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Research on the Period-Doubling Bifurcation of Fractional-Order DCM Buck–Boost Converter Based on Predictor-Corrector Algorithm

Lingling Xie, Jiahao Shi *, Junyi Yao and Di Wan

School of Electrical Engineering, Guangxi University, Nanning 530004, China; xielingling@gxu.edu.cn (L.X.); ww443703528@163.com (J.Y.); m15256968200@163.com (D.W.)
* Correspondence: s13983707513@163.com

Abstract: DC–DC converters are widely used. They are a typical class of strongly nonlinear time-varying systems that show rich nonlinear phenomena under certain working conditions. Therefore, an in-depth study of their nonlinear phenomena, characteristics, and generation mechanism is of great practical significance for gaining a deep understanding of this kind of switching converter, revealing the essence of these nonlinear phenomena and then optimizing the design of this kind of converter. Based on the fact that most of the inductance and capacitance are fractional-order, the nonlinear dynamic characteristics of the fractional-order (FO) DCM buck–boost converter are researched in this paper. The main research work and achievements of this paper include: (1) using the predictor–corrector method of fractional calculus, which is not limited by fractional order and can directly calculate the accurate values of the inductance current and capacitor voltage of the fractional converter; the predictor–corrector model of the FO converter is established. (2) The bifurcation diagrams are obtained based on this model, and the period-doubling bifurcation and chaotic behavior of the FO buck–boost converter are analyzed. (3) The phase diagrams are obtained and verified to the point that period-doubling bifurcation occurs; then, some conclusions are drawn. The results show that under certain operating and parameters conditions, the FO buck–boost converter will appear as a bifurcation and chaotic nonlinear phenomenon. Under the condition of the same circuit parameters, the stability parameter domains of the integer-order buck–boost converter and the FO buck–boost converter are different. Compared with the integer-order converter, the parameter stability region of the FO buck–boost converter is bigger. The FO buck–boost converter is more accurate at describing the nonlinear dynamic characteristics. Furthermore, the predictor–corrector method can also be applied to other FO power converters and provides theoretical guidance for converter parameter optimization and controller design.

Keywords: fractional-order; buck–boost converter; predictor–corrector algorithm; period-doubling bifurcation; chaos

MSC: 37G15

Citation: Xie, L.; Shi, J.; Yao, J.; Wan, D. Research on the Period-Doubling Bifurcation of Fractional-Order DCM Buck–Boost Converter Based on Predictor-Corrector Algorithm. *Mathematics* **2022**, *10*, 1993. https://doi.org/10.3390/math10121993

Academic Editors: Yiming Chen, Aimin Yang, Jiaquan Xie, Yanqiao Wei and António M. Lopes

Received: 28 April 2022
Accepted: 7 June 2022
Published: 9 June 2022

1. Introduction

The nonlinear dynamic characteristics of the DC–DC converter have been studied in past years. The existing research showed that most of the inductance and capacitance in practice are actually fractional-order. It is well known that the inductance and capacitance are the necessary energy storage components in the converter. Therefore, the essence of the converter is fractional order. However, most of the dynamic research results in the converter are based on the integer-order model, which is obviously contrary to its fractional-order nature and unscientific. The integer-order model cannot accurately reflect the fractional-order dynamic characteristics and physical nature of the converter and may even draw the wrong conclusions. With the wide application of the DC–DC converter in new energy power generation and the switching power supply, the working condition of the converter becomes increasingly complex, which makes the nonlinear phenomena of the converter more abundant and the dynamic behavior more complex. At the same time, the application of fractional order in chaotic phenomena has been widely studied [1–3]. Therefore, the accurate analysis of fractional-order nonlinear dynamic behavior and its mechanism of the DC–DC converter is helpful to improve the stability of the system, which provides an important theoretical basis for the optimization and design of the converter.

The authors of [4] modeled and analyzed the Boost converters, and the authors of [5,6] modeled and analyzed the Buck converters by using fractional calculus theory. The FO mathematical model and the FO state average model of the Buck converter and Boost converter are obtained. The influence of fractional order on the performance of FO model is found, but the nonlinear dynamic characteristics are not analyzed. In [7], the influence of fractional order on the output voltage amplitude and the maximum inductance current of the converter based on FO buck–boost converters in DCM mode is discussed, but only within the three orders of 0.7, 0.9, and 1, and the bifurcation behavior of the converter was not analyzed. A time-domain mathematical model of the fractional Buck converter based on the Caputo Fabrizio fractional calculus is proposed, the discrete iterative mapping model of the fractional buck converter in the peak current mode is established in [8], the chaotic behavior of the system is studied through numerical simulation, and the bifurcation diagrams under different fractional orders are studied, but there is a lack of the discussion on stability interval. The FO model of the FO buck–boost converter is derived, and its discrete solution is obtained based on the Adomian decomposition method in [9]. The nonlinear dynamic behaviors such as the bifurcation and chaos of the system are studied by means of the bifurcation diagram and the 0–1 test, but the degree of research is not enough, and some other methods should be used. The authors of [10] established the FO state average model of the buck–boost converter; analyzed the transfer function of the buck–boost converter after theoretical derivation and analysis; and obtained the inductance current bifurcation diagram with a reference current as a parameter by simulation analysis, which selected the fractional-orders of 0.8/0.9 and 1.0/1.0 by oustaloup filter method, but the order selection was not extensive enough. In study [11], the discrete mathematical model of the peak current mode controlled synchronous switching Z-source converter is established by using the predictor–corrector method. The dynamic behavior of the system is studied through the bifurcation diagram, with the reference current as the bifurcation parameter. However, the order selection is single and not representative, and the research on the nonlinear dynamic behavior of the system is not deep enough.

The objective of this paper is to analyze the period-doubling bifurcation and chaos of the FO buck–boost converter based on the nonlinear dynamic system and fractional calculus theory. The predictor–corrector method of fractional calculus is used, which has the advantages of accurate modeling and fast calculation speed, is not limited by fractional-order, and can directly calculate the accurate values of the inductance current and the capacitor voltage of the fractional converter. Therefore, the analysis of the bifurcation would be more accurate and faster. The predictor–corrector model of the FO converter is established by using the predictor–corrector method, and the bifurcation diagrams are obtained based on this model. For verification, the phase diagrams are obtained and compared with the bifurcation diagrams, then some conclusions are drawn.

The paper is organized as follows: first, the mathematical model of the DCM buck–boost converter will be established in the Section 2; then, in Section 3, the bifurcation diagrams of the converter with the reference current, the input voltage, and fractional order as bifurcation parameters are obtained; finally, some conclusions are drawn in Section 4.

2. Predictor–Corrector Model of FO Buck–Boost Converter

2.1. FO DCM Buck–Boost Converter

The FO DCM buck–boost converter with peak current control is shown in Figure 1. It is composed of FO inductance L^{α}, FO capacitance C^{β}, in which the inductance order is α

and the capacitance order is β, where 0 < α, β < 1. switch is S, diode is D, load resistance is R, U_{in} is the input voltage, U_o is the output voltage, and the switching cycle of S is T.

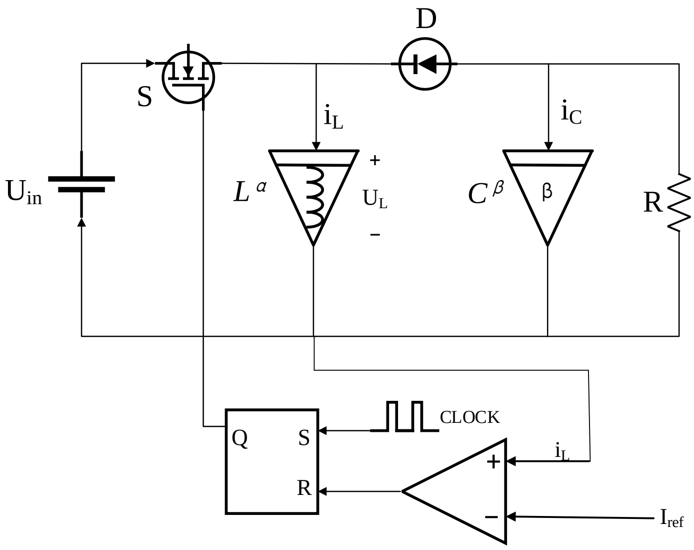


Figure 1. The peak current controlled FO buck–boost converter circuit topology.

The inductance current waveform when the converter operates in DCM mode is shown in Figure 2.

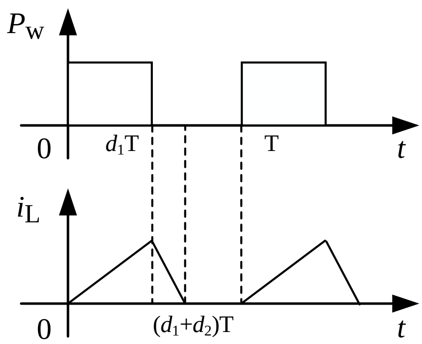


Figure 2. The inductor current waveform in the inductor current discontinuous mode.

The expressions of FO inductive voltage u_L and FO capacitive current i_C are, respectively, that the inductance order is α, and the capacitance order is β:

$$\begin{cases} u_L = L\frac{d^{\alpha} i_L}{dt^{\alpha}} \\ i_C = C\frac{d^{\beta} u_o}{dt^{\beta}} \end{cases} \tag{1}$$

There are three working modes while the converter is in DCM mode.

Mode 1(nT < t < (n + d_1)T): the switch S is open and the diode D is closed

$$\begin{cases} \frac{d^{\alpha} i_L}{dt^{\alpha}} = \frac{V_{in}}{L} \\ \frac{d^{\beta} u_o}{dt^{\beta}} = -\frac{u_o}{RC} \end{cases} \tag{2}$$

Mode 2 ((n + d_1)T < t < (n + d_1 + d_2)T): the switch S is closed and the diode D is open

$$\begin{cases} \frac{d^{\alpha} i_L}{dt^{\alpha}} = \frac{V_{in}}{L} - \frac{u_o}{L} \\ \frac{d^{\beta} u_o}{dt^{\beta}} = \frac{i_L}{C} - \frac{u_o}{RC} \end{cases} \tag{3}$$

Mode 3 ($(n + d_1 + d_2)T < t < (n + 1)T$): the inductor current is equal to 0 before the next high level from power supply

$$\begin{cases} \frac{d^{\alpha} i_{L}}{dt^{\alpha}} = 0 \\ \frac{d^{\beta} u_{o}}{dt^{\beta}} = -\frac{u_{o}}{RC} \end{cases}, \tag{4}$$

where d_1 is the duty cycle and d_2 is the ratio of the time when the inductive current drops from the maximum to zero to the cycle.

2.2. Predictor–Corrector Model

Define the nonlinear switching function S(t). Assume that S(t) = 1 means that the switch S is on, and S(t) = 0 means that the switch S is off.

Then, the expression of S(t) is as follows:

$$S(t) = \begin{cases} 1, nT < t < (n + d_1)T \\ 0, (n + d_1)T < t < (n + 1)T \end{cases} \tag{5}$$

The FO mathematical model in one cycle is obtained from Equations (2)–(4); the expression is:

$$\begin{cases} \frac{d^{\alpha} i_{L}}{dt^{\alpha}} = S\frac{V_{in}}{L} - (1 - S)\frac{u_{o}}{L} \\ \frac{d^{\beta} u_{o}}{dt^{\beta}} = (1 - S)\frac{i_{L}}{C} - \frac{u_{o}}{RC} \end{cases} \tag{6}$$

The predictor–corrector algorithm of fractional calculus was originally proposed by K. Diethelm in 2002. For the fractional calculus equation, this method has the advantages of accurate modeling and fast calculation speed compared with other algorithms. It is not limited by fractional-order and can directly calculate the accurate values of inductance current and capacitor voltage of the fractional converter, which can better analyze the dynamic characteristics of the fractional system.

The predictor formula is as follows [12,13]:

$$y_{h}^{P}(t_{n+1}) = y_0 + \frac{1}{\Gamma(q)} \sum_{j=0}^{n} b_{j,n+1} f(t_j, y_h(t_j)) \tag{7}$$

and the correction formula is

$$\begin{aligned} y_{h}(t_{n+1}) = y_0 + \tfrac{h^q}{\Gamma(q+2)} f(t_{n+1}, y_{h}^{P}(t_{n+1})) \\ + \tfrac{h^q}{\Gamma(q+2)} \sum_{j=0}^{n} a_{j,n+1} f(t_j, y_h(t_j)) \end{aligned} \tag{8}$$

where

$$a_{j,n+1} = \begin{cases} n^{q+1} - (n-q)(n+1)^q, j = 0 \\ (n-j+2)^q + (n-j)^{q+1} - 2(n-j+1)^{q+1}, 1 \le j \le n \\ 1, j = n+1 \end{cases}, \tag{9}$$

$$b_{j,n+1} = \frac{h^q}{q}((n+1-j)^q - (n-j)^q) \tag{10}$$

Use the predictor–corrector algorithm to obtain the discrete model of the FO buck–boost converter. When $t = t_{n+1}$, set $i_{Lh}(t_{n+1}) = i_{n+1}$, $i_{Lh}^{P}(t_{n+1}) = i_{n+1}^{P}$, $u_{oh}(t_{n+1}) = u_{n+1}$, and $u_{oh}^{P}(t_{n+1}) = u_{n+1}^{P}$; the FO predictor–corrector model in switching period T is obtained, as shown below:

$$\begin{aligned} i_{n+1} = i_0 + \tfrac{h^{\alpha}}{\Gamma(\alpha+2)} (S\tfrac{V_{in}}{L} - (1 - S)\tfrac{u_{n+1}^{P}}{L}) \\ + \tfrac{h^{\alpha}}{\Gamma(\alpha+2)} \sum_{i=0}^{n} a_{i,n+1}^{\alpha} (S\tfrac{V_{in}}{L} - (1 - S)\tfrac{u_i}{L}) \end{aligned} \tag{11}$$

$$\begin{aligned} u_{n+1} &= u_0 + \frac{\mathrm{h}^{\beta}}{\Gamma(\beta+2)}\left((1-S)\frac{i_{n+1}^{\mathrm{P}}}{\mathrm{C}} - \frac{u_{n+1}^{\mathrm{P}}}{\mathrm{RC}}\right) \\ &\quad + \frac{\mathrm{h}^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n} a_{i,n+1}^{\beta}\left((1-S)\frac{i_i}{\mathrm{C}} - \frac{u_i}{\mathrm{RC}}\right) \end{aligned} \tag{12}$$

where i_0 and u_0 are the initial values of inductance current and capacitance voltage, respectively, and the expression of inductance current correction coefficient ${a_{i,n+1}}^{\alpha}$ and capacitance voltage correction coefficient ${a_{i,n+1}}^{\beta}$ are shown below:

$$\begin{aligned} a_{i,n+1}^{\alpha} &= \begin{cases} n^{\alpha+1} - (n-\alpha)(n+1)^{\alpha}, i = 0 \\ (n-i+2)^{\alpha} + (n-i)^{\alpha+1} - 2(n-i+1)^{\alpha+1}, 1 \le i \le n \\ 1, i = n+1 \end{cases} \\ a_{i,n+1}^{\beta} &= \begin{cases} n^{\beta+1} - (n-\beta)(n+1)^{\beta}, i = 0 \\ (n-i+2)^{\beta} + (n-i)^{\beta+1} - 2(n-i+1)^{\beta+1}, 1 \le i \le n \\ 1, i = n+1 \end{cases} \end{aligned} \tag{13}$$

The expression of the approximate value of the initial estimation of the inductance current ${i_{n+1}}^{p}$ and the approximate value of the initial estimation of capacitor voltage ${u_{n+1}}^{p}$ are shown below:

$$\begin{cases} i_{n+1}^{p} = i_0 + \frac{1}{\Gamma(\alpha)}\sum\limits_{i=0}^{n} b_{i,n+1}^{\alpha}\left(S\frac{V_{\mathrm{in}}}{\mathrm{L}} - (1-S)\frac{u_i}{\mathrm{L}}\right) \\ u_{n+1}^{p} = u_0 + \frac{1}{\Gamma(\beta)}\sum\limits_{i=0}^{n} b_{i,n+1}^{\beta}\left((1-S)\frac{i_i}{\mathrm{C}} - \frac{u_i}{\mathrm{RC}}\right) \end{cases} \tag{14}$$

3. Period-Doubling Bifurcation of FO Buck–Boost Converter

3.1. Reference Current I_{ref} as the Bifurcation Parameter

The mathematical model of the FO buck–boost converter in DCM is built by using MATLAB/Simulink based on the fractional differential equation of the FO buck–boost converter, derived in Section 2, as shown in Figure 3. The circuit parameters of the converter are listed in Table 1. For details of modules in Figure 3, please refer to Appendix A.

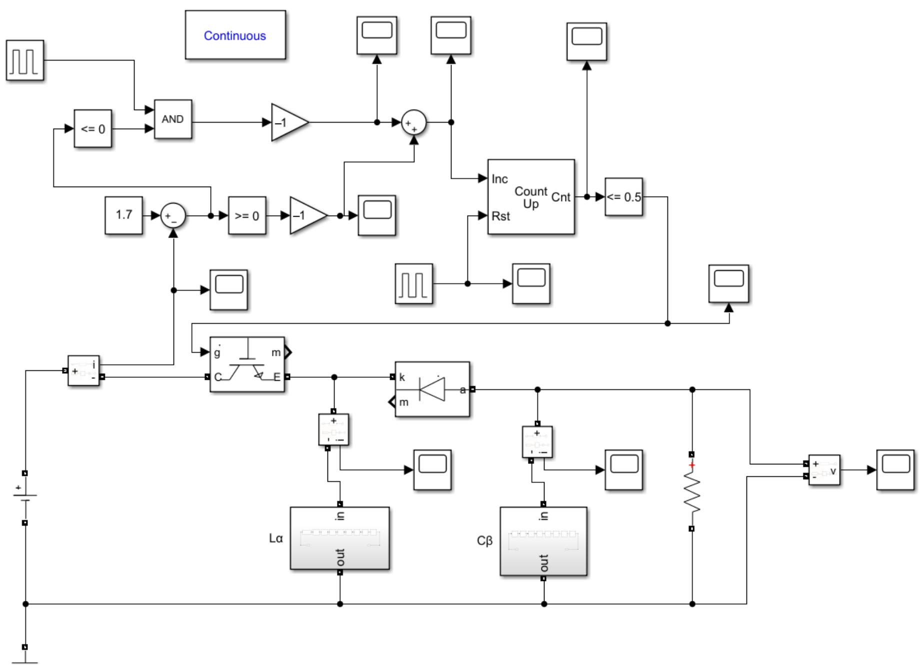
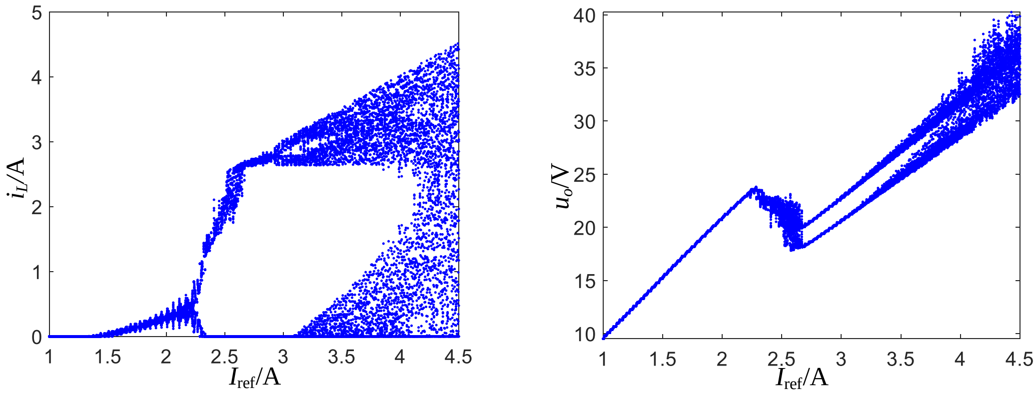


Figure 3. The mathematical model of the FO buck–boost converter.

Table 1. The circuit parameters.

Circuit Components	Values
Resistor (R)	40 Ω
Inductor (L)	14 mH
Capacitor (C)	50 µF
Input voltage	20 V
Reference current	2 A
Switching cycle time	0.05 ms
Inductance order (α)	0.85
Capacitance order (β)	0.85

The bifurcation diagrams with the reference current I_{ref} as the parameter are shown in Figure 4, in which the horizontal direction is the reference current I_{ref} between 1 A and 4.5 A; the vertical direction is the inductive current $i_{L,}$ which ranges from 0 to 4.5 A; and the output voltage is $u_{o,}$ which ranges from 5 to 45 V.



(**a**) (**b**)

Figure 4. A bifurcation diagram of the FO buck–boost converter with I_{ref} as a parameter: (**a**) the inductive current i_L; (**b**) the output voltage u_o.

As shown in Figure 4, the buck–boost converter goes through period-1; period-2; period-4; and, eventually, exhibits chaos. The period-1 solution is stable until I_{ref} = 2.2 A, whereupon a period-doubling bifurcation takes place. The second period-doubling bifurcation occurs, the converter enters a stable period-4 region at I_{ref} = 2.95 A, and the converter enters a stable period-4 region. The converter eventually goes to chaos just above I_{ref} = 3.25 A.

When $\alpha = \beta = 1$, the converter is an integer-order one. Figure 5 shows the bifurcation diagram of the integer-order buck–boost converter with I_{ref} as the bifurcation parameter. The bifurcations and chaotic behavior can be seen in the diagram as I_{ref} varies. When I_{ref} < 1.46 A, the system is stable. The first period-doubling bifurcation occurs at I_{ref} = 1.46 A, and the converter enters a stable period-2 region. As the I_{ref} is continuously increased to 2.01A, the second period-doubling bifurcation occurs and the converter enters a stable period-4 region. The converter eventually exhibits chaos when I_{ref} = 2.59 A.

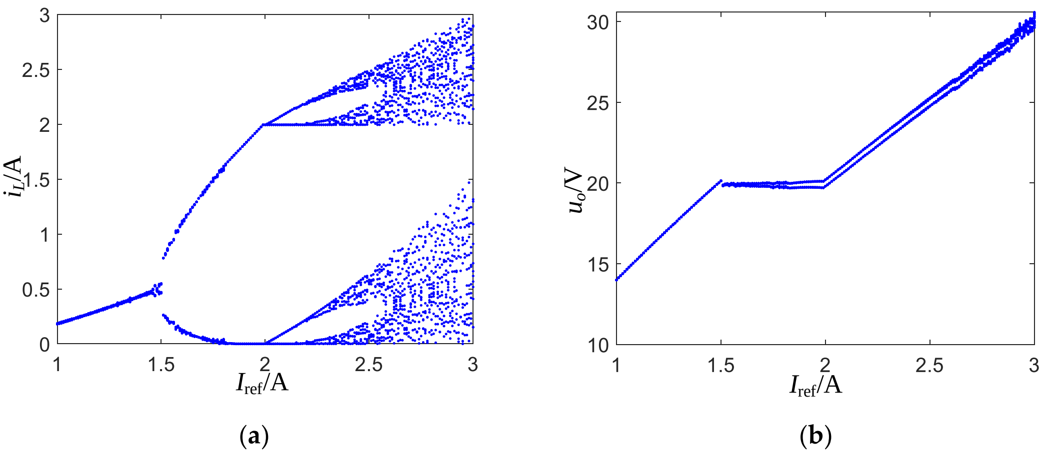


Figure 5. A bifurcation diagram of the integer-order converter with $I_{\rm ref}$ as the bifurcation parameter: (**a**) the inductive current i_L; (**b**) the output voltage u_o.

The reference current value range of fractional-order and integer-order DCM buck–boost converters is shown in Table 2.

Table 2. Reference current value range of fractional-order and integer-order DCM buck–boost converters.

Type of Converter	Stable Region	Period-2 Region	Period-4 Region	Chaotic Region
Fractional-order buck–boost	1.00~2.20 A	2.20~2.95 A	2.95~3.25 A	3.25~4.50 A
Integer-order buck–boost	1.00~1.46 A	1.46~2.01 A	2.01~2.59 A	2.59~3.00 A

From Table 2, it can be seen that the bifurcation phenomenon of the FO converter will lag behind that of the integer-order converter, so that the FO converter system has a larger stability interval.

3.2. Input Voltage U_{in} as the Bifurcation Parameter

Setting the reference current $I_{\rm ref}$ = 2 A, the other circuit parameters are shown in Table 1. The bifurcation diagrams of FO DCM buck–boost converter with input voltage $U_{\rm in}$ as parameter are shown in Figure 6.

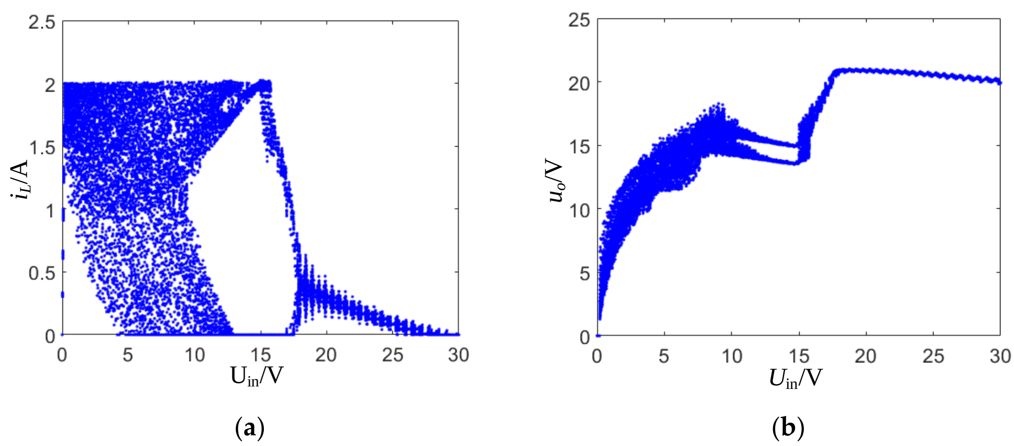
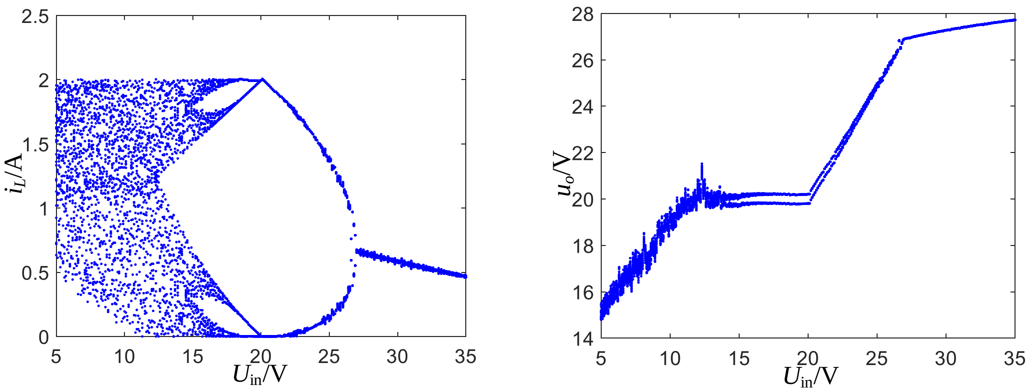


Figure 6. A bifurcation diagram of the FO DCM buck–boost converter with $U_{\rm in}$ as the parameter: (**a**) the inductive current i_L; (**b**) the output voltage u_o.

As shown in Figure 6, when the input voltage $U_{\rm in}$ > 18.65 V, the converter works in the period-1 state. The first period-doubling bifurcation occurs at $U_{\rm in}$ = 18.65 V, and the converter enters a stable period-2 state. As the input voltage decreases, the system enters a stable period-4 state at $U_{\rm in}$ = 15.30 V and eventually enters a chaos region at $U_{\rm in}$ = 12.45 V.

Figure 7 shows the bifurcation diagram of the integer-order buck–boost converter. When the input voltage $U_{\mathrm{in}} > 27$ V, the converter works in the period-1 state. When the input voltage U_{in} is between 27 and 20, the converter settles to the period-2 state via the doubling bifurcation. As the U_{in} decreases to 20, the second period-doubling bifurcation occurs and the converter enters a stable period-4 state. By further decreasing the input voltage, the converter finally goes into a chaotic state.



(**a**) (**b**)

Figure 7. A bifurcation diagram of the integer-order converter with U_{in} as the bifurcation parameter: (**a**) the inductive current i_L; (**b**) the output voltage u_o.

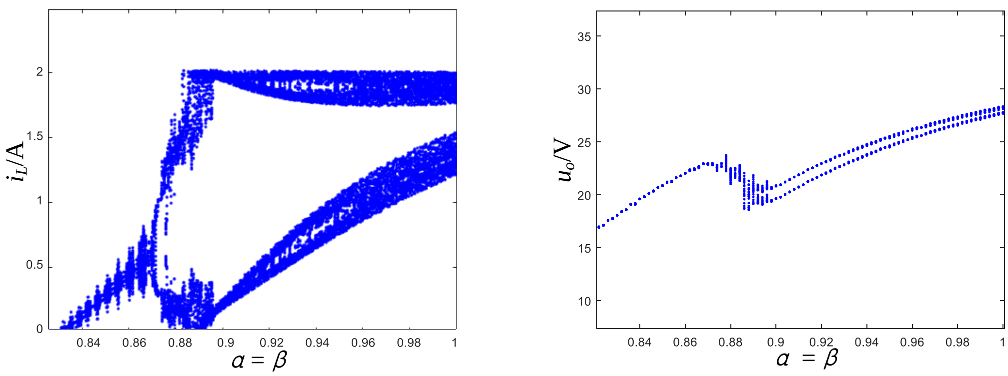
The input voltage value range of the fractional-order and integer-order DCM buck–boost converters is shown in Table 3.

Table 3. The input voltage value range of the fractional-order and integer-order DCM buck–boost converters.

Type of Converter	Stable Region	Period-2 Region	Period-4 Region	Chaotic Region
Fractional-order buck–boost	18.65~35.00 V	15.30~18.65 V	12.45~15.30 V	0~12.45 V
Integer-order buck–boost	27.00~35.00 V	20.00~27.00 V	15.00~20.00 V	5.00~15.00 V

3.3. Fractional Order of Inductance and Capacitance as the Bifurcation Parameter

Setting reference current $I_{\mathrm{ref}} = 2$ A and input voltage $U_{\mathrm{in}} = 20$ V, the other circuit parameters are shown in Table 1. The bifurcation diagrams with the fractional order of inductance and the capacitance order as the bifurcation parameter are obtained, as shown in Figure 8.



(**a**) (**b**)

Figure 8. A bifurcation diagram with the inductance and capacitance order as the bifurcation parameter: (**a**) the inductive current i_L; (**b**) the output voltage u_o.

As is shown in Figure 8, when fractional-order $\alpha = \beta < 0.868$, the converter works in stable period-1 state. When $\alpha = \beta = 0.868$, a period-doubling bifurcation takes place, and the converter goes into the period-2 state. The converter can go to the chaos region at $\alpha = \beta = 0.92$. When $\alpha = \beta = 1$, the converter has entered a chaotic state and the system is unstable. If the integer-order model is still used to describe the buck–boost converter of DCM mode, a wrong conclusion will be drawn.

From Figures 4–8, when the parameters vary, there are some fluctuations in the system, but the whole system is still in the stable range. However, once the parameter exceeds the critical value, the system immediately becomes unstable and begins period-doubling bifurcation.

3.4. Phase Portrait Analysis

Figure 9a,b show that when the reference current $I_{ref} = 1.6$ A, the period-1 solution is stable for the converter, and when the reference current $I_{ref} = 2.26$ A, a period-doubling bifurcation takes place when $\alpha = \beta = 0.85$. When $\alpha = \beta = 1$, the phase portrait of the converter is shown in Figure 9c at $I_{ref} = 1.6$ A, and it can be observed that the converter works in the period-2 state. Lastly, as shown in Figure 9d, when $I_{ref} = 2.26$ A, the phase portrait is period-2.

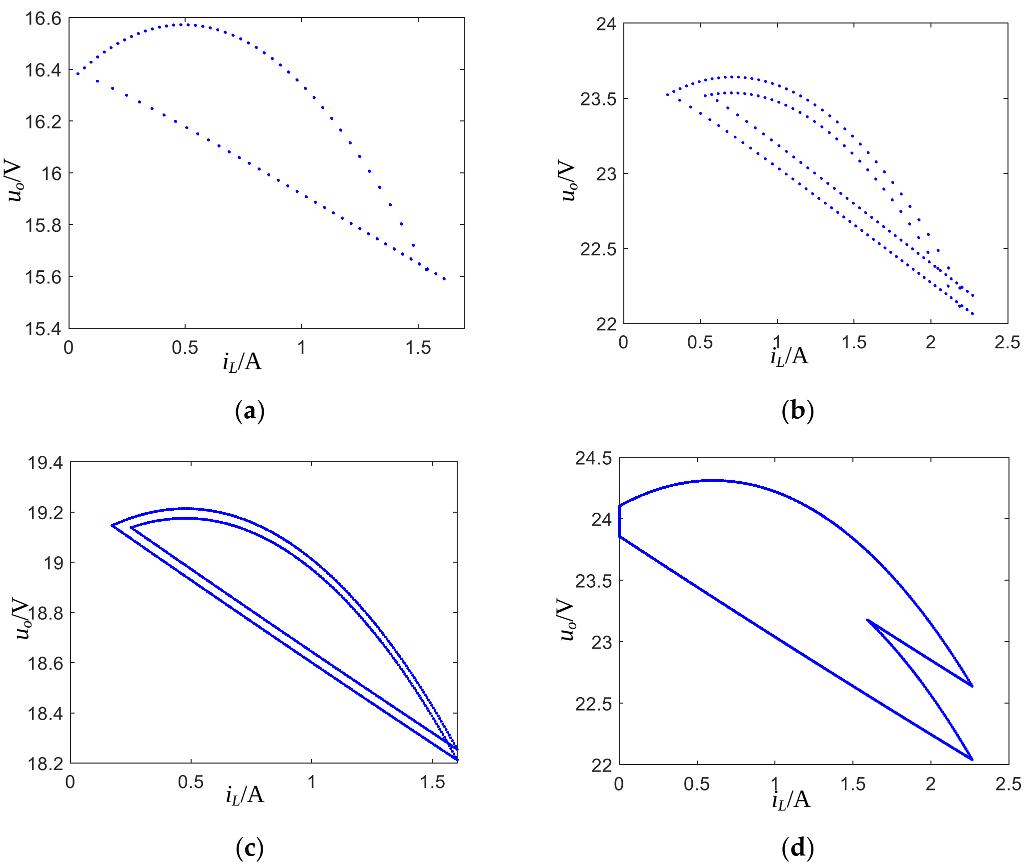


Figure 9. The V-I phase portrait of the different values of the reference current: (**a**) $I_{ref} = 1.6$ A, $\alpha = \beta = 0.85$; (**b**) $I_{ref} = 2.26$ A, $\alpha = \beta = 0.85$; (**c**) $I_{ref} = 1.6$ A, $\alpha = \beta = 1$; and (**d**) $I_{ref} = 2.26$ A, $\alpha = \beta = 1$.

Figure 10a,b show that when the input voltage $U_{in} = 25$ V, the period-1 solution is stable for the converter, and when the input voltage $U_{in} = 16.5$ V, a period-doubling bifurcation takes place when $\alpha = \beta = 0.85$. When $\alpha = \beta = 1$, the phase portrait of the converter is shown in Figure 10c at $U_{in} = 16.5$ V, and it can be observed that the converter works in the period-2 state. Lastly, as shown in Figure 10d, when $U_{in} = 25$ V, the converter works in the period-2 state.

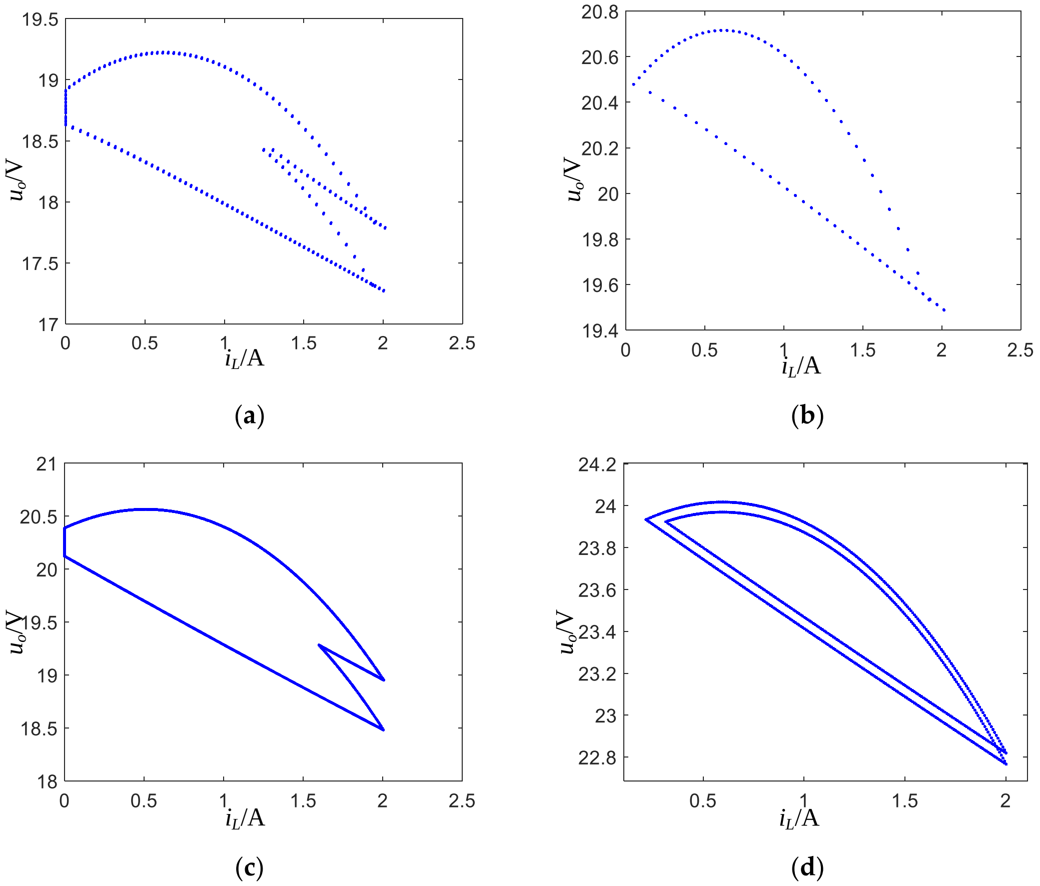


Figure 10. The V-I phase portrait of the different values of the input voltage: (**a**) U_{in} = 16.5 V, $\alpha = \beta = 0.85$; (**b**) U_{in} = 25 V, $\alpha = \beta = 0.85$; (**c**) U_{in} = 16.5 V, $\alpha = \beta = 1$; and (**d**) U_{in} = 25 V, $\alpha = \beta = 1$.

The phase portrait of the FO buck–boost converter are shown in Figure 11.

Figure 11 shows that when fractional-order $\alpha = \beta = 0.72$, the period-1 solution is stable for converter; when fractional-order $\alpha = \beta = 0.902$, a period-doubling bifurcation takes place; when fractional-order $\alpha = \beta = 0.99$, the system enters a chaotic state.

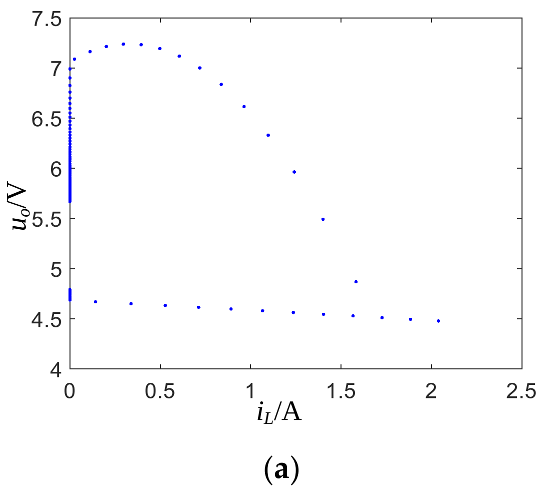
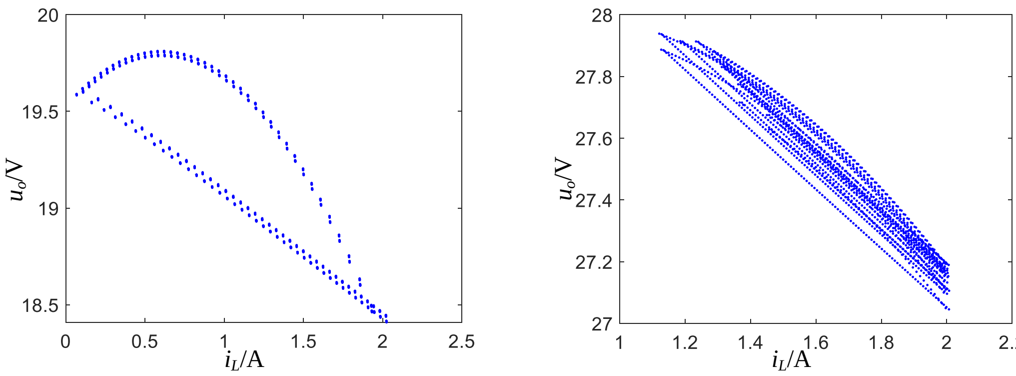


Figure 11. *Cont.*



(**b**) (**c**)

Figure 11. The V-I phase portrait of the FO buck–boost converter with different fractional orders: (**a**) $\alpha = \beta = 0.72$; (**b**) $\alpha = \beta = 0.902$; and (**c**) $\alpha = \beta = 0.99$.

It can be seen from the phase portrait that when other circuit parameters are the same, with the increase of the fractional order, the converter will experience the process from stable state to a period-doubling bifurcation and then enter a chaotic state. Meanwhile, the phase diagrams of the converter with the reference current and the input voltage as parameters are consistent with the bifurcation diagrams. According to the above theoretical analysis, the system is in a stable period-1 for $\alpha = \beta < 0.868$. The first bifurcation will occur at $\alpha = \beta = 0.868$, and it is chaotic for $\alpha = \beta > 0.92$. Figure 11 shows the phase portrait with $\alpha = \beta = 0.72$, 0.902, and 0.99, respectively. In Figure 11, the system goes through period-1 and period-2 and eventually exhibits chaos as the fractional-order varies from 0.78 to 0.99.

In summary, the simulation results support and validate the theoretical analysis.

4. Conclusions

In this paper, the voltage-mode-controlled FO buck–boost converter operating in DCM has been investigated to reveal the nonlinear dynamic behavior by using the predictor–corrector method. The predictor–corrector model, which describes FO buck–boost converters in closed-loop form, has been derived. The analysis of the dynamic evolution course of the period-doubling bifurcation and chaos is made by the bifurcation diagram and the V-I phase diagrams. It can be seen that the voltage-mode-controlled FO buck boost exhibits period-doubling bifurcation for certain values of the reference current, the input voltage, and the fractional order of the inductance and capacitance. The research results show that: (1) the FO buck–boost converters exhibit a wide range of nonlinear behavior, and the system approaches chaos via the period-doubling route as the reference current, input voltage, inductance, and capacitance order varies. (2) Compared with the integer-order buck–boost converter, the period-doubling bifurcation phenomenon of the FO converter will lag, which means the FO system has a larger stability interval. (3) Based on the fact that most of the inductance and capacitance are fractional-order, the model of FO buck–boost converter is more accurate than the integer-order one. Therefore, the results obtained from the FO buck–boost converter can more accurately reveal its nonlinear dynamic characteristics. It can be concluded that under certain parameter conditions, the system exhibits nonlinear dynamic behavior, such as period-doubling bifurcation and chaos. (4) Furthermore, the rationality and operability of the predictor–corrector method are confirmed, so the application of this method to other FO power converters will hopefully be realized and provides theoretical guidance for converter parameter optimization and controller design.

Author Contributions: Investigation, J.S.; Project administration, L.X.; Software, J.Y.; Writing—original draft, J.Y.; Writing—review & editing, D.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

There are some modules in the circuit model established by Simulink, as shown in Figure 3, and the information on them is displayed below.

The module "L^{α}" is the symbol of inductance in fractional order; it means that the order of inductance is 0.85, and the value of the inductance is 14 mH. The specific contents packaged in module "L^{α}" are shown in Figure A1.

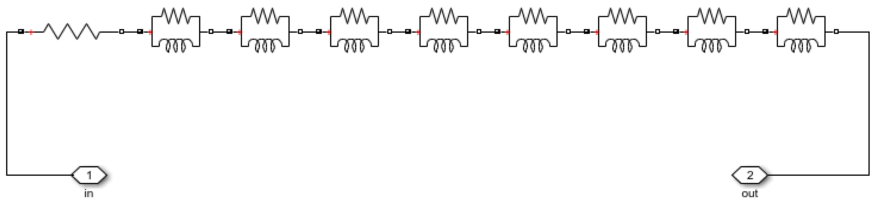


Figure A1. The content packaged in module "L^{α}".

As is shown in Figure A1, the module "L^{α}" is a circuit of the sub-reactor chain that consists of one resistance and seven parallel circuits, which are the combination of the resistance and the inductance, and the whole circuit appears as fractional inductance from the outside.

The module "C^{β}" is the symbol of capacitance in fractional order, and it means the order of capacitance is 0.85, and the value of the capacitance is 50 μF. The specific contents packaged in module "C^{β}" are shown in Figure A2.

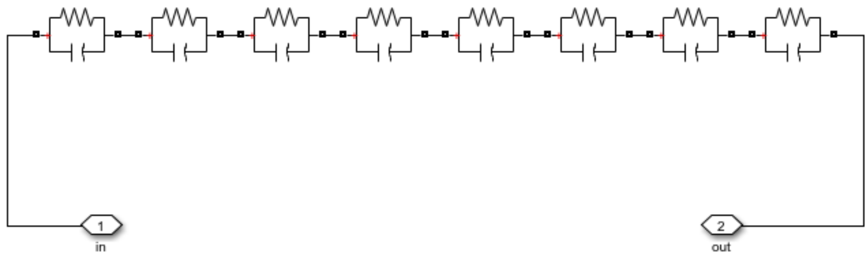


Figure A2. The content packaged in module "C^{β}".

As is shown in Figure A2, the module "C^{β}" is a circuit of a sub-reactor chain that consists of eight parallel circuits that are the combination of the resistance and the capacitance, and the whole circuit appears as fractional capacitance from the outside.

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