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# Scratch as an Environment for Learning the Coordinate System by Elementary School Students

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**Abstract:** The focal point of mathematics education research is to comprehend how students develop and consolidate their abstract mathematical knowledge. As goal of this research, the focus was on exploring the processes that sixth-grade students used to construct mathematical concepts linked to the coordinate system. In this study, a group of eight students was selected for research purposes. Video recordings were used to collect data through observations. The Abstraction in Context (AiC) framework was utilized to analyze the students' processes that took place within the Scratch environment. The research findings suggested that the grade six students successfully constructed most of the anticipated knowledge components, consolidated their previous knowledge related to the zero concept, and began to develop new ones. Specifically, the students were able to construct the concept of zero as a number that is neither positive nor negative, as well as comprehend the X and Y axes.

**Keywords:** Scratch; programming; coordinate system; elementary school students

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## 1. Introduction and Literature Review

The present study examined the processes used by elementary school students to construct mathematical concepts related to the coordinate system in the Scratch environment by means of three core themes: (1) constructing mathematical knowledge, (2) learning the coordinate system, and (3) programming. In the next section, we outline these themes.

### *1.1. Constructing Mathematical Knowledge*

Researchers are interested in the relationship between using technology to learn science and mathematics and students' understanding [1,2]. Abstraction is a multifaceted concept that can occur in various contexts, such as technological, mathematical and educational contexts. The Abstraction in Context (AiC) framework was developed by Dreyfus, Hershkowitz, and Schwarz [3] to describe abstraction processes in different contexts. They characterized abstraction as the process of rearranging former mathematical concepts into a pattern not previously known to the learner in a vertical manner. According to the AiC framework, the involvement in abstraction involves three stages: the recognition of a need for a new construct, the emergence of the new construct, and the consolidation of that construct.

The RBC model is used to describe and analyze the emergence phase of the AiC framework [3]. The three stages of the RBC model are recognizing (R), building with (B), and constructing (C). R involves the recognition that a previously acquired scientific construct is applicable to the current condition. B entails the blend of the distinguished concepts to reach a specific objective, such as the implementation of a plan, engaging in reasoning, or a solution to a problem situation. C refers to the assembly and integration of previous concepts through vertical mathematization to create a new concept. Consolidation is a continuing process that helps students become familiar with a concept, with the use of that concept becoming instant and obvious.

According to research by Dreyfus and Tsamir, as students become more confident in utilizing a new concept, their flexibility in applying it increases [4]. Consolidation of a construct is more likely to occur when it is utilized in subsequent activities after its emergence in a previous activity. Therefore, consolidation establishes a connection between consecutively constructing processes and is heavily influenced by the design of the activity structures that facilitate it.

### 1.2. Students' Learning of the Coordinate System

A coordinate system is a mathematical concept, commonly used in geometry, that employs a combination of numbers, directions, or angles to establish the precise position of a geometric object in either two-dimensional space (a plane) or three-dimensional space. Students need to understand the coordinate system. Skordoulis et al. [5] stated that a poor understanding of the coordinate system could be behind a student's inability to find the dimension of an object. Palupi [6] argued that a student needs to understand that a coordinate system must have a unique origin so that each point in the coordinate system can be uniquely located by this system. In addition, Palupi argued that students are taught coordinate systems primarily on a formal level, requiring them to remember the rules but leaving their mathematical knowledge aside.

Sarama et.al. [7] found that students tended to reverse the $x$-axis and the $y$-axis, especially when the coordinates of a point had a zero, in addition to ignoring the origin. Students' misconceptions related to zero have been reported in the literature [8,9]. For example, Muir [8] reported that elementary students did not conceive zero as a number, while Akdeniz [9] reported significant misconceptions within all the participating students' understanding of zero. Further, researchers have found that students' conceptions of numbers that they developed in elementary school had to be changed when they were introduced to negative numbers [10], which denotes the difficulty of learning the concept of negative numbers. In addition, Fuadiah and Suryadi [11] found that seventh-grade students struggled to understand negative numbers in terms of prerequisite knowledge, terminology, procedures, principles, and problem-solving skills. This suggests that the use of tools, especially technological tools, could assist learners in their conceiving of concepts related to the coordinate system. The technological environment constitutes a meaningful context for students as they visually show these concepts together with the relations among them. This meaningful context, when combined with a teacher's probing questioning, can result in a better understanding of mathematical concepts [12] related to the coordinate system. In the present research, we utilized the Scratch environment to constitute a meaningful context for the student learning of the coordinate system.

Though students can work with the coordinate system at an early age [13], they are formally introduced to it in grade six [6] or grade seven.

### 1.3. Scratch in Mathematics Learning

Programming in education has been attracting the attention of researchers in the technology classroom [14,15], i.e., in the science [16,17] and mathematics classrooms [18,19]. One program that supports student programming is Scratch [20]. Brennan and Resnick [21] described how Scratch could be used to encourage various types of student computational thinking, including computational concepts, computational practices, and computational perspectives. They reported on a wide range of Scratch projects that were carried out, in which seven programming concepts were utilized, that could be transferred to other programming situations, such as sequences, loops, parallelism, events, conditionals, operators, and data types.

Calder [22] said that students can use numeric and symbolic representations in coding blocks to allow for the visualization of these representations and link them dynamically. This linking happens when a student writes code or observes its execution on the screen, in addition to selecting and moving blocks directly on the screen. Specifically, Calder [22] reported the use of Scratch by children to create mathematical learning objects as games, where this creation facilitated their problem-solving processes in geometry.

Daher et al. [23] reported on using Scratch to solve mathematics-based programming problems where the prospective teachers' negotiations supported the development of the students' meta-cognitive processes. The previous results indicated that discussions in a group or with the whole class could assist participants in technological environments in building knowledge and metaknowledge of studied mathematical and programming topics. In the context of the present research, the Scratch environment was used as an illustrator of the coordinate system as it provides the coordinates of a point for each sprite in a Scratch stage. We found Scratch more suitable than other technological environments, such as GeoGebra, due to its ease of use and students' familiarity with it as they had previously used it for coding.

### *1.4. Research Rationale and Goals*

Researchers have studied the role of the coordinate system in understanding mathematical and scientific ideas as algebraic ideas [24]. Few studies have been carried out regarding students' understanding of the coordinate system. Here, we examined how grade six students understand, in a technological environment—specifically, Scratch—the conceptions related to the coordinate system as the coordinates of points and the X and Y axes. The present research was interested in the whole class discussions that led to arguments related to mathematical concepts, where these arguments were based on students' work in the Scratch environment.

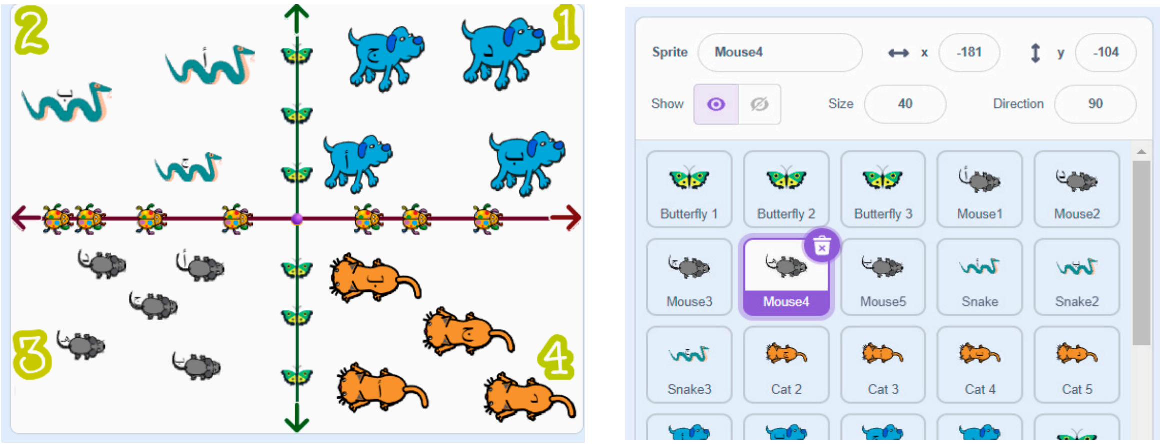
### *1.5. Research Question*

How would sixth-grade students build the concept of the coordinate system in the Scratch environment?

## **2. Materials and Methods**

Nine sixth-grade students participated in the study. The students and their parents agreed to participate in the study. The students' previous knowledge included: "positive numbers" and "the zero as smaller than any positive number". The participating students were high-achieving students in mathematics according to their mathematics teacher.

Two investigative activities were written to enable students to learn the coordinate system in the Scratch environment. In addition, a third activity was designed to assess the knowledge constructs related to the coordinate system. The first activity dealt with 'Points' properties in the four quadrants. The second activity dealt with "the X and Y axes". During the activities, the students worked in three groups of two students and one group of three students. After performing each activity, the students discussed with the teacher in the whole class setting the constructed knowledge elements. In the first and second activities, the Scratch stage was divided using the X and Y axes. Different kinds of animals were placed in the parts of the stage (See Figure 1a). The first activity requested the students to consider the X and Y values of the sprites that laid on each of the quadrants in the Scratch environment. Clicking on a sprite in the Scratch stage, the students could recognize its X and Y coordinate values from the area below the Scratch stage (Figure 1b). Then, they were requested to conclude what distinguished the points in each quadrant. In the second activity, the students examined the X and Y values of the sprites that laid on the "axes" and identified the properties of the points that laid on the X and Y axes. In the third activity, the students were requested to create, using Scratch, a stage that was similar to the stage they worked with in the first two activities (Figure 1a).




(**a**) (**b**)

**Figure 1.** The used Scratch stage and the area below it.

We collected the data using video and voice recordings, as well as the students' answers to the activities. The whole group discussion was also video-recorded. Data analysis was performed deductively to analyze the whole group discussions using the RBC theoretical framework. The expected knowledge elements and sub-elements to be constructed were assumed based upon an a priori analysis. The connections between the knowledge elements are displayed in Figure 2 according to the a priori analysis. We also considered the 'Zero concept' as well as 'the positive numbers' to be previous knowledge elements. An operational definition was created for each element to serve as a guide for the analysis of the knowledge constructing activity, given their crucial roles in the assumed construction processes.

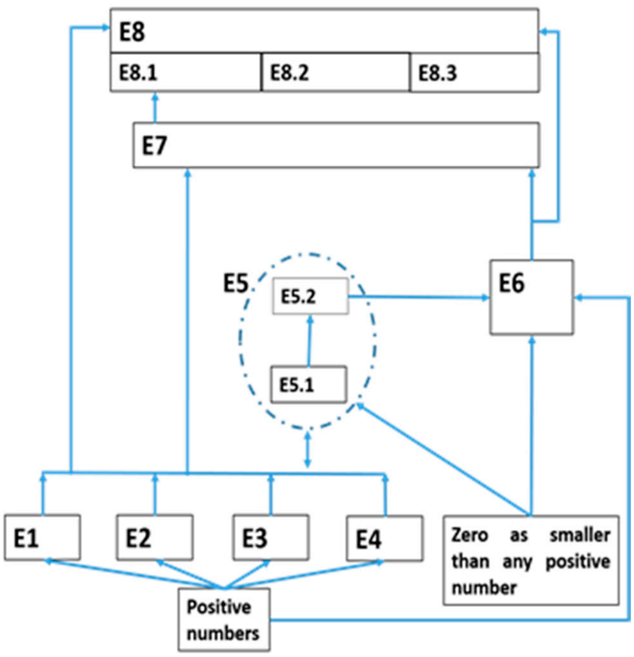



**Figure 2.** The connections between the assumed knowledge elements.

- E1/2/3/4: the point properties in the first/second/third/fourth quadrants, respectively
- E5: negative numbers
- E5.1: negative numbers as numbers that have the "-" sign
    a. E5.2: negative numbers are numbers that are less than zero

- E6: the zero is neither a negative number nor a positive number
- E7: the place of a point with a zero X-value
- E8: the X and Y axes
    a. E8.1: the Y-axis point properties
    b. E8.2: the X-axis point properties
    c. E8.3: the origin of the Cartesian coordinates

Here, we considered the expected knowledge elements to be constructed as (Ei), though when the student constructed that element, we referred to this process as construction (Ci) of the element. In doing so, we followed the authors of the framework, for example, Dreyfus et al. [4].

Table 1 shows an example of the knowledge constructing analysis method used in this research. The following episode occurred when the teacher discussed C1 (the point properties in the first quadrant) with the students after performing the first activity. In the discussion that followed, the teacher asked about positive numbers, negative numbers, and zero. It appeared that she wanted the students to build a new zero construct, C6, where the zero was neither a negative number nor a positive number.

**Table 1.** An example of the analysis method.

| TN | Speaker | Discussion | Knowledge Construction Analysis |
|---|---|---|---|
| 15 | Teacher | What about the zero? Is it positive? Is it negative? | Building a new construct with "Positive numbers" and "Negative" numbers: C6, where the zero is neither a negative number nor a positive number. |
| 17 | Raghad | Middle | Recognizing zero as "between" the positive and negative numbers |
| 18 | Taleen | Equivalent | |
| 19 | Qutaiba | Half and half | |
| 20 | Dima | It is positive and negative at the same time. | The emergence of an incorrect knowledge construct: "the zero is positive and negative at the same time". |
| | Omar | No, it is positive. | The emergence of an incorrect knowledge construct: "the zero is a positive number". |
| 20 | Teacher | What is your opinion, Taleen? | |
| 21 | Taleen | The positive and negative are equal in it. | Recognizing the zero as "equal". |
| 22 | Teacher | So, is it positive or negative? Remember, what are positive numbers? What are negative numbers? | Recognizing the definitions of a "Positive number" and a "Negative number" to build with them a new knowledge construct, C6, where the zero is neither a negative number nor a positive number. |
| 23 | Wadea | I think it is neutral. | The emergence of C6, where the zero is "not positive nor negative number". |
| 24 | Omar | Positive because it does not have a "-" sign. | The emergence of an incorrect knowledge construct: "the zero is a positive number because it does not have "-" sign". |

## 3. Results

The results indicated that the students successfully built the assumed knowledge elements and sub-elements. Here, we describe the process of the knowledge construction after performing the first activity in the Scratch environment, together with the episodes that showed this construction. As mentioned in the methods, we considered the expected knowledge elements to be constructed as (Ei), while when the student constructed that element, we referred to this process as construction (Ci) of the element (see Dreyfus et al. [4]).

Below, we shed light on the process of constructing C7 (the place of a point with a zero X-value) and C8.1 (the properties of coordinate point values that laid on the Y-axis).

In the a priori analysis, we assumed that constructing C1–C4 (the point properties in the first to fourth quadrants) would enable students to construct C5 (the negative numbers concept) and a new concept about zero, C6, where zero was not positive or a negative number. C6 was critical for constructing the other knowledge elements (C7–C8). Below are six episodes. In episodes one to five, the teacher discussed with the students C6 (episode one), C7 (episodes two, three, and four) and C8.1 (episode five). Episode six showed the drawing process of the X axes in the Scratch environment by a pair of students.

### *3.1. The Emergence of C6: Zero as Neither a Positive Number nor a Negative Number*

Episode one showed the emergence of C6 by the students in the whole class setting, where they discussed with the teacher C1–C4, which they built while completing the first activity in the Scratch environment. Before episode one, the teacher discussed with the students C1. In that discussion, the students argued that the coordinates of the points that laid in the first quadrant had no "-" sign. This led the teacher to ask about a name for the numbers that did not have the "-" sign (C5.1) and discuss the definition for negative (C5.2) and positive numbers. It appeared that she wanted the students to build with these constructs the new zero construct, C6.

Episode One: The emergence of C6 after constructing C1–C4 in the Scratch environment

1. Teacher: What about the zero? Is it positive? Is it negative?
2. Ragad: Middle.
3. Taleen: Equivalent.
4. Qutaiba: Half and half.
5. Dema: It is positive and negative at the same time.
6. Omar: No, it is positive.
7. Teacher: What is your opinion, Taleen?
8. Taleen: The positive and negative are equal in it.
9. Teacher: So, is it positive or negative? Remember, what are positive numbers? What are negative numbers?
10. Wadea: I think it is neutral.
11. Omar: Positive because it does not have "-" sign.
12. Qutaiba: The zero does not have a value.
13. Wadea: It is positive and negative at the same time.
14. Dema: I do not know.
15. Teacher: What is your opinion, Abdallah (Abdallah did not respond)?
16. Omar: Positive.
17. Teacher: Why?
18. Omar: I cannot say that the zero is negative because negative numbers have a "-" sign.
19. Teacher: Okay. But what is the definition of positive numbers?
20. Omar: Positive numbers are the zero and above.
21. Teacher: Remember what we said before that positive numbers are numbers bigger than zero.
22. Qutaiba: I know!
23. Mai: It is not negative and not positive.
24. Wadea: It is not negative and not positive. Negative numbers are under the zero. So, zero is not negative. Positive numbers are above the zero. So, zero is not positive.
25. Taleen: Teacher, this what I meant by "Equivalent".

In episode one, we noticed that the teacher focused the attention of her students on recognizing the definitions of negative and positive numbers [9,19]. In the process of constructing C6, the students used several terms in their attempts to describe the characteristic of zero as neither positive nor negative, as follows: middle [2], equivalent [3,25], half and half [4], neutral [10], and 'It is positive and negative at the same time' [13,23]. Gradually, most of the students appeared to be able to construct the "new" zero concept successfully [22–25]. On the other hand, the "-" sign that characterized just negative numbers and the "no" sign of the positive numbers hindered Omar in constructing the new "zero" concept [6,11,16,18,20].

### *3.2. The Emergence of C7: 'The Place of a Point with Zero X Value' Construct*

In episode two, which that occurred after episode one, the teacher triggered the students with a question about the place of (0, 100) on the coordinate system.

Episode Two: The emergence of C7

1. Teacher: Since we have discussed the properties of the zero number and determined the properties of the points that laid on the first quadrant, let us answer the following question: "Where do you think the (0, 100) point lie?". Please describe what distinguishes the points that lie in the first quadrant.
2. Students: The X values are positive and the Y values are positive.
3. Teacher: What did we conclude about the zero?
4. Wadea: The zero is not positive. It divides the positive and negative numbers.
5. Teacher: Okay. If we added another sprite that lies on the first quadrant of the Scratch stage, could the (0, 100) point represent the place of the sprite in the Scratch stage?
6. Mai: No.
7. Teacher: Why?
8. Student: Because of the zero-value of X.
9. Mai: The zero is not positive like the values in the first quadrant.
10. Qutaiba: I agree.
11. Teacher: Okay. Where does this sprite lie?
12. Taleen: In the middle, between the butterflies and ladybugs. Maybe here (pointing at the positive side of the X-axis) or here (pointing at the negative side of the X-axis).
13. Wadea: It does not exist, or there is no place for it.
14. Ragad: Teacher (she went to the board), here (pointing at the Y-axis).
15. Taleen: At the bottom (pointing at the Y-axis).
16. Mai: At the top of the green line (pointing at the Y-axis).
17. Wadea: The x-value is zero, so how could the sprite exist?
18. Qutaiba: It could not.
19. Taleen: Yeah, it could not.
20. Teacher: Okay. We will return to answer this question after discussing the properties of the points that lie in the other quadrants (the teacher discussed and determined with the students the properties of the points that laid in the other quadrants).

Episode two showed that the students recognized C1 [2] and C6 [4] correctly, which enabled some of them to conceive that the point (0, 100) did not lie in the first quadrant [6–10]. Mai explained this by referring to how the points' coordinates in the first quadrant were positive numbers, while the zero was not a positive number [9]. In addition, some students (Ragad, Taleen, and Mai) appeared to conceive that (0, 100) laid in the Y-axis [14–16], while other students could not conceive that this point had a "place" in the Scratch stage [13,17–19]. The teacher thought that the C1–C4 knowledge elements had critical roles in constructing C7. Therefore, she stopped this discussion, asking the students to recognize and discuss C1–C4 [20]. For example, while recognizing C3, the students said: "In the third quadrant, points have negative X values and negative Y values". Then, the teacher continued the discussion related to C7, as shown in episode three below.

### *3.3. Constructing C7: 'The Place of a Point with Zero X Value'*

Episode Three: Constructing C7

1. Teacher: Okay. We return to the question: "where do you think the (0, 100) point lie?"
2. Qutaiba: In no place. All the points have positive or negative numbers.
3. Mai: Not on the Scratch stage.
4. Dema: Not in the quadrants.
5. Ragad: At the lines.

6. Teacher: Which one of them?
7. Taleen: I suspect that it lies on the lines (meaning the x and y axes), but I do not know which one of them.
8. Omar: I think that it lies on the yellow point (meaning (0, 0)).
9. Teacher: The X-value in the point (0, 100) is zero, which is not positive or negative. The Y-value is 100, which is positive (writing on the board (+, +), (−, +), (−, −), (+, −), and (0, +)). What do you think?
10. Taleen: On the ladybugs and on the butterflies (meaning the Y and X axes).
11. Teacher: Where is it on the ladybugs? Where is it on the butterflies?
12. Taleen: Let me see, on the butterflies (the Y-axis).
13. Teacher: Where is it on the butterflies?
14. Taleen: On the upper part (meaning the positive side of the Y-axis).
15. Wadea: On the butterflies (meaning the positive side of the Y-axis). All the butterflies are above the ball (meaning (0, 0)).

Discussing the "ball" point coordinate values

After the teacher's question about the place of (0, 100) [1], Mai, Dema, and Qutaiba declared that (0, 100) did not have a place on the stage [2–4]. Differently, it appeared that Ragad, Taleen, and Omar started constructing C7 by saying that (0, 100) laid in one of the axes [5,7,8]. This caused the teacher to support the students in their knowledge construction by asking them to distinguish between the points that laid on the two lines (the X-axis and the Y-axis). Thus, she asked them to think about which "line" the point (0, 100) laid on [6,11]. In addition, the teacher declared of the previous concept of zero, C6, that "0 is not positive nor negative", and she wrote on the whiteboard the signs that represented C1–C4 ((+, +), (−, +), (−, −) and (+, −) in addition to the sign which represented the zero (0, +) [9]. Then, Taleen and Wadea expressed the "construct" by saying that (0, 100) laid in the positive side of the Y-axis [14,15]. The students' justifications of the constructed element, C7, are shown in episode four. These justifications show the building-with process of C7.

### *3.4. Students' Justifications of C7: 'The Place of a Point with Zero x-Value'*

Episode Four: Students' justifications of C7

1. Teacher (turns to the students): So, why does (0, 100) lie on the upper side of the green line (the positive side of the Y-axis)?
2. Mai: It is between the first and second quadrants. I say that the point (0, 100) lies on the green line (she came to the board and pointed at the upper side of the green line). This is because the zero is between the positive and the negative.
3. Teacher: Mai, can you please say again what you said (the teacher turns to the rest of the students)? If you have another idea, please tell me.
4. Mai: The points in the first quadrant are (+, +) (pointing at the signs that the teacher wrote on the white board). In the second quadrant, they are (−, +). We said that the zero is between the negative and the positive, and so it is on the green line (she points at its upper side).

Mai justified the constructed C7 by using C6, saying that the points on the green line divided the positive and negative numbers, with zero as the divider [2]. In addition, she used the C1–C4 constructs in the process of her C7 justification [4], which showed that these constructs played critical roles in the construction process of C7.

Episode Five: Generalization: From C7 to C8.1

35. Teacher: Can you tell me what is common for all the points that lie on the green line?
36. Mai: Dividers.
37. Taleen: They divide the positive and the negative.
38. Teacher: Which number is this?
39. Wadea: The zero.
40. Teacher: So, what is common to all points on the green line?
41. Students: Zero.

42. Teacher: Let us work with Scratch and verify our conclusion (the students begin working in Scratch).
50. Teacher: What could you conclude?
51. Qutaiba: It is because the value of X is zero. The values of X in the first quadrant are positive, and in the second quadrant, they are negative. So, the blue line divides the positive and the negative. This means that the points on this line have an X-value of zero.
52. Teacher What do you suggest calling the blue line?
53. Taleen: A divider.

Episode five showed that the students conceived that the zero divided the positive and negative numbers [36,37], which was related to the C6 construct. The students were build-with C8.1 by using this property of zero which, as they said, made the Y-axis points have zero in their coordinate values [39,41]. Concluding with the generalization, Qutaiba used C1 and C2 to specify that the X-values of the points that laid on the Y-axis were equal to zero [51].

### *3.5. Drawing the Axes in the Scratch Environment*

After constructing the assumed knowledge elements, the students were requested to draw the X and Y axes using the Scratch environment. Specifically, they were requested to create a stage similar to that shown in Figure 1. Episode six showed the discussion of one pair of students while drawing the *X*-axis, which happened after they drew the positive side of the *Y*-axis.

Episode Six: A pair of students drawing the *X*-axis in the Scratch environment

60. Raghad: How should we draw the red line (the *X*-axis)? I suggest that we move the ball again to the origin (0, 0).
61. Taleen: Okay. We need the "go to x:0 y:0 " block, the "Pen down" block, and the "move step" block.
62. Raghad: Okay. We will move the ball to the origin using "go to x:0 y:0" and then to the right side using "Pen down" and "move 200 step".
63. Taleen: It is not correct! We have to set the direction of the pen using the "Point in direction" block.
64. Raghad: Yes. Okay, now it is correct.
65. Taleen: We drew just the right side of the red line (talking to Raghad). We also have to draw the other side. Move the ball to the origin again.
66. Raghad: No. We do not have to do that. We can use just the "move 500 step" block.
67. Taleen: It does not work. Ahhh ... Again, we forgot to set the direction first. Raghad, use the "Point in direction −90" block and then the move block.

Episode six showed that Raghad and Taleen succeeded in drawing the X and Y axes. In doing so, a mathematical construct related to the axes was dominant in constructing the axes as elements of the coordinate system, where (0, 0) was the intersection point of the X and Y axes [61,62]. At the same time, they encountered some difficulties related to their programming knowledge. As mentioned, they used firstly the 'pen' blocks without a direction [60–62], but afterwards, they paid attention to the need for the direction block [63,67].

One of the groups encountered difficulty in drawing the axis through the 'Pen block' because they did not conceive of the need for a sprite for the drawing process. Afterward, they started to search for a character, and they found a sprite that was a line. Using this sprite, they added a ball and a line as sprites. They moved these sprites to (0, 0). To construct another axis, they used another sprite of the 'line type'. To make the two axes perpendicular, they used the 'turn 15 degrees' block six times. Here, too, the construct "(0, 0) was the intersection point of the x and y axes led the students to build the two axes as elements in the coordinate system.

## 4. Discussion

The present research intended to study how grade six students could construct their knowledge of the coordinate system including points and the two axes. The research results indicated that the Scratch environment enabled the students to construct their knowledge about mathematical concepts related to the coordinate system. For example, in episode one, the Scratch environment constituted a rich environment for the students to explore the point properties in the four quadrants (C1–C4), which led them to extend the known numbers to include negative ones (C5.1). The previous constructs were central to the students in constructing a new zero concept, C6, where "the zero is not negative nor positive", where they mentioned that the zero was 'middle', 'equivalent', 'half-half', 'positive and negative at the same time', 'positive', and 'neutral'. This discussion, which occurred amongst the whole class, together with the questioning of the teacher, supported the students in their construction of C6 (zero is not a negative or a positive number). Thus, features of the Scratch environment and the teacher's directions supported the students in constructing mathematical ideas [12,22].

The programming environment helped the students to suggest different solutions for the mathematical problems. For example, they used different Scratch blocks to draw the X and Y axes. Some groups used the pen-related blocks to do so, while one group used the 'line sprite' and the motion blocks to do so. The previous processes suggested that the Scratch environment could be a creativity-rich environment for mathematical learning. This adds to previous studies that have shown that technological environments could be environments for creativity [1].

In addition, the directions of the teacher supported the students in overcoming difficulties that have been pointed out by researchers as possible when the students learned the concepts of zero and negative numbers. Specifically, researchers have found that introducing negative numbers to students requires them to change their rooted conceptions about numbers that they had built in the elementary school [25]. Here, we found that this happened to the grade six students in conceiving two mathematical notions. The first notion was zero as a number that is not negative or positive and that is between the two types of numbers. The second notion was that of the X and Y axes. At the beginning, the students could not conceive of a point that had zero as its X value and that laid in the coordinate system. The whole class discussion, with the participation of the teacher, allowed the students to consolidate their previously constructed knowledge about the points in the four quadrants, and they used this knowledge to construct E6 and E7. In more detail, during the discussion, the students recognized E1 and E6 correctly, which allowed them build E7, stating that the point (0, 100) did not lie in the first quadrant. It appeared that the students' discussions were triggered by their work using the Scratch stage. Using the Scratch stage, the students ruled out that the point (0, 100) could lie in one of the quadrants, which led them to decide that it lay on the lines that divided the quadrants. The students engaged in the building-with process using the elements E1–E6, where this building-with led to the construction of E7. Here, the language of the students was still influenced by the software environment, and the students talked about the X and Y axes in terms of their colors [26].

We elaborated on the role of the teacher in supporting the students' constructions of their mathematical concepts related to the coordinate system. First, she asked the students direct questions, such as "What about the zero? Is it positive? Is it negative?", in episode one. This type of question caused the students to think about every possibility. Second, the teacher directed her questions to specific students, such as 'What is your opinion Taleen?', in episode one. Such questions could lead to different answers as the teacher was expected to approach different students depending on her previous knowledge of their way of thinking. Third, the teacher asked 'why' questions, as shown in episode two. These questions led the students to verify their answers and justify them. Fourth, the teacher presented Scratch-based mathematical situations, and she questioned the students about them. For example, she presented, in episode two, the following situation: "If we added another sprite that lies on the first quadrant of the Scratch stage, could (0, 100) point represent the place of the sprite in the Scratch stage?". This led the students to rule out the possibility of (0, 100) being in the first quadrant, and so they thought about an alternative place for (0, 100). Fifth, the teacher requested the students to work in the Scratch environment in order to explore the mathematical concept. For example, she approached them and said "Let us work with Scratch and verify our conclusion" in episode five. This work in the Scratch environment helped the students to verify their answers and develop them. The teacher's role included different questioning techniques, such as those described above, and this enabled both the teacher and the students to achieve the learning goals through stimulating the students with cognitive activities [26]. In addition, it was suggested by Burns [27] that the act of asking questions is important for the establishment of an environment in a classroom that is conducive to the development of a student's mathematical reasoning abilities. Here, the teacher supported the building of an environment conductive to mathematical inquiry and led discussion that resulted in the students constructing mathematical concepts related to the coordinate system.

The above argument indicated that the technological tool, Scratch, and a teacher, as escalators of student discussions, could support students in their knowledge construction of complex mathematical elements such as zero and the X and Y axes. Specifically, they could support students in their struggles with difficult scientific concepts such as zero, as this is a struggle that could lead to disequilibrium in a student's understanding of basic mathematical elements such as numbers [28].

In addition to the above, the students, as a collective even in the whole class context, could support the construction of each other's knowledge. For example, when the teacher asked the students where the (0,100) point laid, they answered in the following sequence: nowhere, not in the quadrants, on the lines, and on a specific line. Each answer led to a an additional answer that was a little bit more accurate until they constructed the related knowledge. This showed the power of the collective in education, in general [29], and in constructing mathematical knowledge, in particular [30].

## 5. Conclusions

Technological tools could support a student's construction of mathematical knowledge [30–32]. One such tool that has been adopted by teachers in the mathematics classroom and studied by mathematics education researchers is Scratch [33–36]. In the present research, the Scratch environment proved to enable the consolidation of the zero concept and the construction of coordinate system concepts as the coordinates of points in the four quadrants and as the coordinates of points on the X and Y axes. Teachers can utilize the Scratch environment to provide their students with tools with which they can work to construct concepts related to the coordinate system and to other mathematical concepts [22]. It is important that the utilization of the Scratch environment be accompanied with rich discussion with the teacher and between the students themselves. This rich discussion is intended to complement the work with the technological tool [12]. In the present research, we did not verify the programming utility in Scratch to advance the construction of concepts related to the coordinate system. Future research is needed to verify this issue.

The research findings showed that the Scratch environment could be an environment for the creative learning of mathematics. This is due to the different programming options that the Scratch environment includes. Future research is needed to verify this issue.

## 6. Limitations and Recommendations of the Study

The present study intended to investigate the knowledge construction of mathematical concepts related to the coordinate system by sixth-grade students. The study was performed on nine sixth-grade students. This number is one limitation of the present study, and so it is recommended that future research investigate the construction of mathematical concepts related to the coordinate system using additional students. It is worth investigating whether programming-based tools as Scratch can support elementary school students other than sixth graders in constructing mathematical concepts such as the coordinate system.

The present study did not focus on the coding completed by the sixth-grade students while learning mathematical concepts related to the coordinate system. It is recommended that future research investigate this issue when the students learn mathematical concepts.

The present research showed that Scratch could support elementary school students in building mathematical knowledge related to the coordinate system. Future research is needed to study how other technological tools as GeoGebra could support the construction of mathematical concepts, especially those identified in the literature as being complicated.

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