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W-Boson Mass Anomaly as a Manifestation of Spontaneously Broken Additional $SU(2)$ Global Symmetry on a New Fundamental Scale

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Abstract: Recently, the CDF Collaboration has announced a new precise measurement of the W-boson mass M_W that deviates from the Standard Model (SM) prediction by 7σ. The discrepancy in M_W is about $\Delta_W \simeq 70$ MeV and is probably caused by a beyond the SM physics. Within a certain scenario of extension of the SM, we obtain the relation $\Delta_W \simeq \frac{3\alpha}{8\pi} M_W \simeq 70$ MeV, where α is the electromagnetic fine structure constant. The main conjecture is the appearance of longitudinal components of the W-bosons as the Goldstone bosons of a spontaneously broken additional $SU(2)$ global symmetry at distances much smaller than the electroweak symmetry breaking scale r_{EWSB}. We argue that within this scenario, the masses of charged Higgs scalars can obtain an electromagnetic radiative contribution which enhances the observed value of $M_{W^\pm}$ with respect to the usual SM prediction. Our relation for Δ_W follows from the known one-loop result for the corresponding effective Coleman–Weinberg potential in combination with the Weinberg sum rules.

Keywords: W-bosons; Standard Model; Electroweak corrections

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The CDF Collaboration at Tevatron has recently reported a new precise measurement of the W-boson mass that shows about 7σ deviation from the prediction of the Standard Model (SM) [1]. The newly discovered W-boson mass anomaly caused much excitement among the specialists in Beyond the SM (BSM) physics since it is widely believed that the given discrepancy, if confirmed in future experiments, is related to some new BSM physics.

The new measurement of the W-boson mass announced by the CDF Collaboration is [1]: $M_W^{(\text{CDF})} = 80.4335 \pm 0.0094$ GeV. After combining with the previous Tevatron measurement of M_W, the following final Tevatron result was reported [1],

$$M_W^{(\text{Tevat})} = 80.4274 \pm 0.0089 \text{ GeV}. \tag{1}$$

This value exceeds the SM expectation [2],

$$M_W^{(\text{SM})} = 80.357 \pm 0.006 \text{ GeV}, \tag{2}$$

by

$$\Delta_W = 70 \pm 11 \text{ MeV}. \tag{3}$$

The result (1) can also be combined with other previous measurements of M_W by LEP2, LHC and LHCb experiments, the SM prediction (2) may be updated as well. All these variations are able to change the estimate of discrepancy (3) at the level of 10% (for instance, the updated central values obtained in the global fit of Ref. [3] are $M_W^{(\text{exp})} = 80.413$ GeV and $M_W^{(\text{SM})} = 80.350$ GeV, see also Ref. [4]). It is thus seen that the anomaly in the W-boson mass is certainly present. A more convincing argumentation is given in the original paper [1].

Not surprisingly, the very recent publication by the CDF Collaboration has already caused an avalanche of theoretical papers explaining the observed W-boson mass anomaly with the aid of some tantalizing new BSM physics (see, e.g., [5–9] and numerous references therein). Most of the proposals seem to be centered around the idea of introducing additional fundamental scalar particles, typically a new multiplet of Higgs bosons, which can contribute to the W-boson mass.

We will try to approach the problem partly against the mainstream. Our basic observation is that the magnitude of mass anomaly (3), $\Delta_W \simeq 0.001 M_W$, is of the order of a typical first quantum correction in QED, i.e., of the order of $\mathcal{O}(\alpha/\pi)$, where $\alpha \approx 1/137$ is the fine structure constant (for example, the famous anomalous magnetic moment of the electron, in the first approximation, is $a_e = \frac{\alpha}{2\pi} \approx 0.001$). This observation suggests that Δ_W may have mainly electromagnetic origin and the given electromagnetic correction was missed in the previous SM predictions. Then, the question is how this electromagnetic contribution arises. In the given letter, we propose a possible mechanism that leads to the quantitative prediction (3).

We will consider the electromagnetic correction Δ_W as an effect arising at distances less (possibly, much less) than the Electroweak Symmetry Breaking (EWSB) scale, $r_{\text{EWSB}} \simeq (246\ \text{GeV})^{-1}$, due to certain BSM physics to be guessed. Our working option for BSM physics at distances $r \ll r_{\text{EWSB}}$ will be the following: Along with the standard $SU(2)_L$ gauge symmetry acting on the triplet of gauge bosons (W^+, W^-, W^0) there exists an additional $SU(2)'$ global symmetry acting on the same triplet of gauge bosons. For the derivation of our result, however, it will be convenient to regard $SU(2)'$ as a gauge symmetry acting on the second triplet of gauge bosons (W'^+, W'^-, W'^0) and take the degeneracy limit at the end. We suppose further that the triplet of Higgs scalars (ϕ^+, ϕ^-, ϕ^0) which is eaten by (W^+, W^-, W^0) on the scale r_{EWSB} due to the Higgs mechanism, on a "truly fundamental" level, represents simultaneously the triplet of Goldstone bosons of spontaneously broken $SU(2)'$ part of fundamental symmetry.

Within this scenario, we suggest that the charged scalars ϕ^+ and ϕ^- can obtain an electromagnetic contribution to the mass via the radiative corrections, $\Delta M_\phi = M_{\phi^\pm} - M_{\phi^0} > 0$. This mass difference remains at larger distances, $r \gtrsim r_{\text{EWSB}}$, and, via the Higgs mechanism, eventually translates into

$$\Delta M_\phi = \Delta M_W = M_{W^\pm} - M_{W^0}. \tag{4}$$

The given effect, not taken into account in the SM quantitative predictions, leads then to the observed mass anomaly (3),

$$\Delta_W = \Delta M_W, \tag{5}$$

which seems to be unaffected by the mixing of W^0 with the B-boson of $U(1)_Y$ gauge part in the SM.

Consider the two-point correlation functions of vector currents coupled to the W and W' bosons,

$$\langle J_V^\mu J_V^\nu \rangle = (q^2\eta^{\mu\nu} - q^\mu q^\nu)\Pi_V(q^2), \qquad V = W, W'. \tag{6}$$

The difference of correlators $(\Pi_{W'} - \Pi_W)$ represents an order parameter for the assumed spontaneous symmetry breaking. At large Euclidean momenta $Q^2 = -q^2$, one can write the standard Operator Product Expansion (OPE) for $\Pi_V(Q^2)$. In the field theories based on vector interactions with (initially) massless fermions, it is natural to expect that the first contribution to $\left(\Pi_{W'}(Q^2) - \Pi_W(Q^2)\right)$ arises from four-fermion operators. The case of spontaneous CSB in massless QCD represents a canonical example [10]. Since the four-fermion operators have the mass dimension 6, the OPE leads then to the behavior

$$\left(\Pi_{W'}(Q^2) - \Pi_W(Q^2)\right)_{Q^2\to\infty} \sim \frac{1}{Q^6}. \tag{7}$$

The validity of (7) will be crucial for our scheme.

Next, we apply the method of Weinberg sum rules [11]. This method is based on the saturation of correlators by a narrow resonance contribution plus perturbative continuum equal for both correlators. Omitting the irrelevant subtraction constant, the Weinberg ansatz is

$$\Pi_W(Q^2) = \frac{F_W^2}{Q^2 + M_W^2} + \text{Continuum}, \tag{8}$$

$$\Pi_{W'}(Q^2) = \frac{F_{W'}^2}{Q^2 + M_{W'}^2} + \frac{F_\phi^2}{Q^2} + \text{Continuum}. \tag{9}$$

The corresponding decay constants in residues are defined by

$$\langle 0|J_V^\mu|V\rangle = F_V M_V \epsilon^\mu, \qquad V = W, W', \tag{10}$$

$$\langle 0|J_{W'}^\mu|\phi\rangle = i q^\mu F_\phi. \tag{11}$$

Here, ϵ^μ denotes the polarization vector and ϕ is the triplet of Goldstone Higgs bosons of spontaneously broken $SU(2)'$ symmetry. The parametrization (11) emerges by virtue of the Goldstone theorem. Substituting (8) and (9) into (7) we obtain the relations

$$F_W^2 - F_{W'}^2 = F_\phi^2, \qquad M_W F_W = M_{W'} F_{W'}. \tag{12}$$

The relations (12) are in one-to-one correspondence with the old Weinberg sum rules [11], in which the vector ρ, axial a_1 and pseudoscalar π mesons play the role of W, W' and ϕ, correspondingly.

Initially, the Goldstone bosons $\phi^\pm$ and ϕ^0 are degenerate in mass but one can expect that the photon loops will generate a potential, hence, an electromagnetic mass term for $\phi^\pm$ resulting in a mass splitting $\Delta M_\phi = M_{\phi^\pm} - M_{\phi^0}$. The calculation of ΔM_ϕ in our scenario is the same as the calculation of the electromagnetic mass difference of pseudogoldstone π-mesons, $\Delta M_\pi = M_{\pi^\pm} - M_{\pi^0}$. The one-loop result for the latter is well known,

$$M_{\pi^\pm}^2 - M_{\pi^0}^2 = \frac{3\alpha}{8\pi F_\pi^2} \int_0^\infty dQ^2 Q^2 \Big[\Pi_A(Q^2) - \Pi_V(Q^2)\Big], \tag{13}$$

where Π_V and Π_A are the vector and axial correlators defined as in (6). The result (13) was first derived in 1967 [12] using the current algebra techniques. The modern derivation is based on the method of effective action. The calculation of the corresponding Coleman–Weinberg potential leading to (13) is nicely reviewed in [13]. Importantly, this derivation shows that the relation (13) represents actually a particular case of a more general result: The one-loop radiative correction to the mass of charged Goldstone bosons is proportional to $\int dQ^2 Q^2 (\Pi_{\text{br}} - \Pi_{\text{unbr}})$, where Π_{br} and Π_{unbr} are the two-point correlators of currents corresponding to broken and unbroken generators of a spontaneously broken global symmetry. This is exploited, in particular, in the $SO(5)/SO(4)$ scenario of the composite Nambu–Goldstone Higgs boson to generate the Higgs mass via radiative corrections from hypothetical BSM strong sector (a pedagogical review is given in Ref. [13]).

Using (7)–(9) with the replacements mentioned after (12), one arrives at the relation by Das et al. [12],

$$M_{\pi^\pm}^2 - M_{\pi^0}^2 = \frac{3\alpha}{4\pi} \frac{M_{a_1}^2 M_\rho^2}{M_{a_1}^2 - M_\rho^2} \log\left(\frac{M_{a_1}^2}{M_\rho^2}\right). \tag{14}$$

It should be emphasized that the convergence in (13) is provided by the asymptotic behavior (7) for $\big(\Pi_A(Q^2) - \Pi_V(Q^2)\big)$. The positivity of (14) follows from the fact that the radiative corrections align the vacuum along the direction preserving the $U(1)$ gauge symmetry, i.e., $\langle \pi^+ \rangle = \langle \pi^- \rangle = 0$ in the minimized pion potential so that the photon remains massless.

It is important to note that the relation (14) was derived in the limit of massless pions. When the quark masses are turned on, both $\pi^\pm$ and π^0 obtain a mass becoming pseudogoldstone bosons. The difference $\Delta M_\pi = M_{\pi^\pm} - M_{\pi^0}$, however, remains dominated by electromagnetic correction. This means that the electromagnetic pion mass difference (14) arises at distances much smaller than the scale of spontaneous CSB in QCD, $r_{\text{CSB}} \simeq 0.2$ fm. At distances $r \ll r_{\text{CSB}}$ the pion can be considered as effectively massless. Assuming $M_{\pi^\pm} - M_{\pi^0} \ll M_\pi$, where $M_\pi = M_{\pi^\pm}$ or $M_\pi = M_{\pi^0}$, we can write $M^2_{\pi^\pm} - M^2_{\pi^0} \simeq 2M_\pi \Delta M_\pi$ and obtain the observable value of ΔM_π substituting into

$$\Delta M_\pi \simeq \frac{3\alpha}{8\pi} \frac{M^2_{a_1} M^2_\rho}{M_\pi (M^2_{a_1} - M^2_\rho)} \log\left(\frac{M^2_{a_1}}{M^2_\rho}\right), \tag{15}$$

the observable values of meson masses measured at larger distances, where the meson masses arise from a confinement mechanism. Essentially the same trick we are going to use for the calculation of $\Delta M_W = M_{W^\pm} - M_{W^0}$.

Under our assumptions, we are ready now to write the answer for $M^2_{\phi^\pm} - M^2_{\phi^0}$ directly from (14),

$$M^2_{\phi^\pm} - M^2_{\phi^0} = \frac{3\alpha}{4\pi} \frac{M^2_{W'} M^2_W}{M^2_{W'} - M^2_W} \log\left(\frac{M^2_{W'}}{M^2_W}\right). \tag{16}$$

It should be noted that, if our assumptions are true, the relation (16) can turn out to be much more precise than (14). Indeed, the relation (14) was derived using two rough approximations — infinitely narrow decay width and neglecting contributions of radial excitations. The real ρ and a_1 mesons, however, are broad resonances for which the ratio Γ/M is not small: $\Gamma_\rho \approx 150$ MeV, $M_\rho \approx 775$ MeV, $\Gamma_{a_1} \approx 420$ MeV, $M_{a_1} \approx 1230$ MeV [2]. Quite surprisingly, the theoretical prediction from (15), $\Delta M_\pi^{(\text{th})} \approx 5.8$ MeV, agrees reasonably with the experimentally measured value, $\Delta M_\pi^{(\text{exp})} \approx 4.6$ MeV [2]. Concerning the second approximation, the ρ and a_1 mesons, as all hadrons, are composite systems of quarks bound by strong interactions and this leads to the existence of towers of radially excited ρ and a_1 mesons which are listed in the Particle Data [2]. These excited states contribute to the ρ and a_1 analogues of correlators (8) and (9) via additional pole terms. The resulting modification of (14) seems to improve the quantitative agreement between $\Delta M_\pi^{(\text{th})}$ and $\Delta M_\pi^{(\text{exp})}$ [14]. In the case under consideration, the ratio Γ_W/M_W is smaller by an order of magnitude and the W-boson, as a true elementary particle, does not have radial excitations.

Let us now motivate why we expect the fulfillment of the relation

$$M^2_{W^\pm} - M^2_{W^0} \simeq M^2_{\phi^\pm} - M^2_{\phi^0}. \tag{17}$$

On the scales where the standard Higgs mechanism starts to work, $\phi^\pm$ and ϕ^0 become the longitudinal components of $W^\pm$ and W^0 gauge bosons. The $W^\pm$-bosons produced in the CDF experiment at Tevatron are ultrarelativistic[1]. It is easy to show that the longitudinal polarization ϵ^μ_{L} of such a W-boson becomes increasingly parallel to its four-momentum $k^\mu = (E_W, 0, 0, k)$ as k becomes large (see, e.g., the classical textbook [15]),

$$\epsilon^\mu_{\text{L}}(k) = \frac{k^\mu}{M_W} + \mathcal{O}\left(\frac{M_W}{E_W}\right), \qquad k \to \infty. \tag{18}$$

Since the transverse polarizations $\epsilon^\mu_\perp$ do not grow with k, one can show that the physics of ultrarelativistic W-boson is almost completely determined by its component ϵ^μ_{L}: The amplitude for emission or absorption of such W-bosons becomes equal, at high energy, to the amplitude of emission or absorption of its longitudinal component. This statement constitutes the essence of important *Goldstone boson equivalence theorem*: A relativistically moving, longitudinally polarized massive gauge boson behaves as a Goldstone boson that was eaten by the Higgs mechanism [15]. Since the mass of ultrarelativistic W-boson is also mostly determined by its longitudinal component ϕ, we should expect the relation (17).

Combining (16) and (17) we obtain the expression for $M^2_{W^\pm} - M^2_{W^0}$. Since $\Delta M_W = M_{W^\pm} - M_{W^0} \ll M_W$ one can write $M^2_{W^\pm} - M^2_{W^0} \simeq 2M_W \Delta M_W$. The result for ΔM_W is

$$\Delta M_W \simeq \frac{3\alpha}{8\pi} \frac{M_W M^2_{W'}}{M^2_{W'} - M^2_W} \log\left(\frac{M^2_{W'}}{M^2_W}\right). \tag{19}$$

As in the case of the pion analogue (15), the relation (19) is derived below the scale $r^{(\mathrm{weak})}_{\mathrm{CSB}}$ where all particles are effectively massless. However, the observable value of ΔM_W at larger distances follows after substitution to (19) the values of M_W and $M_{W'}$ at larger distances, where they emerge due to the Higgs mechanism.

Formally, the relation (19) contains only one unknown parameter $M_{W'}$. Another three unknown parameters F_W, $F_{W'}$ and F_ϕ are canceled due to the sum rules (12). Following our suggestion, the last step is to take the degeneracy limit $M_{W'} = M_W$ since W' represents actually the same physical degree of freedom as W. Using the limit $\frac{\log x}{x-1} \to 1$ as $x \to 1$ we obtain from (19) our final result

$$\Delta M_W \simeq \frac{3\alpha}{8\pi} M_W. \tag{20}$$

Substituting the experimental mass of W-boson, the relation (20) predicts $\Delta M_W \simeq 70.0$ MeV. The given value is in perfect agreement with the observed discrepancy (3).

The physical meaning of additional $SU(2)'$ global symmetry above the electroweak scale is an open question. To answer this question, one should elaborate on some other observable consequences of this symmetry. We leave this for the future.

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Note

1 They were produced in proton-antiproton collisions at a center of mass energy $E_{\mathrm{c.m.}} = 1.96$ TeV [1]. One can estimate the average proper energy of each produced W-boson as $E_W \simeq \frac{1}{3} \cdot \frac{1}{2} \cdot E_{\mathrm{c.m.}} \approx 4M_W$, where the factor of $\frac{1}{3}$ takes into account that only one of three available quark-antiquark pairs produces the W-boson and $\frac{1}{2}$ emerges from the well known experimental fact that the quark degrees of freedom carry about half of the momentum of the ultrarelativistic nucleon.

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