

Article

# Can a Chameleon Field Be Identified with Quintessence?

A. N. Ivanov <sup>1,\*</sup> and M. Wellenzohn <sup>1,2</sup>

<sup>1</sup> Atominstitut, Technische Universität Wien, Stadionallee 2, A-1020 Wien, Austria; max.wellenzohn@gmail.com

<sup>2</sup> FH Campus Wien, University of Applied Sciences, Favoritenstraße 226, 1100 Wien, Austria

\* Correspondence: ivanov@kph.tuwien.ac.at

Received: 18 October 2020; Accepted: 22 November 2020; Published: 26 November 2020



**Abstract:** In the Einstein–Cartan gravitational theory with the chameleon field, while changing its mass independently of the density of its environment, we analyze the Friedmann–Einstein equations for the Universe’s evolution with the expansion parameter  $a$  being dependent on time only. We analyze the problem of an identification of the chameleon field with quintessence, i.e., a canonical scalar field responsible for dark energy dynamics, and for the acceleration of the Universe’s expansion. We show that since the cosmological constant related to the relic dark energy density is induced by torsion (*Astrophys. J.* **2016**, *829*, 47), the chameleon field may, in principle, possess some properties of quintessence, such as an influence on the dark energy dynamics and the acceleration of the Universe’s expansion, even in the late-time acceleration, but it cannot be identified with quintessence to the full extent in the classical Einstein–Cartan gravitational theory.

**Keywords:** torsion/Einstein-Cartan; gravity/chameleon

**PACS:** 03.50.-z; 04.25.-g; 04.25.Nx; 14.80.Va

## 1. Introduction

The chameleon field, changing its mass independently of a density of its environment [1,2], has been invented to avoid the problem of the equivalence principle violation [3]. Nowadays it is accepted that the chameleon field, identified with quintessence [4–10], i.e., a canonical scalar field, can be useful for an explanation of the late-time acceleration of the Universe’s expansion [11–14] and may shed light on the origins of dark energy and dark energy dynamics [15–21]. Since the relic dark energy density is closely related to the cosmological constant [15], in contrast to such a hypothesis that the chameleon field might originate the cosmological constant proportionally to the homogeneous static dark energy density, it has been shown at the model-independent level within the Einstein–Cartan gravitational theory [22–38] that the cosmological constant or the relic dark energy density has a geometrical origin caused by torsion [39]. In this case the chameleon field is able only to evolve above the relic background of the dark energy simulating its dynamics and, of course, to make a certain influence on the acceleration of the Universe’s expansion.

For the observation of torsion in the terrestrial laboratories there have been derived potentials of low-energy torsion–neutron interactions [40–42]. In terrestrial laboratories, the extreme smallness of absolute values of torsion was confirmed in different estimates of constraints on the contributions of torsion to observables of elementary particle interactions [43–48], including the qBounce experiments with ultracold neutrons (UCNs) [49–55] (see also [48]).

The chameleon–matter interactions were also intensively investigated in terrestrial laboratories [49–57] in experiments with ultracold and cold neutrons through some effective low-energy chameleon–neutron

potentials [58–61] and by using cold atoms in the atom interferometry [62–66]. However, recently the importance of the chameleon field as quintessence in the late-time acceleration of the Universe has been questioned by Wang et al. [67] and Khoury [68] by pointing out that the conformal factor, relating Einstein’s and Jordan’s frames and defining the chameleon–matter interactions, is essentially constant over the last Hubble time. According to Wang et al. [67] and Khoury [68], this implies a negligible influence of the chameleon field on the late-time acceleration of the Universe’s expansion. To some extent, this should also imply that the chameleon field cannot possess such a property of quintessence as responsibility for the late-time acceleration of the Universe’s expansion [5–7].

Thus, the aim of this paper is to investigate the properties of the chameleon field in comparison to the properties of quintessence. We would like to remind readers that by definition, quintessence is a hypothetical state of dark energy described by a canonical scalar field for an explanation of the observable acceleration of the Universe’s expansion. We have to also emphasize that our analysis is restricted by the classical Einstein–Cartan gravitational theory. Below we show that the chameleon field has no relation to the origin of the cosmological constant, or the relic dark energy density, which is induced by torsion [39]. However, the chameleon field can still influence on the Universe’s expansion even in the late-time acceleration, caused by its evolution above the background of the relic dark energy [39]. By analyzing Einstein’s equations for the flat Universe in spacetime with the Friedmann metric, dependent on the expansion parameter  $a$  [69], we show that conservation of a total energy–momentum tensor of the system, including the chameleon field, radiation and matter (dark and baryon matter), demands the conformal factor to be equal to unity if and only if the dependencies of the radiation  $\rho_r(a)$  and matter  $\rho_m(a)$  densities on the expansion parameter  $a$  do not deviate from their standard forms,  $\rho_r(a) \sim a^{-4}$  and  $\rho_m(a) \sim a^{-3}$  respectively [69]. We obtain the same result by analyzing the first order differential Friedmann–Einstein equation, relating  $\dot{a}^2/a^2$  to the chameleon field, radiation and matter densities, and the second order differential Friedmann–Einstein equation, relating  $\ddot{a}/a$  to the chameleon field, radiation and matter densities and their pressures, where  $\dot{a}$  and  $\ddot{a}$  are the first and second time derivatives of the expansion parameter. Of course, the equality of the conformal factor to unity suppresses any coupling of the chameleon field to a matter density of its environment and makes such a scalar field unhelpful for avoiding the problem of the equivalence principle violation [3]. However, it does not prevent the chameleon field, evolving above the background of the relic dark energy, from a simulation of a dark energy dynamics and having an influence on the acceleration of the Universe’s expansion. Then, we show that the Friedmann–Einstein equation for  $\dot{a}^2/a^2$  is the first integral of the Friedmann–Einstein equation for  $\ddot{a}/a$  if and only if the total energy–momentum of the system, including the chameleon field, radiation and matter, is locally conserved. As a result we infer that (i) if the radiation and matter densities obey their standard dependence on the expansion parameter  $\rho_r(a) \sim a^{-4}$  and  $\rho_m(a) \sim a^{-3}$  the conformal factor is equal to unity and the chameleon field loses the possibility to couple to an environment, and (ii) if the dependencies of the radiation and matter densities deviate from their standard behavior  $\rho_r(a) \sim a^{-4}$  and  $\rho_m(a) \sim a^{-3}$ , the conformal factor is not equal to unity and makes possible interactions of the chameleon field with its environment. In this case, usage of the chameleon field for the problem of equivalence principle violation becomes meaningful. In spite of the fact that the chameleon field does not possess the main property of quintessence in order to be a hypothetical form of dark energy [4], since the relic dark energy density or the cosmological constant has a geometrical origin related to torsion [39], the chameleon field, evolving above the relic dark energy and simulating a dark energy dynamics, might be responsible for an acceleration of the Universe’s expansion.

The paper is organized as follows. In Section 2 we derive Einstein’s equations in the Einstein–Cartan gravitational theory with torsion, chameleon and matter fields. Following [39] we show that the contribution of torsion to the Einstein–Hilbert action is presented in the form of the cosmological constant. Then, following Khoury and Weltman [1] we include the part of the integrand of the Einstein–Hilbert action proportional to the cosmological constant for the potential of the self-interaction of the chameleon field. This implies that the chameleon field has no relation

to an origin of the cosmological constant or the relic dark energy density but can only evolve above such a relic background caused by torsion and simulate dark energy dynamics. In Section 3 in the flat Friedmann spacetime with the standard Friedmann metric  $g_{\mu\nu}$ , i.e.,  $g_{00} = 1$ ,  $g_{0j} = 0$  and  $g_{ij} = a^2(t) \eta_{ij}$  and  $\eta_{ij} = -\delta_{ij}$ , we show that the Einstein equations reduce themselves to the Friedmann–Einstein equations of the Universe’s evolution with the chameleon field, radiation and matter (dark and baryon) densities. Since the Einstein tensor  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ , where  $R_{\mu\nu}$  and  $R$  are the Ricci tensor and scalar curvature, respectively, obey the Bianchi identity  $G^{\mu\nu}{}_{;\mu} = 0$ , where  $G^{\mu\nu}{}_{;\mu}$  is the covariant divergence [69], the total energy–momentum tensor of the system, including the chameleon field, radiation and matter (dark and baryon), should be locally conserved. We find that local conservation of the total energy–momentum tensor imposes the evolution equations for the radiation and matter densities, where the dependence of them on the expansion parameter  $a$  is corrected by the conformal factor in comparison to the standard dependence  $\rho_r(a) \sim a^{-4}$  and  $\rho_a \sim a^{-3}$ , respectively [69]. We show that the Friedmann–Einstein equation for  $\dot{a}^2/a^2$  is the first integral of the Friedmann–Einstein equation for  $\ddot{a}/a$  if and only if the total energy momentum of the system, including the chameleon field, radiation and matter, is locally conserved. In case of the standard dependence of the radiation and matter densities on the expansion parameters  $\rho_r(a) \sim a^{-4}$  and  $\rho_m \sim a^{-3}$  [69], local conservation of the total energy–momentum tensor of the chameleon field, radiation and matter demand the conformal factor to be equal to unity. This suppresses any interaction of the chameleon field with an ambient environment. In Section 4 we discuss experiments to probe torsion in the terrestrial laboratories through effective low-energy torsion-neutron interactions derived in [40–42]. In Section 5 we discuss the results obtained.

## 2. Einstein’s Equations in the Einstein–Cartan Gravitational Theory with Chameleon and Matter Fields

We take the Einstein–Hilbert action of the Einstein–Cartan gravitational theory without chameleon and matter fields in the standard form [27,37,69]:

$$S_{EH} = \frac{1}{2} M_{Pl}^2 \int d^4x \sqrt{-g} \mathcal{R}, \tag{1}$$

where  $M_{Pl} = 1/\sqrt{8\pi G_N} = 2.435 \times 10^{27}$  eV is the reduced Planck mass,  $G_N$  is the Newtonian gravitational constant [70] and  $g$  is the determinant of the metric tensor  $g_{\mu\nu}$ . The scalar curvature  $\mathcal{R}$  is defined by [27,37]

$$\mathcal{R} = g^{\mu\nu} \mathcal{R}^{\alpha}{}_{\mu\alpha\nu} = g^{\mu\nu} \left( \frac{\partial}{\partial x^\nu} \Gamma^{\alpha}{}_{\alpha\mu} - \frac{\partial}{\partial x^\alpha} \Gamma^{\alpha}{}_{\nu\mu} + \Gamma^{\alpha}{}_{\nu\phi} \Gamma^{\phi}{}_{\alpha\mu} - \Gamma^{\alpha}{}_{\alpha\phi} \Gamma^{\phi}{}_{\nu\mu} \right) = g^{\mu\nu} \mathcal{R}_{\mu\nu}, \tag{2}$$

where  $\mathcal{R}^{\alpha}{}_{\mu\beta\nu}$  and  $\mathcal{R}_{\mu\nu}$  are the Riemann and Ricci tensors in the Einstein–Cartan gravitational theory, respectively. Then,  $\Gamma^{\alpha}{}_{\mu\nu}$  is the affine connection

$$\Gamma^{\alpha}{}_{\mu\nu} = \{^{\alpha}{}_{\mu\nu}\} + \mathcal{K}^{\alpha}{}_{\mu\nu} = \{^{\alpha}{}_{\mu\nu}\} + g^{\alpha\sigma} \mathcal{K}_{\sigma\mu\nu}, \tag{3}$$

where  $\{^{\alpha}{}_{\mu\nu}\}$  are the Christoffel symbols [69]

$$\{^{\alpha}{}_{\mu\nu}\} = \frac{1}{2} g^{\alpha\lambda} \left( \frac{\partial g_{\lambda\mu}}{\partial x^\nu} + \frac{\partial g_{\lambda\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\lambda} \right) \tag{4}$$

and  $\mathcal{K}_{\sigma\mu\nu}$  is the contorsion tensor, related to torsion  $\mathcal{T}_{\sigma\mu\nu}$  by  $\mathcal{K}_{\sigma\mu\nu} = \frac{1}{2} (\mathcal{T}_{\sigma\mu\nu} + \mathcal{T}_{\mu\sigma\nu} + \mathcal{T}_{\nu\sigma\mu})$  and  $\mathcal{T}^{\alpha}{}_{\mu\nu} = g^{\alpha\sigma} \mathcal{T}_{\sigma\mu\nu} = \Gamma^{\alpha}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\nu\mu}$  with the following properties:  $\mathcal{K}_{\sigma\mu\nu} = -\mathcal{K}_{\nu\mu\sigma}$  and  $\mathcal{T}_{\mu\nu} = -\mathcal{T}_{\nu\mu}$  [27,37]. The integrand of the Einstein–Hilbert action Equation (1) can be represented in the following form:

$$\sqrt{-g} \mathcal{R} = \sqrt{-g} R + \sqrt{-g} \mathcal{C} + \frac{\partial}{\partial x^\nu} (\sqrt{-g} g^{\mu\nu} \mathcal{K}^{\alpha}{}_{\alpha\mu}) - \sqrt{-g} g^{\mu\nu} \left( \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\alpha} (\sqrt{-g} \mathcal{K}^{\alpha}{}_{\nu\mu}) - \{^{\phi}{}_{\alpha\mu}\} \mathcal{K}^{\alpha}{}_{\nu\phi} - \{^{\alpha}{}_{\nu\phi}\} \mathcal{K}^{\phi}{}_{\alpha\mu} \right), \tag{5}$$

where we have denoted

$$C = g^{\mu\nu}(\mathcal{K}^\varphi_{\alpha\mu} \mathcal{K}^\alpha_{\nu\varphi} - \mathcal{K}^\alpha_{\alpha\varphi} \mathcal{K}^\varphi_{\nu\mu}) \tag{6}$$

and  $R$  is the Ricci scalar curvature of the Einstein gravitational theory, expressed in terms of the Christoffel symbols  $\{\alpha_{\mu\nu}\}$  [69] only. When removing in Equation (5) the total derivatives and integrating by parts, we delete the third term and transcribe the fourth term into the form  $\sqrt{-g} g^{\mu\nu}{}_{;\alpha} \mathcal{K}^\alpha_{\nu\mu}$ , where  $g^{\mu\nu}{}_{;\alpha}$  is the covariant derivative of the metric tensor  $g^{\mu\nu}$ , vanishing because of the metricity condition  $g^{\mu\nu}{}_{;\alpha} = 0$  [69].

*Derivation of Equation (5)*

Let us show that  $\sqrt{-g} \mathcal{R}$  can be presented in the form of Equation (5). Using Equation (3) we get an obvious relation

$$\begin{aligned} \sqrt{-g} \mathcal{R} = & \sqrt{-g} R + \sqrt{-g} C + \sqrt{-g} g^{\mu\nu} \left( \frac{\partial}{\partial x^\nu} \mathcal{K}^\alpha_{\alpha\mu} - \frac{\partial}{\partial x^\alpha} \mathcal{K}^\alpha_{\nu\mu} + \{\alpha_{\nu\varphi}\} \mathcal{K}^\varphi_{\alpha\mu} + \{\varphi_{\alpha\mu}\} \mathcal{K}^\alpha_{\nu\varphi} \right. \\ & \left. - \{\alpha_{\alpha\varphi}\} \mathcal{K}^\varphi_{\nu\mu} - \{\varphi_{\nu\mu}\} \mathcal{K}^\alpha_{\alpha\varphi} \right). \end{aligned} \tag{7}$$

Having replaced the first term in the brackets by the total derivative and by adding the last term in the brackets, we arrive at the expression

$$\begin{aligned} \sqrt{-g} \mathcal{R} = & \sqrt{-g} R + \sqrt{-g} C + \frac{\partial}{\partial x^\nu} (\sqrt{-g} g^{\mu\nu} \mathcal{K}^\alpha_{\alpha\mu}) - \frac{\partial}{\partial x^\nu} (\sqrt{-g} g^{\mu\nu}) \mathcal{K}^\alpha_{\alpha\mu} - \sqrt{-g} g^{\mu\nu} \{\varphi_{\nu\mu}\} \mathcal{K}^\alpha_{\alpha\varphi} \\ & - \sqrt{-g} g^{\mu\nu} \left( \frac{\partial}{\partial x^\alpha} \mathcal{K}^\alpha_{\nu\mu} + \{\alpha_{\alpha\varphi}\} \mathcal{K}^\varphi_{\nu\mu} - \{\alpha_{\nu\varphi}\} \mathcal{K}^\varphi_{\alpha\mu} - \{\varphi_{\alpha\mu}\} \mathcal{K}^\alpha_{\nu\varphi} \right). \end{aligned} \tag{8}$$

Then, the first term in the brackets we rewrite as follows:

$$\begin{aligned} \sqrt{-g} \mathcal{R} = & \sqrt{-g} R + \sqrt{-g} C + \frac{\partial}{\partial x^\nu} (\sqrt{-g} g^{\mu\nu} \mathcal{K}^\alpha_{\alpha\mu}) - \frac{\partial}{\partial x^\nu} (\sqrt{-g} g^{\mu\nu}) \mathcal{K}^\alpha_{\alpha\mu} - \sqrt{-g} g^{\mu\nu} \{\varphi_{\nu\mu}\} \mathcal{K}^\alpha_{\alpha\varphi} \\ & - \sqrt{-g} g^{\mu\nu} \left( \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\alpha} (\sqrt{-g} \mathcal{K}^\alpha_{\nu\mu}) - \frac{1}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial x^\alpha} \mathcal{K}^\alpha_{\nu\mu} + \{\alpha_{\alpha\varphi}\} \mathcal{K}^\varphi_{\nu\mu} - \{\alpha_{\nu\varphi}\} \mathcal{K}^\varphi_{\alpha\mu} - \{\varphi_{\alpha\mu}\} \mathcal{K}^\alpha_{\nu\varphi} \right). \end{aligned} \tag{9}$$

Since  $\sqrt{-g} g^{\mu\nu} \{\varphi_{\nu\mu}\}$  and  $\{\alpha_{\alpha\varphi}\}$  are equal to [69] (see Equation (10.107) and Equation (9.56))

$$\sqrt{-g} g^{\mu\nu} \{\varphi_{\nu\mu}\} = -\frac{\partial}{\partial x^\nu} (\sqrt{-g} g^{\varphi\nu}) \quad , \quad \{\alpha_{\alpha\varphi}\} = \frac{1}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial x^\varphi}, \tag{10}$$

the fourth and fifth terms in the first line in Equation (9) and the second and third terms in the brackets are cancelled out in pairs. This reduces Equation (9) to Equation (5).

Now we may show that the contribution of the fourth term in Equation (5) to the Einstein–Hilbert action reduces to the contribution of the term  $\sqrt{-g} g^{\mu\nu}{}_{;\alpha} \mathcal{K}^\alpha_{\nu\mu}$ . The contribution of the fourth term in Equation (5) to the Einstein–Hilbert action is defined by the integral

$$\int d^4x \sqrt{-g} g^{\mu\nu} \left( \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\alpha} (\sqrt{-g} \mathcal{K}^\alpha_{\nu\mu}) - \{\varphi_{\alpha\mu}\} \mathcal{K}^\alpha_{\nu\varphi} - \{\alpha_{\nu\varphi}\} \mathcal{K}^\varphi_{\alpha\mu} \right). \tag{11}$$

After the integration by parts in the first term we get

$$\oint \sqrt{-g} g^{\mu\nu} \mathcal{K}^\alpha_{\nu\mu} dS_\alpha - \int d^4x \sqrt{-g} \left( \frac{\partial g^{\mu\nu}}{\partial x^\alpha} \mathcal{K}^\alpha_{\nu\mu} + g^{\mu\nu} \{\varphi_{\alpha\mu}\} \mathcal{K}^\alpha_{\nu\varphi} + g^{\mu\nu} \{\alpha_{\nu\varphi}\} \mathcal{K}^\varphi_{\alpha\mu} \right). \tag{12}$$

Having omitted the surface term and having renamed some indices in the second integral in Equation (12), we arrive at the expression

$$- \int d^4x \sqrt{-g} \left( \frac{\partial g^{\mu\nu}}{\partial x^\alpha} + g^{\rho\nu} \{\mu_{\rho\alpha}\} + g^{\nu\rho} \{\mu_{\rho\alpha}\} \right) \mathcal{K}^\alpha_{\nu\mu}, \tag{13}$$

where we have used the property of the Christoffel symbols  $\{\mu_{\alpha\rho}\} = \{\mu_{\rho\alpha}\}$ . The expression in the brackets is the covariant derivative of the metric tensor  $g^{\mu\nu}$  (see, for example, [27,69])

$$g^{\mu\nu}{}_{;\alpha} = \frac{\partial g^{\mu\nu}}{\partial x^\alpha} + g^{\rho\nu} \{\mu_{\rho\alpha}\} + g^{\mu\rho} \{\nu_{\rho\alpha}\}. \tag{14}$$

Thus, the contribution of the fourth term to the Einstein–Hilbert action is proportional to the integral

$$\int d^4x \sqrt{-g} g^{\mu\nu} \left( \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\alpha} (\sqrt{-g} \mathcal{K}^\alpha{}_{\nu\mu}) - \{\varphi_{\alpha\mu}\} \mathcal{K}^\alpha{}_{\nu\varphi} - \{\alpha_{\nu\varphi}\} \mathcal{K}^\varphi{}_{\alpha\mu} \right) = - \int d^4x \sqrt{-g} g^{\mu\nu}{}_{;\alpha} \mathcal{K}^\alpha{}_{\nu\mu}. \tag{15}$$

This confirms our assertion concerning a vanishing contribution of the fourth term in Equation (5) to the Einstein–Hilbert action in case of the metricity condition  $g^{\mu\nu}{}_{;\alpha} = 0$  [69].

Since it has been shown in [39] that  $\mathcal{C} = -2\Lambda_C$ , where  $\Lambda_C$  is the cosmological constant [69,71,72] (see also [15]) or the relic dark energy density, the Einstein–Hilbert action Equation (1) of the Einstein–Cartan gravitational theory with the scalar curvature Equation (2) can be represented in the following form [39]:

$$S_{\text{EH}} = \frac{1}{2} M_{\text{Pl}}^2 \int d^4x \sqrt{-g} (R - 2\Lambda_C). \tag{16}$$

As has been shown in [39], the same result is valid for the Poincaré gauge gravitaitonal theory [73–77] (see also [31–34]). Using Equation (11) the action of the Einstein–Cartan gravitational theory with torsion, chameleon fields and matter fields we take in the form [39]

$$S_{\text{EH}} = \frac{1}{2} M_{\text{Pl}}^2 \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}[\phi] + \int d^4x \sqrt{-\tilde{g}} \mathcal{L}_m[\tilde{g}], \tag{17}$$

where  $\mathcal{L}[\phi]$  is the Lagrangian of the chameleon field

$$\mathcal{L}[\phi] = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \tag{18}$$

and  $V(\phi)$  is the potential of the chameleon self-interaction. In Equation (17), following Khoury and Weltman [1], we have included additively the cosmological constant  $\Lambda_C$  in the form of the relic dark energy density  $\rho_\Lambda = M_{\text{Pl}}^2 \Lambda_C$  into the potential  $V(\phi)$  of the chameleon field self-interaction; i.e.,  $V(\phi) = \rho_\Lambda + \Phi(\phi)$ . This implies that the chameleon field has no relation to the origin of the cosmological constant or the relic dark energy density. It can only evolve above the relic background of the dark energy, caused by torsion.

The matter fields and the radiation [78,79] are described by the Lagrangian  $\mathcal{L}_m[\tilde{g}_{\mu\nu}]$ . The interactions of the matter fields and radiation with the chameleon field are expressed in terms of the metric tensor  $\tilde{g}_{\mu\nu}$  in the Jordan frame [1,2,80], which is conformally related to Einstein’s frame metric tensor  $g_{\mu\nu}$  by  $\tilde{g}_{\mu\nu} = f^2 g_{\mu\nu}$  (or  $\tilde{g}^{\mu\nu} = f^{-2} g^{\mu\nu}$ ) and  $\sqrt{-\tilde{g}} = f^4 \sqrt{-g}$  with  $f = e^{\beta\phi/M_{\text{Pl}}}$ , where  $\beta$  is the chameleon–matter coupling constant [1,2]. The factor  $f = e^{\beta\phi/M_{\text{Pl}}}$  can be interpreted also as a conformal coupling to matter fields and radiation [80] (see also [1,2,81]). For simplicity we have set the chameleon–photon coupling constant  $\beta_\gamma$  [79] to be equal to the chameleon–matter coupling constant  $\beta$ .

By varying the action of Equation (17) with respect to the metric tensor  $\delta g^{\mu\nu}$  (see, for example, [69]), we arrive at Einstein’s equations, modified by the contribution of the chameleon field. We get

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{1}{M_{\text{Pl}}^2} \left( f^2 \tilde{T}_{\mu\nu}^{(m)} + T_{\mu\nu}^{(\phi)} \right), \tag{19}$$

where  $R_{\mu\nu}$  is the Ricci tensor [69];  $\tilde{T}_{\mu\nu}^{(m)}$  and  $T_{\mu\nu}^{(\phi)}$  are the matter (with radiation, which we treat as a radiative fluid [82–86]) and chameleon energy–momentum tensors, respectively, determined by

$$\begin{aligned} \tilde{T}_{\mu\nu}^{(m)} &= \frac{2}{\sqrt{-\tilde{g}}} \frac{\delta}{\delta \tilde{g}^{\mu\nu}} \left( \sqrt{-\tilde{g}} \mathcal{L}[\tilde{g}] \right) = (\tilde{\rho} + \tilde{p}) \tilde{u}_\mu \tilde{u}_\nu - \tilde{p} \tilde{g}_{\mu\nu}, \\ T_{\mu\nu}^{(\phi)} &= \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \left( \sqrt{-g} \mathcal{L}[\phi] \right) = \frac{\partial \phi}{\partial x^\mu} \frac{\partial \phi}{\partial x^\nu} - g_{\mu\nu} \left( \frac{1}{2} g^{\lambda\rho} \frac{\partial \phi}{\partial x^\lambda} \frac{\partial \phi}{\partial x^\rho} - V(\phi) \right). \end{aligned} \tag{20}$$

The factor  $f^2$  appears in front of  $\tilde{T}_{\mu\nu}^{(m)}$  because of the relation

$$\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \left( \sqrt{-g} \mathcal{L}_m[g] \right) = \frac{\sqrt{-\tilde{g}}}{\sqrt{-g}} \frac{\delta \tilde{g}^{\lambda\rho}}{\delta g^{\mu\nu}} \tilde{T}_{\lambda\rho}^{(m)} = f^2 \tilde{T}_{\mu\nu}^{(m)}, \tag{21}$$

where we have used that

$$\frac{\sqrt{-\tilde{g}}}{\sqrt{-g}} = f^4, \quad \frac{\delta \tilde{g}^{\lambda\rho}}{\delta g^{\mu\nu}} = f^{-2} \frac{1}{2} (g^\lambda{}_\mu g^\rho{}_\nu + g^\lambda{}_\nu g^\rho{}_\mu), \tag{22}$$

since  $\tilde{g}^{\lambda\rho} = f^{-2} g^{\lambda\rho}$  [80] and  $\tilde{T}_{\mu\nu}^{(m)} = \tilde{T}_{\nu\mu}^{(m)}$ . Then, the quantities  $\tilde{\rho}$ ,  $\tilde{p}$  and  $\tilde{u}_\mu$  in the Jordan frame are related to the quantities  $\rho$ ,  $p$  and  $u_\mu$  in Einstein’s frame as [80]

$$\tilde{\rho} = f^{-3} \rho, \quad \tilde{p} = f^{-3} p, \quad \tilde{u}_\mu = f u_\mu, \quad \tilde{u}^\mu = f^{-1} u^\mu. \tag{23}$$

This gives  $\tilde{T}_{\mu\nu}^{(m)} = f^{-1} T_{\mu\nu}^{(m)}$ . By plugging Equation (20) with  $\tilde{T}_{\mu\nu}^{(m)} = f^{-1} T_{\mu\nu}^{(m)}$  into Equation (19), we arrive at Einstein’s equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{1}{M_{\text{Pl}}^2} T_{\mu\nu}, \tag{24}$$

where  $T_{\mu\nu}$  is the total energy–momentum tensor equal to

$$T_{\mu\nu} = \left( (\rho + p) u_\mu u_\nu - p g_{\mu\nu} \right) e^{\beta\phi/M_{\text{Pl}}} + \left( \frac{\partial \phi}{\partial x^\mu} \frac{\partial \phi}{\partial x^\nu} - g_{\mu\nu} \left( g^{\lambda\rho} \frac{1}{2} \frac{\partial \phi}{\partial x^\lambda} \frac{\partial \phi}{\partial x^\rho} - V(\phi) \right) \right), \tag{25}$$

where the contribution of torsion  $T_{\mu\nu}^{(\text{tor})} = \rho_\Lambda g_{\mu\nu} = M_{\text{Pl}}^2 \Lambda_C g_{\mu\nu}$  [39] is included additively in the potential  $V(\phi)$  of the self-interactions of the chameleon field. Below we analyze the Einstein equations (Equation (24)) in the cold dark matter (CDM) model [70] in the Friedmann flat spacetime with the line element [69,70]

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu = dt^2 + a^2(t) \eta_{ij} dx^i dx^j, \tag{26}$$

where  $g_{00}(x) = 1$  and  $g_{ij}(x) = a^2(t) \eta_{ij}$  with  $\eta_{ij} = -\delta_{ij}$ . Then,  $a(t)$  is the expansion parameter of the Universe’s evolution [69]. The Christoffel symbols  $\{\alpha{}_{\mu\nu}\}$ , the components of the Ricci tensor  $R_{\mu\nu}$  and the scalar curvature  $R$  are equal to [69]

$$\begin{aligned} \{0{}_{00}\} &= \{0{}_{0j}\} = \{j{}_{00}\} = \{i{}_{kj}\} = 0, \quad \{0{}_{kj}\} = -a\dot{a} \eta_{kj}, \quad \{i{}_{0j}\} = \frac{\dot{a}}{a} \delta^i{}_j, \\ R_{00} &= 3 \frac{\ddot{a}}{a}, \quad R_{0j} = 0, \quad R_{ij} = \left( \frac{\ddot{a}}{a} + 2 \frac{\dot{a}^2}{a^2} \right) g_{ij}, \quad R = 6 \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right), \end{aligned} \tag{27}$$

where  $\eta^{i\ell} \eta_{\ell j} = \delta^i{}_j$  and  $\dot{a}$  and  $\ddot{a}$  are first and second derivatives with respect to time.

### 3. Friedmann–Einstein Equations of the Universe’s Evolution

In Friedmann spacetime, Einstein’s equations (Equation (24)) define the equations of the Universe’s evolution, which are usually called Friedmann’s equations (or the Friedmann–Einstein equations) [69]. They are given by

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3M_{\text{Pl}}^2} (\rho_\phi + (\rho_r + \rho_m)f(\phi)) \tag{28}$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\text{Pl}}^2} (\rho_\phi + 3p_\phi + (\rho_r + 3p_r)f(\phi) + \rho_m f(\phi)), \tag{29}$$

where  $\rho_r$  and  $\rho_m$  are the radiation and matter densities. The scalar field  $\phi$  couples to radiation and matter densities through the conformal factor  $f(\phi) = e^{\beta\phi/M_{\text{Pl}}}$ . Then, the radiation density  $\rho_r$  and pressure  $p_r$  are related by the equation of state  $p_r = \rho_r/3$  [69]. For the description of matter we use the cold dark matter (CDM) model with the pressureless dark and baryon matter [70]. The scalar field density  $\rho_\phi$  and pressure  $p_\phi$  are equal to

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad , \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi). \tag{30}$$

Varying the action Equation (17) with respect to the scalar field  $\phi$  and its derivative one gets the equation of motion for the scalar field [81]. In Friedmann spacetime it reads

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0, \tag{31}$$

where  $V_{\text{eff}}(\phi)$  is the effective potential given by

$$V_{\text{eff}}(\phi) = V(\phi) + \rho_m (f(\phi) - 1). \tag{32}$$

The contribution of the radiation density comes into the effective potential in the form  $(\rho_r - 3p_r)(f(\phi) - 1)$ . As for the equation of state  $p_r = \rho_r/3$ , such a contribution vanishes. Thus, through the interaction with matter density  $\rho_m$  the scalar field can acquire a non-vanishing mass if the effective potential  $V_{\text{eff}}(\phi)$  obeys the constraints

$$\left. \frac{dV_{\text{eff}}(\phi)}{d\phi} \right|_{\phi=\phi_{\text{min}}} = 0 \quad , \quad \left. \frac{d^2V_{\text{eff}}(\phi)}{d\phi^2} \right|_{\phi=\phi_{\text{min}}} > 0, \tag{33}$$

i.e., the effective potential  $V_{\text{eff}}(\phi)$  possesses a minimum at  $\phi = \phi_{\text{min}}$ . An important role for a dependence of a chameleon field mass on a density of an environment is the conformal factor  $f(\phi)$  and its deviation from unity.

#### 3.1. Bianchi Identity, Conservation of Total Energy–Momentum Tensor and Conformal Factor

By using Equation (27) and taking into account that in the Friedmann flat spacetime the non-vanishing components of the Einstein tensor  $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$  are equal to

$$G^{00} = -3\frac{\dot{a}^2}{a^2} \quad , \quad G^{ij} = \left( -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) g^{ij} \tag{34}$$

one may show that Einstein’s tensor  $G_{\mu\nu}$  obeys the Bianchi identity [69]

$$G^{\mu\nu}{}_{;\mu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\rho} (\sqrt{-g} G^{\rho\nu}) + \Gamma^\nu{}_{\mu\rho} G^{\mu\rho} = 0, \tag{35}$$

where  $G^{\mu\nu}{}_{;\mu}$  is a covariant divergence and  $\Gamma^\nu{}_{\mu\rho} = \{\nu{}_{\mu\rho}\}$  are the Christoffel symbols [69]. As a result, the covariant divergence of the total energy–momentum tensor  $T^{\mu\nu}{}_{;\mu}$  should also vanish

$$T^{\mu\nu}{}_{;\mu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\rho} (\sqrt{-g} T^{\rho\nu}) + \Gamma^\nu{}_{\mu\rho} T^{\mu\rho} = 0. \tag{36}$$

Due to time-dependence only Equation (31) takes the form

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial t} (\sqrt{-g} T^{00}) + \Gamma^0{}_{ij} T^{ij} = 0, \tag{37}$$

where we have taken into account Equation (27). Using the non-vanishing components of the total energy momentum tensor

$$T^{00} = \rho_\phi + (\rho_r + \rho_m) f(\phi) \quad , \quad T^{ij} = - (p_\phi + p_r f(\phi)) g^{ij} \tag{38}$$

we transcribe Equation (32) into the form

$$\frac{d}{dt} (\rho_\phi + (\rho_r + \rho_m) f(\phi)) + 3 \frac{\dot{a}}{a} (\rho_\phi + p_\phi + (\rho_r + p_r) f(\phi) + \rho_m f(\phi)) = 0. \tag{39}$$

Since Equation (31) can be rewritten as follows:

$$\frac{d}{dt} (\rho_\phi + \rho_m f(\phi)) = \frac{d}{dt} \rho_m - 3 \frac{\dot{a}}{a} (\rho_\phi + p_\phi), \tag{40}$$

we may remove the contribution of the chameleon field in Equation (39). As result, we get

$$\frac{d}{dt} (\rho_r f(\phi) + \rho_m) + \frac{\dot{a}}{a} (4\rho_r f(\phi) + 3\rho_m f(\phi)) = 0, \tag{41}$$

where we have used the equation of state  $p_r = \rho_r/3$  [69]. Due to independence of radiation and matter densities, Equation (41) can be split into evolution equations of the radiation and matter densities:

$$\begin{aligned} \frac{d}{dt} (\rho_r f(\phi)) + 4 \frac{\dot{a}}{a} (\rho_r f(\phi)) &= 0, \\ \frac{d}{dt} \rho_m + 3 \frac{\dot{a}}{a} \rho_m f(\phi) &= 0. \end{aligned} \tag{42}$$

For the standard dependence of the radiation and matter densities on the expansion parameter  $a(t)$  [69],

$$\rho_r = 3M_{\text{Pl}}^2 H_0^2 \Omega_r \frac{a_0^4}{a^4}, \quad \rho_m = 3M_{\text{Pl}}^2 H_0^2 \Omega_m \frac{a_0^3}{a^3}, \tag{43}$$

where  $a_0, H_0 = 1.438(11) \times 10^{-33}$  eV,  $\Omega_r$  and  $\Omega_m$  are the expansion parameter, the Hubble rate and the relative radiation and matter densities at our time  $t_0 = 1/H_0$  [70], the equations for the radiation and matter densities Equation (42) are satisfied identically for  $f(\phi) = 1$ .

Thus, if the radiation and matter densities depend on the expansion parameter  $a$  as  $\rho_r(a) \sim a^{-4}$  and  $\rho_m(a) \sim a^{-3}$ , local conservation of the total energy–momentum in the Universe can be fulfilled if and only if the conformal factor  $f(\phi)$ , relating Einstein’s and Jordan’s frames and defining the chameleon–matter coupling, is equal to unity; i.e.,  $f(\phi) = 1$ . However, in this case there is no influence of the chameleon field on the evolution of the radiation and matter densities and a dependence of the

chameleon field mass on a density of its environment. In turn, for  $f(\phi) \neq 1$  the evolution equations (Equation (37)) admit some exact solutions. It is convenient to search these solutions independently of the expansion parameter  $a$ . Treating the conformal factor  $f(\phi)$  as a function of the expansion parameter  $a$ , i.e., setting  $f(\phi) = f(a) \neq 1$ , the solutions to Equation (37) can be given by

$$\begin{aligned} \rho_r(a) &= \rho_{r0} \frac{a_0^4}{a^4} \frac{f(a_0)}{f(a)}, \\ \rho_m(a) &= \rho_{m0} \frac{a_0^3}{a^3} \exp\left(3 \int_a^{a_0} \frac{f(a') - 1}{a'} da'\right), \end{aligned} \tag{44}$$

where  $\rho_{r0} = 3M_{\text{Pl}}^2 H_0^2 \Omega_r$  and  $\rho_{m0} = 3M_{\text{Pl}}^2 H_0^2 \Omega_m$  are the radiation and matter densities at out time  $t_0 = 1/H_0$  and  $a(t_0) = a_0$ , i.e., in the era of the late-time acceleration of the Universe’s expansion or the dark energy–dominated era. The integration constants of the first order differential equations (Equation (35)) are fixed by the conditions  $\rho_r(a_0) = \rho_{r0}$  and  $\rho_m(a_0) = \rho_{m0}$ , respectively [69,70]. According to the solutions (Equation (44)), the chameleon field has an influence on the evolution of the radiation and matter densities.

As an example of the conformal factor we may use  $f = e^{\beta\varphi(a)/M_{\text{Pl}}}$  [1,2], where  $\varphi(a)$  is the chameleon field as a function of the expansion parameter  $a$  and the solution to Equation (31), i.e.,  $\phi(t) = \varphi(a)$ . Keeping the linear order contributions in the  $\beta\varphi(a)/M_{\text{Pl}}$  expansion we get

$$\begin{aligned} \rho_r(a) &= \rho_{r0} \frac{a_0^4}{a^4} \left(1 + \frac{\beta}{M_{\text{Pl}}} (\varphi(a_0) - \varphi(a))\right), \\ \rho_m(a) &= \rho_{m0} \frac{a_0^3}{a^3} \left(1 + 3 \frac{\beta}{M_{\text{Pl}}} \int_a^{a_0} \varphi(a') \frac{da'}{a'}\right). \end{aligned} \tag{45}$$

Thus, the deviations of the radiation and matter densities from their standard behavior  $\rho_r(a) \sim a^{-4}$  and  $\rho_m(a) \sim a^{-3}$  are given by

$$\begin{aligned} \delta\rho_r(a) &= \frac{\beta}{M_{\text{Pl}}} \rho_{r0} \frac{a_0^4}{a^4} (\varphi(a_0) - \varphi(a)), \\ \delta\rho_m(a) &= 3 \frac{\beta}{M_{\text{Pl}}} \rho_{m0} \frac{a_0^3}{a^3} \int_a^{a_0} \varphi(a') \frac{da'}{a'}. \end{aligned} \tag{46}$$

Some observations of deviations of the radiation and matter densities in the Universe from their standard form might, in principle, evidence an existence of the chameleon field. Nevertheless, we have to emphasize that the contributions of the conformal factor to the radiation and matter densities at our time are not practically observable. It is seen from the solutions (Equation (44)) that the conformal factor affects the evolution of the radiation and matter densities during the radiation and matter-dominated eras only. Of course, an influence of the chameleon field evolution on the distribution of the radiation density might seem rather questionable, since the evolution equation (Equation (37)) defines an evolution of the product  $\rho_r f(\phi)$ , where one may hardly separate  $\rho_r$  from  $f(\phi)$ . By introducing an effective radiation density  $\rho_r^{(\text{eff})} = \rho_r f(\phi)$  we obtain a canonical radiation density Equation (43), where the contribution of  $f(\phi)$  at  $a = a_0$  is hidden very likely in  $\Omega_r$ .

### 3.2. The Friedmann–Einstein (Equation Equation (28)) as the First Integral of the Friedmann–Einstein Equation (Equation (29))

It is well-known that without the chameleon field and for the conformal factor  $f(a) = 1$  the Friedmann–Einstein differential equation for  $\dot{a}^2/a^2$  is the first integral of the Friedmann–Einstein differential equation for  $\ddot{a}/a$  [69]. However, such a property of Equation (28) with the chameleon field and the conformal factor  $f(a) \neq 1$  to be the first integral of Equation (29) has not so far been investigated and proven in the literature. In order to prove that Equation (28) is the first integral

of Equation (29) with the contributions of the chameleon field and the conformal factor  $f(a) \neq 1$ , we rewrite Equation (28) as follows:

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3M_{\text{Pl}}^2} (\rho_{\text{ch}} + \rho_r f + \rho_m), \tag{47}$$

where  $\rho_{\text{ch}} = \rho_\phi + \rho_m(f - 1) = \frac{1}{2}\dot{\phi}^2 + V_{\text{eff}}(\phi)$  is the chameleon field density, given by Equation (30) with the replacement  $V(\phi) \rightarrow V_{\text{eff}}(\phi)$  (see Equation (32)). In order to find  $\rho_{\text{ch}}$  as a function of the expansion parameter  $a$  we use Equation (31) and transcribe it into the form

$$a \frac{d}{da} \rho_{\text{ch}}(a) + 6\rho_{\text{ch}}(a) = 6V_{\text{eff}}(a), \tag{48}$$

where we have denoted  $V_{\text{eff}}(\phi) = V_{\text{eff}}(a)$ , assuming that  $\phi$  is a function of  $a$ ; i.e.,  $\phi = \phi(a)$ . As a function of the expansion parameter  $a$ , the effective potential  $V_{\text{eff}}(a)$  is given by

$$V_{\text{eff}}(a) = V(a) + \rho_m(a)(f(a) - 1), \tag{49}$$

where  $V(a) = V(\phi) = V(\phi(a))$  with the additive contribution of the relic dark energy density, induced by torsion, and  $f(a) = e^{\beta\phi(a)/M_{\text{Pl}}}$ . The solution to Equation (48) is equal to

$$\rho_{\text{ch}}(a) = \frac{C_\phi}{a^6} + \frac{6}{a^6} \int a^5 V_{\text{eff}}(a) da, \tag{50}$$

where the term  $C_\phi/a^6$  corresponds to the contribution of the kinetic term of a scalar field [87]. The integration constant  $C_\phi$ , we define as follows:  $C_\phi = 3M_{\text{Pl}}^2 H_0^2 \Omega_\phi a_0^6$ , where  $\Omega_\phi$  is the integration constant, having the meaning of a relative density of a scalar field at time  $t_0 = 1/H_0$  [70]. As a result, Equation (47) takes the form

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3M_{\text{Pl}}^2} (\rho_{\text{ch}}(a) + \rho_r(a)f(a) + \rho_m(a)), \tag{51}$$

where in the right-hand-side (r.h.s.) all densities and the conformal factor are functions of the expansion parameter  $a$ . Further, it is convenient to rewrite Equation (29) as follows:

$$\frac{\ddot{a}}{a} = -2\frac{\dot{a}^2}{a^2} + \frac{1}{3M_{\text{Pl}}^2} \rho_r(a)f(a) + \frac{1}{2M_{\text{Pl}}^2} \rho_m(a)f(a) + \frac{1}{M_{\text{Pl}}^2} V(a), \tag{52}$$

where we have used Equation (51). Since the second derivative  $\ddot{a}$  of the expansion parameter  $a$  with respect to time can be given by

$$\ddot{a} = \frac{1}{2} \frac{d\dot{a}^2}{da}, \tag{53}$$

one may transcribe Equation (47) into the form

$$a \frac{d}{da} \dot{a}^2 + 4\dot{a}^2 = \frac{2}{3M_{\text{Pl}}^2} a^2 \rho_r(a)f(a) + \frac{1}{M_{\text{Pl}}^2} a^2 \rho_m(a)f(a) + \frac{2}{M_{\text{Pl}}^2} a^2 V(a). \tag{54}$$

The solution to Equation (54) amounts to

$$\dot{a}^2 = \frac{C}{a^4} + \frac{2}{3M_{\text{Pl}}^2} \frac{1}{a^4} \int a^5 \rho_r(a)f(a) da + \frac{1}{M_{\text{Pl}}^2} \frac{1}{a^4} \int a^5 \rho_m(a)f(a) da + \frac{2}{M_{\text{Pl}}^2} \frac{1}{a^4} \int a^5 V(a) da, \tag{55}$$

where  $C$  is the integration constant. Dividing both sides of Equation (55) by  $a^2$  we arrive at the equation

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3M_{\text{Pl}}^2} \left( \frac{C_\phi}{a^6} + \frac{6}{a^6} \int a^5 U(a) da + \frac{2}{a^6} \int a^5 \rho_r(a) f(a) da + \frac{3}{a^6} \int a^5 \rho_m(a) f(a) da \right), \tag{56}$$

where we have set  $C_\phi = 3M_{\text{Pl}}^2 C = 3M_{\text{Pl}}^2 H_0^2 \Omega_\phi$ . Thus, Equation (51) is the first integral of Equation (29). Making a replacement  $V(a) = V_{\text{eff}}(a) - \rho_m(a)(f(a) - 1)$  we arrive at the expression

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3M_{\text{Pl}}^2} \left( \rho_{\text{ch}}(a) + \frac{2}{a^6} \int a^5 \rho_r(a) f(a) da + \frac{6}{a^6} \int a^5 \rho_m(a) da - \frac{3}{a^6} \int a^5 \rho_m(a) f(a) da \right). \tag{57}$$

Since the radiation and matter densities as functions of  $a$  obey the equations

$$\begin{aligned} a \frac{d}{da} (\rho_r(a) f(a)) &= -4 (\rho_r(a) f(a)), \\ a \frac{d}{da} \rho_m(a) &= -3 \rho_m(a) f(a) \end{aligned} \tag{58}$$

and that  $\rho_r(a) f(a) = \rho_{r0} f(a_0) a_0^4 / a^4$  (see Equation (44)), we transcribe the right-hand-side (r.h.s.) of Equation (57) into the form

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3M_{\text{Pl}}^2} \left( \rho_{\text{ch}}(a) + \rho_r(a) f(a) + \frac{1}{a^6} \int \frac{d}{da} (a^6 \rho_m(a)) da \right) = \frac{1}{3M_{\text{Pl}}^2} (\rho_{\text{ch}}(a) + \rho_r(a) f(a) + \rho_m(a)), \tag{59}$$

This proves that Equation (28) is the first integral of Equation (29) if the total energy–momentum is locally conserved. The evolution of the chameleon field density  $\rho_{\text{ch}}(a)$  independently of the expansion parameter  $a$  is defined by Equation (50), which we rewrite as follows:

$$\rho_{\text{ch}}(a) = \rho_\Lambda + \frac{C_\phi}{a^6} + \frac{6}{a^6} \int a^5 \Phi(a) da + \frac{6}{a^6} \int a^5 \rho_m(a) (f(a) - 1) da, \tag{60}$$

where  $\rho_\Lambda = M_{\text{Pl}}^2 \Lambda_C$ . and the third term in Equation (60) is the model-dependent part of the potential of the self-interaction of the chameleon field  $V(\phi) = \rho_\Lambda + \Phi(\phi)$  [4,10,16,88], taken as a function of the expansion parameter  $a$ , i.e.,  $\Phi(\phi) = \Phi(a)$ . Such a chameleon field density may affect the acceleration of the Universe’s expansion. Setting  $f(a) = 1$  in Equation (60) we get

$$\rho_{\text{ch}}(a) = \rho_\Lambda + \frac{C_\phi}{a^6} + \frac{6}{a^6} \int a^5 \Phi(a) da, \tag{61}$$

where the second and the last terms might still provide an acceleration of the Universe’s expansion additional to that caused by the first term  $\rho_\Lambda$ , which is induced by torsion [39].

#### 4. Torsion–Neutron Low-Energy Interactions

Our analysis carried out above may give an impression that after the absorption of torsion by the cosmological constant, the Einstein–Cartan gravitational theory reduces to Einstein’s gravitational theory [69]. Such an impression can be real only in case of the absence of fermions. As has been shown in [40–42], there is a huge variety of minimal and nonminimal low-energy torsion–neutron interactions. The torsion tensor field  $\mathcal{T}_{\sigma\mu\nu}$ , being a tensor of the third rank and antisymmetric with respect to indices  $\mu$  and  $\nu$ , i.e.,  $\mathcal{T}_{\sigma\mu\nu} = -\mathcal{T}_{\sigma\nu\mu}$ , is defined by 24 independent components: (i) four vectors  $\mathcal{E}^\mu = (\mathcal{E}^0, \vec{\mathcal{E}})$ , (ii) four axial vectors  $\mathcal{B}^\mu = (\mathcal{B}^0, \vec{\mathcal{B}})$  and (iii) 16 tensors  $\mathcal{M}_{\sigma\mu\nu}$  [35,36,44] (see also [40–42]). The effective low-energy torsion–neutron potentials are presented in the form of expansion in powers of  $1/m$ , where  $m$  is the neutron mass, and restricted by the terms of order  $O(1/m)$ , by using the Foldy–Wouthuysen (FW) canonical transformations [89]. The most interesting effective low-energy torsion–neutron interactions are induced in the rotating coordinate systems, which can be used for

experimental probes of torsion in terrestrial laboratories [48,60]. It is important to emphasize that a part of these effective low-energy torsion–neutron interactions provide a violation of time-reversal invariance [42], which can be probed in the terrestrial laboratories.

According to [42], in the coordinate system rotating with an angular velocity  $\vec{\Omega}$  the time component  $\mathcal{E}_0$  of the 4-vector  $\mathcal{E}^\mu$  of the torsion field induces the T-odd, i.e., violating time reversal symmetry, optical potential

$$\Phi_{\text{eff}}^{(\text{T-odd})} = -i \frac{4}{3} \frac{\mathcal{E}_0}{m} \vec{S} \cdot \vec{\Omega}, \tag{62}$$

where  $\vec{S} = \frac{1}{2} \vec{\sigma}$  is the operator of the neutron spin and  $\vec{\sigma}$  are  $2 \times 2$  Pauli matrices [90]. As has been shown in [47], because of the T-odd interaction (Equation (62)), the cross section for low-energy neutron–nucleus scattering, caused by the beam of polarized neutrons passing through a spinning cylinder, should acquire the correction [39]

$$\Delta\sigma_{\text{TV}}(\Omega, p) = \frac{8\pi}{3\sqrt{2}} \mathcal{E}_0 R^2 L \frac{\Omega}{p}, \tag{63}$$

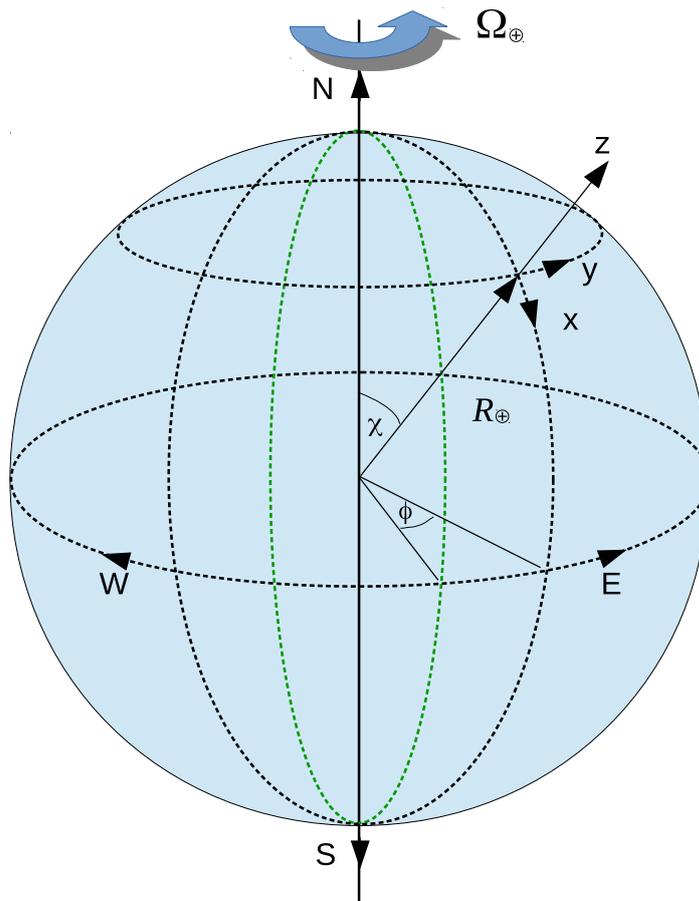
where  $R$  and  $L$  are the radius and length of the spinning cylinder, and  $p$  is a neutron momentum (for a detailed discussion of the  $p$ -dependence of  $\Delta\sigma_{\text{TV}}(\Omega, p)$  we refer to [47] below Equation (5)). The aim of the proposed experiment is a search for the  $\Omega$ -dependent part of the helicity-dependent part of the difference of the cross sections for neutron–nucleus scattering, caused by neutrons polarized parallel and antiparallel to the neutron beam axis coinciding with the axis of a spinning cylinder. For contemporary experimental abilities, such a T-odd correction allows one to probe the time component of the 4-vector part of the torsion field at the level of sensitivity of about  $|\mathcal{E}_0| \sim 10^{-32}$  GeV. This is a few orders of magnitude better in comparison to the estimate obtained in [44].

Another part of the effective low-energy torsion–neutron potentials, which is not proportional to  $1/m$ , can be used for probes of the components of the torsion field in the qBounce experiments dealing with ultracold neutrons (UCNs) bouncing in the gravitational field of the Earth [49–55] (see also [48]). As an example, we consider the effective low-energy potential of the time-component  $\mathcal{B}_0$  (pseudoscalar) of the 4-axial vector  $\mathcal{B}^\mu$  and the time-time-space-components  $(\vec{\mathcal{M}})_k = \mathcal{M}_{00k}$  of the tensor  $\mathcal{M}_{\sigma\mu\nu}$  [42]

$$\Phi_{\text{eff}} = \frac{1}{3} \mathcal{B}_0 \vec{S} \cdot (\vec{\Omega}_\oplus \times (\vec{R}_\oplus + \vec{r})) - \frac{1}{2} \vec{S} \cdot (\vec{\mathcal{M}} \times (\vec{\Omega}_\oplus \times (\vec{R}_\oplus + \vec{r}))), \tag{64}$$

where  $\vec{\Omega}_\oplus$  and  $\vec{R}_\oplus$  are the angular velocity and the radius vector of the Earth as they are shown in Figure 1. Then,  $\vec{r}$  is the radius–vector of the UCN in the laboratory.

The experiments with UCNs, bouncing in the gravitational field of the Earth, are being performed in the laboratory at Institut Laue Langevin (ILL) in Grenoble. The ILL laboratory is fixed to the surface of the Earth in the northern hemisphere. Following [91–95] we choose the ILL laboratory or the standard laboratory frame with coordinates  $(t, x, y, z)$ , where the  $x$ ,  $y$  and  $z$  axes point south, east and vertically upwards, respectively, with northern and southern poles on the axis of the Earth’s rotation with the Earth’s sidereal frequency  $\Omega_\oplus = 2\pi / (23 \text{ hr } 56 \text{ min } 4.09 \text{ s}) = 7.2921159 \times 10^{-5} \text{ rad/s}$ . The position of the ILL laboratory on the surface of the Earth is determined by the angles  $\chi$  and  $\phi$ , where  $\chi = 90^\circ - \theta$  is the colatitude of the laboratory, defined in terms of the latitude  $\theta$ , and  $\phi$  is the longitude of the laboratory measured east of south with the values  $\theta = 45.16667^\circ \text{ N}$  and  $\phi = 5.71667^\circ \text{ E}$  [96], respectively. The beam of UCNs moves from south to north antiparallel to the  $x$ -direction and with energies of UCNs quantized in the  $z$ -direction.



**Figure 1.** The position of the ILL laboratory doing the qBounce experiments on the surface of the Earth.

In the qBounce experiments the contributions of interactions beyond the gravitational interaction of the Earth are measured in terms of the transition frequencies  $\omega_{pq} = E_p - E_q$  of the transitions  $|q\rangle \rightarrow |p\rangle$  between two gravitational states of UCNs  $|q\rangle$  and  $|p\rangle$  [49–55] (see also [59,60]). As of the small values of the components of the torsion field the contribution of the  $\vec{r}$ -dependent part of the effective torsion–neutron potential Equation (64), where the vector  $\vec{r}$  defines a location of the UCN in the coordinate system  $(t, x, y, z)$ , to the transition frequencies between quantum gravitational states of UCNs can be neglected in comparison to the contributions of the terms independent of  $\vec{r}$ . Relative to the axes  $(x, y, z)$  the vectors  $\vec{\Omega}_\oplus$  and  $\vec{R}_\oplus$  are equal to  $\vec{\Omega}_\oplus = (-\Omega_\oplus \sin \chi, 0, \Omega_\oplus \cos \chi)$  and  $\vec{R}_\oplus = (0, 0, R_\oplus)$ , respectively. This allows one to transcribe the effective low-energy torsion–neutron potential Equation (64) into the form

$$\Phi_{\text{eff}} = \Omega_\oplus R_\oplus \sin \chi \left( \frac{1}{3} \mathcal{B}_0 S_y + \frac{1}{2} \mathcal{M}_z S_x - \frac{1}{2} \mathcal{M}_x S_z \right) = 1.1 \times 10^{-6} \left( \frac{1}{3} \mathcal{B}_0 S_y + \frac{1}{2} \mathcal{M}_z S_x - \frac{1}{2} \mathcal{M}_x S_z \right), \quad (65)$$

where  $(S_x, S_y, S_z)$  are operators of the neutron spin  $\vec{S}$ -operator components. Thus, measuring the transition frequencies of spin-flip transitions between gravitational states  $|q \downarrow\rangle \rightarrow |p \uparrow\rangle$  one may measure the contributions of the pseudoscalar  $\mathcal{B}_0$  and tensor  $\mathcal{M}_x = -\mathcal{M}_{00x}$  and  $\mathcal{M}_z = -\mathcal{M}_{00z}$  components of the torsion field. A predictable power of the qBounce experiments we may demonstrate by example of the estimate of the contribution of the pseudoscalar component  $\mathcal{B}_0$  of the torsion field coupled to UCNs. Indeed, according to Lämmerzahl [43], the value of the pseudoscalar component of the torsion field is constrained by  $|\mathcal{B}_0| < 2 \times 10^{-18}$  GeV. Its contribution to the transition frequencies between quantum gravitational states of UCNs  $|q \downarrow\rangle \rightarrow |p \uparrow\rangle$  is of about  $|\Delta\omega_{p\uparrow q\downarrow}| < 7 \times 10^{-16}$  eV. This value is at the level of current experimental sensitivity  $\Delta E < 10^{-15}$  eV [55] and the sensitivity of a nearest future, which is of about  $\Delta E < 10^{-17}$  eV and even  $\Delta E < 10^{-21}$  eV [49,97]. The experiments

discussed in this sections and and many others, which could be carried out by using effective low-energy potentials of torsion–neutron interactions derived in [40–42], might make reliable the geometrical origin of the cosmological constant or the relic dark energy, induced by torsion [39].

## 5. Discussion

We would like to emphasize that our analysis of the chameleon field as a candidate for quintessence is carried out within the classical Einstein–Cartan gravitational theory with the Einstein–Hilbert action linear in the Ricci scalar curvature. By definition [4], quintessence is a hypothetical form of dark energy described by a canonical scalar field for an explanation of the observable acceleration of the Universe’s expansion. The most important that quintessence should be a hypothetical form of dark energy. In this connection in the Einstein–Cartan gravitational theory, when the cosmological constant or the relic dark energy density has the geometrical origin, caused by torsion, the chameleon field possesses no chance to be a hypothetical form of dark energy. In other words having provided a geometrical origin for the cosmological constant or the relic dark energy torsion deprives the chameleon field to have a chance to be quintessence. As a result, the chameleon field is able only to evolve above the relic background of the dark energy, caused by torsion, but not to originate it. Then, as a consequence of conservation of the total energy–momentum of the system, the chameleon field can affect the dark energy dynamics and as well as the Universe’s expansion even also the late-time acceleration. We have shown that such an influence of the chameleon field on the acceleration of the Universe’s expansion retains also even if the conformal factor, relating Einstein’s and Jordan’s frames and defining the interaction of the chameleon field with its ambient matter, is equal to unity (see Equations (60) and (61)). This result is closely related to our proof that for the system, including the chameleon field, radiation and matter (dark and baryon matter), the Friedmann–Einstein equation for  $\dot{a}^2/a^2$  is the first integral for the Friedmann–Einstein equation for  $\ddot{a}/a$ .

We have found that local conservation of the total energy–momentum of the system, including the chameleon field, radiation and matter (dark and baryon matter), leads to the equations of the evolution of the radiation and matter densities, corrected by the conformal factor. Of course, since for radiation the evolution equation defines an evolution of the product  $\rho_r(a)f(\phi)$ , where the conformal factor  $f(\phi)$  is a function of the expansion parameter  $a$ , such a product  $\rho_r(a)f(\phi)$  does not deviate from the standard behavior  $\rho_r(a)f(\phi) \sim a^{-4}$ . Since the radiation density  $\rho_r(a)$  enters to the Friedmann–Einstein equations only in the form of the product  $\rho_r(a)f(\phi)$  one may not probably separate the contribution of the conformal factor above the standard shape proportional to  $a^{-4}$ . In turn, for the matter density  $\rho_m(a)$  the contribution of the conformal factor leads to a deviation from the standard behavior  $\rho_m(a) \sim a^{-3}$  [69]. However, such a deviation might be, in principle, noticeable only during the matter-dominated era. In the dark energy–dominated era that is in our time of the late-time acceleration of the Universe, where the expansion parameter is equal to  $a_0 = a(t_0)$  for the Hubble time  $t_0 = 1/H_0$  [69], the contributions of the conformal factor to the radiation and matter densities in comparison to the standard values  $\rho_r(a_0) = 3M_{\text{Pl}}^2 H_0^2 \Omega_r$  and  $\rho_m(a_0) = 3M_{\text{Pl}}^2 H_0^2 \Omega_m$  are practically unobservable. This agrees well with the constraints on the deviations of the radiation and matter densities from their values at our time to a few parts per million [98], which can be obtained from the constraints on the fifth force caused by the chameleon field in the Galaxy and the Solar system.

The cosmological constant  $\Lambda_C$ , induced by torsion [39], we have included additively to the potential of the self-interaction of the chameleon field as a background of the relic dark energy:  $V(\phi) = \rho_\Lambda + \Phi(\phi)$ . In the chameleon field theory [1,2] the relic dark energy density  $\rho_\Lambda$  is defined as follows:  $\rho_\Lambda = \Lambda^4$ , where the scale  $\Lambda = \sqrt[4]{3M_{\text{Pl}}^2 H_0^2 \Omega_\Lambda} = 2.24(1)$  meV is calculated for the relative dark energy density  $\Omega_\Lambda = 0.685(7)$  [70]. The  $\phi$ -dependent part of the potential of the self-interaction of the chameleon field  $\Phi(\phi)$  is arbitrary to some extent, i.e., model-dependent, and demands a special analysis similar to that carried out in [4,10,16,88]. However, such an analysis goes beyond the scope of our paper. We would like to emphasize that a specific analysis of a dynamics of the chameleon field such as different mechanisms of chameleon screening and a formation of a fifth force, for example, in the

Galaxy and the Solar system is related also to a special choice of the potential of the self-interaction of the chameleon field [16,98]. Such an analysis has been carried out by Brax et al. [16] and Jain et al. [98]. The repetition of such an analysis goes beyond the scope of this paper.

As regards the assertion by Wang et al. [67] and Khoury [68] that since the conformal factor is practically constant during the Hubble time, so the chameleon field is not responsible for the late-time acceleration of the Universe, one may argue that the conformal factor might be, in principle, practically constant (or better to say unity), but such a behavior of the conformal factor does not prohibit the chameleon field, evolving above the relic dark energy background induced by torsion, to take a certain part in dark energy dynamics and, correspondingly, in the acceleration of the Universe's expansion (see Equations (60) and (61)) and even so in the late-time acceleration of the Universe's expansion.

As regards another canonical scalar fields which can be introduced for an explanation of an origin of dark energy and an influence on acceleration of the Universe's expansion such as symmetron and dilaton, we may say the following. Since the dynamics of the symmetron field differs from the dynamics of the chameleon one only by the shape of the potential of self-interaction [99], our conclusion concerning an identification of the chameleon field with quintessence is fully applicable to the symmetron one. In other words the symmetron field cannot be quintessence to full extent. Moreover, we have to mention that an existence of the symmetron field and its importance for an evolution of the Universe might seem to be rather questionable after the qBounce experiments [55] on the transition frequencies between quantum gravitational states of UCNs, which have excluded the existence of the symmetron field.

Then, we have to confess that our analysis of an identification of a canonical scalar field with quintessence, carried out within the classical Einstein–Cartan gravitational theory, can say practically nothing concerning dilaton. Indeed, unlike the gravitational theories with the chameleon and symmetron fields the gravitational theories with dilaton are based on string theory in terms of the non-riemannian structure of space-time [100–107]. Of course, since an inclusion of dilaton as a canonical scalar field is closely related to a requirement of scale invariance of the action of the dynamical system under consideration, the classical Einstein–Cartan gravitational theory can be, in principle, modified by a requirement of scale invariance [108]. However, in such a modified Einstein–Cartan gravitational theory the problem of the geometrical origin of the cosmological constant or the relic dark energy caused by torsion demands a special analysis, which goes beyond the scope of this paper. Nevertheless, if in such a modified Einstein–Cartan gravitational theory torsion would be a geometrical origin of the cosmological constant or the relic dark energy density, our conclusion concerning an impossibility to identify dilation with quintessence might have been valid only within such a modified Einstein–Cartan gravitational theory.

Robust support for the geometrical origin of the cosmological constant or the relic dark energy could be experimental observations of torsion in terrestrial laboratories in terms of its contributions to observables of different physical processes. In Section 4 we have discussed two of these experiments, which can be carried out by using beams of polarized UCNs. We mean the contribution of the T-odd torsion–neutron low-energy interaction to the cross section for the scattering of the beam of polarized neutrons by nucleus in the end of spinning cylinder. This allows one to estimate the value of the time component  $\mathcal{E}_0$  of the 4-vector part of the torsion field at the level of about  $|\mathcal{E}_0| \sim 10^{-32}$  GeV. Another experiment on the probe of torsion can be carried out at ILL by the French–Austrian qBounce Collaboration by using Gravity Resonance Spectroscopy (GRS), a new measuring technique combining quantum measurements and gravity experiments [49–55]. The qBounce experiments, measuring transition frequencies between quantum gravitational states of UCNs, allow one to probe torsion with a sensitivity of about  $\Delta E < 10^{-17}$  eV and even  $\Delta E \sim 10^{-21}$  eV [49,97]. This should improve the existing upper bound on the pseudoscalar  $\mathcal{B}_0$  component of the torsion field by a few orders of magnitude and give new constraints on the tensor  $\mathcal{M}_{00k}$  components of the torsion field [44].

**Author Contributions:** A.N.I.: Conceptualization, Methodology, Data Curation, Investigation, Supervision, Writing—Original draft preparation. M.W.: Formal analysis, Investigation, Methodology, Software, Validation, Writing—Original draft preparation. All authors have read and agreed to the published version of the manuscript.

**Funding:** The work of A. N. Ivanov was supported by the Austrian “Fonds zur Förderung der Wissenschaftlichen Forschung” (FWF) under the contracts P31702-N27, P26636-N20 and P33279-N, and “Deutsche Forschungsgemeinschaft” (DFG) AB 128/5-2. The work of M. Wellenzohn was supported by the MA 23 (FH-Call 16) under the project “Photonik—Stiftungsprofessur für Lehre”.

**Acknowledgments:** We are grateful to Hartmut Abele for stimulating discussions and to Philippe Brax and Alkistis Pourtsidou for fruitful discussions.

**Conflicts of Interest:** The authors declare no conflict of interest.

## References

1. Khoury, J.; Weltman, A. Chameleon cosmology. *Phys. Rev. D* **2004**, *69*, 044026. [[CrossRef](#)]
2. Mota, D.F.; Shaw, D.J. Evading equivalence principle violations, cosmological, and other experimental constraints in scalar field theories with a strong coupling to matter. *Phys. Rev. D* **2007**, *75*, 063501. [[CrossRef](#)]
3. Will, C.M. *Theory and Experiment in Gravitational Physics*; Cambridge University Press: Cambridge, UK, 1993.
4. Ratra, B.; Peebles, P.J.E. Cosmological consequences of a rolling homogeneous scalar field. *Phys. Rev. D* **1988**, *37*, 3406. [[CrossRef](#)] [[PubMed](#)]
5. Peebles, P.J.E.; Vilenkin, A. Quintessential inflation. *Phys. Rev. D* **1999**, *59*, 063505. [[CrossRef](#)]
6. Zlatev, I.; Wang, L.; Steinhardt, P.J. Quintessence, cosmic coincidence, and the cosmological constant. *Phys. Rev. Lett.* **1999**, *82*, 896. [[CrossRef](#)]
7. Steinhardt, P.J. Quintessence and cosmic acceleration. *NATO Sci. Ser. C* **2001**, *565*, 143.
8. Steinhardt, P.J. The quintessential Universe. *AIP Conf. Proc.* **2001**, *586*, 279.
9. Steinhardt, P.J. A quintessential introduction to dark energy. *Philos. Trans. R. Soc. Lond. A* **2003**, *361*, 2497. [[CrossRef](#)]
10. Tsujikawa, S. Quintessence: A review *Class. Quantum Gravity* **2013**, *30*, 21400. [[CrossRef](#)]
11. Perlmutter, S.; Aldering, G.; Deustua, S.; Fabbro, S.; Goldhaber, G.; Groom, D.E.; Kim, A.G.; Kim, M.Y.; Knop, R.A.; Nugent, P.; et al. Cosmology from type Ia supernovae. *Bull. Am. Astron. Soc.* **1997**, *29*, 1351.
12. Riess, A.G.; Filippenko, A.V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliland, R.L.; Hogan, C.J.; Jha, S.; Kirshner, R.P.; et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.* **1998**, *116*, 1009–1038. [[CrossRef](#)]
13. Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R.A.; Nugent, P.; Castro, P.G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D.E.; et al. Measurements of  $\Omega$  and  $\Lambda$  from 42 high redshift supernovae. *Astron. J.* **1999**, *517*, 565. [[CrossRef](#)]
14. Goobar, A.; Perlmutter, S.; Goldhaber, G.; Knop, R.A.; Nugent, P.; Castro, P.G.; Deustua, S.; Fabbro, S.; Groom, D.E.; Hook, I.M.; et al. The acceleration of the universe: Measurements of cosmological parameters from type Ia supernovae. *Phys. Scr. T* **2000**, *85*, 47. [[CrossRef](#)]
15. Peebles, P.J.E.; Ratra, B. The cosmological constant and dark energy. *Rev. Mod. Phys.* **2003**, *75*, 559–606. [[CrossRef](#)]
16. Brax, P.; van de Bruck, C.; Davis, A.C.; Khoury, J.; Weltman, A. Detecting dark energy in orbit: The cosmological chameleon. *Phys. Rev. D* **2004**, *70*, 123518. [[CrossRef](#)]
17. Copeland, E.J.; Sami, M.; Tsujikawa, S. Dynamics of dark energy. *Int. J. Mod. Phys. D* **2006**, *15*, 1753–1936. [[CrossRef](#)]
18. Frieman, J.A.; Turner, M.S.; Huterer, D. Dark energy and the accelerating Universe. *Annu. Rev. Astron. Astrophys.* **2008**, *46*, 385. [[CrossRef](#)]
19. Jain, B.; Joyce, A.; Thompson, R.; Upadhye, A.; Battat, J.; Brax, P.; Davis, A.; de Rham, C.; Dodelson, S.; Erickcek, A.; et al. Novel probes of gravity and dark energy. *arXiv* **2013**, arXiv:1309.5389.
20. Brax, P.; Davis, A.-C. Casimir, gravitational, and neutron tests of dark energy. *Phys. Rev. D* **2015**, *91*, 063503. [[CrossRef](#)]
21. Pignol, G. Probing dark energy models with neutrons. *Int. J. Mod. Phys. A* **2015**, *30*, 1530048. [[CrossRef](#)]
22. Cartan, É. Sur une généralisation de la notion de courbure de Riemann et les espaces à torsion. *C. R. Acad. Sci.* **1922**, *174*, 593.

23. Cartan, É. Sur les variétés à connexion affine et la relativité généralisée (première partie). *Ann. Ec. Norm.* **1923**, *40*, 325. [[CrossRef](#)]
24. Cartan, É. Sur les variétés à connexion affine et la relativité généralisée (première partie). *Ann. Ec. Norm.* **1924**, *41*, 1. [[CrossRef](#)]
25. Cartan, É. Sur les variétés à connexion affine et la relativité généralisée (deuxième partie). *Ann. Ec. Norm.* **1925**, *42*, 17. [[CrossRef](#)]
26. Cartan, É.; Einstein, A. *Letters of Absolute Parallelism*; Princeton University Press: Princeton, NJ, USA, 1975.
27. Schrödinger, E. *Space-Time Structure*; Cambridge at the University Press: Cambridge, UK, 1950.
28. Hehl, F.W.; Kerlick, G.D.; van der Heyde, P. General relativity with spin and torsion and its deviations from Einstein's theory. *Phys. Rev. D* **1974**, *10*, 1066–1069. [[CrossRef](#)]
29. Hehl, F.W.; van der Heyde, P.; Kerlick, G.D.; Nester, J.M. General relativity with spin and torsion: Foundations and prospects. *Rev. Mod. Phys.* **1976**, *48*, 393–416. [[CrossRef](#)]
30. Hehl, F.W.; Obukhov, Y.N. Elie Cartan's torsion in geometry and in field theory, an essay. *Ann. Fond. Louis Broglie* **2007**, *32*, 157–194.
31. Hehl, F.W. Gauge Theories of Gravity and Spacetime. *arXiv* **2012**, arXiv:1204.3672v2.
32. Blagojević, M.; Hehl, F.W. (Eds.) *Gauge Theories of Gravitation: A Reader with Commentaries*; Imperial College Press: London, UK, 2013.
33. Hehl, F.W.; Obukhov, Y.N. Conservation of energy-momentum of matter as the basis for the gauge theory of gravitation. *arXiv* **2019** arXiv:1909.01791.
34. Obukhov, Y.N.; Hehl, F.W. General relativity as a special case of Poincaré gauge gravity. *Phys. Rev. D* **2020**, *102*, 044058. [[CrossRef](#)]
35. Shapiro, I.L. Physical aspects of the space-time torsion. *Phys. Rep.* **2002**, *357*, 113–213. [[CrossRef](#)]
36. Hammond, R.T. Torsion gravity. *Rep. Prog. Phys.* **2002**, *65*, 599–649. [[CrossRef](#)]
37. Kostelecký, V.A. Gravity, Lorentz violation, and the standard model. *Phys. Rev. D* **2004**, *69*, 105009. [[CrossRef](#)]
38. Ni, W.-T. Reports on Progress in Physics Searches for the role of spin and polarization in gravity. *Rep. Prog. Phys.* **2010**, *73*, 056901. [[CrossRef](#)]
39. Ivanov, A.N.; Wellenzohn, M. Einstein–Cartan gravity with torsion field serving as an origin for the cosmological constant or dark energy density. *Astrophys. J.* **2016**, *829*, 47. [[CrossRef](#)]
40. Ivanov, A.N.; Wellenzohn, M. Nonrelativistic approximation of the Dirac equation for slow fermions coupled to the chameleon and torsion fields in the gravitational field of the Earth. *Phys. Rev. D* **2015**, *92*, 065006. [[CrossRef](#)]
41. Ivanov, A.N.; Wellenzohn, M. Effective low-energy potential for slow Dirac fermions in Einstein–Cartan Gravity with torsion and chameleon field. *Phys. Rev. D* **2015**, *92*, 125004. [[CrossRef](#)]
42. Ivanov, A.N.; Wellenzohn, M. Spin precession of slow neutrons in Einstein–Cartan gravity with torsion, chameleon, and magnetic field. *Phys. Rev. D* **2016**, *93*, 045031. [[CrossRef](#)]
43. Lämmerzahl, C. Constraints on space-time torsion from Hughes–Drever experiments. *Phys. Lett. A* **1997**, *228*, 223–231. [[CrossRef](#)]
44. Kostelecký, V.A.; Russell, N.; Tasson, J.D. Constraints on torsion from bounds on Lorentz violation. *Phys. Rev. Lett.* **2008**, *100*, 111102. [[CrossRef](#)]
45. Obukhov, Y.N.; Silenko, A.J.; Teryaev, O.V. Spin-torsion coupling and gravitational moments of Dirac fermions: Theory and experimental bounds. *Phys. Rev. D* **2014**, *90*, 124068. [[CrossRef](#)]
46. Lehnert, R.; Snow, W.M.; Yan, H. A first experimental limit on in-matter torsion from neutron spin rotation in liquid  $^4\text{He}$ . *Phys. Lett. B* **2014**, *730*, 353–356; Erratum in **2015**, *744*, 415. [[CrossRef](#)]
47. Ivanov, A.N.; Snow, W.M. Parity-even and time-reversal-odd neutron optical potential in spinning matter induced by gravitational torsion. *Phys. Lett. B* **2017**, *764*, 186–189. [[CrossRef](#)]
48. Ivanov, A.N. Presented at the Workshop Dark Energy in the Laboratory, Chicheley Hall, UK, 20–22 April 2016.
49. Abele, H.; Jenke, T.; Leeb, H.; Schmiedmayer, J. Ramsey's method of separated oscillating fields and its application to gravitationally induced quantum phase shifts. *Phys. Rev. D* **2010**, *81*, 065019. [[CrossRef](#)]
50. Jenke, T.; Geltenbort, P.; Lemmel, H.; Abele, H. Realization of a gravity-resonance-spectroscopy technique. *Nat. Phys.* **2011**, *7*, 468–472. [[CrossRef](#)]
51. Abele, H.; Jenke, T.; Stadler, D.; Geltenbort, P. QuBounce: The dynamics of ultra-cold neutrons falling in the gravity potential of the Earth. *Nucl. Phys. A* **2009**, *827*, 593c–595c. [[CrossRef](#)]

52. Jenke, T.; Stadler, D.; Abele, H.; Geltenbort, P. Q-BOUNCE—Experiments with quantum bouncing ultracold neutrons. In *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*; Particle Physics with Slow Neutrons; Elsevier: Amsterdam, The Netherlands, 2009; Volume 611, pp. 318–321.
53. Abele, H.; Leeb, H. Gravitation and quantum interference experiments with neutrons. *New J. Phys.* **2012**, *14*, 055010. [[CrossRef](#)]
54. Jenke, T.; Cronenberg, G.; Bürgdorfer, J.; Chizhova, L.A.; Geltenbort, P.; Ivanov, A.N.; Lauer, T.; Lins, T.; Rotter, S.; Saul, H.; et al. Gravity resonance spectroscopy constrains dark energy and dark matter scenarios. *Phys. Rev. Lett.* **2014**, *112*, 151105. [[CrossRef](#)]
55. Cronenberg, G.; Brax, P.; Filter, H.; Geltenbort, P.; Jenke, T.; Pignol, G.; Pitschmann, M.; Thalhammer, M.; Abele, H. Acoustic Rabi oscillations between gravitational quantum states and impact on symmetron dark energy. *Nat. Phys.* **2018**, *14*, 1022–1026. [[CrossRef](#)]
56. Lemmel, H.; Brax, P.; Ivanov, A.N.; Jenke, T.; Pignol, G.; Pitschmann, M.; Potocar, T.; Wellenzohn, M.; Zawisky, M.; Abele, H. Neutron interferometry constrains dark energy chameleon fields. *Phys. Lett. B* **2015**, *743*, 310–314. [[CrossRef](#)]
57. Li, K.; Arif, M.; Cory, D.G.; Haun, R.; Heacock, B.; Huber, M.G.; Nsofini, J.; Pushin, D.A.; Saggiu, P.; Sarenac, D.; et al. Neutron limit on the strongly-coupled chameleon field. *Phys. Rev. D* **2016**, *93*, 062001. [[CrossRef](#)]
58. Brax, P.; Pignol, G. Strongly coupled chameleons and the neutronic quantum bouncer. *Phys. Rev. Lett.* **2011**, *107*, 111301. [[CrossRef](#)] [[PubMed](#)]
59. Ivanov, A.N.; Höllwieser, R.; Jenke, T.; Wellenzohn, M.; Abele, H. Influence of the chameleon field potential on transition frequencies of gravitationally bound quantum states of ultracold neutrons. *Phys. Rev. D* **2013**, *87*, 105013. [[CrossRef](#)]
60. Ivanov, A.N.; Cronenberg, G.; Höllwieser, R.; Jenke, T.; Pitschmann, M.; Wellenzohn, M.; Abele, H. Exact solution for chameleon field, self-coupled through the Ratra-Peebles potential with  $n = 1$  and confined between two parallel plates. *Phys. Rev. D* **2016**, *94*, 085005. [[CrossRef](#)]
61. Ivanov, A.N.; Pitschmann, M. Nonrelativistic approximation of the Dirac equation for slow fermions in static metric spacetimes. *Phys. Rev. D* **2014**, *90*, 045040. [[CrossRef](#)]
62. Burrage, C.; Copeland, E.J.; Hinds, E.A. Probing dark energy with atom interferometry. *JCAP* **2015**, *2015*, 42. [[CrossRef](#)]
63. Burrage, C.; Copeland, E.J. Using atom interferometry to detect dark energy. *Contemp. Phys.* **2016**, *57*, 164–176. [[CrossRef](#)]
64. Hamilton, P.; Jaffe, M.; Haslinger, P.; Simmons, Q.; Müller, H.; Khoury, J. Atom-interferometry constraints on dark energy. *Science* **2015**, *349*, 849–851. [[CrossRef](#)]
65. Elder, B.; Khoury, J.; Haslinger, P.; Jaffe, M.; Müller, H.; Hamilton, P. Chameleon dark energy and atom interferometry. *Phys. Rev. D* **2016**, *94*, 044051. [[CrossRef](#)]
66. Burrage, C.; Sakstein, J. Tests of chameleon gravity. *Living Rev. Relat.* **2018**, *21*, 1. [[CrossRef](#)]
67. Wang, J.; Hui, L.; Khoury, J. No-go theorems for generalized chameleon field theories. *Phys. Rev. Lett.* **2012**, *109*, 241301. [[CrossRef](#)] [[PubMed](#)]
68. Khoury, J. Chameleon field theories. *Class. Quantum Gravity* **2013**, *30*, 214004. [[CrossRef](#)]
69. Rebhan, E. *Theoretische Physik: Relativitätstheorie und Kosmologie*; Springer: Berlin/Heidelberg, Germany, 2012.
70. Particle Data Group. Review of Particle Physics. *Prog. Theor. Exp. Phys.* **2020**, *2020*, 083C01. [[CrossRef](#)]
71. Weinberg, S. The cosmological constant problem. *Rev. Mod. Phys.* **1989**, *61*, 1. [[CrossRef](#)]
72. Peebles, P.J.E. Evolution of the cosmological constant. *Nature* **1999**, *398*, 25. [[CrossRef](#)]
73. Kibble, T.W.B. Lorentz invariance and the gravitational field. *J. Math. Phys.* **1961**, *2*, 212. [[CrossRef](#)]
74. Utiyama, R. Invariant theoretical interpretation of interaction. *Phys. Rev.* **1956**, *101*, 1597. [[CrossRef](#)]
75. Sciama, D.W. On the interpretation of the Einstein-Schrodinger unified field theory. *J. Math. Phys.* **1961**, *2*, 472. [[CrossRef](#)]
76. Sciama, D.W. The physical structure of general relativity. *Rev. Mod. Phys.* **1964**, *36*, 463–469. [[CrossRef](#)]
77. Blagojević, M. *Gravitation and Gauge Symmetries*; Series in High-Energy Physics, Cosmology and Gravitation; Foster, B., Grishchuk, L., Kolb, E.W., MacCallum, M.A.H., Perkins, D.H., Schutz, B.F., Eds.; Institute of Physics Publishing: Bristol, UK; Philadelphia, PA, USA, 2001.

78. Davis, A.-C.; Schelpe, C.A.O.; Shaw, D.J. Effect of a chameleon scalar field on the cosmic microwave background. *Phys. Rev. D* **2009**, *80*, 064016. [[CrossRef](#)]
79. Baum, S.; Cantatore, G.; Hoffmann, D.H.H.; Karuza, M.; Semertzidis, Y.K.; Upadhye, A.; Zioutas, K. Detecting solar chameleons through radiation pressure. *Phys. Lett. B* **2014**, *739*, 167–173. [[CrossRef](#)]
80. Dicke, R.H. Mach's principle and invariance under transformation of units. *Phys. Rev.* **1962**, *125*, 2163. [[CrossRef](#)]
81. Ivanov, A.N.; Wellenzohn, M. Standard electroweak interactions in gravitational theory with chameleon field and torsion. *Phys. Rev. D* **2015**, *91*, 085025. [[CrossRef](#)]
82. Tomas, L.H. The radiation field in a fluid in motion. *Q. J. Math.* **1930**, *1*, 239. [[CrossRef](#)]
83. Weinberg, S. Entropy generation and the survival of photo-galaxies in an expanding Universe. *Astrophys. J.* **1971**, *168*, 175. [[CrossRef](#)]
84. Straumann, N. On radiative fluids. *Helv. Phys. Acta* **1976**, *49*, 269.
85. Schweizer, M.A. Transient and transport coefficients for radiative fluids. *Astrophys. J.* **1982**, *258*, 798–811. [[CrossRef](#)]
86. Schweizer, M.A. Relativistic radiative hydrodynamics. *Ann. Phys.* **1988**, *183*, 80. [[CrossRef](#)]
87. Steinhardt, P.J.; Turok, N. Cosmic evolution in a cyclic universe. *Phys. Rev. D* **2002**, *65*, 126003. [[CrossRef](#)]
88. Copeland, E.J.; Mizuno, S.; Shaeri, M. Dynamics of a scalar field in Robertson-Walker spacetimes. *Phys. Rev. D* **2009**, *79*, 103515. [[CrossRef](#)]
89. Foldy, L.L.; Wouthuysen, S.A. On the Dirac theory of spin 1/2 particle and its nonrelativistic limit. *Phys. Rev.* **1950**, *78*, 29. [[CrossRef](#)]
90. Itzykson, C.; Zuber, J.-B. *Quantum Field Theory*; McGraw-Hill Inc.: New York, NY, USA, 1980.
91. Bluhm, R.; Kostelecký, V.A.; Lane, C.D.; Russell, N. Clock-comparison tests of Lorentz and CPT symmetry in space. *Phys. Rev. Lett.* **2002**, *88*, 090801. [[CrossRef](#)] [[PubMed](#)]
92. Kostelecký, V.A.; Mewes, M. Signals for Lorentz violation in electrodynamics. *Phys. Rev. D* **2002**, *66*, 056005. [[CrossRef](#)]
93. Bluhm, R.; Kostelecký, V.A.; Lane, C.D.; Russell, N. Probing Lorentz and CPT violation with space-based experiments. *Phys. Rev. D* **2003**, *68*, 125008. [[CrossRef](#)]
94. Kostelecký, V.A.; Mewes, M. Electrodynamics with Lorentz-violating operators of arbitrary dimension. *Phys. Rev. D* **2009**, *80*, 015020. [[CrossRef](#)]
95. Ding, Y.; Kostelecký, V.A. Lorentz-violating spinor electrodynamics and Penning traps. *Phys. Rev. D* **2016**, *94*, 056008. [[CrossRef](#)]
96. GPS Coordinates of Grenoble. Available online: <https://latitude.to/map/fr/france/cities/grenoble> (accessed on 26 November 2020).
97. Ivanov, A.N.; Wellenzohn, M.; Abele, H. Probing of violation of Lorentz invariance by ultracold neutrons in the Standard Model Extension. *Phys. Lett. B* **2019**, *797*, 134819. [[CrossRef](#)]
98. Jain, B.; Vikram, V.; Sakstein, J. Astrophysical tests of modified gravity: Constraints from distance indicators in the nearby Universe. *Astrophys. J.* **2013**, *779*, 39. [[CrossRef](#)]
99. Hinterbichler, K.; Khoury, J. Symmetron fields: Screening Long-range forces through local symmetry restoration. *Phys. Rev. Lett.* **2010**, *104*, 231301. [[CrossRef](#)]
100. Damour, T.; Polyakov, A.M. The string dilation and a least coupling principle. *Nucl. Phys. B* **1994**, *423*, 532–558.
101. Gasperini, M.; Piazza, F.; Veneziano, G. Quintessence as a runaway dilaton. *Phys. Rev. D* **2001**, *65*, 023508. [[CrossRef](#)]
102. Damour, T.; Piazza, F.; Veneziano, G. Violations of the equivalence principle in a dilaton-runaway scenario. *Phys. Rev. D* **2002**, *66*, 046007. [[CrossRef](#)]
103. Damour, T.; Piazza, F.; Veneziano, G. Runaway dilaton and equivalence principle violations. *Phys. Rev. Lett.* **2002**, *89*, 081601. [[CrossRef](#)] [[PubMed](#)]
104. Fujii, Y. Mass of the dilaton and the cosmological constant. *Prog. Theor. Phys.* **2003**, *110*, 433–439. [[CrossRef](#)]
105. Gasperini, M. Dilaton cosmology and phenomenology. *Lect. Notes Phys.* **2008**, *737*, 787–844.
106. Guendelman, E.I.; Kaganovich, A.B. Absence of the fifth force problem in a model with spontaneously broken dilatation symmetry. *Ann. Phys.* **2008**, *323*, 866–882. [[CrossRef](#)]

107. Brax, P.; van de Bruck, C.; Davis, A.-C. The dilaton and modified gravity. *Phys. Rev. D* **2010**, *82*, 063519. [[CrossRef](#)]
108. Saa, A. Einstein-Cartan theory of gravity revisited. *arXiv* **1993**, arXiv:9309027.

**Publisher's Note:** MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



© 2020 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).