

Cracking Strange Stars by Torsional Oscillations

Francesco Tonelli ^{1,2,†}  and Massimo Mannarelli ^{1,*,†}¹ Laboratori Nazionali del Gran Sasso, Istituto Nazionale di Fisica Nucleare, I-67100 L'Aquila, Italy;

francesco.tonelli@lngs.infn.it

² Dipartimento di Scienze Fisiche e Chimiche, Università di L'Aquila, I-67010 L'Aquila, Italy

* Correspondence: massimo@lngs.infn.it

† These authors contributed equally to this work.

Received: 8 December 2017; Accepted: 29 January 2018; Published: 17 February 2018

Abstract: Strange stars are one of the possible compact stellar objects formed in the core collapse of supernovae. These hypothetical stars are made by deconfined quark matter and are selfbound. In our study, we focus on the torsional oscillations of a non bare strange star, i.e., a strange star with a thin crust made of standard nuclear matter. We construct a theoretical model assuming that the inner parts of the star are in two different phases, namely the color flavour locked phase and the crystalline colour superconducting phase. Since the latter phase is rigid, with a large shear modulus, it corresponds to a first stellar crust. Above this crust a second small crust made by standard nuclear matter is suspended thanks to a strong electromagnetic dipolar moment. We focus on the electromagnetically coupled oscillations of the two stellar crusts. Notably, we find that if a small fraction of the energy of a glitch event like a typical Vela glitch is conveyed in torsional oscillations, the small nuclear crust will likely break. This is due to the fact that in this model the maximum stress, due to torsional oscillations, is likely located near the star surface.

Keywords: neutron stars; star oscillations

1. Introduction

The properties of hadronic matter at densities higher than the nuclear saturation density are under intense theoretical and experimental inspection [1,2]. The high temperature regime is studied in relativistic heavy ion experiments [1], leading to the production and identification of the quark gluon plasma. The low temperature regime ($T \lesssim 1$ MeV) is relevant in the physics of compact stellar objects (CSOs), originating from the collapse of a supernova. The CSOs can be divided into three classes: neutron stars, hybrid stars and strange stars. Neutron stars are the widely studied class of CSOs and are mainly made of nucleons, electrons and muons (to ensure the charge neutrality of the star). If deconfined quarks are present in the core, we are in the presence of the second class of CSOs, the hybrid stars. Strange stars are instead almost completely made of deconfined quark matter [3,4]. The astronomical observations indicate that the mass of CSOs is between $1.2 M_{\odot}$ and $2 M_{\odot}$, where M_{\odot} is the solar mass. The estimated radius is of the order of ten kilometers. Unfortunately using these observed values it is not possible to determine the nature of the CSOs because strange stars and hybrid stars can *masquerade* as standard neutron stars [5].

The existence of strange stars is based on the hypothesis of Bodmer [6] and Witten [7] that standard nuclei are in a metastable state. According to this hypothesis, the real ground state of hadronic matter is a configuration that corresponds to an hypothetical short range free-energy minimum of the strong interaction. This is a *collapsed state* of matter and we can imagine a strange star as a huge collapsed state of hadrons. The interaction that binds the star is the strong interaction, i.e., the star is self bound, with gravity playing a role only for very massive stars. So strange stars have no lower limit on mass and can be arbitrarily small.

Unfortunately, assuming that strange matter is the ground state of hadronic matter does not unambiguously define the properties of the system, even at the densities reachable in the CSOs. The essential point is that Quantum Chromodynamics (QCD) is not in the perturbative regime, so it is not under quantitative control. Therefore, we have to use some approximate scheme. Analysis with various methods indicates that strange matter should be in a superconducting phase [8], i.e., a phase in which quarks form Cooper pairs and break the SU(3) color gauge symmetry. In the inner part of the star, at high density, the color flavour locked (CFL) phase should be favored [9]. In the CFL phase u, d, s quarks of all colors pair coherently in a BCS-like state, maximizing the free-energy gain. However, it is conceivable that CFL is not the unique color superconducting phase realized in strange stars. In fact, at lower densities, the chemical potential of the strange quark becomes comparable with its mass, so coherent pairing cannot happen. For that reason a different superconducting phase can be favored; one possibility is the crystalline color superconducting (CCSC) phase [10,11]. One important feature is that the CCSC phase is characterized by a periodic modulation of the diquark pairing. The periodic modulation of the pairing implies that the CCSC phase is mechanically rigid and it turns out that it has an extremely large shear modulus [11,12], which is a key ingredient for torsional oscillations [13,14]. Indeed, the existence of a phase with a large shear modulus suggests that torsional oscillations of large amplitude can be sustained by this structure. Torsional oscillations of strange stars with a CCSC crust have been first analyzed in [15], while in [16] the coupled oscillations of the CCSC crust and of the ionic crust have been studied. In the present contribution to the proceedings of the CSQCD VI conference we report on the latter study.

2. The Model

2.1. Background Configuration

The star composition we have hypothesized in [16] is shown in Figure 1.

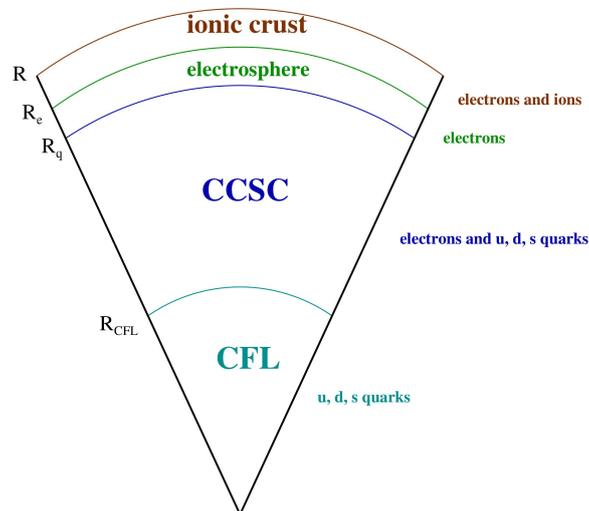


Figure 1. Schematic picture (not in scale) of the stellar model we propose. The mass of the structure is around $1.4 M_{\odot}$, and the total radius is $R \simeq 9.2$ km. The electrosphere has a thickness of few hundred fm, while the ionic crust has a thickness of about 200 m [16].

The inner part of the strange star, called the *quarkosphere*, is populated by deconfined quarks (u, d and s). In the core, at the highest density, the CFL phase is realized and above it, at a radius R_{CFL} , there is a transition to the CCSC phase. The actual radius R_{CFL} is unknown, so we will consider it as a parameter of the model. In the CCSC phase, due to the lack of strange quarks, electrons are present to guarantee the charge neutrality. Above the quarkosphere, there is a very small layer (about a hundred fermi thick) populated only by electrons, forming the so-called *Electrosphere*. This is possible because

quarks are confined within the quark sphere of radius R_q by the strong interaction, but electrons do not feel the strong interaction and can therefore get outside the quark sphere. They are only bound by the electrostatic interaction on a range of hundreds of fm. On the top of this structure there is a small standard ionic crust, which is electromagnetically suspended due to the positive charge present at the surface of the quark sphere [3].

To determine the mass and the radius of our structure, we solve the Tolman Oppenheimer Volkov (TOV) equations that are a generalization of the hydrostatic equilibrium in non rotating spherical metric. For that reason we have to choose an Equation of State (EoS) that can describe our system. Since QCD is not perturbative, it is not possible to determine the actual EoS. The strange star temperature is much lower than the typical scale of QCD, thus it could be considered equal to zero. This means that we can consider our system as a Fermi liquid at zero temperature. To take into account the strong interaction we use a Taylor expansion of the grand potential as a function of the average baryonic chemical potential, μ , as proposed in [5]:

$$\Omega_{QM} = -\frac{3}{4\pi^2} a_4 \mu^4 + \frac{3}{4\pi^2} a_2 \mu^2 + B_{\text{eff}} \tag{1}$$

where a_4 , a_2 and B_{eff} are independent of μ . We use the set of parameters $a_4 = 0.7$, $a_2 = (200 \text{ MeV})^2$ and $B_{\text{eff}} = (165 \text{ MeV})^4$ discussed in [15].

We assume that the previous EoS describes the whole quark sphere and is matched with an EoS valid for the standard ionic crust at R_q , corresponding to a pressure of the quark sphere equal to the pressure corresponding to the neutron drip in the standard ionic crusts. In fact if the density exceeds the neutron drip density, neutrons in the ionic crust are ripped off from the nuclei and fall down into the quark sphere, where they are eventually converted in deconfined light quarks. For the ionic crust we assume that it consists of a Coulomb crystal embedded in a degenerate electron gas, and we use the data reported in [17].

Setting the central density to $\rho_c = 1.5 \times 10^{15} \text{ (g/cm}^3\text{)}$ we obtain a star of $1.4 M_\odot$ with a radius of $R \simeq 9.2 \text{ km}$. The ionic crust is about 200 m thick, and so the radius at which we have the transition between quark matter and standard nuclear matter is $R_q \simeq 9 \text{ km}$.

2.2. Torsional Oscillation

The two considered crusts have a nonvanishing shear modulus and so non radial modes can be excited. We briefly review the equations governing the non radial modes. Defining as \vec{u} the displacement vector, the non radial modes obey to $\nabla \cdot \vec{u} = 0$, with a vanishing radial component, that is $u_r = 0$. Furthermore, assuming that $u = e^{i\sigma t} \zeta(r)$, in the newtonian limit and considering the fluid description, according to [13] we can write the Euler's equation in spherical coordinates as follows

$$\sigma^2 W_i(r) = v_i^2 \left[-\frac{d \log v_i}{dr} \left(\frac{dW_i}{dr} - \frac{W_i}{r} \right) - \frac{1}{r} \frac{d}{dr} \left(r^2 \frac{dW_i}{dr} \right) + \frac{l(l+1)}{r} W_i \right], \tag{2}$$

where $i = 1, 2$ characterizes the two crusts, v_i are the corresponding shear moduli and W is a function of r defined as

$$\zeta_\theta = W \frac{1}{\sin\theta} \frac{\partial Y_{lm}}{\partial \phi}, \quad \zeta_\phi = -W \frac{\partial Y_{lm}}{\partial \theta}. \tag{3}$$

To solve this equation in both the two crusts we have to know the shear modulus of the CCSC crust, of the ionic crust and set the appropriate boundary conditions.

The rigidity of the CCSC crust has been studied in [12]. The shear modulus calculated for the CCSC matter is:

$$\nu \simeq \nu_0 \left(\frac{\Delta}{10 \text{ MeV}} \right)^2 \left(\frac{\mu_q}{400 \text{ MeV}} \right)^2, \tag{4}$$

where Δ is the pairing energy of the condensate, μ_q is the quark chemical potential and the reference value is:

$$v_0 = 2.47 \frac{\text{MeV}}{\text{fm}^3}. \tag{5}$$

This is the value that we will use in our study, because we expect that $\Delta \sim 10$ MeV and the quark chemical potential is roughly constant within the CCSC phase, see [15] for a discussion. The actual value of the CCSC shear modulus can be different from the reference value also because the procedure for its evaluation relies on a number of approximations, see [11]. However, the most important aspect for our discussion is that the shear modulus of the CCSC matter is much larger than the shear modulus of standard nuclear matter. Note also that the effect of the pairing gap in the EoS is effectively included in the a_2 coefficient of Equation (1), see [5] and the discussion in [15].

The shear modulus of the ionic crust strongly depends on the particular crystalline structure of the crust and on the plane of application. Actually, we do not know the crystalline structure of the ionic crust, but we can calculate an effective shear modulus [18] as:

$$v_{\text{eff}} = c \frac{n_N(r)(Z(r)e)^2}{a_N(r)}, \tag{6}$$

where $n_N(r)$ is the density of nucleons as a function of the radial coordinate r , $Z(r)$ is the number of protons in the nuclei, and $a_N(r) = (3/(4\pi n_N))^{(1/3)}$ is the average inter-ion spacing. The constant c has been evaluated in [18,19] and it is $c \simeq 0.1$.

The boundary conditions that we impose are the followings

$$\left(\frac{dW_1}{dr} - \frac{W_1}{r} \right) \Big|_{R=R_{\text{CFL}}} = 0 \tag{7}$$

$$v_1 \left(\frac{dW_1}{dr} - \frac{W_1}{r} \right) \Big|_{R=R_q} = v_2 \left(\frac{dW_2}{dr} - \frac{W_2}{r} \right) \Big|_{R=R_q} \tag{8}$$

$$W_1(R_q) = W_2(R_q) \tag{9}$$

$$\left(\frac{dW_2}{dr} - \frac{W_2}{r} \right) \Big|_{R=R_{\text{ocean}}} = 0, \tag{10}$$

where $R_{\text{ocean}} \simeq 9.15$ km and the ocean is the region in which the density is less than 10^7 g/cm³. The first, the second and the last equations are no-traction conditions, while the third one is a no-slip condition. The no-traction condition means that there is no force acting between two adjacent layers. The no-slip condition means that the displacement at the interface of two layers is the same. See [16] for more details on these boundary conditions.

3. Results

In our study we focus on the $l = 1$ modes that we call ${}_1t_n$, where n indicates the number of nodes, corresponding to a twist of the two crusts. We numerically solve the equation for the non radial modes, considering the density inside the CCSC crust as a constant (the estimated error made with this approximation is less than 10%) and a realistic radial density dependence in the ionic crust. Since the transition between the CFL and CCSC phase depends on the unknown pressure difference between the two phases, we define $R_{\text{CFL}} = aR_q$ with a a parameter that varies between 0 and 1, and we study the problem varying a .

In Figure 2 we show the obtained frequencies of the ${}_1t_1, {}_1t_2, {}_1t_3$ modes as a function of a . As we can easily see, the typical frequencies are of the order of 10 kHz and we can identify two different behaviors. There is one kind of oscillations, dependent on the parameter a , associated to a non radial coupled crusts oscillation (CCSO), in which both crusts are sensibly displaced. A second kind of oscillations is almost independent of a and is associated to a sensible displacement of only the ionic

crust (ICO). To show the amplitude of the oscillation we assume that the energy of the order of the one released in a Vela-like glitch ($E \sim 5 \times 10^{42}$ erg) is conveyed to the ${}_1t_n$ modes.

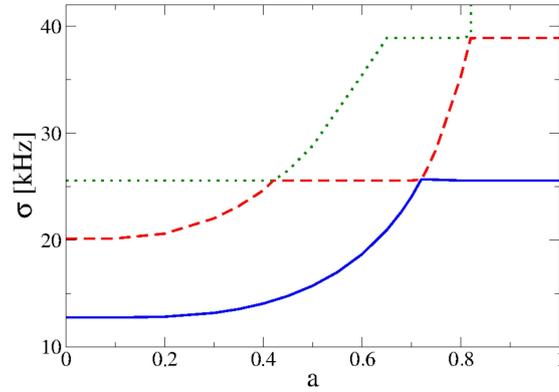


Figure 2. Frequencies of the modes ${}_1t_1$ (solid blue), ${}_1t_2$ (dashed red) and ${}_1t_3$ (dotted green) as a function of $a = R_q/R_{CFL}$. As it is possible to see, we have two kind of features. There are oscillations whose frequencies depends on a and oscillations with frequencies independent of a [16].

In Figure 3 we show the amplitude of ${}_1t_1$ oscillations in two different cases. In the left panel we choose $a = 0.4$ while for the right panel we choose $a = 0.8$. For $a = 0.4$ the energy is divided by the CCSC crust and the ionic crust, corresponding to a CCSO-type mode. The maximum amplitude of the oscillation is of the order of 20 cm. In the right panel we consider the $a = 0.8$ case. In this case all the energy is conveyed on the ionic crust, corresponding to a ICO-type mode, and the amplitude of the oscillation is of the order of the km.

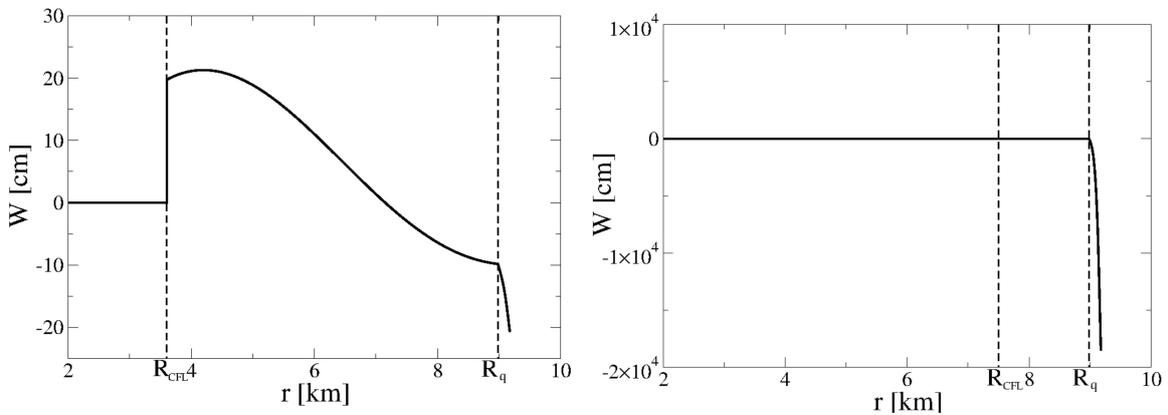


Figure 3. Radial dependence of the amplitude of the ${}_1t_1$ mode oscillation inside our strange star model. **Left:** Oscillation of the coupled crusts oscillation (CCSO)-type (see text) obtained choosing $a = 0.8$. The amplitude of the oscillation is of the order of 20 cm. **Right:** Oscillation of the ISO-type (see text) obtained choosing $a = 0.4$. In the nuclear crust the amplitude can reach values of the order of few km [16].

To better understand our results we study the deformation of the solid crusts due to the torsional oscillation. A measure of the deformation is the shear strain. For our study, due to symmetry of the problem, we restrict our analysis to the radial component of the strain, that is defined as:

$$|s| = \left| \frac{dW}{dr} - \frac{W}{r} \right|. \tag{11}$$

For the shear strain in the case $a = 0.8$ we obtain a maximum near the surface of the star. This is a particularly relevant result because in previous analysis, such as [13], the maximum strain is far inside the inner crust and it is impossible to break the crust. For the ionic crust, despite the high uncertainties, the maximum strain should be between 10^{-4} and 10^{-2} [20] but in perfect crystals values of the order of 10^{-1} could be appropriate [21]. Since in our model we obtain maximum strains larger than these values, of the order of unity, this probably means that in our model it is possible to break the ionic crust during a glitch or in any other event releasing a comparable amount of energy.

4. Conclusions

We have considered a model of a nonbare strange star comprising a quark sphere of superconducting quark matter surmounted by a standard nuclear matter crust [16]. The quark sphere is in two different phases: the CFL phase and the rigid CCSC phase. The CCSC crust and the ionic crust are separated by an electron layer a few hundred fm thick. We have solved the TOV equations using a simple parameterization of the EoS of quark matter in function of the baryon chemical potential matched with a realistic EoS for the description of the ionic crust.

Both the CCSC and the ionic crusts are rigid, so electromagnetically coupled torsional oscillations are possible. We have found two types of oscillations, the first involves the two crusts with comparable amplitude (CCSO-type), and the second confined in the ionic crust (ICO-type). If the CCSC is thinner than around 2 km, ICOs are the only relevant oscillations.

We have studied in detail the $l = 1$ torsional modes, obtaining frequencies of the order of 10 kHz. These modes correspond to oscillatory twists of the crust and do not conserve angular momentum. For that reason we have assumed that they are triggered by a pulsar glitch and following this idea we have assumed that the energy of a Vela-like glitch is conveyed to the strange star crust. If the CCSC crust is sufficiently thin, then ISOs are triggered and it is possible to break the ionic crust. Indeed, computing the strain as a function of the radial coordinate, we find that its maximum is located a few tens of meters below the stellar surface and is of the order of unity. This probably means that within our model it is possible to break the ionic crust during a glitch or any other stellar event conveying a comparable amount of energy to the torsional oscillations.

Possible observables related to torsional oscillations are giant Gamma Ray Burst (see [22]) and Quasi Periodic Oscillations (see [23–27]).

Author Contributions: These authors contributed equally to this work.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Shuryak, E. Strongly coupled quark-gluon plasma in heavy ion collisions. *Rev. Mod. Phys.* **2017**, *89*, 035001.
2. Alford, M.G.; Schmitt, A.; Rajagopal, K.; Schafer, T. Color superconductivity in dense quark matter. *Rev. Mod. Phys.* **2008**, *80*, 1455–1515.
3. Alcock, C.; Farhi, E.; Olinto, A. Strange stars. *Astrophys. J.* **1986**, *310*, 261.
4. Haensel, P.; Zdunik, J.L.; Schaeffer, R. Strange quark stars. *Astron. Astrophys.* **1986**, *160*, 121–128.
5. Alford, M.; Braby, M.; Paris, M.W.; Reddy, S. Hybrid stars that masquerade as neutron stars. *Astrophys. J.* **2005**, *629*, 969.
6. Bodmer, A. Collapsed nuclei. *Phys. Rev. D* **1971**, *4*, 1601–1606.
7. Witten, E. Cosmic separation of phases. *Phys. Rev. D* **1984**, *30*, 272–285.
8. Rajagopal, K.; Wilczek, F. The Condensed Matter Physics of QCD. In *At the frontier of particle physics. Handbook of QCD*; Shifman, M., Ed.; World Scientific: Hackensack, NJ, USA, 2000.
9. Alford, M.; Rajagopal, K.; Wilczek, F. The Minimal CFL-Nuclear Interface. *Nucl. Phys. B* **2000**, *537*, 443–458.
10. Alford, M.; Bowers, J.A.; Rajagopal, K. Crystalline Color Superconductivity. *Phys. Rev. D* **2001**, *63*, 074016.
11. Anglani, R.; Casalbuoni, R.; Ciminale, M.; Ippolito, N.; Gatto, R.; Mannarelli, M.; Ruggieri, M. Crystalline color superconductors. *Rev. Mod. Phys.* **2014**, *86*, 509–561.

12. Mannarelli, M.; Rajagopal, K.; Sharma, R. The rigidity of crystalline color superconducting quark matter. *Phys. Rev. D* **2007**, *76*, 074026.
13. McDermott, P.N.; van Horne, H.M.; Hansen, C.J. Nonradial oscillations of neutron stars. *Astrophys. J.* **1988**, *325*, 725–748.
14. Samuelsson, L.; Andersson, N. Neutron Star Asteroseismology. Axial Crust Oscillations in the Cowling Approximation. *Mon. Not. R. Astron. Soc.* **2007**, *374*, 256–268.
15. Mannarelli, M.; Pagliaroli, G.; Parisi, A.; Pilo, L. Electromagnetic signals from bare strange stars. *Phys. Rev. D* **2014**, *89*, 103014.
16. Mannarelli, M.; Pagliaroli, G.; Parisi, A.; Pilo, L.; Tonelli, F. Torsional oscillation of non bare strange stars. *Astrophys. J.* **2015**, *815*, 81.
17. Haensel, P.; Pichon, B. Experimental nuclear masses and the ground state of cold dense matter. *Astron. Astrophys.* **1994**, *283*, 313–318.
18. Strohmayer, T.; van Horn, H.M.; Ogata, S.; Iyetomi, H.; Ichimaru, S. The shear modulus of the neutron star crust and nonradial oscillations of neutron stars. *Astrophys. J.* **1991**, *375*, 679–686.
19. Hoffman, K.; Heil, J. Mechanical properties of non-accreting neutron star crusts. *Mon. Not. R. Astron. Soc.* **2012**, *426*, 2404–2412.
20. Horowitz, C.J.; Kadau, K. The Breaking Strain of Neutron Star Crust and Gravitational Waves. *Phys. Rev. Lett.* **2009**, *102*, 191102.
21. Kittel, C. *Introduction to Solid State Physics*; Wiley: New York, NY, USA, 1976.
22. Thompson, C.; Duncan, R.C. The Giant Flare of 1998 August 27 from SGR 1900+14: II. Radiative Mechanism and Physical Constraints on the Source. *Astrophys. J.* **2001**, *561*, 980.
23. Israel, G.; Belloni, T.; Stella, L.; Rephaeli, Y.; Gruber, D.E.; Casella, P.; Dall’Osso, S.; Rea, N.; Persic, M.; Rothschild, R.E. Discovery of Rapid X-ray Oscillations in the Tail of the SGR 1806-20 Hyperflare. *Astrophys. J. Lett.* **2005**, *628*, L53–L56.
24. Watts, A.L.; Strohmayer, T.E. Detection with RHESSI of high frequency X-ray oscillations in the tail of the 2004 hyperflare from SGR 1806-20. *Astrophys. J. Lett.* **2005**, *632*, L111.
25. Glampedakis, K.; Samuelson, L.; Anderson, N. Elastic or magnetic? A toy model for global magnetar oscillations with implications for QPOs during flares. *Mon. Not. R. Astron. Soc. Lett.* **2006**, *371*, L74–L77.
26. Watts, A.L.; Reddy, S. Magnetar oscillations pose challenges for strange stars. *Mon. Not. R. Astron. Soc.* **2007**, *379*, L63–L66.
27. Watts, A.L. Neutron starquakes and the dynamic crust. *arXiv* **2011**, arXiv:1111.0514.



© 2018 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).