

Text S1. Calculation of the coefficient of coincidence between the two adjacent SNP intervals for the female parent in an F₁ population of *Populus deltoides* and *Populus simonii*

Considering three ordered SNPs of 1, 2, and 3 that segregate in $ab \times aa$, the counts and frequencies of the genotypes at the three SNPs in an F₁ hybrid population of *P. deltoides* and *P. simonii* under different linkage phases are listed in Table S2, where r_{ij} denotes the frequency of crossovers that occur ($i = 1$) or not ($i = 0$) in the first interval between SNPs 1 and 2 and that occur ($j = 1$) or not ($j = 0$) in the second interval between SNPs 2 and 3.

Let C denote the coefficient of coincidence between the first and second intervals. Then, under the linkage phase of case 1 (Table S2), the crossover frequencies can be expressed as following

$$\begin{cases} r_{11} = r_1 r_2 C \\ r_{01} = r_2 - r_1 r_2 C \\ r_{10} = r_1 - r_1 r_2 C \\ r_{00} = 1 - r_1 - r_2 + r_1 r_2 C \end{cases} \quad (S1)$$

where r_1 and r_2 are the recombination fractions of the first and second intervals, respectively. When r_1 and r_2 are known, the likelihood of the coefficient of coincidence can be written as

$$L(C) = \frac{n!}{\prod_{j=1}^8 n_j!} \left(\frac{1}{2} r_{00}\right)^{n_1+n_8} \left(\frac{1}{2} r_{01}\right)^{n_2+n_7} \left(\frac{1}{2} r_{10}\right)^{n_4+n_5} \left(\frac{1}{2} r_{11}\right)^{n_3+n_6} \quad (S2)$$

where $n = n_1 + n_2 + \dots + n_8$.

The derivatives of the log-likelihood with respect to r_1 , r_2 , and C can be derived to be

$$\begin{cases} \frac{\partial \ln L}{\partial r_1} = \frac{n_3 + n_6}{r_1} + \frac{n_4 + n_5}{r_1} - \frac{(n_2 + n_7)C}{1 - r_1 C} - \frac{(n_1 + n_8)(1 - r_2 C)}{1 - r_1 - r_2 + r_1 r_2 C} \\ \frac{\partial \ln L}{\partial r_2} = \frac{n_3 + n_6}{r_2} - \frac{(n_4 + n_5)C}{1 - r_2 C} + \frac{n_2 + n_7}{r_2} - \frac{(n_1 + n_8)(1 - r_1 C)}{1 - r_1 - r_2 + r_1 r_2 C} \\ \frac{\partial \ln L}{\partial C} = \frac{n_3 + n_6}{C} - \frac{(n_4 + n_5)r_2}{1 - r_2 C} - \frac{(n_2 + n_7)r_1}{1 - r_1 C} + \frac{(n_1 + n_8)r_1 r_2}{1 - r_1 - r_2 + r_1 r_2 C} \end{cases} \quad (S3)$$

If we set the derivatives above to be zero, then the MLEs of r_1 , r_2 , and C can be solved as following

$$\begin{cases} \hat{r}_1 = \frac{1}{n} (n_3 + n_4 + n_5 + n_6) \\ \hat{r}_2 = \frac{1}{n} (n_2 + n_3 + n_6 + n_7) \\ \hat{C} = \frac{n_3 + n_6}{n \hat{r}_1 \hat{r}_2} \end{cases} \quad (S4)$$

For the other cases of linkage phase (Table S2), the MLE of the parameters can be similarly derived as following,

$$\begin{cases} \hat{r}_1 = \frac{1}{n} (n_3 + n_4 + n_5 + n_6) \\ \hat{r}_2 = \frac{1}{n} (n_1 + n_4 + n_5 + n_8) \\ \hat{C} = \frac{n_4 + n_5}{n \hat{r}_1 \hat{r}_2} \end{cases} \quad \text{for case 2} \quad (S5)$$

$$\begin{cases} \hat{r}_1 = \frac{1}{n}(n_1 + n_2 + n_7 + n_8) \\ \hat{r}_2 = \frac{1}{n}(n_1 + n_4 + n_5 + n_8) \\ \hat{C} = \frac{n_1 + n_8}{n\hat{r}_1\hat{r}_2} \end{cases} \quad \text{for case 3} \quad (S6)$$

$$\begin{cases} \hat{r}_1 = \frac{1}{n}(n_1 + n_2 + n_7 + n_8) \\ \hat{r}_2 = \frac{1}{n}(n_2 + n_3 + n_6 + n_7) \\ \hat{C} = \frac{n_2 + n_7}{n\hat{r}_1\hat{r}_2} \end{cases} \quad \text{for case 4} \quad (S7)$$

Table S2. The counts and frequencies of genotypes at SNPs 1, 2, and 3 that segregate in $ab \times aa$ under different linkage phases.

Linkage Phase			Genotype		Count	Frequency
			SNPs	1 2 3		
Case 1			aa aa aa		n_1	$r_{00}/2$
			aa aa ab		n_2	$r_{01}/2$
SNP 1	$a \mid b$	$b \mid a$	aa ab aa		n_3	$r_{11}/2$
SNP 2	$a \mid b$ or $b \mid a$		aa ab ab		n_4	$r_{10}/2$
SNP 3	$a \mid b$	$b \mid a$	ab aa aa		n_5	$r_{10}/2$
			ab aa ab		n_6	$r_{11}/2$
			ab ab aa		n_7	$r_{01}/2$
			ab ab ab		n_8	$r_{00}/2$
Case 2			aa aa aa		n_1	$r_{01}/2$
			aa aa ab		n_2	$r_{00}/2$
SNP 1	$a \mid b$	$b \mid a$	aa ab aa		n_3	$r_{10}/2$
SNP 2	$a \mid b$ or $b \mid a$		aa ab ab		n_4	$r_{11}/2$
SNP 3	$b \mid a$	$a \mid b$	ab aa aa		n_5	$r_{11}/2$
			ab aa ab		n_6	$r_{10}/2$
			ab ab aa		n_7	$r_{00}/2$
			ab ab ab		n_8	$r_{01}/2$
Case 3			aa aa aa		n_1	$r_{11}/2$
			aa aa ab		n_2	$r_{10}/2$
SNP 1	$a \mid b$	$b \mid a$	aa ab aa		n_3	$r_{00}/2$
SNP 2	$b \mid a$ or $a \mid b$		aa ab ab		n_4	$r_{01}/2$
SNP 3	$a \mid b$	$b \mid a$	ab aa aa		n_5	$r_{01}/2$
			ab aa ab		n_6	$r_{00}/2$
			ab ab aa		n_7	$r_{10}/2$
			ab ab ab		n_8	$r_{11}/2$
Case 4			aa aa aa		n_1	$r_{10}/2$
			aa aa ab		n_2	$r_{11}/2$
SNP 1	$a \mid b$	$b \mid a$	aa ab aa		n_3	$r_{01}/2$
SNP 2	$b \mid a$ or $a \mid b$		aa ab ab		n_4	$r_{00}/2$
SNP 3	$b \mid a$	$a \mid b$	ab aa aa		n_5	$r_{00}/2$
			ab aa ab		n_6	$r_{01}/2$
			ab ab aa		n_7	$r_{11}/2$
			ab ab ab		n_8	$r_{10}/2$