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Estimation of Dynamic Panel Data Models with Stochastic Volatility Using Particle Filters

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Abstract: Time-varying volatility is common in macroeconomic data and has been incorporated into macroeconomic models in recent work. Dynamic panel data models have become increasingly popular in macroeconomics to study common relationships across countries or regions. This paper estimates dynamic panel data models with stochastic volatility by maximizing an approximate likelihood obtained via Rao-Blackwellized particle filters. Monte Carlo studies reveal the good and stable performance of our particle filter-based estimator. When the volatility of volatility is high, or when regressors are absent but stochastic volatility exists, our approach can be better than the maximum likelihood estimator which neglects stochastic volatility and generalized method of moments (GMM) estimators.

Keywords: dynamic panel data models; stochastic volatility; particle filters; state space modeling

JEL Classification: C13; C15; C23

1. Introduction

Dynamic panel data models characterize the dynamic adjustment processes which are common in economic relationships. For estimation and inference, the literature has focused on the generalized method of moments (GMM) approach (e.g., Arellano and Bond [1]; Blundell and Bond [2]) and the likelihood approach (e.g., Hsiao, Pesaran, and Tahmiscioglu [3]; Hayakawa and Pesaran [4]). In this paper, we estimate dynamic panel data models with stochastic volatility by particle filters in a likelihood approach.

It has been well documented that there is time-varying volatility in macroeconomic data. Kim and Nelson [5], McConnell and Perez-Quiros [6] and Blanchard and Simon [7], for instance, studied the moderated volatility in the U.S. real GDP growth. Stock and Watson [8] found that the decline in volatility was common among many U.S. macroeconomic time series. They argued that the moderation was associated more with a decrease in the magnitude of unforecastable disturbances than with the propagation mechanism of those disturbances. Fernández-Villaverde and Rubio-Ramírez [9] provided an updated documentation of the great moderation in the U.S. economy. Besides, they showed the presence of time-varying volatility of the Emerging Markets Bond Index+ spread reported by J.P. Morgan [10]. Blanchard and Simon [7] and Stock and Watson [8] revealed that the time variation in volatility also existed in other developed countries. Traditionally homoscedasticity is assumed for the innovations of macroeconomic time series. Recently, motivated by the studies mentioned above, some papers incorporated time-varying volatility in various models. Fernández-Villaverde and Rubio-Ramírez [11] used the particle filter to estimate the DSGE models with stochastic volatility on the structural shocks. Koop and Korobilis [12] discussed the time-varying parameter vector autoregressive models with multivariate stochastic volatility. Hamilton [13] argued that time-varying volatility should be considered even when the conditional mean is the direct object of interest in that statistical efficiency gains can be obtained by incorporating appropriate features of time-varying volatility into estimation of the conditional mean.

Dynamic panel data models have become increasingly popular in macroeconomics to study common relationships across countries or regions, such as growth convergence (e.g., Islam [14]), purchasing power parity (e.g., Frankel and Rose [15]) and mean reversion of interest rates (Wu and Chen [16]). One example of incorporating time-varying volatility into dynamic panel models is the study of uncertainty shocks, which are often modeled as time variations in the volatility of business conditions or policy rules and can influence individual firm's decisions and macroeconomic aggregates [17–19]. To study the impact of such shocks on employment, output and other economic outcomes, one might use firm-level panel data in a standard model of the firm extended by introducing time-varying volatility of demand and productivity [17]. The model's solution involves estimating a dynamic panel data model with time-varying volatility in the innovations, which might be parameterized through stochastic volatility [11,20]. In the panel models with stochastic volatility, there is no closed-form expression for the likelihood. Consequently, some simulation-based method is required for efficient estimation.

Particle filters, also known as sequential Monte Carlo methods, are simulation-based techniques to estimate the posterior density of state variables for nonlinear and non-Gaussian state space models.[1] The methods approximate the continuous distribution by a discrete distribution made of weighted draws called particles. There are various algorithms which differ mainly in the choices of the incremental importance distribution and the resampling algorithm aimed to improve the level of statistical efficiency in terms of Monte Carlo variation. We apply the Rao-Blackwellized particle filter (e.g., Chen and Liu [22], Doucet, Godsill, and Andrieu [23], Andrieu and Doucet [24], Karlsson, Schon, and Gustafsson [25]) in particular to the state-space representation of dynamic panel data models in first differences and maximize the simulated likelihood using the approximation method in Olsson and Rydén [26]. When estimating the coefficients in the panel equation, Monte Carlo studies show that our estimator can be more precise than the maximum likelihood estimator which neglects stochastic volatility [3] and GMM estimators (e.g., Arellano and Bond [1]; Blundell and Bond [2]) in the presence of stochastic volatility especially in the case of high volatility of volatility. The advantage of the particle filter-based estimator is larger in dynamic panel data models with no regressors.

The paper is organized as follows. Section 2 presents the dynamic panel data models with stochastic volatility and the state-space representation. Section 3 introduces the particle filter-based estimator. Section 4 reports some Monte Carlo simulations to study finite sample properties. Section 5 concludes the paper.

2. The Models and State Space Representation

In general, we study the dynamic panel data models with time-varying volatility given by

$$y_{it} = B(L)y_{i,t-1} + \gamma \mathbf{x}_{it} + \mu_i + \sigma_t v_{it}, \tag{1}$$

where $i = 1, ..., N$ and $t = 1, ..., T$; $B(L)$ denotes a polynomial of lag operators; $\mathbf{x}_{it}$ denotes a vector of strictly exogenous regressors; μ_i is the individual-specific effect on which we impose no restriction under the fixed effects specification; ν_{it} is independently, identically distributed (i.i.d.) across both i and t and is independent of the volatility σ_t. We parameterize σ_t^2 through the stochastic volatility process to model conditional heteroskedasticity. The standard specification is

$$\log(\sigma_t^2) = (1 - \phi)\kappa + \phi \log(\sigma_{t-1}^2) + \eta_t. \tag{2}$$

[1] See Creal [21] for a survey of particle filters for economic applications.

Furthermore, ν_{it} and η_t follow a joint normal distribution

$$\begin{pmatrix} v_{it} \\ \eta_t \end{pmatrix} \sim N(0, \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \theta^2 \end{bmatrix}), \tag{3}$$

in which σ_v^2 is set to 1 for identifiability reasons. Various extensions of the basic specification can be made. For example, it is possible to incorporate dependence between ν_{it} and η_t, which is called the leverage effect suggested by the evidence of stock returns; it is also possible to model error terms using a fat-tailed distribution.

The focus of the paper is on estimation of the coefficients in (1)–(3) in a likelihood approach. Because of stochastic volatility, the tractable expression for the exact likelihood function is unknown. Consequently, we will compute the likelihood by simulation in the state space form of the transformed model. For ease of exposition, we consider as the benchmark model the AR(1) dynamic panel model with an exogenous regressor x_{it}:

$$y_{it} = \beta y_{i,t-1} + \gamma x_{it} + \mu_i + \sigma_t v_{it}. \tag{4}$$

The state space form of the first difference of the model can be written as

$$\Delta y_{it} = \begin{bmatrix} 1 & 1 & 0 & \Delta x_{it} \end{bmatrix} \alpha_{it} \tag{5}$$

$$\alpha_{it} = \begin{bmatrix} \beta & \beta & 0 & \beta\Delta x_{i,t-1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \alpha_{i,t-1} + \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \sigma_t \nu_{it}, \tag{6}$$

where the state vector is $\alpha_{it} = \begin{bmatrix} \beta\Delta y_{i,t-1} \\ \sigma_t \nu_{it} - \sigma_{t-1}\nu_{i,t-1} \\ -\sigma_t \nu_{it} \\ \gamma \end{bmatrix}$. (5) is the observation equation, while (2) and (6) are the state equations. This form is not the only state space version of the model, but all versions have the same joint likelihood of $\Delta y_i = (\Delta y_{i2}, \ldots, \Delta y_{iT})$ given Δy_{i1} and $\Delta x_i = (\Delta x_{i2}, \ldots, \Delta x_{iT})$, which we will use for parameter estimation. In the special case where there is no regressor, the state space form is simplified to

$$\Delta y_{it} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \tilde{\alpha}_{it}$$

$$\tilde{\alpha}_{it} = \begin{bmatrix} \beta & \beta & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tilde{\alpha}_{i,t-1} + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \sigma_t \nu_{it},$$

where the state vector is $\tilde{\alpha}_{it} = \begin{bmatrix} \beta\Delta y_{i,t-1} \\ \sigma_t \nu_{it} - \sigma_{t-1}\nu_{i,t-1} \\ -\sigma_t \nu_{it} \end{bmatrix}$.

3. Estimation by Particle Filters

We aim to estimate the model in a likelihood approach. Since both σ_t and v_{it} are stochastic, the model in first difference is non-Gaussian and a closed-form expression for the exact likelihood of $(\Delta y_1, \ldots, \Delta y_N)$ given $(\Delta y_{11}, \ldots, \Delta y_{N1})$ and $(\Delta x_1, \ldots, \Delta x_N)$ does not exist. Conditional on the volatility process $\{\sigma_t\}$, however, the resulting system including (3), (5) and (6) is a linear Gaussian state space model. It follows that the likelihood of $(\Delta y_1, \ldots, \Delta y_N)$ given $(\Delta y_{11}, \ldots, \Delta y_{N1})$, $(\Delta x_1, \ldots, \Delta x_N)$ and $(\sigma_2, \ldots, \sigma_T)$ can be evaluated analytically by the Kalman filter. This constitutes the basic idea of

Rao-Blackwellized particle filters, e.g., Chen and Liu [22], Doucet, Godsill, and Andrieu [23], Andrieu and Doucet [24], Karlsson, Schon, and Gustafsson [25].

Particle filters can be regarded as the extension of the Kalman filter to address nonlinear and non-Gaussian state space models in which the posterior density of state variables seldom has the closed-form expression. They are simulation-based techniques to obtain filtered estimates of the states as well as an unbiased estimate of the likelihood. Particle filters can be implemented in several ways, which vary with the choices of the incremental importance distribution and the resampling algorithm. The model containing (2), (3), (5) and (6) belongs to a class of models suitable for Rao-Blackwellized particle filters, which integrate out a subset of state variables (α_{it} in our case) in order to reduce the Monte Carlo variation of the simulation-based estimators.

We apply the algorithm of Rao-Blackwellized particle filters to the panel data models. Although we focus on the AR(1) dynamic panel model, the algorhtim can be adapted for a general model straightforwardly. Let $\mathcal{I}_{it}$ denote the information set containing all observations of y_{it} and x_{it} up to t. $m_{it|t-1} = \mathrm{E}(\alpha_{it}|\mathcal{I}_{i,t-1}, \sigma_t^2)$ and $\Sigma_{it|t-1} = \mathrm{Var}(\alpha_{it}|\mathcal{I}_{i,t-1}, \sigma_t^2)$ represent prediction mean and variance of α_{it}, respectively. The algorithm produces M simulated state variables and corresponding weights $\{\sigma_t^{2(j)}, m_{it|t-1}^{(j)}, \Sigma_{it|t-1}^{(j)}, \hat{w}_{t|t-1}^{(j)}\}_{j=1}^M$ for $t = 2, \ldots, T$ and $i = 1, \ldots, N$. Given the values of parameters $\omega = (\beta, \gamma, \kappa, \phi, \theta)$, the algorithm is:

(1) For $j = 1, \ldots, M$, draw $\log(\sigma_2^{2(j)})$ from the stationary distribution

$$\log(\sigma_2^{2(j)}) \sim N(\kappa, \frac{\theta^2}{1-\phi^2}),$$

and for $i = 1, ..., N$, set the initial values of prediction means and variances equal to expectations and variances respectively conditional on Δy_{i1}, $\sigma_1^{2(j)}$ and $\sigma_2^{2(j)}$:[2]

$$m_{i2|1}^{(j)} = \begin{bmatrix} \beta \Delta y_{i1} \\ 0 \\ 0 \\ \gamma \end{bmatrix}$$

$$\Sigma_{i2|1}^{(j)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\sigma_2^{2(j)} & -\sigma_2^{2(j)} & 0 \\ 0 & -\sigma_2^{2(j)} & \sigma_2^{2(j)} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Compute the conditional log-likelihood of $(\Delta y_{12}, \ldots, \Delta y_{N2})$ as $l_2^{(j)} = \sum_{i=1}^N l_{i2}^{(j)}$ where

$$l_{i2}^{(j)} = -0.5\log|V_{i2}^{(j)}| - 0.5 v_{i2}^{(j)} (V_{i2}^{(j)})^{-1} v_{i2}^{(j)},$$

with the forecast error $v_{i2}^{(j)} = \Delta y_{i2} - \begin{bmatrix} 1 & 1 & 0 & \Delta x_{i2} \end{bmatrix} m_{i2|1}^{(j)}$ and forecast variance $V_{i2}^{(j)} = \begin{bmatrix} 1 & 1 & 0 & \Delta x_{i2} \end{bmatrix} \Sigma_{i2|1}^{(j)} \begin{bmatrix} 1 \\ 1 \\ 0 \\ \Delta x_{i2} \end{bmatrix}$.

The log importance weight is $w_{2|1}^{(j)} = 0$ and the normalized importance weight is $\hat{w}_{2|1}^{(j)} = \frac{1}{M}$.

For $t = 3, ..., T$, $i = 1, ..., N$ and $j = 1, ..., M$,

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2 We assume $\sigma_1^2 \simeq \sigma_2^2$, so $\mathrm{var}(\sigma_2 \nu_{i2} - \sigma_1 \nu_{i1}|\sigma_1, \sigma_2) = \sigma_1^2 + \sigma_2^2 \simeq 2\sigma_2^2$.

(2) Update the weight by $w_{t|t-1}^{(j)} = w_{t-1|t-2}^{(j)} + l_{t-1}^{(j)}$ to incorporate the new likelihood [27] and $\hat{w}_{t|t-1}^{(j)} = \frac{\exp(w_{t|t-1}^{(j)})}{\sum_{j=1}^{M}\exp(w_{t|t-1}^{(j)})}$.

(3) Draw

$$\log(\sigma_t^{2(j)}) = (1-\phi)\kappa + \phi\log(\sigma_{t-1}^{2(j)}) + \theta\eta_t^{(j)},$$

where $\eta_t^{(j)} \sim N(0,1)$ is serially independent.

(4) Run the Kalman filter to obtain $m_{it|t-1}^{(j)}$ and $\Sigma_{it|t-1}^{(j)}$. Let $m_{it|t-1,2}^{(j)}$ denote the entry in the 2nd row of $m_{it|t-1}^{(j)}$ and $\Sigma_{it|t-1,22}^{(j)}$ denote the entry in the 2nd row and 2nd column of $\Sigma_{it|t-1}^{(j)}$. After some algebra (see the Appendix A), we have

$$m_{it|t-1}^{(j)} = \begin{bmatrix} \beta\Delta y_{i,t-1} \\ m_{it|t-1,2}^{(j)} \\ 0 \\ \gamma \end{bmatrix}, \tag{7}$$

and

$$\Sigma_{it|t-1}^{(j)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Sigma_{it|t-1,22}^{(j)} & -\sigma_t^{2(j)} & 0 \\ 0 & -\sigma_t^{2(j)} & \sigma_t^{2(j)} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{8}$$

such that

$$m_{it|t-1,2}^{(j)} = -\frac{\sigma_{t-1}^{2(j)}}{\Sigma_{i,t-1|t-2,22}^{(j)}}(\Delta y_{i,t-1} - \beta\Delta y_{i,t-2} - \gamma\Delta x_{i,t-1} - m_{i,t-1|t-2,2}^{(j)}), \tag{9}$$

and

$$\Sigma_{it|t-1,22}^{(j)} = \sigma_t^{2(j)} + \sigma_{t-1}^{2(j)} - \frac{\sigma_{t-1}^{4(j)}}{\Sigma_{i,t-1|t-2,22}^{(j)}}. \tag{10}$$

(5) Compute the conditional log-likelihood of $(\Delta y_{1t}, \ldots, \Delta y_{Nt})$ by $l_t^{(j)} = \sum_{i=1}^{N} l_{it}^{(j)}$ such that

$$l_{it}^{(j)} = -0.5\log|V_{it}^{(j)}| - 0.5 v_{it}^{(j)} (V_{it}^{(j)})^{-1} v_{it}^{(j)},$$

where $v_{it}^{(j)} = \Delta y_{it} - \begin{bmatrix} 1 & 1 & 0 & \Delta x_{it} \end{bmatrix} m_{it|t-1}^{(j)}$ and $V_{it}^{(j)} = \begin{bmatrix} 1 & 1 & 0 & \Delta x_{it} \end{bmatrix} \Sigma_{it|t-1}^{(j)} \begin{bmatrix} 1 \\ 1 \\ 0 \\ \Delta x_{it} \end{bmatrix}$.

(6) Resample with replacement (discussed later) M particles $\sigma_{t-1}^{2(j)}, \sigma_t^{2(j)}, m_{it|t-1}^{(j)}$ and $\Sigma_{it|t-1}^{(j)}$ with the weight $\hat{w}_{t|t-1}^{(j)}$ every three increments.[3] Then reset $w_{t|t-1}^{(j)} = 0$ and $\hat{w}_{t|t-1}^{(j)} = \frac{1}{M}$.

The simulation-based estimate of the joint log-likelihood of $(\Delta y_1, \ldots, \Delta y_N)$ given $(\Delta y_{11}, \ldots, \Delta y_{N1})$ and $(\Delta x_1, \ldots, \Delta x_N)$ is

[3] It is an ad-hoc choice for stability of the algorithm, see Shephard [27].

$$\hat{l}(\omega) = \sum_{i=1}^{N}\sum_{t=2}^{T}\log[\sum_{j=1}^{M}\hat{w}_{t|t-1}^{(j)}\exp(l_{it}^{(j)})]. \tag{11}$$

Note that this algorithm is appropriate when x_{it} is strictly exogenous. If it is predetermined or endogenous, the likelihood function would have a different form rather than the one given above, because the conditioning set assuming strict exogeneity includes all values of x_{it} from $t = 1$ to T.

In practice, one particle's normalized importance weight converges to one while the others converge to zero over time. In other words, the discrete distribution made of weighted draws would become degenerate. Resampling is crucial to stabilize the algorithm by eliminating the particles which have low importance weights and multiplying the heavily weighted particles. The simplest unbiased algorithm is multinomial resampling introduced in Gordon et al. [28]. It draws new particles $\{\tilde{\sigma}_{t-1}^{2(j)}, \tilde{\sigma}_{t}^{2(j)}, \tilde{m}_{it|t-1}^{(j)}, \tilde{\Sigma}_{it|t-1}^{(j)}\}_{j=1}^{M}$ from the point mass distribution $\{\sigma_{t-1}^{2(j)}, \sigma_{t}^{2(j)}, m_{it|t-1}^{(j)}, \Sigma_{it|t-1}^{(j)}, \hat{w}_{t|t-1}^{(j)}\}_{j=1}^{M}$. We employ multinomial resampling as it is a requirement for good asymptotic performance of the approximation method in Olsson and Rydén [26] used later.

Although (11) is an unbiased estimator of the exact log-likelihood under some regularity conditions, direct maximization for parameter estimation suffers from discontinuity induced from the generalized inverse operation at the resampling stage, which makes invalid the common gradient-based optimization methods. Olsson and Rydén [26] approximated the likelihood by means of step functions or B-spline interpolation and maximized the approximate likelihood. They showed consistency and asymptotic normality of the estimators under some assumptions. This seems the only work that studies asymptotic properties of parameter estimators in the particle filter literature. One of the assumptions requires a compact state space which is obviously not true in our case. We however still use this method for two reasons. One is that the compactness assumption can be potentially released given new results of uniform convergence properties in time dimension, although the full proof of the extension is beyond the scope of this paper; the other reason is the good finite sample performance shown below.

Specifically, Olsson and Rydén [26] discretized the parameter space Ω by a grid $\bar{\Omega} = \{\omega_g\}_{g=1}^{G} \subseteq \Omega$. Let $[\omega]$ denote the closest point in the grid to $\omega \in \Omega$.[4] The grid-based approximation of the likelihood using piecewise constant functions is given by

$$\hat{l}(\omega) \simeq \hat{l}([\omega]). \tag{12}$$

The approximation can also be made via spline interpolation, which is more efficient than piecewise constant functions but suffers from higher computational costs as the dimension of parameter space grows. Consequently, we maximize (12) to obtain the parameter estimates.

4. Monte Carlo Studies

In this section, we investigate the finite sample performance of particle filter-based estimators. The baseline data generating process in our Monte Carlo studies is an AR(1) dynamic panel model with an exogenous regressor

$$\begin{aligned} y_{it} &= \beta y_{i,t-1} + \gamma x_{it} + (1-\beta)\mu_i + \sigma_t v_{it} \\ x_{it} &= b x_{i,t-1} + \lambda \epsilon_{it} \\ \mu_i &= \sqrt{\tau}(\frac{q_i - 1}{\sqrt{2}})\varsigma_i \end{aligned}$$

[4] If there is more than one point having the smallest distance from ω, the point with lowest index g will be chosen.

$$\log(\sigma_t^2) = (1-\phi)\log(\mu) + \phi\log(\sigma_{t-1}^2) + \eta_t,$$

where $q_i \sim \chi_1^2$; $v_{it}, \epsilon_{it}, \varsigma_i \sim N(0,1)$ and $\eta_t \sim N(0,\theta^2)$; q_i, v_{it}, ϵ_{it}, ς_i and η_t are all i.i.d. within series and independent of each other.[5] We experiment with $\beta = 0.8$, $\gamma = 0.7$, $b = 0.5$, $\lambda = 2$ and $\tau = 1$. $N = 50, 100$ and $T = 50, 100$ are typical in macroeconomic applications. According to the empirical results of Fernández-Villaverde and Rubio-Ramírez [11], we consider $\mu = 0.002$, $\phi = 0.99$ and $\theta = 0.5$. We also check the performance of the estimators in the absence of stochastic volatility ($\phi = \theta = 0$) as well as in the case of higher volatility of volatility ($\phi = 0.99$, $\theta = 1$). We set $y_{i0} = 0$ and discard the first 100 observations of simulated data in order that the series is long enough to eliminate the initial effect. The number of the particles is set to 400, as we experimented with more particles up to 1000 but found few extra benefits. We carry out 1000 replications for each experiment. The quality of the estimators are evaluated by biases and root mean square errors (RMSE).

When estimating β and γ, we compare our estimator (PF) with some popular estimators in the literature, including the maximum likelihood estimator based on the model in first differences (FDML) [3], GMM [1] and system GMM (SGMM) [2]. Let ε_{it} denote the disturbance in the level equation. GMM exploits a set of linear orthogonality conditions $\mathrm{E}(y_{i,t-s}\Delta\varepsilon_{it}) = 0$ $(t \geq 2;\ s \geq 2)$ and $\mathrm{E}(x_{is}\Delta\varepsilon_{it}) = 0(t \geq 2;\ s \geq 1)$ (strict exogeneity) for the equation in first differences. We select a subset of these conditions often used in practice to improve the finite sample performance [4]. That is, $\mathrm{E}(y_{i,t-s-2}\Delta\varepsilon_{it}) = 0$ for $t = 2$, $s = 0$ and $t \geq 3$, $s = 0, 1$. SGMM uses extra moment conditions $\mathrm{E}(\Delta y_{i,t-1}((1-\beta)\mu_i + \varepsilon_{it})) = 0$ $(t \geq 2)$ and $\mathrm{E}(\Delta x_{it}((1-\beta)\mu_i + \varepsilon_{it}) = 0(t \geq 2)$ for the level equation. We use one-step GMM instead of two-step GMM for its better finite sample performance. We use the pattern search method in Matlab to find the maximum of the approximate likelihood function. PF is implemented at a quite low computational cost, although still slower than the other methods considered here.

Table 1 lists the estimation results for β and γ. The results seem quite mixed and no single method is dominant in all designs. While FDML or GMM is most precise when stochastic volatility is absent, PF can be preferred in the presence of stochastic volatility especially when the volatility of volatility is high. As the volatility of volatility grows, FDML, GMM and SGMM become much worse at estimating γ. This is not surprising as they neglect the information contained in stochastic volatility. Table 2 reports the estimates of ϕ, θ and μ in the stochastic volatility equation. The estimator $\hat{\phi}$ in the models without stochastic volatility has large bias. The results also show that the bias of $\hat{\mu}$ increases in the volatility of volatility. We also estimate an AR(1) panel model with no exogenous regressor x_{it} (the data generating processes for the other variables are same as above). The estimates of β are listed in Table 3. The absence of the regressor makes PF favoured when stochastic volatility exists. The estimates of ϕ, θ and μ in the model with no regressors reported in Table 4 are similar to those in Table 2. In sum, as PF takes correct specification into account, its performance is generally good in finite samples and relatively stable across designs.

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5 τ, which measures the degree of cross-section to time-series variation, can influence the finite sample properties of GMM-type estimators.

Table 1. Estimates of β and γ. In each design, the upper two rows give the biases and root mean square errors (RMSE) (in brackets) of $\hat{\beta}$, while the lower two rows give those of $\hat{\gamma}$. Numbers in bold font indicate the smallest biases and RMSE.

				$T = 50$							$T = 100$		
N	ϕ	θ	**FDML**	**GMM**	**SGMM**	**PF**	N	ϕ	θ	**FDML**	**GMM**	**SGMM**	**PF**
50	0	0	**0.000**	**0.000**	0.001	0.003	50	0	0	**−0.000**	**0.000**	0.002	0.003
			(0.000)	(0.001)	(0.002)	(0.003)				**(0.000)**	**(0.000)**	(0.002)	(0.003)
			−0.000	**0.000**	**0.000**	0.003				**0.000**	**0.000**	**0.000**	0.003
			(0.000)	**(0.000)**	(0.001)	(0.003)				**(0.000)**	**(0.000)**	(0.001)	(0.003)
	0.99	0.5	−0.003	−0.002	**0.000**	0.002		0.99	0.5	−0.001	−0.002	**−0.000**	0.003
			(0.009)	(0.012)	(0.012)	**(0.007)**				**(0.005)**	(0.009)	(0.010)	**(0.005)**
			0.001	**0.001**	**0.001**	0.004				**0.001**	**0.001**	**0.001**	0.004
			(0.017)	(0.017)	(0.016)	**(0.013)**				**(0.009)**	(0.013)	(0.013)	(0.011)
	0.99	1	−0.011	−0.007	−0.002	**0.000**		0.99	1	−0.006	−0.006	**−0.002**	**0.002**
			(0.022)	(0.035)	(0.023)	(0.037)				**(0.015)**	(0.023)	(0.023)	(0.029)
			0.078	**0.002**	−0.037	−0.008				−0.010	−0.042	−0.062	**−0.001**
			(1.444)	(0.983)	(1.909)	**(0.106)**				(1.664)	(1.800)	(2.418)	**(0.082)**
100	0	0	0.016	**0.000**	0.001	0.003	100	0	0	**−0.000**	**0.000**	0.001	0.003
			(0.028)	**(0.000)**	(0.001)	(0.003)				**(0.000)**	**(0.000)**	(0.001)	(0.003)
			−0.007	**0.000**	**−0.000**	0.003				**0.000**	**−0.000**	**−0.000**	0.003
			(0.012)	**(0.000)**	(0.001)	(0.003)				**(0.000)**	**(0.000)**	**(0.000)**	(0.003)
	0.99	0.5	0.016	−0.001	**−0.000**	0.002		0.99	0.5	−0.001	−0.001	**0.000**	0.003
			(0.033)	**(0.008)**	**(0.008)**	(0.016)				**(0.004)**	(0.006)	(0.006)	**(0.004)**
			−0.007	**0.001**	**0.001**	0.003				0.001	**0.000**	**0.000**	0.003
			(0.018)	**(0.017)**	**(0.017)**	(0.027)				(0.017)	(0.010)	**(0.008)**	(0.018)
	0.99	1	0.009	−0.004	**−0.002**	**−0.002**		0.99	1	−0.007	−0.003	**−0.001**	0.003
			(0.039)	(0.024)	**(0.019)**	(0.048)				**(0.013)**	(0.019)	(0.017)	(0.037)
			0.050	−0.054	−0.057	**−0.007**				**0.004**	0.007	−0.005	−0.008
			(1.134)	(2.978)	(3.046)	**(0.103)**				(0.942)	(1.510)	(1.591)	**(0.103)**

Table 2. Estimates of ϕ, θ and μ. In each circumstance, the first row gives the bias and the second row gives RMSE in brackets.

			$T = 50$						$T = 100$		
N	ϕ	θ	$\hat{\phi}$	$\hat{\theta}$	$\hat{\mu}$	N	ϕ	θ	$\hat{\phi}$	$\hat{\theta}$	$\hat{\mu}$
50	0	0	0.968	0.242	0.077	50	0	0	0.969	0.160	0.019
			(0.968)	(0.275)	(0.172)				(0.969)	(0.167)	(0.055)
	0.99	0.5	−0.052	0.082	0.254		0.99	0.5	−0.029	0.063	0.233
			(0.080)	(0.184)	(0.383)				(0.038)	(0.149)	(0.366)
	0.99	1	−0.058	−0.091	0.447		0.99	1	−0.040	−0.086	0.454
			(0.091)	(0.186)	(0.571)				(0.062)	(0.185)	(0.576)
100	0	0	0.968	0.317	0.155	100	0	0	0.968	0.182	0.054
			(0.969)	(0.357)	(0.266)				(0.968)	(0.204)	(0.149)
	0.99	0.5	−0.051	0.112	0.324		0.99	0.5	−0.031	0.076	0.261
			(0.081)	(0.208)	(0.441)				(0.042)	(0.164)	(0.381)
	0.99	1	−0.060	−0.096	0.476		0.99	1	−0.046	−0.089	0.505
			(0.091)	(0.191)	(0.588)				(0.066)	(0.181)	(0.609)

Table 3. Estimates of β in an AR(1) panel with no regressors. In each design, the first row gives the biases and the second row gives the RMSE in brackets. Numbers in bold font indicate the smallest biases and RMSE.

			$T = 50$							$T = 100$			
N	ϕ	θ	FDML	GMM	SGMM	PF	N	ϕ	θ	FDML	GMM	SGMM	PF
50	0	0	**−0.002**	−0.090	0.183	−0.035	50	0	0	**0.000**	−0.038	0.181	−0.011
			(0.015)	(0.123)	(0.184)	(0.042)				**(0.009)**	(0.053)	(0.182)	(0.014)
	0.99	0.5	**0.020**	−0.057	0.085	−0.028		0.99	0.5	0.026	−0.032	0.076	**−0.008**
			(0.059)	(0.110)	(0.121)	**(0.042)**				(0.066)	(0.066)	(0.111)	**(0.022)**
	0.99	1	0.030	−0.068	0.065	**−0.009**		0.99	1	0.035	−0.034	0.048	**0.004**
			(0.090)	(0.280)	(0.115)	**(0.060)**				(0.089)	(0.084)	(0.099)	**(0.054)**
100	0	0	**−0.001**	−0.058	0.178	−0.033	100	0	0	**0.000**	−0.021	0.173	−0.009
			(0.010)	(0.083)	(0.179)	(0.041)				**(0.007)**	(0.032)	(0.174)	(0.016)
	0.99	0.5	0.025	−0.038	0.079	**−0.021**		0.99	0.5	0.033	−0.021	0.076	**−0.003**
			(0.063)	(0.083)	(0.112)	**(0.040)**				(0.074)	(0.053)	(0.109)	**(0.030)**
	0.99	1	0.036	−0.037	0.061	**−0.001**		0.99	1	0.045	−0.028	0.053	**0.009**
			(0.093)	(0.096)	(0.107)	**(0.073)**				(0.097)	(0.081)	(0.098)	**(0.059)**

Table 4. Estimates of ϕ, θ and μ in an AR(1) panel with no regressors. In each circumstance, the first row gives the bias and the second row gives RMSE in brackets.

			$T = 50$						$T = 100$		
N	ϕ	θ	$\hat{\phi}$	$\hat{\theta}$	$\hat{\mu}$	N	ϕ	θ	$\hat{\phi}$	$\hat{\theta}$	$\hat{\mu}$
50	0	0	0.955	0.292	0.022	50	0	0	0.968	0.176	0.014
			(0.956)	(0.367)	(0.034)				(0.968)	(0.197)	(0.022)
	0.99	0.5	−0.109	0.227	0.167		0.99	0.5	−0.073	0.211	0.151
			(0.136)	(0.303)	(0.309)				(0.089)	(0.280)	(0.278)
	0.99	1	−0.092	−0.012	0.321		0.99	1	−0.067	0.007	0.353
			(0.123)	(0.101)	(0.488)				(0.086)	(0.082)	(0.512)
100	0	0	0.935	0.431	0.033	100	0	0	0.960	0.231	0.021
			(0.936)	(0.530)	(0.047)				(0.960)	(0.303)	(0.032)
	0.99	0.5	−0.128	0.251	0.151		0.99	0.5	−0.084	0.273	0.149
			(0.165)	(0.316)	(0.279)				(0.104)	(0.328)	(0.272)
	0.99	1	−0.104	−0.022	0.313		0.99	1	−0.075	0.001	0.330
			(0.137)	(0.109)	(0.476)				(0.098)	(0.090)	(0.497)

5. Conclusions

Motivated by time-varying volatility in macroeconomic data and the growing popularity of dynamic panel data models in macroeconomics, we propose a particle filter-based method to estimate the dynamic panel data models with stochastic volatility. Specifically, we represent the transformed model in the state space form and compute the simulated likelihood by Rao-Blackwellized particle filters. The parameter estimates are obtained by maximizing the likelihood approximated by piecewise constant functions. Monte Carlo results show that our estimator is relatively stable across scenarios and has good performance in finite samples, especially when the volatility of volatility is high or when regressors are absent.

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Appendix A. Deriving the Equations (7)–(10)

For ease of notations, we omit the superscript (j). We apply the standard Kalman filter recursive equations (see e.g., Durbin and Koopman [29]) to our model:

$$\begin{bmatrix} m_{i,t+1|t,1} \\ m_{i,t+1|t,2} \\ m_{i,t+1|t,3} \\ m_{i,t+1|t,4} \end{bmatrix} = \begin{bmatrix} \beta & \beta & 0 & \beta\Delta x_{it} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_{it|t-1,1} \\ m_{it|t-1,2} \\ m_{it|t-1,3} \\ m_{it|t-1,4} \end{bmatrix} + \begin{bmatrix} K_{t,1} \\ K_{t,2} \\ K_{t,3} \\ K_{t,4} \end{bmatrix} (\Delta y_{it} - m_{it|t-1,1} - m_{it|t-1,2} - \triangle x_{it} m_{it|t-1,4})$$

$$\begin{aligned} &\begin{bmatrix} \Sigma_{i,t+1|t,11} & \Sigma_{i,t+1|t,12} & \Sigma_{i,t+1|t,13} & \Sigma_{i,t+1|t,14} \\ \Sigma_{i,t+1|t,21} & \Sigma_{i,t+1|t,22} & \Sigma_{i,t+1|t,23} & \Sigma_{i,t+1|t,24} \\ \Sigma_{i,t+1|t,31} & \Sigma_{i,t+1|t,32} & \Sigma_{i,t+1|t,33} & \Sigma_{i,t+1|t,34} \\ \Sigma_{i,t+1|t,41} & \Sigma_{i,t+1|t,42} & \Sigma_{i,t+1|t,43} & \Sigma_{i,t+1|t,44} \end{bmatrix} \\ = &\begin{bmatrix} \beta & \beta & 0 & \beta\Delta x_{it} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Sigma_{it|t-1,11} & \Sigma_{it|t-1,12} & \Sigma_{it|t-1,13} & \Sigma_{it|t-1,14} \\ \Sigma_{it|t-1,21} & \Sigma_{it|t-1,22} & \Sigma_{it|t-1,23} & \Sigma_{it|t-1,24} \\ \Sigma_{it|t-1,31} & \Sigma_{it|t-1,32} & \Sigma_{it|t-1,33} & \Sigma_{it|t-1,34} \\ \Sigma_{it|t-1,41} & \Sigma_{it|t-1,42} & \Sigma_{it|t-1,43} & \Sigma_{it|t-1,44} \end{bmatrix} \\ &\left(\begin{bmatrix} \beta & 0 & 0 & 0 \\ \beta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \beta\Delta x_{it} & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \\ \Delta x_{it} \end{bmatrix} \begin{bmatrix} K_{t,1} & K_{t,2} & K_{t,3} & K_{t,4} \end{bmatrix} \right) \\ &+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{t+1} & -\sigma^2_{t+1} & 0 \\ 0 & -\sigma^2_{t+1} & \sigma^2_{t+1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \end{aligned} \tag{A1}$$

where

$$\begin{aligned} \begin{bmatrix} K_{t,1} \\ K_{t,2} \\ K_{t,3} \\ K_{t,4} \end{bmatrix} = &\begin{bmatrix} \beta & \beta & 0 & \beta\Delta x_{it} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Sigma_{it|t-1,11} & \Sigma_{it|t-1,12} & \Sigma_{it|t-1,13} & \Sigma_{it|t-1,14} \\ \Sigma_{it|t-1,21} & \Sigma_{it|t-1,22} & \Sigma_{it|t-1,23} & \Sigma_{it|t-1,24} \\ \Sigma_{it|t-1,31} & \Sigma_{it|t-1,32} & \Sigma_{it|t-1,33} & \Sigma_{it|t-1,34} \\ \Sigma_{it|t-1,41} & \Sigma_{it|t-1,42} & \Sigma_{it|t-1,43} & \Sigma_{it|t-1,44} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ \triangle x_{it} \end{bmatrix} \\ &[\begin{bmatrix} 1 & 1 & 0 & \triangle x_{it} \end{bmatrix} \begin{bmatrix} \Sigma_{it|t-1,11} & \Sigma_{it|t-1,12} & \Sigma_{it|t-1,13} & \Sigma_{it|t-1,14} \\ \Sigma_{it|t-1,21} & \Sigma_{it|t-1,22} & \Sigma_{it|t-1,23} & \Sigma_{it|t-1,24} \\ \Sigma_{it|t-1,31} & \Sigma_{it|t-1,32} & \Sigma_{it|t-1,33} & \Sigma_{it|t-1,34} \\ \Sigma_{it|t-1,41} & \Sigma_{it|t-1,42} & \Sigma_{it|t-1,43} & \Sigma_{it|t-1,44} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ \triangle x_{it} \end{bmatrix}]^{-1}. \end{aligned}$$

If

$$\begin{bmatrix} \Sigma_{it|t-1,11} & \Sigma_{it|t-1,12} & \Sigma_{it|t-1,13} & \Sigma_{it|t-1,14} \\ \Sigma_{it|t-1,21} & \Sigma_{it|t-1,22} & \Sigma_{it|t-1,23} & \Sigma_{it|t-1,24} \\ \Sigma_{it|t-1,31} & \Sigma_{it|t-1,32} & \Sigma_{it|t-1,33} & \Sigma_{it|t-1,34} \\ \Sigma_{it|t-1,41} & \Sigma_{it|t-1,42} & \Sigma_{it|t-1,43} & \Sigma_{it|t-1,44} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Sigma_{it|t-1,22} & -\sigma_t^2 & 0 \\ 0 & -\sigma_t^2 & \sigma_t^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

such as when $t = 2$,

$$\Sigma_{i2|1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\sigma_2^2 & -\sigma_2^2 & 0 \\ 0 & -\sigma_2^2 & \sigma_2^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

then

$$\begin{aligned} \begin{bmatrix} K_{t,1} \\ K_{t,2} \\ K_{t,3} \\ K_{t,4} \end{bmatrix} &= \begin{bmatrix} \beta & \beta & 0 & \beta\Delta x_{it} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Sigma_{it|t-1,22} & -\sigma_t^2 & 0 \\ 0 & -\sigma_t^2 & \sigma_t^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ \triangle x_{it} \end{bmatrix} \\ & \quad [\begin{bmatrix} 1 & 1 & 0 & \triangle x_{it} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Sigma_{it|t-1,22} & -\sigma_t^2 & 0 \\ 0 & -\sigma_t^2 & \sigma_t^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ \triangle x_{it} \end{bmatrix}]^{-1} \\ &= \begin{bmatrix} \beta \\ -\sigma_t^2/\Sigma_{it|t-1,22} \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \tag{A2}$$

Substituting (A2) into (A1) yields

$$\begin{bmatrix} \Sigma_{i,t+1|t,11} & \Sigma_{i,t+1|t,12} & \Sigma_{i,t+1|t,13} & \Sigma_{i,t+1|t,14} \\ \Sigma_{i,t+1|t,21} & \Sigma_{i,t+1|t,22} & \Sigma_{i,t+1|t,23} & \Sigma_{i,t+1|t,24} \\ \Sigma_{i,t+1|t,31} & \Sigma_{i,t+1|t,32} & \Sigma_{i,t+1|t,33} & \Sigma_{i,t+1|t,34} \\ \Sigma_{i,t+1|t,41} & \Sigma_{i,t+1|t,42} & \Sigma_{i,t+1|t,43} & \Sigma_{i,t+1|t,44} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_{t+1}^2 + \sigma_t^2 - \sigma_t^4/\Sigma_{it|t-1,22} & -\sigma_{t+1}^2 & 0 \\ 0 & -\sigma_{t+1}^2 & \sigma_{t+1}^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

which implies (8) and (10).

Further, if

$$\begin{bmatrix} m_{it|t-1,1} \\ m_{it|t-1,2} \\ m_{it|t-1,3} \\ m_{it|t-1,4} \end{bmatrix} = \begin{bmatrix} \beta\Delta y_{i,t-1} \\ m_{it|t-1,2} \\ 0 \\ \gamma \end{bmatrix},$$

such as when $t = 2$,

$$m_{i2|1} = \begin{bmatrix} \beta\Delta y_{i1} \\ 0 \\ 0 \\ \gamma \end{bmatrix},$$

then we will have

$$\begin{bmatrix} m_{i,t+1|t,1} \\ m_{i,t+1|t,2} \\ m_{i,t+1|t,3} \\ m_{i,t+1|t,4} \end{bmatrix} = \begin{bmatrix} \beta\Delta y_{it} \\ -\sigma_t^2(\Delta y_{it} - \beta\Delta y_{i,t-1} - m_{it|t-1,2} - \triangle x_{it}\gamma)/\Sigma_{it|t-1,22} \\ 0 \\ \gamma \end{bmatrix},$$

which implies (7) and (9).

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