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Mathematics and Language: A One-to-One Correspondence in Bilingual Environments

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Abstract: Previous research has shown that language is an essential part of the development of mathematical skills and, specifically, in solving verbal problems. We know that using a different language from one's mother tongue is highly beneficial for students, and that language plurality and bilingualism is more and more habitual and present in current educational environments. However, what is still not clear is how it influences certain tasks, especially the most naturalistic ones, such as the formation of the concept of numbers and in tasks with a greater verbal component, such as solving problems in the early ages. The present research examined the problem-solving performance of first- and second-grade elementary education students in bilingual environments, comparing the problem solving of students whose language of instruction (LI) is the same as their mother tongue (MT) and those whose MT differs from the LI. Through an analysis of variance, the results showed that there exist differences in change and combination problems. Discrepancies in performance were also found, depending on the evolutionary moment. These results suggest that it is necessary to study how the LI can shape mathematical skills in the early years.

Keywords: mathematics teaching; bilingualism; language of instruction (LI); problem solving; cognitive development

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1. Introduction

In recent decades, mathematics teaching has been studied from various perspectives, and understanding how children develop mathematical concepts and the variables that influence this construction is a crucial issue that has lately gained greater relevance. One of the greatest challenges is to understand how language impacts the teaching of mathematics, as well as its influence on learning and how mathematical thinking develops.

The relevance of this complex question increases when educational programs use a second language, different from the mother tongue (MT), as the language of instruction (LI), to serve as the vehicle for educational experiences. Morgan [1] argues that there is currently no consensus on which language practices should be enhanced or improved in this context.

From this overview, there are three predominant approaches to relating language with mathematics, as stated by [2]: (1) the politics of language and linguistic diversity; (2) the various modes of communication and their linguistic representation; and (3) the interactionist aspect of language in classroom discourse.

This research will focus on the last approach, which aims to investigate how the language of instruction (LI) affects the development of mathematical concepts and competence in problem-solving tasks in schools. Particularly, the focus lies in the implementation of CLIL (Content and Language Integrated Learning) at early stages of education.

1.1. Content and Language Integrated Learning (CLIL)

In schools, bilingualism was born with the idea of acquiring a second language while learning various contents related to different subjects. From this perspective, different teaching models have been developed that focus on that duality, that is, teaching and learning the subject in a second language as a means of communication, such as Content and Language Integrated Learning (CLIL), one of the best-known methods [3]. Based on the literature, the CLIL approach is twofold, depending on the educational stage. For the upper levels, it can be implemented as a partial immersion process to teach half of the curriculum in another language. According to Bently [4], this is known as "hard CLIL". Alternatively, it can be used from a holistic and interdisciplinary perspective mainly to teach content to children in the early years and the first stages of primary school, known as "soft or weak CLIL" [5]. According to García [6], through this pedagogy, students acquire specific knowledge or content related to curricular areas.

Cenoz [7] differentiated CBI (Content-Based Instruction) from CLIL, and although they are pedagogical approaches and share the same characteristics, CBI is more well known in another context, that is, in the USA and Canada, while CLIL is more widespread around Europe [8]. For our study, which is carried out in these two different contexts, it is more relevant to consider these programs not from a contextual perspective but from the fact that they are promoting language proficiency, and to observe if academic content, such as mathematics, is someway affected by the LI. There is evidence that CLIL emphasizes problem solving and "know-how" by which learners' motivation increases as they develop problem-solving skills and carry out tasks in languages different from their own [9].

Concerning children's motivation, Lasagabaster and Sierra [3] also claim that it is greatly enhanced, and the curriculum content is effectively obtained. Most importantly, classroom activities designed in CLIL foster cooperation, collaboration, and opportunities to use materials along with the correct application of learning strategies and scaffolding. The authors also point out that integrating language to teach curricular contents also facilitates the appearance of opportunities to develop learners' cognitive skills and creative thinking. We can also say that there are contents whose low complexity allow them to be worked into other languages without major difficulty, but controversy arises when we introduce content that is a language in itself. For this reason, there are authors proposing that research on mathematics education and language needs to be established as an "area of study" [10], (p. 3).

Likewise, Vygotsky's sociocultural theory highlights the use of language as the natural vehicle of thought, because it is learned through social interactions in real contexts and gives meaning to words and expressions [11]. In this framework, learning mathematics in a second or foreign language implies being able to establish teaching models that allow the acquisition of knowledge and LI improvement.

For Moschkovich [12], there are three differentiated perspectives when teaching mathematics in bilingual contexts: vocabulary acquisition, meaningful construction, and participation in interactions in the discussion group. We must keep in mind that, to a greater or lesser extent, the success of bilingual learning will depend on the role of language in learning mathematics, paying special attention to the theoretical perspective adopted by the teacher [13]. Therefore, it is important that teachers consider the need to work on linguistic aspects specific to each subject, as well as the exchange of ideas between students, so that they obtain an adequate understanding of the concepts.

Language is an essential tool in academic fields such as history, biology, or mathematics that learners will have to cope with during their school lives and later; learning to use these discourse types, along with the skills each requires, is the key to academic and professional success. Furthermore, for the post-modern generation, learning to use a language different from one's native language has become a crucial ability among other competences necessary to survive in the workplace, that is, the exact areas in which CLIL specializes [14].

In order to improve the teaching of mathematics in a different LI from the MT, Erath et al. [15] propose the following six principles regarding the methodology and activities used in the classroom: (1) involving students in the exchange of ideas, practices or forms of resolution; (2) establishing several mathematical language routines; (3) connecting multiple language forms and multimodal representations; (4) including students' multilingual resources; (5) using scaffolding to sequence and combine language and mathematical learning opportunities; and (6) comparing elements of language (form, function, etc.) to increase students' grammatical construction.

When teaching takes place in the early ages, some of these principles would be more relevant than others; for example, the comparison of language elements can help establish greater phonological awareness in students that would help them in solving problems. For example, if the teacher establishes activities where they reflect on the meaning of "he has 7 more candies" or "he has 7 candies more than", it will allow students to establish differences between change and comparison problems.

It is, therefore, necessary for the speaker to develop metalinguistic skills to help him acquire and differentiate with greater or lesser success between more than one language. The child will refer to an object with different words or sounds depending on the language, and this does not depend on the characteristics of the words but on the language itself, because the same object can have different words that define it [16] (19). Volterra and Taeschner (20) had previously already announced the hypothesis of the unitary system, in which the lexicon or semantic system is the one that prevails and develops throughout the first years before the separation of syntactic linguistic systems in bilinguals.

Most of the USA bilingual programs are known as Transitional Bilingual Education (TBE), which are mainly designed for ELLs (English Learning Learners) and can serve any non-English language group, although they are generally addressed to Spanish-speaking students. Students can use their "native languages to help them transition to English" [17] (p. 6). However, more recently, Dual Language Education (DLE) is gaining popularity and replacing TBE programs, which are considered subtractive and assimilationist. DLE's main goal is to develop bilingualism, in the sense of speaking two languages fluently, educating in biliteracy, and achieving bilingual academic success [17]. Dual programs offer all students the possibility of learning at least two languages and content area subjects in both languages, the MT and the LI.

The improvement of linguistic skills and the development of mathematical concepts will also lead us to value the concept of translanguaging as a pedagogical tool, which, unlike code-switching and codemixing, is not simply about speakers changing languages but about constructing and using complex and original interrelated discursive practices [18,19]. Taking advantage of both the MT and the LI as a pedagogical tool will help the learner increase their understanding of complex linguistic mathematical concepts when necessary.

1.2. Learning Mathematics at Early Ages

All research studies highlight the importance of understanding the most basic mathematical concepts such as numerical processing, basic arithmetic operations, etc., since these can be applied and connected with the most complex concepts [20–22]. Friso-van den Bos [23] emphasizes the acquisition of numerical processing as one of the most important predictors of mathematical abilities in adults, and longitudinal studies have shown that children who are able to integrate the numerical sense of numbers and designate their signifier with their meaning have greater mathematical ability as adults. This develops during the early ages and is a long and complex process that covers all grades of elementary education. In the 1990s, it was studied by various neuropsychologists, such as Dehaene [24], and provides a model that still remains valid to this day.

The Triple Code Model proposes the existence of the following three main codes for numerical processing in the brain: (1) the quantity code, which is an innate system that allows human beings to estimate and compare approximate quantities without the need to use numerical symbols (approximate representation of numerical magnitude); (2) the digit code or system related to the processing of exact numbers, which is based on the use of numerical symbols (digits) to represent specific quantities (exact representation of numbers using symbols); and (3) the verbal quantity code, which is a system that connects the representation of numbers with a verbal form, that is, words (verbal representation of numbers).

All these systems are interrelated in the brain and work together to facilitate our understanding and processing of numbers. In order to read numbers aloud and be able to write them down, the child must carry out transcoding, that is, the process of transforming one format of a number into another format of the same number. It is, indeed, one of the most complex processes performed at the early ages [25,26]. The lexical processing of a number corresponds with the processing of the symbolic part of the number, such as the symbol for 7 or its name, "seven". Syntactic processing involves the rules that the child must learn to understand a group of numbers all together, which will allow him to establish a cardinality. It means that the child must know that the number is made up of both the value of its figures and the place they occupy within the number [27]. According to these studies, the child first needs to acquire the syntactic structure of the number as a framework within which he will later insert the names of the numbers in the corresponding places [28].

García Sala and Villagrán [29] found out that there are certain variables that, at early ages, make transcoding difficult: (1) the number of digits that the number has; this will affect the errors it provides, that is, the larger the number, the more errors are committed; (2) the position of zero, if it is in the last position; and (3) the difficulty in using a number due to its difficulty and/or irregularity; for example, 11 compared with 16 when naming it.

As we can observe, language has great weight in giving sense to the formation of numbers since it is made up of a piece of verbal and written language. However, it not only has that function, since any language allows us to distinguish between assimilating a number, establishing how precise it is, and differentiating the numerical capacity that individuals possess compared with other species [24].

When mathematics learning takes place in a language different from the MT, it brings about cognitive implications since those tasks that involve retrieval are carried out more efficiently in the LI that students have been instructed in than in the individuals' native language [30] (33). This indicates that the LI has strong dominance in the establishment of the concept, and when executive functions such as working memory are being used, it is interesting to know if this happens in the same way in tasks containing a greater linguistic component, such as verbal problems.

There are also models that deal with the influence of phonological efficiency in problem solving and with the choice of resolution strategies, depending on the phonological loop [31]. We need to specify that the role of the phonological loop is to encode and store the verbal codes used in calculations and is also involved in the temporary storage of intermediate results. That means that those strategies requiring less cognitive demand will always be performed; therefore, if the instruction is carried out in a different language from the MT, encoding and retrieval will tend to be enhanced in that language. However, this does not mean that the effectiveness will be the same as that of those students who received instruction in their MT.

Considering that problem solving involves textual understanding, and, consequently, that textual decoding and understanding are based on the phonological system, the LI becomes especially relevant [32].

These models suggest that the information phonologically analyzed at both the word and number level will be transferred to the working memory, which, in turn, will pass the information on to the processing system to free up space, and it will subsequently take on more information that allows it to keep on working on the task. The more difficult the information is, the more limited to higher levels the flow of information will be [33].

Solving verbal problems is one of the most comprehensive tasks because a child needs, from a very early age, to carry out processes such as planning, programming, using different strategies, listening, and verifying results, among others [34].

Research has demonstrated that problem solving is a reliable indicator of mathematical competence and cognitive skills in early years [35], as that involves the integration of fundamental cognitive skills such as attention, memory, and perception, making it a practical tool for monitoring children's development. It is expressed through motor skills. However, it is important to note that problem solving should not be the sole indicator of mathematical competence or cognitive skills. This ability is not static and varies with the exposure to tasks, development of logical skills, construction of the concept of number, and learning. Bermejo [36] considers the first years of primary school as crucial stages in studying problem solving. This is because problem solving is the most complex mathematical task that can be developed at that age, and it provides information about the construction of mathematical concepts and reveals erroneous approaches through the errors committed.

Furthermore, it is essential to emphasize that problem solving is a vital aspect when designing intervention programs and provides insight into areas that require improvement without being influenced by the MT language used. For instance, mathematical problem solving is an effective indicator for assessing the skills of both English and Spanish students, as it has been demonstrated to be independent of the language used to carry out the activity. For this reason, it is particularly relevant in research studies that combine language and mathematics. This is an aspect supported by Walker [35] for all languages.

We want to highlight that language use is especially relevant for the student because it will pave the way to finding a solution, explaining what process has been followed, and using it socially when sharing with his peers. Therefore, knowing the differences between students who work in their MT as well as the LI will allow us to establish teaching methodological guidelines in classrooms that will improve the increasingly widespread bilingual programs throughout the world.

2. Materials and Methods

2.1. Objectives

The purpose of the present study is twofold. Firstly, we aim to analyze the possible statistical differences in mathematical problems depending on whether or not the LI used in the mathematics teaching coincides with the MT. The discrete languages used are Spanish (Spa) and English (Eng). Secondly, we aim to determine the possible statistically significant differences in the different mathematical problems used (i.e., type of problem, location of the unknown, type of operation, and general score), depending on the course grade (first/second) of elementary education.

2.2. Design

This preliminary study employs an ex post facto research design, which is used to investigate the causes and awareness of a phenomenon that has already occurred [37–40].

2.3. Sampling

A sample of 241 individuals (129 girls) was used through non-probabilistic convenience sampling, all of them belonging to schools with bilingual programs. Two out of the three centers are located in Spain and implement a language immersion program in English. However, one of them teaches in English, and the other one uses Spanish as the LI. The third school is located in the USA, with a Spanish immersion program. In all the schools, we found students whose MT coincides with the LI as well as those who did not have a matching mother language, regardless of the program they were enrolled in.

The descriptive statistical analyses (see Table 1) indicate a mean (M) age of 6.98 and a standard deviation (SD) of 0.80, with a range between 5 and 8 years.

Table 1. Sample distribution according to age, dichotomized by LI and MT.

	LI											
	Spanish				English				Total			
	M	SD	n	%	M	SD	n	%	M	SD	n	%
	7.01	0.76	88	36.51	6.98	0.80	72	29.88	6.99	0.79	160	66.39
Mother Tongue (MT)	6.95	0.82	77	31.95	7.12	0.86	4	1.66	6.95	0.83	81	33.61
	6.98	0.80	165	68.46	7.01	0.81	76	31.54	6.98	0.80	241	100.00

This was a convenience sample, where the number of participants equaled the number of students in the first and second grades who were part of the bilingual school in which the study was conducted. In these multicultural and multilingual schools, there were students from many different nations, such as Brazil, the USA, Portugal, England, and Italy, among others; all these students were part of the sample.

We must also note that the sample analyzed was not segmented according to the country of origin; rather, it depended on whether or not the MT coincided with the LI. For example, in the sample of children whose MT was Spanish and the LI was also Spanish, we had students from Spanish and American schools. In Table 2, we show the coincidence of the language with respect to the country and the school in which they were enrolled.

Table 2. Distribution of the coincidence of the LI according to the country.

			LI **English**	**Spanish**	**Overall Total**
Country	ESP				
	Mother tongue	English	4	21	25
	Mother tongue	Spanish	72	72	144
Country	USA				
	MT	English		56	56
	MT	Spanish		16	16
		Overall Total	76	165	241

2.4. Instrument

In our study we used the following tests:

- The Raven Colored Progressive Matrices test [41]. We used it to assess whether there were significant IQ differences among the participants.
- Finally, each participant was individually administered 20 addition and subtraction word problems. These were administered orally and simultaneously shown in written form using the language in which the instruction had been carried out by means of cards. They were read at least twice and repeated as many times as was required by the students. In any case, no more than twenty problems were provided per participant. The answer was recorded as correct when the student knew how to explain the answer. When the answers were given at random, these were considered invalid. The selection of problems was based on the TEDI-MATH scale test, but since they did not cover all types of problems for assessment, they were complemented with problems extracted from the research carried out by Bermejo et al. [36]. The first-grade students solved problems that did not exceed a cardinality of ten. While the second-year students solved the same problems, the cardinality of the numbers was increased to a maximum of twenty. The problems were sequenced in increasing order of difficulty following the classification established in Bermejo et al. [42] (see Table 3):

Table 3. Classification of problems.

	Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Type	Change	x	x	x	x		x	x			x	x									
	Equalization					x			x									x	x		
	Combination									x										x	
	Comparison												x	x	x	x	x				x
Unknown	Result	x	x	x	x	x			x												x
	Medium						x	x		x							x	x	x	x	
	Beginning										x	x	x	x	x	x					
Operation	Addition	x		x		x	x			x			x		x			x		x	x
	Subtraction		x		x			x	x		x	x		x		x	x		x		
Total Score		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x

2.5. Procedure

We must note that the data collection was based on the principle of non-intervention, through which we were seeking to analyze the complete independence of the population, for which total freedom to give consent was provided. It resulted in the voluntary participation of the subjects with parental consent and without financial compensation for being part of the study, and it respects the anonymous and confidential nature of the participants.

In addition, informed consent was provided for the transfer of data to be processed solely for research purposes, seeking to respect the ethical principles regarding scientific research that are included in the Ethical Declaration of Helsinki.

Once we obtained the authorization to carry out the research, a presentation letter of the study was sent to the management teams of all the schools involved, who subsequently informed the families after its approval by the management team. Bearing in mind that the participants were minors, they were sent an informed consent letter for their parent/guardian to sign. All the data were obtained anonymously and treated confidentially.

2.6. Data Analysis

Before data exploration, the assumption of normality was verified using the Kolmogorov–Smirnov test (p K-S > 0.05). Based on the results, parametric statistics were used to analyze the data related to the course; specifically, the Student's t hypothesis contrast test with Cohen's d test as an estimator of the measured effect size. For the analyses in language coincidence, the ANOVA test was carried out based on the estimate of the size of the eta-squared effect (η^2), using version 26.0 of the statistical analysis software SPSS (Statistical Package for Social Sciences).

3. Results

Addressing the first research objective, Table 4 shows the descriptive analyses of the sample of mathematical problems differentiated by mother tongue (MT) and the language of instruction (LI).

Table 4. Hypothesis contrast test for independent samples of mathematical problems differentiated by grade and language coincidence (yes/no).

	No Language Coincidence				**Language Coincidence**			
	t	**gl**	***p***	***d***	***t***	**gl**	***p***	***d***
Type-Change	2.94	147	0.004 **	0.483	0.85	90	0.393	0.179
Type-Equalization	2.60	147	0.010 *	0.427	1.35	90	0.179	0.283
Type-Combination	2.79	147	0.006 **	0.458	3.47	90	<0.001 ***	0.725
Type-Comparison	3.50	147	<0.001 ***	0.574	2.89	90	0.005 **	0.604
Unknown-Result	2.90	147	0.004 **	0.475	2.37	90	0.020 *	0.495
Unknown-Medium	3.50	147	<0.001 ***	0.575	1.83	90	0.070	0.383
Unknown-Beginning	2.87	147	0.005 **	0.472	2.32	90	0.022 *	0.484
Operation-Addition	3.74	147	<0.001 ***	0.614	3.72	90	<0.001 ***	0.777
Operation-Subtraction	3.19	147	0.002 **	0.523	0.87	90	0.384	0.183
Total Score	3.65	147	<0.001 ***	0.599	2.558	90	0.012 *	0.534

Note: * $p < 0.050$, ** $p < 0.010$, *** $p < 0.001$.

When there were no language coincidences, all the previously mentioned statistically significant differences (t > 2.50, $p < 0.05$) were maintained in favor of the second-year subjects compared with the first-year subjects. However, when there was a language coincidence, these differences were diluted in some cases: Type–Change (t = 0.85, $p > 0.05$), Type—Equalization (t = 1.35, $p > 0.05$), Unknown—Medium (t = 1.83, $p > 0.05$) and Operation—Subtraction (t = 0.87, $p > 0.05$).

The results indicate a clear influence of the LI on problem solving when it aligns with the MT. That is the reason the difference in problem solving between the first and second grades is less significant when the languages do coincide, with only a few cases showing significant differences, such as in comparison and combination problems, where the grammatical structure is more complex.

The descriptive analyses of the sample of mathematical problems differentiated by language coincidence resulted in the data presented in Table 5.

Table 5. Descriptive statistics of mathematical problems differentiated by language coincidence (yes/no).

	Coincidence	n	M	SD
Type-Change	No	149	5.812	1.783
	Yes	92	6.67	1.293
Type-Equalization	No	149	1.913	1.294
	Yes	92	2.17	1.210
Type-Combination	No	149	0.785	0.785
	Yes	92	1.09	0.821
Type-Comparison	No	149	2.322	1.872
	Yes	92	2.90	1.767
Unknown-Result	No	149	4.906	1.486
	Yes	92	5.20	1.242
Unknown-Medium	No	149	3.436	2.355
	Yes	92	4.30	2.152
Unknown-Beginning	No	149	2.490	1.679
	Yes	92	3.34	1.578
Operation-Addition	No	149	5.154	2.387
	Yes	92	6.05	2.420
Operation-Subtraction	No	149	5.678	2.684
	Yes	92	6.78	2.193
Total Score	No	149	10.832	4.810
	Yes	92	12.84	4.170

Therefore, we analyzed the possible statistically significant differences in the mathematical problems depending on whether MT coincided or not with the LI used in the mathematics teaching (see Table 6).

Table 6. Hypothesis contrast test for independent samples of mathematical problems differentiated by language coincidence (yes/no).

	t	gl	p	d
Type-Change	4.03	239	<0.001 ***	0.534
Type-Equalization	1.56	239	0.120	0.207
Type-Combination	2.85	239	0.005 **	0.378
Type-Comparison	2.39	239	0.018 *	0.317
Unknown-Result	1.56	239	0.120	0.207
Unknown-Medium	2.87	239	0.004 **	0.381
Unknown-Beginning	3.89	239	<0.001 ***	0.516
Operation-Addition	2.83	239	0.005 **	0.375
Operation-Subtraction	3.32	239	0.001 **	0.440
Total Score	3.30	239	0.001 **	0.438

Note: * $p < 0.050$, ** $p < 0.010$, *** $p < 0.001$.

As the previous test denotes, there are statistically significant differences in most of the factors evaluated, which are always with higher values in language coincidence compared with non-coincidence and with a small or moderate effect size. Specifically, language coincidence is superior in the categories of Type—Change, Type—Combination, Type—Comparison, Unknown—Medium, Unknown—Beginning, Operation—Addition, Operation—Subtraction, and Total Score.

The findings reveal noteworthy distinctions in almost all instances, with heightened academic proficiency observed among those students whose MT matches with the LI. This trend, however, is not consistently upheld in problems pertaining to equalization and those wherein the unknown variable corresponds to the outcome.

Concerning the second objective, apart from language coincidences, we analyzed the possible differences between the combinations of LI and MT (Spa–Spa, Eng–Eng, Spa–Eng, and Eng–Spa). Table 7 shows the descriptive statistics:

Table 7. Descriptive statistics of mathematical problems differentiated by LI and MT.

Coincidence LI-MT	**Yes**				**No**			
	Spa-Spa (n = 88)		**Eng-Eng (n = 4)**		**Spa-Eng (n = 77)**		**Eng-Spa (n = 72)**	
	M	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
Type-Change	6.70	1.27	6.00	1.83	6.08	1.82	5.53	1.71
Type-Equalization	2.19	1.20	1.75	1.50	1.90	1.35	1.93	1.24
Type-Combination	1.08	0.82	1.25	0.96	0.83	0.80	0.74	0.77
Type-Comparison	2.92	1.78	2.50	1.73	2.01	1.85	2.65	1.85
Unknown-Result	5.22	1.25	4.75	1.26	5.03	1.45	4.78	1.52
Unknown-Medium	4.31	2.14	4.25	2.75	3.39	2.43	3.49	2.29
Unknown-Beginning	3.38	1.57	2.50	1.73	2.40	1.81	2.58	1.54
Operation-Addition	6.06	2.44	6.00	2.16	5.16	2.51	5.15	2.26
Operation-Subtraction	6.84	2.15	5.50	3.11	5.66	2.79	5.69	2.58
Total Score	12.9	4.14	11.5	5.20	10.8	5.04	10.8	4.59

For this reason, we analyzed the possible statistically significant differences in the mathematical problems based on MT and LI (see Tables 8 and 9). It should be noted that in the post hoc contrasts, only those that show statistically significant differences are presented.

Table 8. Hypothesis contrast test for independent samples of mathematical problems differentiated by LI and MT.

	F	**gl_1**	**gl_2**	***p***	**η^2**	**Decisions**
Type-Change	7.17	3	237	<0.001 ***	0.083	Spa-Spa > Eng-Spa ***
Type-Equalization	0.97	3	237	0.407	0.012	
Type-Combination	2.92	3	237	0.035 *	0.036	Spa-Spa > Eng-Spa *
Type-Comparison	3.51	3	237	0.016 *	0.043	Spa-Spa > Spa-Eng **
Unknown-Result	1.34	3	237	0.261	0.017	
Unknown-Medium	2.75	3	237	0.053	0.034	
Unknown-Beginning	5.55	3	237	0.001 **	0.066	Spa-Spa > Eng-Spa * ∧ Spa-Spa > Spa-Eng **
Operation-Addition	2.64	3	237	0.081	0.032	
Operation-Subtraction	4.02	3	237	0.008 **	0.049	Spa-Spa > Eng-Spa * ∧ Spa-Spa > Spa-Eng *
Total Score	3.73	3	237	0.012 *	0.045	Spa-Spa > Eng-Spa * ∧ Spa-Spa > Spa-Eng *

Note: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 9. Subsequent contrasts of the tests on mathematical problems differentiated by LI and MT.

		Spa-Eng				**Eng-Spa**			
	LI-MT	***t***	**gl**	***p***	***d***	***t***	**gl**	***p***	***d***
Type-Change	Spa-Spa					4.62	237	<0.001 ***	0.776
Type-Combination	Spa-Spa					2.70	237	0.037 *	0.427
Type-Comparison	Spa-Spa	3.19	237	0.009 **	0.501				
Unknown-Beginning	Spa-Spa	3.79	237	0.001 **	0.533	3.03	237	0.014 *	0.514

Table 9. *Cont.*

	LI-MT	Spa-Eng t	Spa-Eng gl	Spa-Eng p	Spa-Eng d	Eng-Spa t	Eng-Spa gl	Eng-Spa p	Eng-Spa d
Operation-Subtraction	Spa-Spa	3.00	237	0.015 *	0.474	2.87	237	0.023 *	0.484
Total Score	Spa-Spa	2.90	237	0.021 *	0.455	2.81	237	0.027 *	0.480

Note: * $p < 0.050$, ** $p < 0.010$, *** $p < 0.001$.

The analysis of bilingual programs across diverse countries reveals a consistent trend in the competence displayed by students whose MT differs from the LI, resulting in closely aligned mean scores. Conversely, deviations from this trend are apparent in the response patterns of students whose MT corresponds with the LI. This contrast is substantiated in the subsequent tables, illustrating the disparity in the mean scores.

Considering the results, in most of the factors analyzed, there are statistically significant differences, with a small or moderate effect ($t > 2.50$, $p < 0.05$, $\eta^2 > 0.035$), and always with higher values in the Spa–Spa compared with the Eng–Spa (4 elements) and Spa–Eng (5 elements) groups; in some cases, being both together (3 elements). Specifically, Spa–Spa students obtained better scores than Spa–Eng in the categories of Type—Comparison, Unknown—Beginning, Operation—Subtraction and Total Score. Similarly, Spa–Spa showed statistically higher scores than Eng–Spa in the Type—Change, Type—Combination, Unknown—Beginning, Operation—Subtraction, and Total Score categories.

In order to address the second research objective, the descriptive analyses of the sample of mathematical problems differentiated by course resulted in the data presented in Table 10.

Table 10. Descriptive statistics of mathematical problems differentiated by grade.

	Grade	n	M	SD
Type-Change	1st	124	5.83	1.61
	2nd	117	6.47	1.65
Type-Equalization	1st	124	1.78	1.23
	2nd	117	2.26	1.26
Type-Combination	1st	124	0.68	0.76
	2nd	117	1.13	0.79
Type-Comparison	1st	124	2.03	1.78
	2nd	117	3.09	1.76
Unknown-Result	1st	124	4.69	1.33
	2nd	117	5.36	1.39
Unknown-Medium	1st	124	3.21	2.22
	2nd	117	4.36	2.27
Unknown-Beginning	1st	124	2.42	1.62
	2nd	117	3.22	1.66
Operation-Addition	1st	124	4.73	2.22
	2nd	117	6.31	2.39
Operation-Subtraction	1st	124	5.59	2.52
	2nd	117	6.63	2.49
Total Score	1st	124	10.33	4.40
	2nd	117	12.94	4.58

Once these data were extracted in response to the first research objective, we analyzed the possible statistically significant differences in the mathematical problems depending on the course (see Table 11).

Table 11. Hypothesis contrast test for independent samples of mathematical problems differentiated by grade.

	t	gl	*p*	*D*
Type-Change	3.03	239	0.003 **	0.391
Type-Equalization	2.95	239	0.003 **	0.380
Type-Combination	4.40	239	<0.001 ***	0.567
Type-Comparison	4.60	239	<0.001 ***	0.593
Unknown-Result	3.78	239	<0.001 ***	0.488
Unknown-Medium	3.97	239	<0.001 ***	0.512
Unknown-Beginning	3.75	239	<0.001 ***	0.483
Operation-Addition	5.29	239	<0.001 ***	0.682
Operation-Subtraction	3.20	239	0.002 **	0.412
Total Score	4.51	239	<0.001 ***	0.581

Note: ** $p < 0.010$, *** $p < 0.001$.

Once the hypothesis contrast tests were carried out, statistically significant differences were found in all the dimensions analyzed, always with higher values in the measurements for the second-grade students compared to the first-grade students and with an effect size, according to the interpretation of López-Martín and Ardura-Martínez [43], from small ($d > 0.200$) to moderate ($d > 0.500$). Specifically, they were in the following categories: Type—Change, Type—Equalization, Type—Combination, Type—Comparison, Unknown—Result, Unknown—Medium, Unknown—Beginning, Operation—Addition, Operation—Subtraction, and Total Score.

It is evident that the resolution patterns are identical in both the first and second grades. Mathematical competence is determined by the placement of the unknown, with the students in the second grade consistently demonstrating greater competence than those in the first grade. After analyzing these data, the sample was dichotomized between subjects who had a coincidence of LI and MT (Eng–Eng or Spa–Spa) or not (Eng–Spa or Spa–Eng). The descriptive statistics of the subsamples can be seen in Table 12.

Table 12. Descriptive statistics of mathematical problems differentiated by grade and language coincidence (yes/no).

		No Language Coincidence			Language Coincidence		
	Grade	**n**	**M**	**SD**	**n**	**M**	**SD**
Type-Change	1st	79	5.41	1.71	45	6.55	1.11
	2nd	70	6.25	1.76	47	6.79	1.44
Type-Equalization	1st	79	1.65	1.28	45	2.00	1.12
	2nd	70	2.20	1.25	47	2.34	1.27
Type-Combination	1st	79	0.62	0.75	45	0.80	0.78
	2nd	70	0.97	0.78	47	1.36	0.76
Type-Comparison	1st	79	1.83	1.78	45	2.37	1.74
	2nd	70	2.87	1.82	47	3.40	1.65
Unknown-Result	1st	79	4.58	1.41	45	4.88	1.19
	2nd	70	5.27	1.49	47	5.49	1.23
Unknown-Medium	1st	79	2.82	2.24	45	3.88	2.02
	2nd	70	4.12	2.29	47	4.70	2.21
Unknown-Beginning	1st	79	2.12	1.59	45	2.95	1.56
	2nd	70	2.90	1.68	47	3.70	1.51
Operation-Addition	1st	79	4.49	2.21	45	5.15	2.18
	2nd	70	5.90	2.36	47	6.91	2.33
Operation-Subtraction	1st	79	5.03	2.56	45	6.57	2.14
	2nd	70	6.40	2.64	47	6.98	2.24
Total Score	1st	79	9.53	4.49	45	11.73	3.91
	2nd	70	12.30	4.76	47	13.89	4.17

The presented averages indicate that first-year students have lower proficiency than second-year students. However, when considering the coincidence of LI with MT, students whose languages coincide perform better. The differences between first and second-year students tend to be greater when their languages do not coincide, but narrower when they do.

4. Discussion

One of the main objectives of the present research was to determine whether the language of instruction influences problem solving and whether or not the type of LI conditions these outcomes; that is, whether it has a greater effect when the MT is English and the students are being taught in Spanish or whether it is not relevant. We can see how competence does not depend so much on this aspect. In this respect, when dealing with problems of change and combination, the students whose MT is English and LI is Spanish, their LI shows greater performance. However, in the problems of equalization and comparison, it would be the other way around, meaning that performance would be higher in students whose MT is Spanish and whose LI is English.

If we focus on comparing students whose LI is the same or not, we see that in the problems of change, students whose MT and LI is Spanish show better resolution, followed by those whose MT is English and LI is Spanish. Significant differences are also established between the rest of the groups. In contrast, concerning combination, equalization, and comparison problems, we did not find significant differences between the groups in terms of their LI and MT. This may be due to the high variability of answers in the change problems and also because these are the least difficult problems for the youngest students. Moreover, we could observe that there were students with good understanding and planning capabilities compared with other structures, and that in comparison problems, students obtained a higher failure rate because they are more complex, and the differences diminish.

Nevertheless, we can also see that, depending on the place where the unknown is located, there are many more differences, particularly when it is located at the beginning. This can happen because only a small proportion of the students can understand the situations posed by this type of problem, as it requires a greater capacity for abstraction. On this occasion, those whose coincident language is Spanish would show greater resolution, followed by students with the LI in English and whose MT is Spanish.

One of the noteworthy findings pertains to the striking similarity observed in the problem-solving patterns among students whose MT does not correspond with their LI, regardless of whether the instructional language is English or Spanish. This underscores a clear influence of the language of instruction on problem resolution, irrespective of the specific language and academic course under consideration [19,44–48].

This leads us to propose that the language of instruction (LI) is a variable that influences the development of mathematical concepts and cognitive processes such as comprehension, planning, and execution, irrespective of the methodology employed. It is essential to note that teaching methodologies in the United States and in Spain differ; for example, methodologies in bilingual programs in the United States utilize peer discussion and hands-on manipulation, whereas Spanish programs do the same but on a minor scale. In light of the results, we can assert that language exerts a more substantial impact than the methodology itself, as evidenced by the analogous patterns observed among students enrolled in bilingual programs whose MT does not coincide with the LI, in contrast to those with congruent linguistic backgrounds.

The second main objective of this study aimed at discovering if there were significant differences between the first- and second-grade groups regarding problem solving. As we can see, second-grade students show greater competence in problem solving regardless of the type of semantic structure shown. This reaffirms those studies [44,49,50] that confirm evolutionary learning; that is, the more often the students do a task, the greater mastery they will acquire. One can observe that second-year students show better reading comprehension compared with first-year students. However, considering that the presentation of the problems was orally carried out, this reduces the correlation between problem solving and reading comprehension.

We must consider that in early childhood education, the problems that students mainly tackle are those of change and combination with the unknown in the result, and it is not until the first grade of elementary education that they face other verbal structures, such as comparison and equalization. However, in these problems, the first- and second-grade students did not show similar performance either, so regardless of whether the exposure to those problems was greater or less in previous stages, we can see how the resolution improves as students move up through the grades.

This could happen because second-grade students tend to improve their planning processes regarding the resolution and leave behind the more immediate responses in which the child only plans an operation with the numbers that arise in the problem without knowing how to explain what and why he did it.

Regarding the place of the unknown, we observe the same tendency as that occurring in accordance with the type of problem in the sense that second-grade students show better competence. As we can see, the resolution pattern is similar in both courses, since the problems that present the unknown in the result are the ones that have been solved best compared with those that present the unknown in one of the terms of the equation. Additionally, the problems with the unknown at the beginning have been those that present worse resolution, thus confirming all previous studies [19,42,51–53]. This allows us to conclude that problems with the unknown at the beginning are the ones that present the greatest difficulty compared with problems that have the unknown in the result, which students find the easiest of all to solve.

When focusing on the LI, we observe how the response pattern remains the same, with the second-grade students showing greater competence in problem solving, but the more we observe the problem, the grade, and the LI, the more differences we find, since students whose language (MT) is coincident have better competence compared with students whose language is not. It means that they can solve a higher number of problems. Moreover, it indicates that, although the presentation of the problems has been both oral and written, language proficiency will prevent the student from being able to carry out a process of decoding the language at the same time as transcoding the number at an early age, thus making resolution of the problems difficult.

If we observe the mean scores, we can see that in problems with a simpler structure, such as change problems, first-year students with a coincident language (MT) have an average of $\dot{X} = 6.55$ compared with second-year students with a non-coincidence language (non-MT), where we find that $\dot{X} = 6.25$. If we analyze the rest of the problems, the way they are solved is very similar. For example, in the most complex problems, such as comparison problems, we find an average of 2.37 for first-grade students using their MT and 2.87 for second-grade students using their non-MT. If we consider the differences that exist between first- and second-grade students in the change problems, we can observe that the younger students show a better resolution if the language is coincident; the average for MT = 6.55, while that for non-MT is 5.41. In the second grade, these differences are reduced; when they solve in their MT, the average is 6.79, while in their non-MT, it is 6.25. This makes us think that when the LI is not the same and the grade is lower, there exists a greater difference in the resolution of problems, and if the structure of the problem is simpler, the difference becomes increasingly smaller as students course through the upper grades. However, in problems where the structure is more complex, such as in comparison problems, the differences continue to remain. First-grade students (MT) show an average of 2.37 while those who solve in their non-MT have an average of 1.83. Secondly, we observe that the mean differences remain similar (MT, 3.40; non-MT, 2.87).

If we look at Table 7, we will see that when the LI is different from the MT, there are significant differences with respect to all types of problems, regardless of the place the unknown occupies or whether they are addition or subtraction problems. However, when the LI is the same, the differences do not appear in all problems. We can observe that in the simplest problems, such as change or equalization, there are no significant differences. We do not even see differences when the unknown is found at the end or in one of the other terms, since these problems are less difficult than when the unknown is located at the beginning.

This allows us to conclude that the student needs more time to decode and understand a problem in a language other than the MT, but as he becomes familiar with the structure and understands the situation, the gap narrows. It implies that LI has an initial effect in a direct way, but after being exposed to it for longer, the student acquires sufficient strategies to compensate for the difficulty that is found when the LI does not coincide with the MT. This makes us think that the mathematical language will develop independently of the LI at the beginning, as stated by Van Rinsveld et al. [47,52].

Considering all the results obtained, it becomes evident that the language of instruction functions as a variable with an influence analogous to the developmental stage of the student. The patterns shown by second-grade students with a non-coincident language closely resemble those exhibited by first-grade students with coincident language. It can be concluded that the LI may result in a gap in mathematical competence compared with the level attained in the MT during the early years. This leads to the idea that teaching mathematics in a language other the mother tongue may have a comparable impact to that of age on problem-solving dynamics [19]. Consequently, a new thorough investigation has been opened to ascertain whether these disparities tend to diminish over time and culminate in the attainment of comparable proficiency, mirroring the evolutionary trajectory of the student.

One of the limitations of this study is that the sample size of students whose MT is English and LI is English is very limited. This is because the students that were taught in English came from only one group, and the number of students with English as their MT was smaller. The rest of the groups analyzed were made up of a larger number of students, with close to eighty students each. Ideally, the sample should reflect the wider population; however, the reliance on a convenience sample from institutions with bilingual programs determined the number of students across the respective groups.

Additionally, we have also controlled different variables such as the homogeneity of the groups and the intervention of the teachers, considering that the groups belonged to different educational centers, and that the students analyzed in terms of their LI and MT did not come from the same center. Students whose MT and LI are Spanish came from both Spain and the USA, a methodological variable we have controlled. However, there are other variables that cannot be controlled because these are specific limitations of a non-experimental design.

At the same time, it is important to highlight that due to the distribution of the sample, which sometimes did not meet the requirements of normality, the analyses carried out have been obtained through non-parametric tests, thus limiting the robustness of the conclusions.

In regards to the educational impact related to the results obtained from this study, we can highlight two main facts related to teaching mathematics in bilingual programs. In the first place, we must recognize the influence it exerts on problem-solving processes at early ages. Therefore, it would be convenient to set teaching routines that allow the understanding of the situation proposed as developed by Bruner [53], that is, applying various mathematical teaching methods such as the enactive, pictorial, and symbolic steps. The first two methods allow the child to have a better transcoding of the number and, therefore, a better understanding of the shown situation [54].

Secondly, the relationship of teaching mathematics in an LI presents several aspects for analysis that are ignored. One of those generally neglected issues is that teaching in a language other than the MT must have specific aspects that would not be necessary in the MT, such as a systematic teaching of vocabulary. If we support early-years teaching with images that allow students to relate words to their signifiers and aid with the systematic teaching of key words and vocabulary they can use to decipher information, comparison situations such as “more than” or “less than” will help them to create strategies to solve these specific problems. An effective approach could involve the use of code-switching techniques that can effectively support bilingual students in the mathematics classroom, including translanguaging, code-switching for clarification, code-switching for scaffolding, and code-switching for vocabulary development [55].

5. Conclusions

Considering the results of our study as a reference, we can conclude that there is an effect of the LI on the resolution of verbal problems at early ages. Moreover, this effect is independent of both the LI used and the students' MT. There exists an evident uniformity in problem-solving patterns across bilingual programs conducted in both Spanish and English, where linguistic disparity exists. This uniformity contrasts with the distinct resolution patterns observed among students whose LI coincides with their MT.

We can also observe how the resolution patterns are repeated concerning the difficulty of the problems, whether the LI coincides with the MT or not, and also how an evolutionary learning of these problematic situations is set, finding better performance in the older students vs. younger students. The distinguishable difference in response patterns between first- and second-grade students corresponds to the differences observed in the coincident languages. For greater clarity, second-grade students with a non-coincident language display a resolution pattern reminiscent of their first-grade counterparts with coincident language. This calls for further research into these patterns at an advanced academic level, facilitating the empirical verification of whether language-induced differentials tend to attenuate longitudinally, analogous to age-related trends.

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