.004	0.004	0.004
5.00	ARL	1.02	1.04	1.06	1.11	1.20	1.32	1.49	1.02	1.03	1.06	1.11	1.19	1.29	1.42
	SDRL	0.13	0.19	0.25	0.32	0.40	0.47	0.50	0.14	0.18	0.24	0.31	0.39	0.4564	0.50
	SERL	0.001	0.0013	0.0017	0.0022	0.003	0.003	0.004	0.001	0.001	0.002	0.002	0.0028	0.0032	0.004

Table 2. ARLs, SDRLs, and SERLs property of proposed $AEWMA^1_{PCA}$ control chart at $ARL_0 = 200$, $k = 1.00$, $p = 3$, and PCA = 2.

$d(\mu)$	λ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10	0.10	0.10
	γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	1.00	1.50	2.00	2.50	3.00	3.50	4.00
	$h_{AEWMA^1_{PCA}}$	17.7	11.2	5	2.9	2.6	2.6	2.55	11.95	7.95	4.55	3.65	3.48	3.42	3.44
0.00	ARL	198	200	201	203	202	205	202	199	202	199	204	202	197	197
	SDRL	219	211	190	190	181	182	178	206	208	197	198	191	190	187
	SERL	1.548	1.493	1.340	1.340	1.276	1.284	1.261	1.459	1.473	1.392	1.400	1.352	1.346	1.322
0.25	ARL	121.97	123.68	128.73	119.88	120.23	121.85	121.00	127.54	131.04	124.33	121.59	118.09	117.67	116.10
	SDRL	136.95	132.97	108.80	108.80	102.48	103.78	102.64	134.14	133.83	121.72	116.71	110.39	110.61	107.48
	SERL	0.968	0.940	0.769	0.769	0.725	0.734	0.726	0.949	0.946	0.861	0.825	0.781	0.782	0.760
0.50	ARL	47.73	50.61	52.59	49.82	50.37	50.58	50.33	52.07	53.94	50.39	48.14	47.49	46.74	46.98
	SDRL	54.32	54.08	41.25	41.25	38.87	38.70	38.18	54.84	54.82	47.89	42.87	42.11	40.66	40.33
	SERL	0.384	0.382	0.292	0.292	0.275	0.274	0.270	0.388	0.388	0.339	0.303	0.298	0.288	0.285
0.75	ARL	19.38	21.42	22.72	23.02	23.76	24.26	24.09	22.05	23.39	22.32	21.78	21.55	21.55	21.62
	SDRL	21.12	21.90	17.64	17.64	16.77	16.76	16.65	22.90	23.11	19.68	17.68	16.95	16.94	16.89
	SERL	0.149	0.155	0.125	0.125	0.119	0.119	0.118	0.162	0.163	0.139	0.125	0.120	0.120	0.119
1.00	ARL	9.51	10.73	11.79	12.71	13.25	13.78	13.83	10.73	11.49	11.62	11.74	11.86	11.91	12.09
	SDRL	9.37	9.81	9.10	9.10	8.69	8.68	8.52	10.33	10.33	9.51	8.55	8.35	8.23	8.31
	SERL	0.066	0.069	0.064	0.064	0.061	0.061	0.060	0.073	0.073	0.067	0.061	0.059	0.058	0.059
1.25	ARL	5.79	6.43	7.14	7.87	8.54	8.93	9.09	6.31	6.83	7.20	7.54	7.71	7.75	7.88
	SDRL	4.93	5.12	5.18	5.18	4.99	4.89	4.86	5.36	5.44	5.19	4.89	4.69	4.60	4.57
	SERL	0.035	0.036	0.037	0.037	0.035	0.035	0.034	0.038	0.039	0.037	0.035	0.033	0.033	0.032
1.50	ARL	3.96	4.46	4.92	5.55	6.14	6.53	6.71	4.24	4.64	5.00	5.35	5.60	5.72	5.76
	SDRL	2.90	3.10	3.30	3.30	3.21	3.11	2.97	3.10	3.22	3.19	3.03	2.90	2.83	2.82
	SERL	0.021	0.022	0.023	0.023	0.023	0.022	0.021	0.022	0.023	0.023	0.021	0.021	0.020	0.020

Table 2. *Cont.*

$d(\mu)$															
	λ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10	0.10	0.10
	γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	1.00	1.50	2.00	2.50	3.00	3.50	4.00
$d(\mu)$	$h_{AEWMA^1_{PCA}}$	17.7	11.2	5	2.9	2.6	2.6	2.55	11.95	7.95	4.55	3.65	3.48	3.42	3.44
1.75	ARL	2.99	3.36	3.72	4.21	4.71	5.12	5.34	3.19	3.50	3.78	4.14	4.40	4.55	4.60
	SDRL	1.96	2.07	2.21	2.21	2.21	2.17	2.04	2.08	2.13	2.16	2.08	1.97	1.90	1.85
	SERL	0.014	0.015	0.016	0.016	0.016	0.015	0.015	0.015	0.015	0.015	0.015	0.014	0.014	0.013
2.00	ARL	2.40	2.70	2.99	3.42	3.82	4.18	4.47	2.55	2.79	3.02	3.36	3.64	3.82	3.91
	SDRL	1.40	1.47	1.64	1.64	1.66	1.64	1.57	1.51	1.53	1.54	1.53	1.48	1.40	1.38
	SERL	0.010	0.010	0.012	0.012	0.012	0.012	0.011	0.011	0.011	0.011	0.011	0.011	0.010	0.010
2.25	ARL	2.01	2.26	2.51	2.84	3.19	3.54	3.84	2.13	2.34	2.53	2.84	3.12	3.32	3.42
	SDRL	1.07	1.14	1.23	1.23	1.28	1.32	1.27	1.14	1.18	1.18	1.20	1.17	1.12	1.07
	SERL	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.008	0.008	0.008	0.009	0.008	0.008	0.008
2.50	ARL	1.73	1.96	2.17	2.47	2.77	3.08	3.35	1.82	1.99	2.18	2.46	2.72	2.92	3.06
	SDRL	0.84	0.91	0.98	0.98	1.03	1.08	1.06	0.88	0.93	0.95	0.99	0.96	0.93	0.89
	SERL	0.006	0.006	0.007	0.007	0.007	0.008	0.008	0.006	0.007	0.007	0.007	0.007	0.007	0.006
2.75	ARL	1.54	1.73	1.91	2.17	2.45	2.72	2.97	1.61	1.77	1.93	2.17	2.43	2.63	2.76
	SDRL	0.70	0.76	0.82	0.82	0.86	0.88	0.89	0.74	0.78	0.79	0.81	0.83	0.78	0.75
	SERL	0.005	0.005	0.006	0.006	0.006	0.006	0.006	0.005	0.006	0.006	0.006	0.006	0.006	0.005
3.00	ARL	1.41	1.56	1.73	1.95	2.21	2.44	2.68	1.46	1.58	1.74	1.95	2.19	2.39	2.54
	SDRL	0.60	0.65	0.70	0.70	0.73	0.74	0.79	0.62	0.66	0.69	0.71	0.72	0.69	0.67
	SERL	0.004	0.005	0.005	0.005	0.005	0.005	0.006	0.004	0.005	0.005	0.005	0.005	0.005	0.005
4.00	ARL	1.09	1.16	1.25	1.40	1.60	1.80	1.98	1.11	1.17	1.24	1.40	1.58	1.77	1.94
	SDRL	0.29	0.37	0.51	0.51	0.54	0.51	0.48	0.32	0.39	0.44	0.51	0.54	0.52	0.46
	SERL	0.002	0.003	0.004	0.004	0.004	0.004	0.003	0.002	0.003	0.003	0.004	0.004	0.004	0.003
5.00	ARL	1.01	1.02	1.04	1.10	1.22	1.38	1.57	1.01	1.02	1.05	1.10	1.21	1.35	1.53
	SDRL	0.10	0.14	0.30	0.30	0.41	0.49	0.50	0.11	0.15	0.21	0.31	0.40	0.48	0.51
	SERL	0.001	0.001	0.002	0.002	0.003	0.003	0.004	0.001	0.001	0.002	0.002	0.003	0.003	0.004
	λ	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	1.00	1.50	2.00	2.50	3.00	3.50	4.00
$d(\mu)$	$h_{AEWMA^1_{PCA}}$	8.24	6.04	4.64	4.24	4.14	4.14	4.14	6.74	5.34	4.64	4.44	4.42	4.42	4.42
0.00	ARL	203	200	203	198	199	199	200	206	198	202	201	204	201	202
	SDRL	209	203	203	197	195	194	195	209	200	200	198	201	198	198
	SERL	1.477	1.437	1.436	1.391	1.376	1.375	1.377	1.476	1.413	1.418	1.401	1.418	1.404	1.397
0.25	ARL	130.66	130.19	125.39	120.89	119.82	119.94	119.37	132.39	126.77	124.68	121.59	123.72	122.94	125.42
	SDRL	132.57	131.41	122.87	117.32	116.10	116.01	116.41	132.72	126.62	123.46	120.25	122.86	121.03	122.49
	SERL	0.937	0.929	0.869	0.830	0.821	0.820	0.823	0.938	0.895	0.873	0.850	0.869	0.856	0.866
0.50	ARL	55.27	54.57	50.49	48.91	47.53	48.16	48.15	56.56	53.09	50.57	50.31	49.29	49.88	49.68
	SDRL	56.34	53.92	48.56	46.24	44.07	44.78	45.40	56.58	51.81	48.65	48.05	47.49	47.28	47.79
	SERL	0.398	0.381	0.343	0.327	0.312	0.317	0.321	0.400	0.366	0.344	0.340	0.336	0.334	0.338
0.75	ARL	23.55	23.51	22.08	21.46	20.94	21.32	21.22	24.31	23.03	22.27	21.69	21.82	21.87	21.88
	SDRL	23.09	22.61	19.85	18.58	17.98	18.51	18.33	23.33	21.41	20.15	19.73	19.53	19.73	19.80
	SERL	0.163	0.160	0.140	0.131	0.127	0.131	0.130	0.165	0.151	0.143	0.140	0.138	0.140	0.140
1.00	ARL	11.77	11.85	11.56	11.31	11.37	11.30	11.43	12.06	11.66	11.40	11.33	11.31	11.54	11.38
	SDRL	10.81	10.32	9.40	8.76	8.57	8.62	8.77	10.75	10.01	9.61	8.99	9.14	9.26	9.25
	SERL	0.076	0.073	0.067	0.062	0.061	0.061	0.062	0.076	0.071	0.068	0.064	0.065	0.066	0.065
1.25	ARL	6.81	7.00	7.09	7.21	7.16	7.23	7.28	7.08	7.05	7.06	7.12	7.07	7.11	7.06
	SDRL	5.56	5.37	4.99	4.82	4.67	4.69	4.76	5.75	5.37	5.10	4.96	4.88	4.86	4.81
	SERL	0.039	0.038	0.035	0.034	0.033	0.033	0.034	0.041	0.038	0.036	0.035	0.035	0.034	0.034
1.50	ARL	4.56	4.79	5.02	5.12	5.17	5.21	5.25	4.76	4.84	4.92	5.00	5.04	5.08	5.01
	SDRL	3.27	3.25	3.09	2.90	2.79	2.86	2.85	3.39	3.19	3.03	2.97	2.90	2.95	2.88
	SERL	0.023	0.023	0.022	0.021	0.020	0.020	0.020	0.024	0.023	0.021	0.021	0.021	0.021	0.020
1.75	ARL	3.40	3.61	3.79	4.01	4.07	4.10	4.13	3.53	3.63	3.79	3.87	3.95	3.93	3.96
	SDRL	2.14	2.14	2.05	1.98	1.89	1.85	1.86	2.19	2.14	2.05	1.93	1.91	1.87	1.91
	SERL	0.015	0.015	0.015	0.014	0.013	0.013	0.013	0.016	0.015	0.015	0.014	0.014	0.013	0.014
2.00	ARL	2.70	2.87	3.09	3.28	3.40	3.45	3.46	2.81	2.92	3.06	3.19	3.27	3.30	3.28
	SDRL	1.54	1.56	1.54	1.44	1.38	1.36	1.34	1.57	1.54	1.47	1.40	1.37	1.36	1.33
	SERL	0.011	0.011	0.011	0.010	0.010	0.010	0.010	0.011	0.011	0.010	0.010	0.010	0.010	0.009
2.25	ARL	2.24	2.39	2.59	2.81	2.95	3.00	3.02	2.32	2.44	2.60	2.73	2.83	2.85	2.84
	SDRL	1.18	1.20	1.18	1.15	1.08	1.06	1.04	1.18	1.19	1.17	1.09	1.06	1.02	1.02
	SERL	0.008	0.009	0.008	0.008	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.008	0.007	0.007
2.50	ARL	1.92	2.05	2.23	2.45	2.61	2.68	2.69	2.00	2.09	2.26	2.41	2.49	2.53	2.55
	SDRL	0.93	0.95	0.96	0.95	0.89	0.85	0.82	0.95	0.96	0.95	0.90	0.85	0.81	0.81
	SERL	0.007	0.007	0.007	0.007	0.006	0.006	0.006	0.007	0.007	0.007	0.006	0.006	0.006	0.006
2.75	ARL	1.71	1.81	1.97	2.17	2.34	2.42	2.46	1.77	1.84	1.99	2.14	2.26	2.32	2.33
	SDRL	0.78	0.80	0.81	0.81	0.76	0.71	0.69	0.80	0.80	0.81	0.78	0.73	0.68	0.67
	SERL	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.006	0.006	0.006	0.006	0.005	0.005	0.005
3.00	ARL	1.53	1.61	1.77	1.95	2.12	2.24	2.28	1.57	1.66	1.78	1.94	2.06	2.15	2.16
	SDRL	0.66	0.68	0.71	0.70	0.67	0.62	0.57	0.67	0.70	0.71	0.70	0.64	0.58	0.57
	SERL	0.005	0.005	0.005	0.005	0.005	0.004	0.004	0.005	0.005	0.005	0.005	0.005	0.004	0.004
4.00	ARL	1.13	1.18	1.26	1.39	1.56	1.72	1.85	1.16	1.20	1.28	1.40	1.54	1.68	1.69
	SDRL	0.35	0.39	0.45	0.51	0.53	0.51	0.43	0.37	0.41	0.46	0.51	0.53	0.50	0.49
	SERL	0.003	0.003	0.003	0.004	0.004	0.004	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.004
5.00	ARL	1.02	1.03	1.05	1.10	1.19	1.32	1.46	1.02	1.03	1.05	1.10	1.18	1.29	1.31
	SDRL	0.14	0.17	0.22	0.30	0.39	0.47	0.50	0.15	0.18	0.23	0.30	0.39	0.45	0.46
	SERL	0.001	0.001	0.002	0.002	0.003	0.003	0.004	0.001	0.001	0.002	0.002	0.003	0.003	0.003

Table 3. ARLs, SDRLs, and SERLs property of proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart at $\text{ARL}_0 = 200$, $k = 1.50$, $p = 3$, and PCA = 2.

$d(\mu)$															
	λ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10	0.10	0.10
	γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	1.00	1.50	2.00	2.50	3.00	3.50	4.00
	$\text{H}_{\text{AEWMA}^1_{\text{PCA}}}$	15.12	4.82	2.34	2.14	2.14	2.14	2.14	10.14	4.44	3.06	2.98	2.98	2.98	2.98
1.75	ARL	4.14	4.72	5.21	5.48	5.57	5.60	5.57	4.36	4.6358	4.73	4.85	4.82	4.87	4.84
	SDRL	3.35	3.58	3.24	3.10	3.02	3.05	3.01	3.57	3.35	2.99	2.95	2.88	2.95	2.94
	SERL	0.024	0.025	0.023	0.022	0.021	0.022	0.021	0.025	0.024	0.021	0.021	0.020	0.021	0.021
2.00	ARL	3.06	3.48	3.96	4.23	4.37	4.38	4.38	3.17	3.4729	3.62	3.76	3.76	3.76	3.76
	SDRL	2.24	2.37	2.26	2.12	2.09	2.06	2.05	2.34	2.30	2.07	2.00	1.97	2.00	1.97
	SERL	0.016	0.017	0.016	0.015	0.015	0.015	0.015	0.017	0.016	0.015	0.014	0.014	0.014	0.014
2.25	ARL	2.41	2.75	3.18	3.45	3.58	3.61	3.62	2.48	2.7136	2.94	3.08	3.09	3.10	3.11
	SDRL	1.60	1.69	1.67	1.57	1.52	1.50	1.47	1.64	1.67	1.51	1.47	1.41	1.44	1.42
	SERL	0.011	0.012	0.012	0.011	0.011	0.011	0.010	0.012	0.012	0.011	0.010	0.010	0.010	0.010
2.50	ARL	1.98	2.23	2.61	2.93	3.06	3.11	3.13	2.03	2.2565	2.48	2.63	2.66	2.66	2.66
	SDRL	1.18	1.25	1.30	1.23	1.16	1.12	1.11	1.22	1.26	1.18	1.12	1.08	1.08	1.07
	SERL	0.008	0.009	0.009	0.009	0.008	0.008	0.008	0.009	0.009	0.008	0.008	0.008	0.008	0.008
2.75	ARL	1.69	1.92	2.23	2.51	2.69	2.76	2.78	1.73	1.9049	2.14	2.30	2.33	2.35	2.35
	SDRL	0.91	1.00	1.04	0.99	0.94	0.90	0.88	0.93	0.98	0.96	0.88	0.84	0.86	0.86
	SERL	0.006	0.007	0.007	0.007	0.007	0.006	0.006	0.007	0.007	0.007	0.006	0.006	0.006	0.006
3.00	ARL	1.49	1.66	1.95	2.23	2.42	2.50	2.52	1.52	1.6569	1.88	2.07	2.12	2.12	2.12
	SDRL	0.72	0.80	0.86	0.86	0.80	0.74	0.71	0.75	0.79	0.80	0.75	0.70	0.72	0.71
	SERL	0.005	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.006	0.006	0.005	0.005	0.005	0.005
4.00	ARL	1.11	1.17	1.31	1.52	1.72	1.89	1.98	1.11	1.1724	1.30	1.48	1.54	1.55	1.55
	SDRL	0.32	0.39	0.49	0.56	0.56	0.49	0.39	0.33	0.39	0.48	0.53	0.53	0.54	0.53
	SERL	0.002	0.003	0.004	0.004	0.004	0.003	0.003	0.002	0.003	0.003	0.004	0.004	0.004	0.004
5.00	ARL	1.01	1.02	1.07	1.15	1.29	1.4703	1.67	1.01	1.0269	1.06	1.14	1.179	1.19	1.18
	SDRL	0.11	0.15	0.25	0.36	0.46	0.50	0.48	0.12	0.16	0.24	0.34	0.38	0.39	0.39
	SERL	0.001	0.001	0.002	0.0025	0.003	0.004	0.003	0.0008	0.0011	0.0017	0.002	0.003	0.003	0.003
$d(\mu)$	λ	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	1.00	1.50	2.00	2.50	3.00	3.50	4.00
	$\text{H}_{\text{AEWMA}^1_{\text{PCA}}}$	7.62	4.65	3.83	3.75	3.75	3.75	3.75	6.75	4.75	4.26	4.16	4.16	4.16	4.16
0.00	ARL	196	201	201	199	200	202	200	205	198	202	197	196	200	197
	SDRL	199	201	199	195	194	198	197	204	197	199	195	195	197	194
	SERL	1.409	1.422	1.410	1.381	1.374	1.400	1.393	1.442	1.393	1.406	1.379	1.379	1.392	1.371
0.25	ARL	151.06	150	141.20	141.67	142	141.53	141.80	155.79	146.12	146.81	141.88	143.41	141.64	144.00
	SDRL	152	150	140	137	139	138	138	156	146	143	139	141	139	141
	SERL	1.071	1.058	0.989	0.970	0.982	0.979	0.973	1.105	1.029	1.012	0.986	0.998	0.985	0.997
0.50	ARL	83.87	77.17	70.04	67.46	67.89	69.22	67.85	85.70	74.84	71.82	69.79	69.97	71.32	70.23
	SDRL	84.13	76.28	67.92	64.62	65.32	66.14	65.56	85.92	73.83	69.66	68.52	68.84	69.414	67.99
	SERL	0.595	0.539	0.480	0.457	0.462	0.468	0.464	0.608	0.522	0.493	0.485	0.487	0.491	0.481
0.75	ARL	40.94	37.17	32.42	32.18	32.21	31.94	32.05	42.38	36.57	34.56	33.63	33.00	33.37	33.66
	SDRL	41.46	35.75	30.46	29.68	29.69	29.32	29.87	42.46	35.38	32.78	31.72	31.30	31.5244	31.85
	SERL	0.293	0.253	0.215	0.210	0.210	0.207	0.211	0.300	0.250	0.232	0.224	0.221	0.223	0.225
1.00	ARL	20.55	18.90	16.59	16.66	16.73	16.64	16.81	21.27	18.79	17.65	17.44	17.17	17.28	17.34
	SDRL	20.22	17.42	14.59	14.59	14.43	14.43	14.57	20.73	17.49	15.92	15.83	15.40	15.6818	15.74
	SERL	0.143	0.123	0.103	0.103	0.102	0.102	0.103	0.147	0.124	0.113	0.112	0.109	0.111	0.111
1.25	ARL	11.25	10.56	9.74	9.75	9.72	9.70	9.69	11.66	10.43	10.05	9.87	9.92	9.84	9.88
	SDRL	10.55	8.90	7.89	7.89	7.82	7.76	7.77	10.84	9.02	8.39	8.30	8.30	8.207	8.24
	SERL	0.075	0.063	0.056	0.056	0.055	0.055	0.055	0.077	0.064	0.059	0.059	0.059	0.058	0.058
1.50	ARL	6.78	6.73	6.35	6.35	6.30	6.36	6.35	7.11	6.56	6.37	6.40	6.38	6.41	6.37
	SDRL	5.96	5.33	4.62	4.65	4.60	4.66	4.62	6.13	5.20	4.83	4.88	4.86	4.8534	4.84
	SERL	0.042	0.038	0.033	0.033	0.033	0.033	0.033	0.043	0.037	0.034	0.035	0.034	0.034	0.034
1.75	ARL	4.61	4.61	4.53	4.53	4.55	4.52	4.55	4.78	4.59	4.49	4.52	4.52	4.51	4.46
	SDRL	3.71	3.31	3.00	2.96	2.93	2.95	2.90	3.78	3.29	3.05	3.07	3.05	3.0535	3.00
	SERL	0.026	0.023	0.021	0.021	0.021	0.021	0.021	0.027	0.023	0.022	0.022	0.022	0.022	0.021
2.00	ARL	3.32	3.44	3.46	3.51	3.46	3.50	3.49	3.49	3.41	3.45	3.41	3.43	3.42	3.42
	SDRL	2.38	2.22	2.04	2.05	1.99	2.01	2.02	2.51	2.22	2.08	2.05	2.05	2.0452	2.08
	SERL	0.017	0.016	0.014	0.015	0.014	0.014	0.014	0.018	0.016	0.015	0.015	0.015	0.015	0.015
2.25	ARL	2.61	2.73	2.79	2.85	2.84	2.84	2.83	2.67	2.68	2.78	2.75	2.75	2.78	2.76
	SDRL	1.72	1.62	1.48	1.46	1.46	1.47	1.45	1.72	1.56	1.48	1.48	1.47	1.5054	1.46
	SERL	0.012	0.012	0.011	0.010	0.010	0.010	0.010	0.012	0.011	0.010	0.011	0.010	0.011	0.010
2.50	ARL	2.12	2.26	2.37	2.40	2.40	2.39	2.40	2.18	2.26	2.36	2.32	2.33	2.34	2.34
	SDRL	1.27	1.24	1.14	1.12	1.11	1.12	1.11	1.29	1.23	1.14	1.12	1.13	1.1392	1.13
	SERL	0.009	0.009	0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.008	0.008	0.008	0.008	0.008
2.75	ARL	1.80	1.92	2.05	2.09	2.10	2.10	2.10	1.85	1.92	2.04	2.03	2.04	2.02	2.03
	SDRL	0.98	0.97	0.91	0.89	0.90	0.90	0.90	1.00	0.98	0.90	0.90	0.92	0.897	0.91
	SERL	0.007	0.007	0.006	0.006	0.006	0.006	0.006	0.007	0.007	0.006	0.006	0.007	0.006	0.006
3.00	ARL	1.57	1.67	1.83	1.87	1.87	1.87	1.87	1.61	1.69	1.81	1.79	1.80	1.80	1.79
	SDRL	0.78	0.80	0.76	0.74	0.75	0.76	0.75	0.79	0.80	0.76	0.75	0.76	0.7517	0.76
	SERL	0.006	0.006	0.005	0.005	0.005	0.005	0.005	0.006	0.006	0.005	0.005	0.005	0.005	0.005
4.00	ARL	1.13	1.18	1.28	1.32	1.32	1.31	1.31	1.15	1.19	1.27	1.26	1.27	1.27	1.27
	SDRL	0.35	0.40	0.47	0.49	0.48	0.48	0.48	0.37	0.41	0.46	0.46	0.46	0.4585	0.46
	SERL	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.0026	0.003	0.003	0.003	0.003	0.003	0.003
5.00	ARL	1.02	1.02	1.06	1.07	1.07	1.0658	1.0685	1.02	1.0272	1.05	1.05	1.06	1.05	1.05
	SDRL	0.12	0.16	0.23	0.25	0.25	0.25	0.25	0.14	0.16	0.22	0.22	0.23	0.2177	0.23
	SERL	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.0016	0.002	0.0016	0.002	0.002
1.75	ARL	4.61	4.61	4.53	4.53	4.55	4.52	4.55	4.78	4.59	4.49	4.52	4.52	4.51	4.46
	SDRL	3.71	3.31	3.00	2.96	2.93	2.95	2.90	3.78	3.29	3.05	3.07	3.05	3.0535	3.00
	SERL	0.026	0.023	0.021	0.021	0.021	0.021	0.021	0.027	0.023	0.022	0.022	0.022	0.022	0.021

Table 3. *Cont.*

$d(\mu)$	λ / γ / $H_{AEWMA^1_{PCA}}$	0.20 / 1.00 / 7.62	0.20 / 1.50 / 4.65	0.20 / 2.00 / 3.83	0.20 / 2.50 / 3.75	0.20 / 3.00 / 3.75	0.20 / 3.50 / 3.75	0.20 / 4.00 / 3.75	0.30 / 1.00 / 6.75	0.30 / 1.50 / 4.75	0.30 / 2.00 / 4.26	0.30 / 2.50 / 4.16	0.30 / 3.00 / 4.16	0.30 / 3.50 / 4.16	0.30 / 4.00 / 4.16
	ARL	3.32	3.44	3.46	3.51	3.46	3.50	3.49	3.49	3.41	3.45	3.41	3.43	3.42	3.42
2.00	SDRL	2.38	2.22	2.04	2.05	1.99	2.01	2.02	2.51	2.22	2.08	2.05	2.05	2.0452	2.08
	SERL	0.017	0.016	0.014	0.015	0.014	0.014	0.014	0.018	0.016	0.015	0.015	0.015	0.015	0.015
	ARL	2.61	2.73	2.79	2.85	2.84	2.84	2.83	2.67	2.68	2.78	2.75	2.75	2.78	2.76
2.25	SDRL	1.72	1.62	1.48	1.46	1.46	1.47	1.45	1.72	1.56	1.48	1.48	1.47	1.5054	1.46
	SERL	0.012	0.012	0.011	0.010	0.010	0.010	0.010	0.012	0.011	0.010	0.011	0.010	0.011	0.010
	ARL	2.12	2.26	2.37	2.40	2.40	2.39	2.40	2.18	2.26	2.36	2.32	2.33	2.34	2.34
2.50	SDRL	1.27	1.24	1.14	1.12	1.11	1.12	1.11	1.29	1.23	1.14	1.12	1.13	1.1392	1.13
	SERL	0.009	0.009	0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.008	0.008	0.008	0.008	0.008
	ARL	1.80	1.92	2.05	2.09	2.10	2.10	2.10	1.85	1.92	2.04	2.03	2.04	2.02	2.03
2.75	SDRL	0.98	0.97	0.91	0.89	0.90	0.90	0.90	1.00	0.98	0.90	0.90	0.92	0.897	0.91
	SERL	0.007	0.007	0.006	0.006	0.006	0.006	0.006	0.007	0.007	0.006	0.006	0.007	0.006	0.006
	ARL	1.57	1.67	1.83	1.87	1.87	1.87	1.87	1.61	1.69	1.81	1.79	1.80	1.80	1.79
3.00	SDRL	0.78	0.80	0.76	0.74	0.75	0.76	0.75	0.79	0.80	0.76	0.75	0.76	0.7517	0.76
	SERL	0.006	0.006	0.005	0.005	0.005	0.005	0.005	0.006	0.006	0.005	0.005	0.005	0.005	0.005
	ARL	1.13	1.18	1.28	1.32	1.32	1.31	1.31	1.15	1.19	1.27	1.26	1.27	1.27	1.27
4.00	SDRL	0.35	0.40	0.47	0.49	0.48	0.48	0.48	0.37	0.41	0.46	0.46	0.46	0.4585	0.46
	SERL	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.0026	0.003	0.003	0.003	0.003	0.003	0.003
	ARL	1.02	1.02	1.06	1.07	1.07	1.0658	1.0685	1.02	1.0272	1.05	1.05	1.06	1.05	1.05
5.00	SDRL	0.12	0.16	0.23	0.25	0.25	0.25	0.25	0.14	0.16	0.22	0.22	0.23	0.2177	0.23
	SERL	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.0016	0.002	0.0016	0.002	0.002

5. Results and Discussion

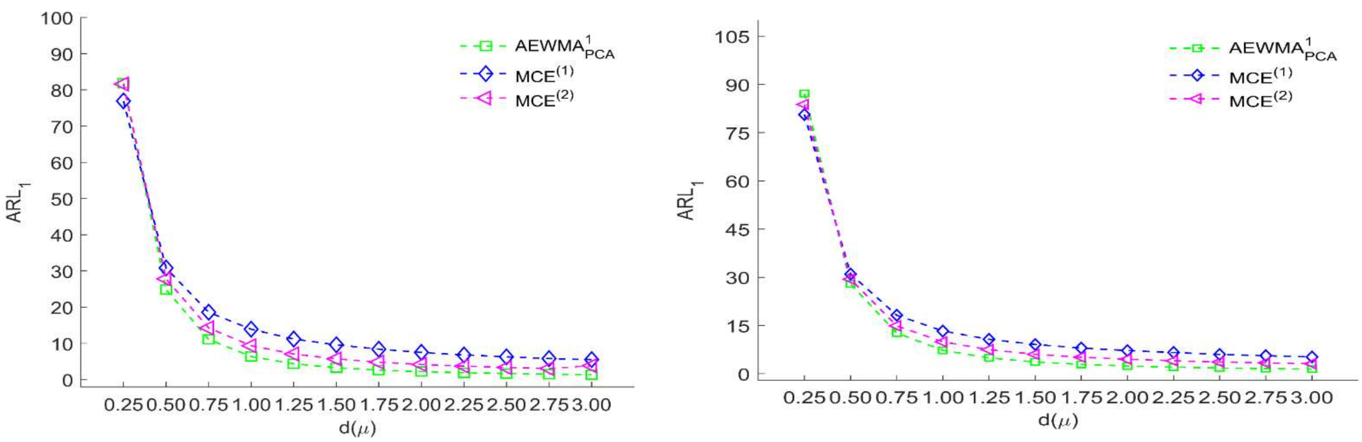
This section provides a comprehensive assessment of the proposed $AEWMA^1_{PCA}$ control chart against the $MCE^{(1)}$ and $MCE^{(2)}$ control charts designed by [12], the MCUSUM control chart offered by [7], MC_1 control chart proposed by [8], MC_1^{PCA} and $MCUSUM_{PCA}$ control charts suggested by [23], PC-chart recommended by [21], Hotelling's T^2 presented by [5], $MEC^{(1)}$ and $MEC^{(2)}$ initiated by [56], and EWMA-PC by [14].

5.1. Proposed vs. $MCE^{(1)}$ and $MCE^{(2)}$ Control Charts

The proposed $AEWMA^1_{PCA}$ control chart has superior performance as compared to the $MCE^{(1)}$ and $MCE^{(2)}$ control charts. For example, at $d(\mu) = 0.50$, $\lambda = 0.05$, $k = 0.50$, and $\gamma = 1$, the proposed $AEWMA^1_{PCA}$ control chart produces ARL_1 24.46, while the $MCE^{(1)}$ and $MCE^{(2)}$ control charts keep ARL_1 30.8 and 27.8, respectively (see Table 1 vs. Table 4 and Figure 2a). Similarly, as λ increases (i.e., $\lambda > 0.05$), the proposed $AEWMA^1_{PCA}$ control chart still identifies out-of-control signals earlier against the $MCE^{(1)}$ and $MCE^{(2)}$ control charts. For instance, the 27.89 is the ARL_1 value of the proposed $AEWMA^1_{PCA}$ control chart when $d(\mu) = 0.50$, $\lambda = 0.10$, $k = 0.50$, and $\gamma = 1$ (see Table 1 vs. Table 4 and Figure 2b), while the $MCE^{(1)}$ and $MCE^{(2)}$ control charts have larger ARL_1 values for the same amount of parameters values. In short, the proposed control chart outperforms when $d(\mu) \geq 0.50$. Additionally, the SERL and SDRL of the proposed $AEWMA^1_{PCA}$ and the $MCE^{(1)}$ and $MCE^{(2)}$ control charts are also given in the respective tables to gain more understanding about their behavior. Similarly, the proposed $AEWMA^1_{PCA}$ control chart also shows superior performance in terms of overall assessment. For instance, at $\lambda = 0.50$, $\gamma = 1$, and $k = 0.50$, the 7.70, 1.00, and 1.00 are EQL, RARL, and PCI, respectively, of the proposed $AEWMA^1_{PCA}$ control chart, and these are smaller than the EQL, RARL, and PCI of the $MCE^{(1)}$ and $MCE^{(2)}$ control charts (see Table 5). Even though, for $\gamma = 1.5, \ldots, 4$ and $\lambda = 0.10$, the proposed $AEWMA^1_{PCA}$ control chart still produces smaller EQL, RARL, and PCI relative to the $MCE^{(1)}$ and $MCE^{(2)}$ control charts (see Table 5).

Table 4. ARL/SERL values of $\mathrm{MCE}^{(1)}$ and $\mathrm{MCE}^{(2)}$ control charts at $k = 0.50d(\mu)$.

$p = 3$	λ	0.05	0.1	0.25	0.5	0.75
	$H_{MEC^{(1)}}$	9	7.94	7.44	5.64	4.52
$d(\mu)$	$\mu_1(\delta, 0, 0)$	$\mathrm{MCE}^{(1)}$				
0.00	0.00	202.8/1.906	199.3/1.916	199.6/1.908	199/1.860	205/1.884
0.25	0.25	76.9/0.598	80.6/0.692	84.5/0.754	85.3/0.741	87.9/0.763
0.50	0.50	30.8/0.175	31.0/0.196	31.4/0.211	31.5/0.215	31.7/0.224
0.75	0.75	18.6/0.073	18.2/0.077	17.3/0.085	16.9/0.090	16.9/0.091
1.00	1.00	13.9/0.042	13.3/0.042	12.1/0.043	11.4/0.049	11.3/0.048
1.25	1.25	11.2/0.028	10.7/0.27	9.5/0.028	8.7/0.030	8.5/0.030
1.50	1.50	9.6/0.021	9.1/0.020	8.0/0.019	7.1/0.021	6.9/0.0216
1.75	1.75	8.4/0.016	8.0/0.016	7.0/0.015	6.1/0.016	5.8/0.016
2.00	2.00	7.5/0.013	7.2/0.013	6.2/0.012	5.3/0.021	5.0/0.012
2.25	2.25	6.8/0.011	6.6/0.011	5.6/0.010	4.8/0.010	4.4/0.010
2.50	2.50	6.3/0.010	6.0/0.009	5.2/0.009	4.4/0.008	4.0/0.008
2.75	2.75	5.8/0.009	5.6/0.008	4.8/0.008	4.0/0.007	3.7/0.007
3.00	3.00	5.5/0.008	5.2/0.007	4.5/0.007	3.8/0.007	3.4/0.006
p = 3 and PC = 2	λ	0.05	0.1	0.25	0.5	0.75
	$h_{MCE^{(2)}}$	3.60	4.32	4.25	3.9	3.33
$d(\mu)$	$\mu_1(\delta, 0, 0)$	$\mathrm{MCE}^{(2)}$				
0.00	0.00	199/1.98	199/1.973	201/1.94	198/1.97	200/2.041
0.25	0.25	81.6/0.763	83.8/0.780	88.4/0.84	90.4/0.855	91.2/0.868
0.50	0.50	27.8/0.218	29.4/0.232	30.4/0.24	30.7/0.248	31.5/0.254
0.75	0.75	14.27/0.089	14.9/0.092	15.2/0.09	14.9/0.097	15.2/0.100
1.00	1.00	9.35/0.047	9.92/0.046	9.88/0.047	9.50/0.051	9.97/0.052
1.25	1.25	7.06/0.030	7.47/0.029	7.37/0.029	6.97/0.030	6.80/0.030
1.50	1.50	5.71/0.021	6.05/0.020	5.97/0.02	5.55/0.020	5.47/0.021
1.75	1.75	4.81/0.016	5.14/0.015	5.06/0.015	4.70/0.015	4.45/0.015
2.00	2.00	4.17/0.012	4.50/0.012	4.46/0.012	4.07/0.011	3.84/0.012
2.25	2.25	3.72/0.010	4.01/0.010	3.99/0.01	3.64/0.009	3.39/0.010
2.50	2.50	3.34/0.009	3.65/0.009	3.62/0.008	3.28/0.008	3.06/0.008
2.75	2.75	3.06/0.008	3.36/0.007	3.34/0.007	3.03/0.007	2.79/0.007
3.00	3.00	3.83/0.007	3.10/0.006	3.10/0.007	2.80/0.006	2.58/0.006



(**a**) (**b**)

Figure 2. (**a**): ARL comparison of $\mathrm{AEWMA}^{1}_{\mathrm{PCA}}$, $\mathrm{MCE}^{(1)}$, and $\mathrm{MCE}^{(2)}$ control charts at $\lambda = 0.05$ when $\mathrm{ARL}_0 = 200$. (**b**) ARL comparison of $\mathrm{AEWMA}^{1}_{\mathrm{PCA}}$, $\mathrm{MCE}^{(1)}$, and $\mathrm{MCE}^{(2)}$ control charts at $\lambda = 0.10$ when $\mathrm{ARL}_0 = 200$.

Table 5. EQL, RARL, and PCI of control chart when $ARL_0 = 200$, $p = 3$, and PCA = 2.

	$AEWMA^1_{PCA}$ at $k = 0.50$ & $\lambda = 0.05$							$MCE^{(1)}$	$MCE^{(2)}$
γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	$k = 0.50$	$k = 0.50$
EQL	7.70	8.73	9.61	10.29	10.92	11.80	12.80	22.84	13.73
RARL	1.00	1.12	1.22	1.30	1.37	1.47	1.59	2.66	1.65
PCI	1.00	1.13	1.25	1.34	1.42	1.53	1.66	2.96	1.78
	$AEWMA^1_{PCA}$ at $k = 0.50$ & $\lambda = 0.10$							$MCE^{(1)}$	$MCE^{(2)}$
γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	$k = 0.50$	$k = 0.50$
EQL	8.42	9.28	9.92	10.54	11.29	12.10	12.96	21.96	14.37
RARL	1.00	1.09	1.15	1.22	1.30	1.38	1.47	2.35	1.59
PCI	1.00	1.10	1.18	1.25	1.34	1.44	1.54	2.61	1.71
	$AEWMA^1_{PCA}$ at $k = 0.50$ & $\lambda = 0.05$							$MCUSUM_{PC}$	$MCPC^{(1)}$
γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	$k = 0.50$	$k = 0.50$
EQL	7.70	8.73	9.61	10.29	10.92	11.80	12.80	13.27	12.19
RARL	1.00	1.12	1.22	1.30	1.37	1.47	1.59	1.63	1.52
PCI	1.00	1.13	1.25	1.34	1.42	1.53	1.66	1.72	1.58
	$AEWMA^1_{PCA}$ at $k = 1.00$ & $\lambda = 0.05$							$MCUSUM_{PC}$	$MCPC^{(1)}$
γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	$k = 1.00$	$k = 1.00$
EQL	9.87	10.87	11.80	12.76	13.96	14.96	15.77	12.54	12.75
RARL	1.00	1.10	1.19	1.28	1.40	1.49	1.57	1.26	1.28
PCI	1.00	1.10	1.20	1.29	1.42	1.52	1.60	1.27	1.29
	$AEWMA^1_{PCA}$ at $k = 1.50$ & $\lambda = 0.05$							$MCUSUM_{PC}$	$MCPC^{(1)}$
γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	$k = 1.50$	$k = 1.50$
EQL	14.00	15.37	15.66	16.31	16.72	16.82	16.87	15.55	16.60
RARL	1.00	1.10	1.14	1.19	1.22	1.23	1.23	1.12	1.17
PCI	1.00	1.10	1.12	1.17	1.19	1.20	1.20	1.11	1.19
	$AEWMA^1_{PCA}$ at $k = 0.50$ & $\lambda = 0.05$							$MEC^{(1)}$	$MEC^{(2)}$
γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	$k = 0.50$ $\lambda = 0.25$	$k = 0.50$ $\lambda = 0.25$
EQL	7.70	8.73	9.61	10.29	10.92	11.80	12.80	28.18	17.49
RARL	1.00	1.12	1.22	1.30	1.37	1.47	1.59	3.24	2.09
PCI	1.00	1.13	1.25	1.34	1.42	1.53	1.66	3.66	2.27
	$AEWMA^1_{PCA}$ at $k = 0.50$ & $\lambda = 0.30$							$MEC^{(1)}$	$MEC^{(2)}$
γ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	$k = 0.50$ $\lambda = 0.25$	$k = 0.50$ $\lambda = 0.25$
EQL	9.27	9.78	10.38	11.02	11.66	12.15	12.54	28.18	17.49
RARL	1.00	1.05	1.10	1.16	1.21	1.26	1.29	2.69	1.74
PCI	1.00	1.05	1.12	1.19	1.26	1.31	1.35	3.04	1.89

5.2. Proposed vs. $MCUSUM_{PC}$ Control Chart

The proposed $AEWMA^1_{PCA}$ control chart also shows earlier detection ability against the $MCUSUM_{PC}$ control chart. For example, at $\lambda = 0.05$, $k = 0.50$, $p = 3$, and PCA $= 2$, the $AEWMA^1_{PCA}$ control chart ARL_1 are 81.79, 24.87, and 11.10 for $d(\mu) = 0.25,\ 0.50,$ and 1.00, respectively, while the 83.20, 29.80, and 15.10 are the ARL_1 values of the $MCUSUM_{PC}$ control chart. This demonstrates that the $AEWMA^1_{PCA}$ control chart has smaller ARL_1 values relative to the $MCUSUM_{PC}$ control chart (see Tables 1–3 vs. Table 6 and Figure 3a). Similarly, as $k > 0.50$ such as $k = 1.00$, the $MCUSUM_{PC}$ control chart depicts larger ARL_1 values as compared to the proposed $AEWMA^1_{PCA}$ control chart (see Tables 1–3 vs. Table 6 and Figure 3a). Correspondingly, the superiority of the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart for $k = 1.50$ can be observed in Figure 3b and by comparing Tables 1–3 vs. Table 6. In terms of the overall presentation assessment, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart also gives better performance against the $\text{MCUSUM}_{\text{PC}}$ control chart by showing the smallest EQL, RARL, PCI values at $k = 0.50,\ 1.00,$ and 1.50. For example, at $k = 0.50$, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart EQL, RARL, and PCI are 7.70, 1.00, and 1.00, respectively, for $\gamma = 1.00$, while the $\text{MCUSUM}_{\text{PC}}$ control chart keeps larger EQL, RARL, and PCI (see Table 5). Additionally, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart persists in this superiority for $\gamma = 1.50, \ldots, 4.00$ and $k = 1.00$ and 1.50 as well (see Table 5).

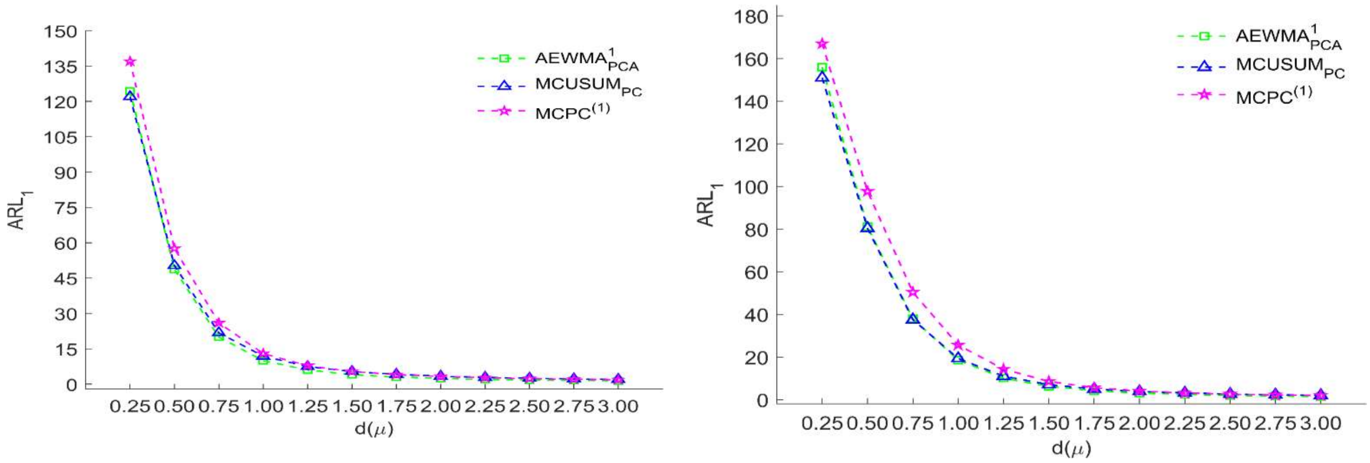
Table 6. ARL properties of the $\text{MCUSUM}_{\text{PC}}$ and $\text{MCPC}^{(1)}$ control charts at different choices of k when $\text{ARL}_0 = 200$.

					$\text{MCUSUM}_{\text{PC}}$									
	p				3.00			p			4.00			
	PC		2.00			3		PC		4.00			2.00	
	k	0.50	1.00	1.50	0.50	1.00	1.50	k	0.50	1.00	1.50	0.50	1.00	1.50
	$H_{\text{MCUSUM}_{\text{PC}}}$	5.50	3.01	1.94	6.92	3.76	2.42	$H_{\text{MCUSUM}_{\text{PC}}}$	8.20	4.50	2.89	5.50	3.01	1.94
$d(\mu)$	$\mu(\delta, 0, 0)$							$\mu(\delta, 0, 0, 0)$		MCUSUM				
0.00	0.00	201	202	202	204	197	200	0.00	206	199.1	197	202	199	206
0.25	0.25	83.2	122	156	89.2	122	151	0.25	89.5	124	149	83.9	124	155
0.50	0.50	29.8	50.3	79.3	31.8	50.1	80.4	0.50	33.8	51.3	79.1	29.5	50.2	79.2
0.75	0.75	15.1	21.8	39.0	16.9	22.7	37.5	0.75	18.3	22.5	37.9	15.2	22.7	38.2
1.00	1.00	9.77	11.9	19.5	11.3	12.3	19.3	1.00	12.5	12.8	18.9	9.81	11.9	19.4
1.25	1.25	7.29	7.48	11.1	8.38	7.98	11.0	1.25	9.43	8.61	11.0	7.27	7.47	11.0
1.50	1.50	5.78	5.38	6.98	6.75	5.82	7.00	1.50	7.58	6.37	7.13	5.77	5.34	6.93
1.75	1.75	4.81	4.16	4.81	5.65	4.59	4.99	1.75	6.38	5.01	5.17	4.81	4.19	4.88
2.00	2.00	4.10	3.37	3.63	4.85	3.78	3.81	2.00	5.53	4.19	4.05	4.13	3.38	3.63
2.25	2.25	3.62	2.86	2.88	4.30	3.24	3.13	2.25	4.85	3.60	3.29	3.63	2.86	2.86
2.50	2.50	3.23	2.51	2.39	3.85	2.83	2.59	2.50	4.36	3.14	2.80	3.24	2.50	2.40
2.75	2.75	2.94	2.22	2.06	3.48	2.55	2.27	2.75	3.94	2.82	2.46	2.95	2.23	2.04
3.00	3.00	2.69	2.04	1.81	3.19	2.31	2.01	3.00	3.64	2.57	2.19	2.69	2.03	1.80

							MC_1^{PCA}							
	p	3.00	p	4.00		p	3.00	p	4.00		p	3.00	p	4.00
	PC	2.00	3	PC		PC	2.00	3	PC		PC	2.00	3	PC
	k	0.50	1.00	1.50		k	0.50	1.00	1.50		k	0.50	1.00	1.50
	$H_{\text{MC}_1^{\text{PCA}}}$	4.75	2.72	1.82		$H_{\text{MC}_1^{\text{PCA}}}$	4.75	2.72	1.82		$H_{\text{MC}_1^{\text{PCA}}}$	4.75	2.72	1.82
$d(\mu)$	$\mu(\delta, 0, 0)$				$d(\mu)$	$\mu(\delta, 0, 0)$				$d(\mu)$	$\mu(\delta, 0, 0)$			
0.00	0.00	198	202	198	0.00	0.00	198	202	198	0.00	0.00	198	202	198
0.25	0.25	89.6	137	156	0.25	0.25	89.6	137	156	0.25	0.25	89.6	137	156
0.50	0.50	31.1	57.6	87.0	0.50	0.50	31.1	57.6	87.0	0.50	0.50	31.1	57.6	87.0
0.75	0.75	15.0	26.0	44.7	0.75	0.75	15.0	26.0	44.7	0.75	0.75	15.0	26.0	44.7
1.00	1.00	9.32	13.0	22.8	1.00	1.00	9.32	13.0	22.8	1.00	1.00	9.32	13.0	22.8
1.25	1.25	6.66	7.84	12.6	1.25	1.25	6.66	7.84	12.6	1.25	1.25	6.66	7.84	12.6
1.50	1.50	5.20	5.38	7.56	1.50	1.50	5.20	5.38	7.56	1.50	1.50	5.20	5.38	7.56
1.75	1.75	4.29	4.02	5.11	1.75	1.75	4.29	4.02	5.11	1.75	1.75	4.29	4.02	5.11
2.00	2.00	3.68	3.24	3.74	2.00	2.00	3.68	3.24	3.74	2.00	2.00	3.68	3.24	3.74
2.25	2.25	3.25	2.73	2.88	2.25	2.25	3.25	2.73	2.88	2.25	2.25	3.25	2.73	2.88
2.50	2.50	2.89	2.34	2.38	2.50	2.50	2.89	2.34	2.38	2.50	2.50	2.89	2.34	2.38
2.75	2.75	2.62	2.10	2.00	2.75	2.75	2.62	2.10	2.00	2.75	2.75	2.62	2.10	2.00
3.00	3.00	2.42	1.88	1.75	3.00	3.00	2.42	1.88	1.75	3.00	3.00	2.42	1.88	1.75

5.3. Proposed vs. $\text{MCPC}^{(1)}$ Control Chart

The proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart also illustrates the earlier identification of the signal over the $\text{MCPC}^{(1)}$ control charts. For example, at $\lambda = 0.05$, $k = 0.50$, $p = 3$, and $\text{PCA} = 2$, the 89.60, 31.10, and 15.00 are the ARL_1 values of the $\text{MCPC}^{(1)}$ control chart, whereas the $\text{AEWMA}^1_{\text{PCA}}$ control chart ARL_1 values are 81.79, 24.87, and 11.10 for $d(\mu) = 0.25,\ 0.50,$ and 1.00, respectively. These are smaller than the ARL_1 values of the $\text{MCPC}^{(1)}$ control chart (see Tables 1–3 vs. Table 6). Likewise, as $k > 0.50$, such as $k = 1.00$, the $\text{MCPC}^{(1)}$ control chart also portrays larger ARL_1 values in comparison to the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart (see Tables 1–3 vs. Table 6 and Figure 3a). Correspondingly, the superiority of the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart for $k = 1.50$ can be observed in Figure 3b and by comparing Tables 1–3 vs. Table 6, too. With respect to the holistic performance calculation, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart provides better performance against the $\text{MCPC}^{(1)}$ control chart by showing the smallest EQL, RARL, PCI values at $k = 0.50,\ 1.00,$ and 1.50 (see Table 5). For example, at $k = 0.50$, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart EQL, RARL, and PCI values are 7.70, 1.00, and 1.00, respectively, for $\gamma = 1.00$, while the $\text{MCPC}^{(1)}$ control chart retains larger values of EQL, RARL, and PCI. Additionally, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart persists in its superiority for $\gamma = 1.50, \ldots, 4.00$ and $k = 1.00$ and 1.50 (see Table 5).



(**a**) (**b**)

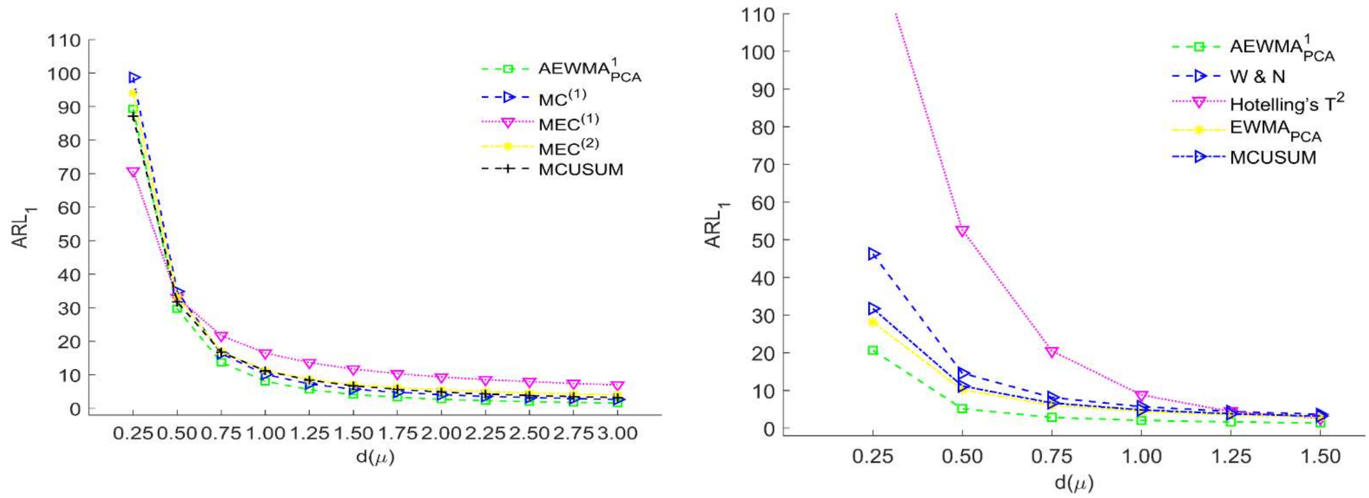
Figure 3. (**a**): ARL comparison of $\text{AEWMA}^1_{\text{PCA}}$, $\text{MCUSUM}_{\text{PC}}$, and $\text{MCPC}^{(1)}$ control charts at $\lambda = 0.05$, $k = 1.00$, $p = 3$ and $PCA = 2$ when $\text{ARL}_0 = 200$. (**b**) ARL comparison of $\text{AEWMA}^1_{\text{PCA}}$, $\text{MCUSUM}_{\text{PC}}$, and $\text{MCPC}^{(1)}$ control charts at $\lambda = 0.05$, $k = 1.50$, $p = 3$, and $PCA = 3$ when $\text{ARL}_0 = 200$.

5.4. Proposed vs. $MEC^{(1)}$ and $MEC^{(2)}$ Control Charts

The proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart keeps smaller ARL_1 values against the $\text{MEC}^{(1)}$ and $\text{MEC}^{(2)}$ control charts. For example, at $k = 0.50$ and $\lambda = 0.25$, the 94.00, 33.37, and 16.89 are ARL_1 values of the $\text{MEC}^{(2)}$ control chart for $d(\mu) = 0.25,\ 0.50,$ and 0.75, respectively, whereas, for $k = 0.50,\ \gamma = 1,$ and $\lambda = 0.30$, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart ARL_1 values are 89.31, 29.70, and 13.20; these are smaller than the $\text{MEC}^{(2)}$ control chart ARL_1 values (see Table 1 vs. Table 7 and Figure 4a). In the comparison of the $\text{MEC}^{(1)}$ control chart, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart also identifies earlier out-of-control signals. For instance, at $k = 0.50$ and $\lambda = 0.25$, the 70.83, 33.08, and 21.68 are the ARL_1 values of the $\text{MEC}^{(1)}$ control chart for $d(\mu) = 0.25,\ 0.50,$ and 0.75, respectively. In contrast, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart shows smaller ARL_1 values except for $d(\mu) = 0.25$ (see Table 1 vs. Table 7 and Figure 4a). Besides, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart also demonstrates efficient comprehensive performance against the $\text{MEC}^{(1)}$ and $\text{MEC}^{(2)}$ control charts. For instance, at $k = 0.50$ and $\lambda = 0.05$, the values of 7.70, 1.00, and 1.00 are the EQL, RARL, and PIC values of the $\text{AEWMA}^1_{\text{PCA}}$ control chart, while, at $k = 0.50$ and $\lambda = 0.25$, the $\text{MEC}^{(1)}$ and $\text{MEC}^{(2)}$ control charts retain larger EQL, RARL, and PIC values (see Table 5). Similarly, at $k = 0.50$ and $\lambda = 0.30$, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart persists in its superiority relative to the $\text{MEC}^{(1)}$ and $\text{MEC}^{(2)}$ control charts in terms of overall performance (see Table 5).

Table 7. ARL values of other control charts when $ARL_0 = 200$ and $p = 3$.

			$MEC^{(1)}$	$MEC^{(2)}$	**MCUSUM**	**W & N**	**Hoteling-χ^2**	**EWMA-PCPCA = 2**
$d(\mu)$	$\mu_1(\delta,0,0)$	$MC^{(1)}$ $H_{MC^{(1)}} = 5.48$ $k = 0.50$	$\lambda = 0.25$ $H_{MEC^{(1)}} = 28.31$ $k = 0.50$	$\lambda = 0.25$ $H_{MEC^{(2)}} = 8.21$ $k = 0.50$	H = 6.88	$H_{W\ \&\ N} = 5.22$ $k = 0.50$	UCL = 12.85	
0.00	0.00	200	200	200	200	200.27	201.1	200
0.25	0.25	98.71	70.83	94.00	87.11			
0.50	0.50	34.84	33.08	33.37	31.74	46.28	129.8	28.1
0.75	0.75	16.30	21.68	16.89	16.75			
1.00	1.00	10.12	16.47	11.31	11.16	14.58	52.6	10.2
1.25	1.25	7.26	13.63	8.64	8.39			
1.50	1.50	5.69	11.70	7.10	6.68	8.16	20.5	6.12
1.75	1.75	4.75	10.33	6.19	5.64			
2.00	2.00	4.07	9.33	5.52	4.83	5.69	8.8	4.41
2.25	2.25	3.53	8.54	5.00	4.27			
2.50	2.50	3.19	7.96	4.59	3.81	4.42	4.4	3.51
2.75	2.75	2.88	7.39	4.28	3.46			
3.00	3.00	2.64	6.96	3.99	3.17	3.65	2.6	2.92



(**a**) (**b**)

Figure 4. (**a**): ARL comparison of $AEWMA^1_{PCA}$, $MC^{(1)}$, $MEC^{(1)}$,$MEC^{(1)}$, and MCUSUM control charts at $\lambda = 0.05$, $k = 0.50$, $p = 3$ and $PCA = 2$ when $ARL_0 = 200$. (**b**): ARL comparison of $AEWMA^1_{PCA}$, Hotelling's T^2, W & N, and $EWMA_{PCA}$ control charts at $\lambda = 0.05$, $k = 0.50$, $p = 3$, and $PCA = 2$ when $ARL_0 = 200$.

5.5. Proposed vs. Other Control Charts

The proposed $AEWMA^1_{PCA}$ control chart also gives an earlier detection signal as compared to the $MC^{(1)}$ control chart. For example, at $k = 0.50$, the $MC^{(1)}$ control chart produces 98.71, 34.84, and 16.30 for $d(\mu) = 0.25,\ 0.50,$ and 0.75, respectively. In contrast, at $\lambda = 0.05$, $k = 0.50$, $p = 3$, and $PCA = 2$, the values of 81.79, 24.87, and 11.10 are the ARL_1 values of the proposed $AEWMA^1_{PCA}$ control chart for the same size of the shift (see Table 1 vs. 7 and Figure 4b). Similarly, the MCUSUM control chart [7] also gives an inferior detection ability relative to the proposed $AEWMA^1_{PCA}$ control chart. For instance, the values of 87.11, 31.74, and 16.75 are the ARL_1 values of the MCUSUM control chart, which are larger against the proposed $AEWMA^1_{PCA}$ control chart's ARL_1 values (see Tables 1–3 vs. 7 and Figure 4b). Likewise, the W & N control chart also depicts lower performance as compared to the proposed $AEWMA^1_{PCA}$ control chart. For example, the values of 46.28, 14.58, and 8.16 are the ARL_1 values of the W & N control chart at $d(\mu) = 0.50,\ 1.00,$ and 1.50, respectively, and these are larger than the proposed $AEWMA^1_{PCA}$ control chart (see Tables 1–3 vs. Table 7 and Figure 4b). Moreover, the superior performance of the proposed $AEWMA^1_{PCA}$ control chart against the Hotelling's T^2 can be seen in Tables 1–3 vs. Table 7 and Figure 4b. The behavior of the EWMA_{PCA}c control chart also shows a delay detection ability as well.

6. Real-Life Example

The implementation of the control charts with real-life data to show the practical aspects to practitioners, engineers, and researchers is vital in these respective industries. Therefore, this section introduces the application of the proposed $\text{AEWMA}^1_{\text{PCA}}$ along with other control charts such as $\text{MCE}^{(1)}$, Hotelling's T^2, $\text{MC}^{(1)}$, and MEWMA control charts with real-life data to demonstrate the practical features of the study [12,23]. References [12,23] used the wind turbine output data which is given in Table 8 to show the importance of their studies. The wind turbine output data were collected at Juaymah meteorological station in Saudi Arabia in 2007. In brief, there were four quality variables whose wind turbine output were measured based on wind speed and height. At every ten minutes' interval, the average wind speed was observed at a height of 10 m (meters per second (m/s)), 20 m, 30 m, and 40 m. Additionally, as described in the Section 3 to Section 5, the proposed control charts required assumptions that the data should follow the multivariate standard normal distribution to produce effective results against other control charts.

Table 8. Numerical example of $\text{AEWMA}^1_{\text{PCA}}$ along $\text{MCE}^{(1)}$, Hotelling's T^2, $\text{MC}^{(1)}$, and MEWMA control charts.

Real-Life Data					Control Charts					Real-Life Data					Control Charts				
SN	**WS10** $X_{1(i)}$	**WS20** $X_{2(i)}$	**WS30** $X_{3(i)}$	**WS40** $X_{4(i)}$	$AEWMA^1_{PCA}$ $h_{AEWMA^1_{PCA}}$ **= 4.55**	$MCE^{(1)}$ $H_{MCE^{(1)}}=6.6$	MC^1 $H_{MC^1}=6.18$	**Hotelling's** T^2 $\text{UCL}_{T^2}=14.86$	**MEWMA** $H_{ME}=13.86$	**SN**	**WS10** $X_{1(i)}$	**WS20** $X_{2(i)}$	**WS30** $X_{3(i)}$	**WS40** $X_{4(i)}$	$AEWMA^1_{PCA}$ $h_{AEWMA^1_{PCA}}$ **= 4.55**	$MCE^{(1)}$ $H_{MCE^{(1)}}=6.6$	MC^1 $H_{MC^1}=6.18$	**Hotelling's** T^2 $UCL_{T^2}=14.86$	**MEWMA** $H_{ME}=13.86$
1	5.4	6.5	7.5	8.5	0.18	0.42	1.71	9.09	2.21	32	2.9	4	4.7	5.4	2.42	3.12	3.52	4.88	4.59
2	5.6	6.6	7.7	8.6	0.53	1.19	3.64	9.32	3.27	33	2.6	3.5	4.3	5.2	2.82	2.67	5.64	4.34	5.17
3	3	3.7	3.9	4.3	0.39	0.85	0.25	4.10	0.16	34	2.7	3.5	4.4	5.3	3.48	2.14	7.64	4.46	5.53
4	3.6	4.6	5.5	6.2	0.39	0.67	0.00	6.00	0.64	35	2.4	3.2	4.1	5	4.43	1.96	10.11	4.00	6.11
5	4	5.1	5.8	6.3	0.35	0.38	0.00	6.50	0.77	36	2.8	3.9	4.7	5.4	5.43	2.46	11.75	4.80	6.00
6	3.8	4.9	5.8	6.6	0.34	0.26	0.11	6.47	0.96	37	5.1	6	6.8	7.3	5.87	2.81	10.15	8.04	3.77
7	3.4	3.8	3.9	4.3	0.55	0.24	2.15	4.33	2.41	38	5.8	6.8	7.5	8	5.69	2.86	7.43	9.17	1.38
8	3.3	4.3	5.2	6	0.89	0.48	2.99	5.58	2.71	39	5.8	6.6	7.4	7.9	5.05	2.77	4.87	9.01	0.48
9	3.3	4.4	5.4	6.5	1.30	0.78	3.59	5.90	2.77	40	5.9	6.8	7.4	7.9	3.96	2.48	2.19	9.13	1.75
10	3.8	4.9	5.9	7	1.62	1.19	3.47	6.67	2.36	41	5.8	6.7	7.3	7.9	2.91	1.98	0.00	9.01	2.69
11	4	5.1	6.1	7.1	1.84	1.65	3.11	6.94	1.92	42	6.5	7.4	8	8.5	2.63	1.36	2.63	10.06	4.08
12	3.7	4.8	5.8	6.8	2.04	2.15	3.24	6.48	1.90	43	6.1	7	7.7	8.4	2.77	0.81	4.79	9.59	4.82
13	3.5	4.6	5.5	6.5	2.30	2.66	3.72	6.09	2.08	44	6	6.9	7.7	8.2	3.18	0.80	6.79	9.43	5.27
14	6.1	6.5	6.6	6.8	1.98	2.59	1.58	8.41	0.46	45	7	8	8.7	9.2	4.10	1.29	10.39	11.02	6.66
15	6.6	7	7.1	7.3	1.48	1.95	0.00	9.18	1.26	46	3.7	4.3	4.8	5.5	4.42	1.65	8.34	5.39	3.97
16	6.5	6.9	7.1	7.4	1.50	1.31	1.80	9.13	2.46	47	4.1	4.6	5.1	5.8	4.39	1.91	6.81	5.89	2.31
17	6.5	7.1	7.3	7.4	1.92	0.52	3.75	9.29	3.46	48	4.5	5	5.5	6.2	4.21	2.21	5.90	6.51	1.49
18	6.6	7	7.2	7.4	2.65	0.66	5.67	9.25	4.20	49	3.5	4	4.5	5.1	3.64	2.47	3.45	4.93	0.59
19	6.9	7.2	7.4	7.6	3.77	1.46	7.95	9.60	4.99	50	3.3	3.7	4.1	4.7	2.73	2.46	0.60	4.44	1.97
20	6.8	7.3	7.6	7.8	5.75	2.41	10.32	9.74	5.64	51	2.3	2.8	3.1	3.7	2.57	2.24	0.00	2.94	4.12
21	6.8	7.2	7.4	7.5	8.00	3.45	12.52	9.53	6.02	52	1.6	1.9	2.2	2.7	3.19	2.38	4.89	1.66	6.65
22	6.9	7.3	7.5	7.6	10.36	4.58	14.87	9.68	6.40	53	2.3	2.6	2.9	3.3	4.26	2.99	8.76	2.68	7.88
23	3.3	4.4	5.1	5.7	10.49	5.32	12.92	5.46	3.84	54	4.4	5.5	6.7	7.8	4.86	3.92	7.41	7.76	5.34
24	3.3	4.3	5	5.7	10.18	5.73	10.91	5.38	1.91	55	4.4	5.4	6.5	7.5	5.18	5.00	6.35	7.51	3.63
25	3.3	4.3	4.9	5.5	9.52	6.00	8.85	5.26	0.69	56	4.4	4.6	4.7	4.9	5.40	5.86	7.30	5.59	3.70
26	3.3	4.3	5	5.6	8.60	6.19	6.86	5.34	1.04	57	4.5	4.8	5.1	5.2	5.51	6.69	7.92	5.95	3.55
27	3.5	4.4	5.2	5.8	7.53	6.16	5.12	5.61	1.60	58	4.5	4.7	4.8	5	5.56	7.38	8.83	5.74	3.66
28	3.5	4.4	5.1	5.8	6.24	5.92	3.39	5.57	2.08	59	4.6	4.9	5.1	5.2	5.52	8.00	9.46	6.03	3.55
29	3.3	4.2	5	5.8	4.28	5.34	1.45	5.38	2.59	60	4.8	5.2	5.3	5.4	5.35	8.27	9.79	6.38	3.30
30	2.7	3.6	4.4	5.3	2.73	4.49	0.00	4.50	3.56	61	5.1	5.4	5.5	5.6	5.01	8.47	9.84	6.74	2.99
31	2.7	3.6	4.4	5.3	2.40	3.59	1.96	4.50	4.30	62	5.2	5.6	5.6	5.8	4.50	8.35	9.72	6.96	2.69
63	5.1	5.5	5.6	5.7	3.91	8.23	9.73	6.84	2.55	82	3.1	4	4.9	5.6	3.29	2.30	4.97	5.11	0.90
64	5.1	5.5	5.6	5.8	3.26	8.13	9.72	6.87	2.44	83	2.7	3.6	4.4	5	2.47	1.90	2.10	4.38	2.20
65	5.8	6.8	7.6	8.4	2.44	7.94	7.04	9.36	1.68	84	2.3	3.2	4	4.5	2.31	1.47	0.00	3.72	3.75
66	5.3	6.4	7.3	8.2	1.98	7.58	4.77	8.82	2.02	85	2.9	3.8	4.6	5	2.49	1.31	1.82	4.61	4.34
67	4.6	5.7	6.7	7.4	1.59	7.29	3.40	7.74	1.94	86	3.7	4.8	5.8	6.7	2.68	1.34	1.89	6.43	3.64
68	4.7	5.6	6.3	7.2	1.25	6.83	2.36	7.50	1.81	87	3.9	5	6	6.7	2.81	1.58	1.76	6.66	2.97
69	5.3	6.3	7.2	8.2	1.13	6.27	0.25	8.74	2.55	88	3.5	4.6	5.6	6.2	3.01	1.95	2.26	6.00	2.89
70	5.1	6.3	7.1	7.9	1.15	5.42	0.00	8.51	2.97	89	3.6	4.8	5.7	6.3	3.23	2.14	2.58	6.20	2.70
71	5.2	6.3	7.1	7.9	1.28	4.50	1.14	8.55	3.31	90	4.3	5.3	6.2	6.7	3.28	2.43	2.08	7.00	2.03
72	4.8	5.9	6.7	7.8	1.40	3.48	1.82	8.05	3.25	91	4.3	5.5	6.4	7.1	3.20	2.48	1.39	7.32	1.42
73	6.5	7.6	8.4	9.1	1.96	2.63	4.87	10.51	4.80	92	4.2	5.4	6.5	7.1	3.04	2.62	0.85	7.28	1.14
74	6.3	7.2	7.8	8.5	2.78	2.05	7.24	9.82	5.50	93	4.5	5.8	6.8	7.5	2.72	2.57	0.00	7.82	1.04
75	5.5	6.6	7.4	8.1	3.59	1.86	8.79	8.97	5.48	94	4.5	5.7	6.7	7.5	2.31	2.42	0.44	7.74	1.25
76	5.2	6.5	7.4	8.2	4.36	2.08	10.25	8.86	5.41	95	4.8	5.9	6.8	7.6	1.76	2.21	1.05	8.01	1.57
77	4.3	5.4	6.4	7	4.80	2.38	10.09	7.24	4.29	96	4.9	5.9	7	7.7	1.32	2.31	1.83	8.16	1.98
78	4.1	5	5.8	6.4	4.92	2.53	9.22	6.54	2.97	97	4.6	5.7	6.8	7.5	0.99	2.64	2.32	7.82	2.14
79	4	5	5.9	6.4	4.79	2.53	8.36	6.54	2.01	98	4.4	5.6	6.7	7.4	0.74	3.02	2.65	7.63	2.17
80	4.1	5.3	6.2	6.8	4.53	2.56	7.96	6.97	1.61	99	5	6.3	7.2	7.9	0.66	3.06	3.76	8.51	2.67
81	3.9	5	5.9	6.6	4.10	2.55	7.18	6.58	1.12										

Therefore, the multivariate normality test's Royston value is applied to the data to ensure normality after it is standardized. The Royston's statistic value is 4.63 and p-value = 0.0660, while the given significance level is 0.05; this shows the normality of the data. Moreover, a suitable number of principal components [10] is identified (see Figure 5) for the proposed control chart. Therefore, two numbers of PC are taken to implement the proposed control chart by using 99 observations. The proposed $\text{AEWMA}^1_{\text{PCA}}$ along the $\text{MCE}^{(1)}$, Hotelling's T^2, $\text{MC}^{(1)}$, and MEWMA control charts are implemented. For a fair comparison, the $\lambda = 0.25$ and $k = 0.50$ parameters' values are chosen, while $\text{ARL}_0 = 200$ for all control charts. The graphical presentation is provided in Figure 5. Here, the findings with real-life data also demonstrate the superiority of the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart against other control charts. For example, at $\gamma = 1.00$, 1.50, 2.00, 2.50, 3.0, 3.50, and 4.00, the proposed $\text{AEWMA}^1_{\text{PCA}}$ control chart detects a total of 34, 36, 37, 38, 38, and 40 out-of-control points out of 90, respectively. In contrast, the MC^1, Hotelling's T^2, MEWMA, and MCE^1 control charts identify 38, 0, 0, and 12 out-of-control points (see Table 8 and

Figure 6a,b), respectively. Moreover, the proposed $AEWMA^1_{PCA}$ control chart identifies earlier first single at 19th order against the Hotelling's T^2, MEWMA, and MCE^1 control charts. As a result, practitioners, engineers, and researchers can gain the benefits from these proposed control charts to distinguish the special causes in different industries such as manufacturing, banking, medical, etc. Consequently, it may affirm that the proposed $AEWMA^1_{PCA}$ and $AEWMA^2_{PCA}$ control charts will be a good addition in SPC literature as well.

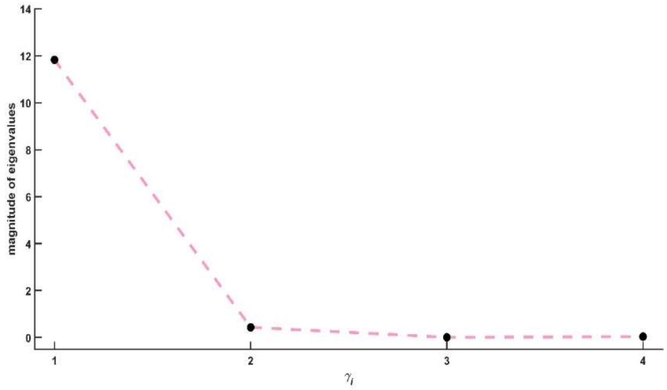
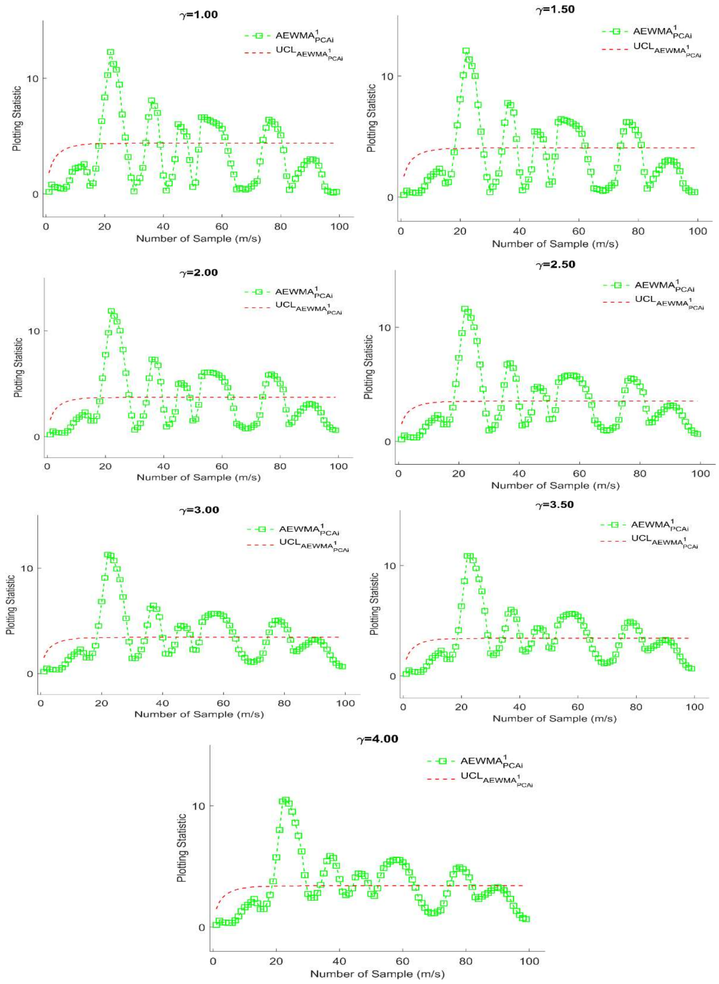
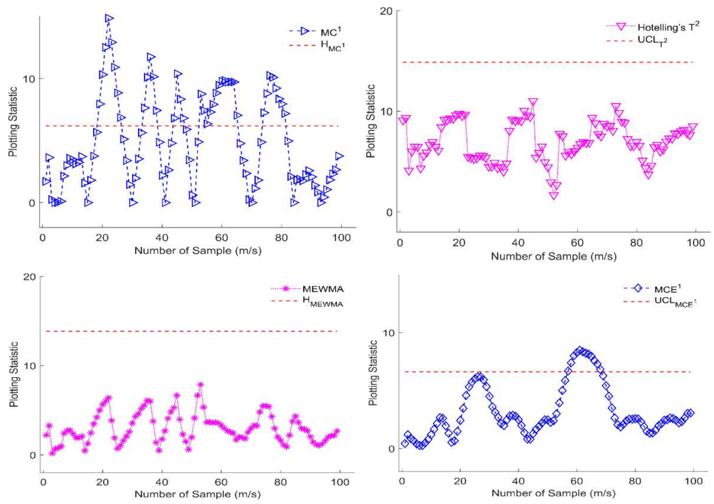


Figure 5. A scree plot of an eigen values.



(a)

Figure 6. *Cont.*



(**b**)

Figure 6. (**a**): Graphical presentation of proposed control charts with real-life data (m/s) at different values of *y*. (**b**): Graphical presentation of other control charts with real-life data (m/s).

7. Summary, Conclusions, and Recommendations

This study offers two adaptive exponential weighted moving average (AEWMA) control chats to detect different sizes of a shift in the process mean vector. The proposed AEWMA control charts accept the plotting statistic of the MC_1^{PCA} control chart as an input, whereas as the MC_1^{PCA} control chart [23] is the modified form of the multivariate $MC^{(1)}$ [8] control chart based on based on principal component analysis (PCA) statistic instead of the variable's vector. The score (Huber and Bi-square) functions are used in the structures of the proposed control charts to introduce the adaptive concept and to enable the proposed control charts to distinguish the different sizes of a shift. The originality and/or novelty is to reduce data reduction techniques such as PCA along the incorporation of the adaptive method to improve performance and to identify various sizes of a shift the in the process mean vector. Algorithms are designed in MATLAB to obtain run length (RL) properties through a Monte Carlo simulation for the proposed control charts to analyze the performance against $MCE^{(1)}$, $MCE^{(2)}$, $MCUSUM_{PC}$, $MCPC^{(1)}$, $MEC^{(1)}$, $MEC^{(2)}$, $MC^{(1)}$, MCUSUM, W and N, Hotelling's T^2, and the $EWMA_{PCA}$ control charts. Performance measures such as an average of RL, standard deviation of RL, and standard error of RL are used to assess the performance for a single shift, while, for the overall performance, extra quadratic loss, relative ARL, and the performance comparison index performance measures are also used. The evaluation shows the dominance of the proposed control charts against the mentioned control charts. Additionally, to highlight the application process and the benefits of the proposed control charts, a real-life example of the wind turbine process is involved. This study performs better when underline process characteristics follow simple random sampling (SRS) and multivariate normal distribution (MND), other than the SRS and MND, which required more investigation to authenticate the proposed control chart's ability; otherwise, the results will be unreliable. Further, the same study idea can be extended for the variance–covariance matrix shift along with auxiliary information and ranked set techniques.

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Abbreviations

Acronym/Symbol	**Description**
ARL	Average Run Length
ARL_0	Out-of-Control Average Run Length
ARL_1	In-Control ARL_0
$ARL_{d(\mu)}$	ARL_1 and/or ARL_0 at Specific Value of $d(\mu)$
$ARL_{\mathrm{bmk}(d(\mu))}$	ARL_1 and/or ARL_0 at Specific Value of $d(\mu)$ of a Benchmark Control Chart
ACUSUM	Adaptive CUSUM
AEWMA	Adaptive EWMA (AEWMA)
$\mathrm{AEWMA}_{\mathrm{PCA}}$	Proposed Control Charts
$\mathrm{AEWMA}^1_{\mathrm{PCA}}$	$\mathrm{AEWMA}_{\mathrm{PCA}}$ Based on Huber
$\mathrm{AEWMA}^2_{\mathrm{PCA}}$	$\mathrm{AEWMA}_{\mathrm{PCA}}$ Based on Bi-square
$\mathrm{AEWMA}^2_{\mathrm{PCAi}}$	Plotting Statistic of $\mathrm{AEWMA}^2_{\mathrm{PCA}}$
$AEWMA^1_{PCAi}$	Plotting Statistic of $\mathrm{AEWMA}^1_{\mathrm{PCA}}$
$AEWMA^2_{PCAi}$	Plotting Statistic of $\mathrm{AEWMA}^2_{\mathrm{PCA}}$
CUSUM	Cumulative Sum
$C_{1(i)}$	Accumulates Information of X_{ji}
$\mathrm{cov}(y_i, y_l)$	Covariance
$d(\mu)$	Non-Centrality Parameter
$d(\mu)_{\max}$	Maximum value of $d(\mu)$
$d(\mu)_{\min}$	Minimum value of $d(\mu)$
EQL	Extra Quadratic Loss
EWMA	Exponentially Weighted Moving Average
$e'_1,\ e'_2, \ldots, e'_p$	Eigenvectors of Σ
e_i	Error
E_{i-1}	Plotting Statistic of EWMA
H_0	Null Hypothesis
H_1	Alternative Hypothesis
$\mathrm{H}_{\mathrm{MC}_1}$	UCL of MC_1 Control Chart
$\mathrm{H}_{\mathrm{MC}_1^{\mathrm{PCA}}}$	UCL of $\mathrm{MC}_1^{\mathrm{PCA}}$
$\mathrm{h}_{\mathrm{AEWMA}^1_{\mathrm{PCA}}}$	Constant of $\mathrm{AEWMA}^1_{\mathrm{PCA}}$ Control Chart
$\mathrm{h}_{\mathrm{AEWMA}^2_{\mathrm{PCA}}}$	Constant of $\mathrm{AEWMA}^2_{\mathrm{PCA}}$ control chart
i	Sample of jth Quality Characteristic
j^{th}	Number of Quality Characteristic
K/k	Constant
UCL	Upper Control Limit
UCL_{T^2}	UCL Hotelling's T^2 Control Chart
$UCL_{\mathrm{AEWMA}^2_{\mathrm{PCA}}i}$	UCL of $\mathrm{AEWMA}^2_{\mathrm{PCA}}$

$\mathrm{UCL}_{\mathrm{PCM}}$	UCL of PC-chart
MC_1	MCUSUM
$UCL_{\mathrm{AEWMA}^1_{\mathrm{PCA}}i}$	UCL of $\mathrm{AEWMA}^1_{\mathrm{PCA}}$
MCUSUM	Multivariate CUSUM
MEWMA	Multivariate EWMA
MD	Mahalanobis Distance
MC_{1i}	Plotting Statistic of MC_1
$\mathrm{MC}_1^{\mathrm{PCA}}$	MC_1 Control Chart Based on PCA
MC_{1i}^{PCA}	Plotting Statistic of $\mathrm{MC}_1^{\mathrm{PCA}}$
MD_i^2	Plotting Statistic of Hotelling's T^2 Control Chart
n_i	Time Varying Sample
PCA	Principal Component Analysis
PCA_i	Plotting Statistic of PC-Chart
PCI	Performance Comparison Index
RARL	Relative Average Run Length
γ	Constant
γ_m or $\gamma_{(i)}$	Eigenvalues of Σ
SDRL	Standard Deviation of RL
SERL	Standard Error of RL
SPC	Statistical Process Control
$w_h(e_i)$	Time Varying Parameter of $\mathrm{AEWMA}^1_{\mathrm{PCA}}$
$w_b(e_i)$	Time Varying Parameter of $\mathrm{AEWMA}^2_{\mathrm{PCA}}$
$\chi^2_{\alpha,p}$	Chi-square Distribution
$\mathrm{X}_{p\times 1}/X_{ji}$	Process Variables
$y_1, y_2, \ldots, y_p$	Principal Components
$\varnothing_h(.)$	Huber function
$\varnothing_b(e_i)$	Bi-square function
φ	Constant and $\varphi > 0$
μ	Mean Vector
Σ	Variance-Covariance Matrix
$\mu_{\mathrm{MC}_{1i}^{\mathrm{PCA}}}$	Empirical Time Varying Mean of $\mathrm{MC}_1^{\mathrm{PCA}}$
$\sigma_{\mathrm{MC}_{1i}^{\mathrm{PCA}}}$	Empirical Time Varying Standard Deviation of $\mathrm{MC}_1^{\mathrm{PCA}}$

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