universe

MDPI

Article

Lemaître Class Dark Energy Model for Relaxing Cosmological Constant

Irina Dymnikova [1,2,*], Anna Dobosz [2,†] and Bożena Sołtysek [2,†]

1 A.F. Ioffe Physico-Technical Institute of the Russian Academy of Sciences, Polytekhnicheskaja 26, 194021 St. Petersburg, Russia
2 Department of Mathematics and Computer Science, University of Warmia and Mazury, Słoneczna 54, 10-710 Olsztyn, Poland; dobosz@uwm.edu.pl (A.D.); bozena@matman.uwm.edu.pl (B.S.)
* Correspondence: irina@uwm.edu.pl; Tel.: +7-812-292-7912
† These authors contributed equally to this work.

Academic Editors: Manuel Krämer, Mariusz P. Dąbrowski and Vincenzo Salzano
Received: 20 January 2017; Accepted: 2 May 2017; Published: 4 May 2017

Abstract: Cosmological constant corresponds to the maximally symmetric cosmological term with the equation of state $p = -\rho$. Introducing a cosmological term with the reduced symmetry, $p_r = -\rho$ in the spherically symmetric case, makes cosmological constant intrinsically variable component of a variable cosmological term which describes time-dependent and spatially inhomogeneous vacuum dark energy. Relaxation of the cosmological constant from the big initial value to the presently observed value can be then described in general setting by the spherically symmetric cosmology of the Lemaître class. We outline in detail the cosmological model with the global structure of the de Sitter spacetime distinguished by the holographic principle as the only stable product of quantum evaporation of the cosmological horizon entirely determined by its quantum dynamics. Density of the vacuum dark energy is presented by semiclassical description of vacuum polarization in the spherically symmetric gravitational field, and its initial value is chosen at the GUT scale. The final non-zero value of the cosmological constant is tightly fixed by the quantum dynamics of evaporation and appears in the reasonable agreement with its observational value.

Keywords: dark energy; de Sitter vacuum; spacetime symmetry; relaxing cosmological constant

1. Introduction

The biggest challenge may be the cosmological vacuum energy.
John Ellis, 2003

One of the most challenging issues of modern cosmology is to describe the positive late time acceleration through a single self-consistent theoretical scheme.
A. Aviles, A. Bravetti, S. Capozziello and O. Luongo, 2014

According to observational data, the present state of our universe is dominated by a dark energy with the equation of state $p = w\rho$; the parameter w satisfies the condition $w < -1/3$ which guarantees accelerated expansion.The value $w = -1$ represents the cosmological constant λ corresponding to the vacuum density ρ_{vac} by $\lambda = 8\pi G\rho_{vac}$. CMB (Cosmic Microwave Background) measurements [1] combined with BAO (Baryon Acoustic Oscillations) data [2] and SNe (SuperNovae) Ia data [3] give the value $w = -1.06 \pm 0.06$ at 68% CL [2]. On the other hand, much bigger value of a cosmological constant is required to trigger the early inflationary stage. The fact that it must be constant creates the first aspect of *the problem of the cosmological constant*. Second aspect concerns the difference, by 123 orders of magnitude, between its observational value and the value predicted by the quantum field theory, which leads to the fine-tuning problem (see, e.g., [4]).

Various models have been developed with a dark energy of non-vacuum origin which mimics cosmological constant when necessary (for a review [5,6]), and confronted with cosmography tests [7,8]. Model-independent evidence for dark energy evolution from BAO data prefers theories in which cosmological constant Λ relaxes from a large initial value [9].

In this paper we review our results on relaxing cosmological constant obtained in the frame of the model-independent self-consistent approach which allows to make cosmological constant intrinsically variable. In this approach a vacuum dark energy is presented as a single physical entity on the basis of the algebraic classification of stress-energy tensors and related to it spacetime symmetry (detailed description in [10]).

The Einstein cosmological term $\Lambda\delta^i_k$ corresponds to the maximally symmetric de Sitter vacuum $T^i_k = \rho_{vac}\delta^i_k$ with $\rho_{vac} = const$ by virtue of the contracted Bianchi identities $T^i_{k;i} = 0$. The model-independent way to make Λ variable, based on the algebraic classification of stress-energy tensors, consists in reducing the maximal symmetry of the Einstein cosmological term $T^i_k = \rho_{vac}\delta^i_k$ (with $\rho_{vac} = const$), while keeping its vacuum identity, i.e., the Lorentz-invariance in a certain direction(s). A stress-energy tensor T^i_k with a reduced symmetry describes a vacuum dark fluid defined by the equation of state $p_\alpha = -\rho$ valid only in the distinguished direction(s) [11,12], which makes it intrinsically anisotropic. The behavior of pressure(s) p_β with $\beta \neq \alpha$ is determined from $T^i_{k;i} = 0$. This generalizes the description of the cosmological constant Λ to the density component $\Lambda^t_t = 8\pi G\rho_{vac} \neq const$ of the variable cosmological term $\Lambda^i_k = 8\pi G T^i_k$ [13] which makes a vacuum energy intrinsically dynamical, i.e., time evolving and spatially inhomogeneous.

In the spherically symmetric case vacuum dark fluid is described by stress-energy tensors specified by $T^t_t = T^r_r$ ($p_r = -\rho$), which are intrinsically anisotropic: $p_\perp = -\rho - r\rho'/2$ by virtue of $T^i_{k;i} = 0$ where $p_\perp(r) = -T^\theta_\theta = -T^\phi_\phi$ is the transversal pressure [13,14]. For the case of non-negative energy density for any observer, they generate regular spacetimes with the obligatory de Sitter center $T^i_k = \Lambda\delta^i_k$ at $r = 0$ [14,15]. In the case of two vacuum scales, they connect smoothly $T^i_k = \Lambda\delta^i_k$ at $r = 0$ with $T^i_k = \lambda\delta^i_k$ at $r \to \infty$, with $\lambda < \Lambda$ [11,12], and describe intrinsic relaxation of a large initial $\Lambda = 8\pi G\rho_\Lambda$ towards a small $\lambda = 8\pi G\rho_\lambda$. A mechanism for relaxation of cosmological constant to a needed non-zero value is provided by spacetime symmetry (detailed explanation in [10]).

The key point is that the cosmological models with vacuum dark energy presented by the variable cosmological term must be anisotropic, since in the isotropic FLRW cosmology ρ_{vac} must be constant by virtue of $T^i_{k;i} = 0$. In the spherically symmetric case the intrinsic relaxation of the cosmological constant is described in the frame of the most general Lemaître class models with anisotropic perfect fluid. The Lemaître class includes the FLRW model as the particular case of the full isotropy and homogeneity and, in consequence, of $\lambda = 8\pi G\rho_{vac}$ = const. Cosmological Lemaître models with vacuum dark energy asymptotically approach the isotropic FLRW models at the earliest and present (eventually also intermediate) stages when the symmetry of a source term is restored to the de Sitter vacuum [11,16]. In the case $\Omega = 1$ de Sitter center represents in the Lemaître coordinates (R, τ) a non-singular non-simultaneous de Sitter bang from the surface $r(R, \tau) = 0$ [11,16].

Regular spacetimes with the de Sitter center contain a special class of spacetimes with the same global structure as for the de Sitter spacetime. Such a spacetime is distinguished by the holographic principle as the only stable product of quantum evaporation of the cosmological horizon, with basic physical parameters tightly fixed by quantum dynamics of the cosmological horizon [17]. Particular model of this special class [18] we outline here in more detail.

This paper is organized as follows. In Section 2 we show how the holographic principle picks out a proper spacetime including the values of its basic parameters. Section 3 is devoted to the detailed description of the related Lemaître dark energy model with the parameters tightly fixed by dynamics of the cosmological horizon, and in Section 4 we summarize and discuss the results.

2. Holographic Principle and Triple-Horizon Spacetimes

The holographic principle, formulated originally as the requirement to constrain the number of independent quantum degrees of freedom contained in a spatial region by its surface area [19], leads to the conjecture that a physical system can be entirely determined by the data stored on its boundary which is frequently also referred to as the holographic principle [20]. Some basic information on application of the holographic principle in quantum gravity can be found in [17]. Here we show how the holographic principle singles out the special class of one-horizon spacetimes, parametrized by one function (a density profile), in which dynamical evolution is entirely determined by the quantum evaporation of the cosmological horizon and proceeds towards triple-horizon spacetimes with the basic parameters tightly fixed by the quantum dynamics of the horizon [17].

For the source terms which satisfy $T_t^t = T_r^r$, the weak energy condition (which requires non-negative density for any observer preferable for cosmological models) leads to a monotonically decreasing density profile $T_t^t = \rho(r)$ [15]. In the de Sitter space with the background vacuum density $\rho_\lambda = (8\pi G)^{-1}\lambda$ we can introduce $T_t^t(r) = \rho(r) + \rho_\lambda$, where $\rho(r)$ is a dynamical vacuum density decreasing from $\rho_\Lambda = (8\pi G)^{-1}\Lambda$ at $r = 0$ to zero at infinity. The metric has the form [11]

$$ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2; \;\; g(r) = 1 - \frac{2G\mathcal{M}(r)}{r} - \frac{\lambda r^2}{3}; \quad \mathcal{M}(r) = 4\pi \int_0^r \rho(x)x^2 dx, \qquad (1)$$

and is asymptotically de Sitter with λ as $r \to \infty$ and with $(\Lambda + \lambda)$ as $r \to 0$.

Geometry has three basic length scales, the gravitational radius $r_g = 2GM$ $(M = 4\pi \int_0^\infty \rho r^2 dr)$, the de Sitter radius related to the de Sitter interior, $r_\Lambda = \sqrt{3/\Lambda}$, and de Sitter radius related to the background vacuum, $r_\lambda = \sqrt{3/\lambda}$. The characteristic parameter relating the dynamical vacuum density at the center ρ_Λ with the background vacuum density ρ_λ reads

$$q = r_\lambda / r_\Lambda = \sqrt{\Lambda/\lambda} = \sqrt{\rho_\Lambda/\rho_\lambda}. \qquad (2)$$

In the case of two vacuum scales the spacetime can have at most three horizons defined by $g(r) = 0$ [11]: the internal horizon r_a, the event horizon $r_b > r_a$ of a regular cosmological black (white) hole whose mass is restricted within $M_{cr1} < M < M_{cr2}$, and the cosmological horizon $r_c > r_b$. The values M_{cr1} and M_{cr2} correspond to the double-horizon states, $r_a = r_b$ and $r_b = r_c$, respectively, and depend on the parameter q (more detail in [10]). Dependence of horizons radii on the mass is shown in Figure 1 left for the case $q = 50$. In Figure 1 right we plotted the double horizon $r_b = r_a$ (solid line denoted as r_b) and the cosmological horizon r_c (dashed line) dependently on the parameter q. For the certain values of q_{cr} and M_{cr}, three horizons coincide at the triple horizon r_t (Figure 1 right, the point where dashed and solid line meet), defined by three algebraic equations $g(r_t) = 0$, $g'(r_t) = 0$ and $g''(r_t) = 0$ [17,21].

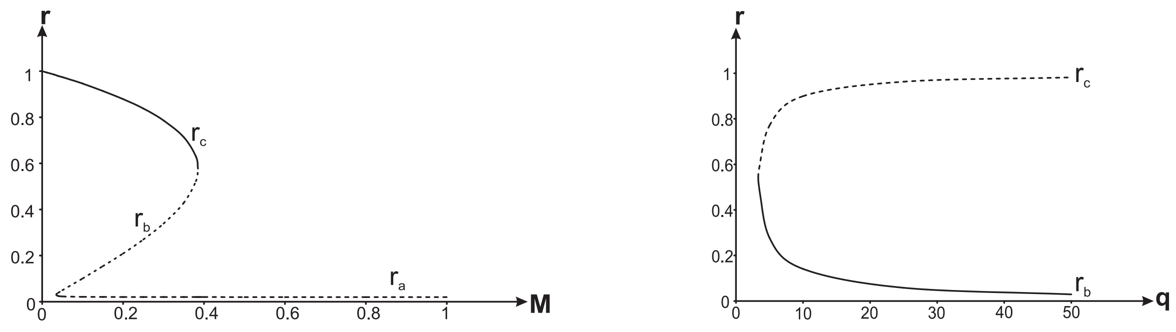

Figure 1. Horizons r_a, r_b and r_c for $q = 50$ (**left**); double horizon $r_b = r_a$ and cosmological horizon r_c coinciding at the triple horizon (**right**) at $q = q_{cr}$. Distances are normalized to r_λ.

The triple-horizon spacetime arises as a result of quantum evolution of a one-horizon spacetime described by the metric function $g(r)$ which has the inflection point r_i defined by $g'(r_i) = 0$, $g''(r_i) = 0$, and the mass parameter $M > M_{cr}$ [17]. This metric is shown in Figure 2(left) [21]. Evolution is governed by quantum evaporation of the cosmological horizon of spacetime with the inflection point and goes toward the triple-horizon spacetime with $q = q_{cr}$; $M = M_{cr}$ shown in Figure 2(right).

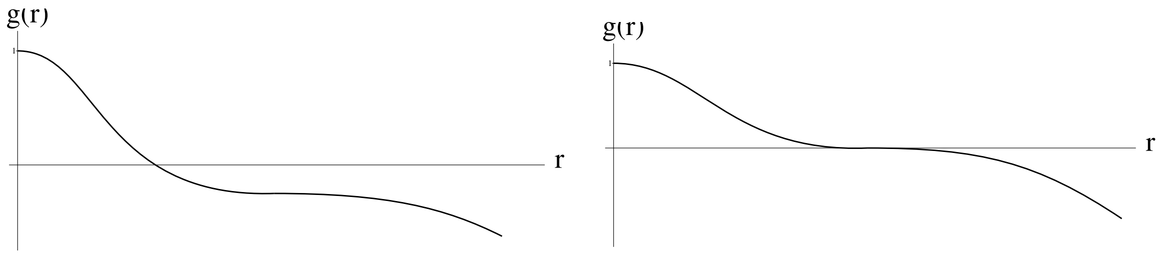

Figure 2. Metric function with the inflection point (**left**) and with the triple-horizon (**right**).

The Gibbons-Hawking temperature on the horizon $r = r_h$ [22] and its specific heat [23] are given by

$$kT_h = \frac{\hbar}{4\pi c}|g'(r_h)|; \;\; C_h = \frac{2\pi r_h}{g'(r_h) + g''(r_h) r_h}. \tag{3}$$

In the course of evaporation the cosmological horizon (Figure 2 left) moves outwards unless the system achieves the triple-horizon state $r_h = r_t$ (Figure 2 right) corresponding to $M = M_{cr}$ [17]. Specific heat of this horizon is always positive and tends to infinity at the triple horizon [21], so that the triple-horizon spacetime is the thermodynamically stable final product of evaporation of the cosmological horizon. Evaporation stops completely at $T_h = 0$ and $C_h \to \infty$. Three algebraic equations which specify the triple-horizon state ($g(r_t) = 0$; $g'(r_t) = 0$; $g''(r_t) = 0$) define uniquely the basic parameters M_{cr}, r_t, and $q_{cr} = \rho_\Lambda / \rho_\lambda$ which gives the tightly fixed non-zero present value of a vacuum dark energy density ρ_λ for the given value ρ_Λ [17].

We see that the evolution of a one-horizon spacetime with the inflection point (shown in Figure 2 left) is governed by the quantum dynamics of surrounding it surface (cosmological horizon) and goes towards the triple-horizon spacetime, whose basic physical parameters, M_{cr}, r_t and q_{cr}, are entirely defined by the data stored on its boundary (triple-horizon surface) - in agreement with the basic sense of the holographic principle [17].

Applying description of the vacuum density by the density profile [14]

$$\rho(r) = \rho_\Lambda e^{-r^3/r_\Lambda^2 r_g}, \tag{4}$$

obtained in the semiclassical model for the vacuum polarization in the spherically symmetric gravitational field [15], we obtain [17]

$$M_{cr} = 2.33 \times 10^{56}\text{g}; \; q_{cr}^2 = 1.37 \times 10^{107}; \; r_t = 5.4 \times 10^{28}\text{cm}. \tag{5}$$

To evaluate the vacuum dark energy density from $q_{cr}^2 = \rho_\Lambda / \rho_\lambda$, we adopt $\rho_\Lambda = \rho_{GUT}$. The Grand Unification scale is estimated as $M_{GUT} \sim 10^{15} - 10^{16}$ GeV. This gives the value of ρ_λ within the range 1.7×10^{-30} g $\cdot$ cm^{-3} – 1.7×10^{-26} g $\cdot$ cm^{-3}, respectively. The observational value $\rho_{\lambda\ (obs)} \simeq 6.45 \times 10^{-30}$ g $\cdot$ cm^{-3} [24] corresponds, in the considered context, to $M_{GUT} \simeq 1.4 \times 10^{15}$ GeV. This gives $\rho_{GUT} = 8.8 \times 10^{77}$ g $\cdot$ cm^{-3} and $r_\Lambda = 1.8 \times 10^{-25}$ cm. For this scale q_{cr}^2 gives the value of the present vacuum density ρ_λ in agreement with its observational value.

3. Lemaître Model for Relaxing Cosmological Constant

3.1. Basic Equations

A Lemaître cosmology is described by the metric

$$ds^2 = d\tau^2 - e^{2\nu(R,\tau)}dR^2 - r^2(R,\tau)d\Omega^2. \tag{6}$$

Here coordinates R and τ are the Lagrange (comoving) coordinates. The Einstein equations read [25]

$$8\pi G p_r = \frac{1}{r^2}\left(e^{-2\nu}r'^2 - 2r\ddot{r} - \dot{r}^2 - 1\right), \tag{7}$$

$$8\pi G p_\perp = \frac{e^{-2\nu}}{r}(r'' - r'\nu') - \frac{\dot{r}\dot{\nu}}{r} - \ddot{\nu} - \dot{\nu}^2 - \frac{\ddot{r}}{r}, \tag{8}$$

$$8\pi G\rho = -\frac{e^{-2\nu}}{r^2}\left(2rr'' + r'^2 - 2rr'\nu'\right) + \frac{1}{r^2}\left(2r\dot{r}\dot{\nu} + \dot{r}^2 + 1\right), \tag{9}$$

$$8\pi G T_t^r = \frac{e^{-2\nu}}{r}\left(2\dot{r}' - 2r'\dot{\nu}\right) = 0. \tag{10}$$

The component T_t^r vanishes in the comoving reference frame, and the Equation (10) gives [25]

$$e^{2\nu} = \frac{r'^2}{1 + f(R)}. \tag{11}$$

The function $f(R)$ is an arbitrary integration function. Applying (11) in the Equation (7) results in the equation of motion [11]

$$\dot{r}^2 + 2r\ddot{r} + 8\pi G p_r r^2 = f(R). \tag{12}$$

For the vacuum dark energy specified by $p_r = -\rho$, the first integration in (12) yields

$$\dot{r}^2 = \frac{2G\mathcal{M}(r)}{r} + f(R) + \frac{F(R)}{r}. \tag{13}$$

A second integration function $F(R)$ should be chosen equal to zero for models regular at $r = 0$, since $\mathcal{M}(r) \to 0$ as r^3 for $r \to 0$ where $\rho(r) \to \rho_\Lambda < \infty$ [11,15]. The second integration in (12) yields

$$\tau - \tau_0(R) = \int \frac{dr}{\sqrt{2G\mathcal{M}(r)/r + f(R)}}. \tag{14}$$

The third integration function $\tau_0(R)$ is called "the bang-time function" [26].

3.2. Model with the Relaxing Cosmological Constant

For the case of vacuum dark fluid specified by $T_t^t = T_r^r (p_r = -\rho)$, the basic properties of the Lemaître class models are determined by the basic properties of the function $g(r)$ in (1) via transition to the geodesic coordinates (R,τ), where R is the congruence parameter of the family of radial timelike geodesics and τ is the proper time. The matrix of the transition from the mapping $[r,t]$ to the mapping $[R,\tau]$ reads $\dot{t} = \frac{E(R)}{g(r)}; r' = \sqrt{E^2(R) - g(r)};\ \dot{r} = \pm r';\ t' = \pm\frac{E^2(R) - g(r)}{E(R)g(r)}$, where dots and primes denote ∂_τ and ∂_R. The resulting metric [16]

$$ds^2 = d\tau^2 - \frac{[E^2(R) - g(r(R,\tau))]}{E^2(R)}dR^2 - r^2(R,\tau)d\Omega^2 \tag{15}$$

corresponds to (6) with $f(R) = E^2(R) - 1$, and $E^2(R) - g(r) = [r'(R,\tau)]^2 = [\dot{r}(R,\tau)]^2$.

For small values of r the solution (14) reduces to

$$\tau - \tau_0(R) = \int \frac{dr}{\sqrt{r^2/r_\Lambda^2 + f(R)}}. \tag{16}$$

For expanding models $\dot{r} = r'$ and hence r is a function of $(R + \tau)$. We can thus choose $\tau_0(R) = -R$.

For the case $f(R) = 0$, preferred by observational data ($\Omega = 1$), we have $E^2 = 1$, and evolution starts from the time-like regular surface $r(R, \tau) = 0$. At approaching this surface the Equation (16) gives the expansion law

$$r = r_\Lambda e^{(\tau+R)/r_\Lambda}; \;\; e^{2\nu} = (r')^2 = r^2/r_\Lambda^2. \tag{17}$$

With using the coordinate transformation $q = e^{R/r_\Lambda}$, the metric (6) reduces to the standard FLRW form

$$ds^2 = d\tau^2 - r_\Lambda^2 e^{2\tau/r_\Lambda}\left(dq^2 + q^2 d\Omega^2\right), \tag{18}$$

with the de Sitter scale factor (17), and describes a non-singular non-simultaneous de Sitter bang from the surface $r(\tau + R \to -\infty) = 0$ [11]. Further evolution is determined by dynamics of pressures $p_r = -\rho - \rho_\lambda$; $p_\perp = -\rho - \rho_\lambda - r\rho'/2$ [11]. The weak energy condition is satisfied and leads to monotonically decreasing density [15]. As a result the radial pressure $p_r = -\rho$ is monotonically increasing. Transversal pressure evolves from the value $p_\perp = -\rho_\Lambda - \rho_\lambda$ at the inflation to the final value $p_\perp = -\rho_\lambda$, through one maximum somewhere in between ([16] and references therein). Typical behavior of pressures in shown in Figure 3. The variable $(\tau + R)$ is normalized to the GUT time $t_{GUT} = r_\Lambda/c \simeq 0.6 \times 10^{-35}$s.

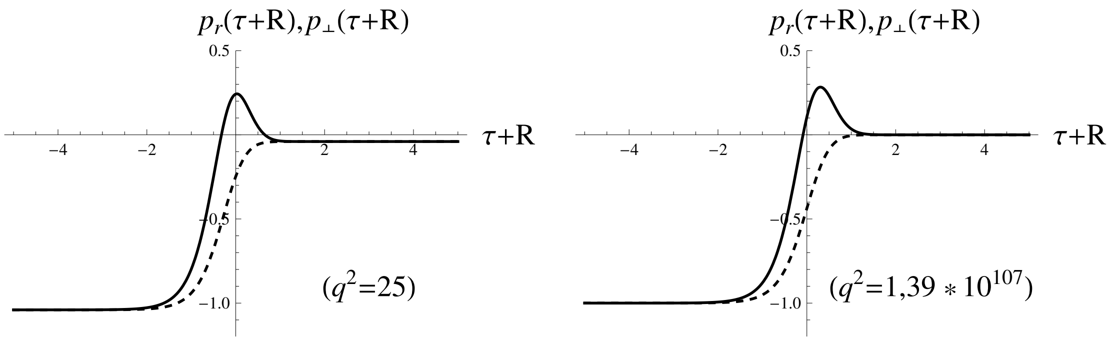

Figure 3. Typical behavior of pressures. The radial pressure p_r is plotted with the dashed line and the transversal pressure $p_\perp$ is plotted by the solid line.

The first inflationary stage is followed by the essentially anisotropic Kasner-like stage. It is easily to see from the metric (15) that typical for this stage is the expansion in the transversal direction with $\partial_\tau r > 0$ and shrinking in the radial direction where $\partial_R |g_{RR}| < 0$ until $dg(r)/dr < 0$. For $E^2 = 1$ the metric during this stage ($r_\Lambda \ll r \ll r_\lambda$) takes the form [11]

$$ds^2 = d\tau^2 - (\tau + R)^{-2/3} N(R) dR^2 - D(\tau + R)^{4/3} d\Omega^2, \tag{19}$$

where $N(R)$ is a smooth regular function and D is a constant, with $N(R)$ and D dependent on the choice of the particular model for the vacuum density in (1).

The line element (6) can be written in the form with two scale factors, $r(\tau, R)$ and $b(\tau, R) \equiv e^{\nu(R,\tau)}$ in accordance with (11). This results in the metric

$$ds^2 = d\tau^2 - b^2(\tau, R) dR^2 - r^2(\tau, R) d\Omega^2, \tag{20}$$

The evolution of two scale factors in the course of the evolution is shown in Figure 4 (Figure 4a, the upper curve for r(t, R), the lower for b(t, R)), where we also show the behavior of their derivatives at the early stage (Figure 4b) [18]. Figure 4 is plotted with the density profile (4) and the model parameters (5). Distances are normalized to $r_* = (r_\Lambda^2 r_g)^{1/3} = 1.26 \times 10^{-7}$ cm, with $r_g = 2GM_t = 3.49 \times 10^{28}$ cm in accordance with (5), and $r_\Lambda \simeq 2.4 \times 10^{-25}$ cm corresponding to $M_{GUT} \simeq 10^{15}$ GeV.

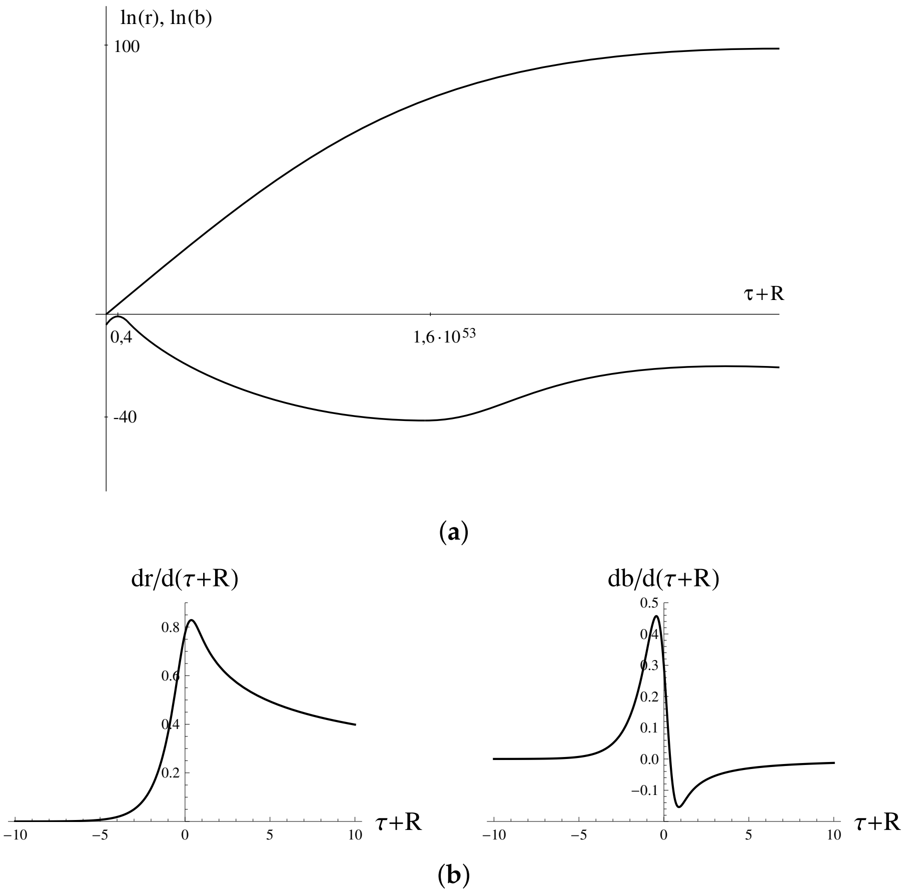

Figure 4. Behavior of the scale factors (**a**), and their derivatives $dr/d\tau$, $db/d\tau$ (**b**).

At the first inflationary stage and the stage of the present accelerated expansion the behavior of two scale factors is similar (curves run parallel and differ only by constant) and corresponds to the flat de Sitter cosmology. The maximum of the scale factor $b(\tau, R)$ at $\tau + R \simeq 0,4$ corresponds to the maximum of the transversal pressure at $r \simeq 1.2 r_*$ in Figure 3(right).

According to observational data, the present vacuum density starts to dominate at the age of about 3×10^9 years. In this limit Equation (16) gives the expansion law $r = r_\lambda e^{(\tau+R)/r_\lambda}$, with $r_\lambda = \sqrt{3/\lambda}$, Equation (11) gives for the second scale factor in (6) $e^{2\nu} = r^2/r_\lambda^2$, and the metric (6) takes the de Sitter FLRW form $ds^2 = d\tau^2 - r_\lambda^2 e^{2c\tau/r_\lambda} \left(dq^2 + q^2 d\Omega^2\right)$, where $q = e^{R/r_\lambda}$ [16].

Intrinsic anisotropy of the Lemaître class cosmological models can be described by the mean anisotropy parameter [27]

$$A = \frac{1}{3H^2} \sum_{i=1}^{3} H_i^2 - 1; \quad H_i = \frac{\dot{a}_i(\tau)}{a_i(\tau)}; \quad H = \frac{H_1 + H_2 + H_3}{3}, \tag{21}$$

where a_i are the scale factors, and H_i are the directional Hubble parameters. For the spherically symmetric models the scale factors are $a_1 = b = r'$, $a_2 = a_3 = r$, and [16]

$$A = 2\frac{(\dot{\mathcal{M}}/\mathcal{M}(r) - 3\dot{r}/r)^2}{(\dot{\mathcal{M}}/\mathcal{M} + 3\dot{r}/r)^2}. \tag{22}$$

In Figure 5 we plotted the behavior of the anisotropy parameter at the early stage and at the stage close to the present epoch of the accelerated expansion.

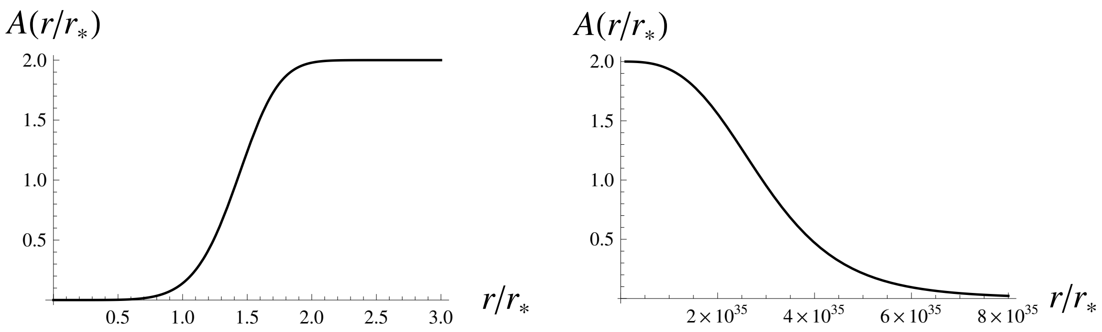

Figure 5. Behavior of the anisotropy parameter at the early stage and at the stage close to the present epoch.

The anisotropy parameter grows quickly at the postinflationary stage. At the maximum of $p_\perp$ and $b(\tau + r)$, it takes the value $A \simeq 0.4$, grows further achieving $A = 2$ at $r \simeq 2.5 \times 10^{-7}$ cm. Slow decreasing starts at $r \simeq 6 \times 10^{27}$ cm. Discussion of anisotropy is presented in the next section.

4. Summary and Discussion

In the applied here model-independent approach vacuum dark energy is presented by the variable cosmological term, introduced on the basis of the algebraic classification of stress-energy tensors and spacetime symmetry. It is intrinsically anisotropic which makes possible model-independent relaxation of initial value of the cosmological constant Λ to its present value λ in the frame of the Lemaître cosmological models, intrinsically anisotropic and asymptotically de Sitter at the early and late time. For the certain class of one-horizon Lemaître models, parametrized by the density profile, evolution is governed by the quantum evaporation of the cosmological horizon and goes towards the triple-horizon state. The basic physical parameters of the final state, the mass, radius and the relation $q^2 = \Lambda/\lambda = \rho_\Lambda/\rho_\lambda$, are tightly fixed by the data stored at its boundary (the cosmological horizon) in agreement with the basic sense of the holographic principle. The choice the density profile due to gravitational vacuum polarization and of its initial value ρ_Λ at the GUT scale, gives the present value of the cosmological constant in reasonable agreement with observations.

Recent observations suggest that our Universe can be deviated from isotropy [28–30]. Anisotropy of the Universe was constrained at the magnitude level of 2%–5% by SNe Ia data [29], and at the level of 4.4% by the Union2 dataset and high-redshift gamma-ray bursts [30].

The Lemaître dark energy model allows for detailed analysis of the universe anisotropy against observations, with the special attention to the question of bounds on the anisotropies in the primordial universe which requires a comprehensive analysis. It cannot be done on the basis of the above anisotropy parameter which tells us only about anisotropy of the universe filled with the vacuum dark energy. Lemaître metric can be used as the background metric in the self-consistent analysis, similar to that presented in the classical paper [31], where the extended collision-time anisotropy formalism has been developed on the basis of the multicomponent multicollision time approximation to the Boltzmann equation for the neutrinos, with taking into account that each neutrino flavor contributes to the anisotropic stresses [31]. As the background metric the Bianchi type-I was applied,

$ds^2 = -dt^2 + e^{2\alpha}e^{2\beta_{ij}}dx^i dx^j$. The Einstein equations involve contributions from neutrinos, photons and electrons ($\rho_{\nu e}, \rho_{\nu\mu}, \rho_{\nu\tau}, \rho_{(\gamma+e)}$ in what follows) and read $\dot{\alpha}^2 = (8\pi G/3)[\rho_{(\gamma+e)} + \rho_{\nu e}V(\beta_e) + \rho_{\nu\mu}V(\beta_\mu) + \rho_{\nu\tau}V(\beta_\tau)] + \dot{\beta}_+^2$; $\ddot{\beta}_+ = -3\dot{\alpha}\dot{\beta}_+ - (4\pi G/3)[\rho_{\nu e}(\partial V(\beta_e)/\partial\beta_e) + \rho_{\nu\mu}(\partial V(\beta_\mu)/\partial\beta_\mu) + \rho_{\nu\tau}(\partial V(\beta_\tau)/\partial\beta_\tau]$, where $V(\beta)$ are functions of neutrino contributions to anisotropy, and $\dot{\beta}_+$ is the shear velocity responsible for the metric anisotropy. The measure of anisotropy during nucleosynthesis $\beta_1 = [(3/8\pi G)\dot{\beta}_+^2 + \rho_{\nu e}V_e + \rho_{\nu\mu}V_\mu + \rho_{\nu\tau}V_\tau]/(\rho_\gamma + \rho_\nu)$ is calculated numerically with the Boltzmann equation [31]. Similar approach should be applied with the Lemaître metric for vacuum dark energy whose basic parameters have also to be found through a self-consistent analysis in the frame of the extended collision-time anisotropy formalism. This will be done (we hope) and presented in a separate paper. For analysis of collisions involving neutrinos, it can be essential that masses of neutrinos can involve de Sitter vacuum and can be related to breaking of spacetime symmetry from the de Sitter group [32]. This relation, obligatory for the cases satisfying $T_t^t = T_r^r$ [15], is suggested by the fact that the Higgs field participate in mass generation in its false vacuum state with $p = -\rho$. This relation allows to explain the observable effect of negative mass-square differences for neutrinos by calculating the particles masses as the eigenvalues of the Casimir operator in the de Sitter space, and to estimate gravito-electroweak scale from the neutrino data [32].

Author Contributions: All authors contributed equally to this work.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Ade, P.A.; Aikin, R.W.; Barkats, D.; Benton, S.J.; Bischoff, C.A.; Bock, J.J.; Brevik, J.A.; Buder, I.; Bullock, E.; Duband, L.; et al. Detection of B-Mode Polarization at Degree Angular Scales by BICEP2. *Phys. Rev. Lett.* **2014**, *112*, 241101.
2. Anderson, L.; Aubourg, E.; Bailey, S.; Beutler, F.; Bhardwaj, V.; Blanton, M.; Bolton, A.S.; Brinkmann, J.; Brownstein, J.R.; Chuang, C.H.; et al. The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: Baryon acoustic oscillations in the Data Releases 10 and 11 Galaxy samples. *Mon. Not. R. Astron. Soc.* **2014**, *441*, 24–62.
3. Suzuki, N.; Rubin, D.; Lidman, C.; Aldering, G.; Amanullah, R.; Barbary, K.; Barrientos, L.F.; Botyanszki, J.; Brodwin, M.; Dawson, K.S.; et al. The The Hubble Space Telescope Cluster Supernova Survey. V. Improving the dark-energy constraints above z > 1 and building an early-type-hosted supernova sample. *Astrophys. J.* **2012**, *746*, 85.
4. Copeland, E.J.; Sami, M.; Tsujikawa, S. Dynamics of dark energy. *Int. J. Mod. Phys. D* **2006**, *15*, 1753–1936.
5. Bamba, K.; Capozziello, S.; Nojiri, S.; Odintsov, S.D. Dark energy cosmology: The equivalent description via different theoretical models and cosmography tests. *Astrophys. Space Sci.* **2012**, *342*, 155–228.
6. Rivera, A.B.; Farieta, J.G. Exploring the Dark Universe: Constraint on dynamical dark energy models from CMB, BAO and Growth Rate Measurements. *arXiv* **2016**, arXiv:1605.01984.
7. Aviles, A.; Bravetti, A.; Capozziello, S.; Luongo, O. Precision cosmology with Padé rational approximations: Theoretical predictions versus observational limits. *Phys. Rev. D* **2014**, *90*, 043531.
8. De la Cruz-Dombriz, A.; Dunsby, P.K.S.; Luongo, O.; Reverberi, L. Model-independent limits and constraints on extended theories of gravity from cosmic reconstruction techniques. *J. Cosmol. Astropart. Phys.* **2016**, *2016*, 042.
9. Sahni, V.; Shafiello, A.; Starobinsky, A.A. Model-independent Evidence for Dark Energy Evolution from Baryon Acoustic Oscillations. *Astrophys. J.* **2014**, *793*, L40.
10. Dymnikova, I. Dark energy and spacetime symmetry. *Universe* **2017**, *3*, 20.
11. Bronnikov, K.A.; Dobosz, A.; Dymnikova, I. Nonsingular vacuum cosmologies with a variable cosmological term. *Class. Quantum Gravity* **2003**, *20*, 3797–3814.
12. Dymnikova, I.; Galaktionov, E. Vacuum dark fluid. *Phys. Lett. B* **2007**, *645*, 358–364.
13. Dymnikova, I. The algebraic structure of a cosmological term in spherically symmetric solutions. *Phys. Lett. B* **2000**, *472*, 33–38.
14. Dymnikova, I. Vacuum nonsingular black hole. *Gen. Relativ. Gravit.* **1992**, *24*, 235–242.

15. Dymnikova, I. The cosmological term as a source of mass. *Class. Quantum Gravity* **2002**, *19*, 725–740.
16. Bronnikov, K.A.; Dymnikova, I.; Galaktionov, E. Multihorizon spherically symmetric spacetimes with several scales of vacuum energy. *Class. Quantum Gravity* **2012**, *29*, 095025.
17. Dymnikova, I. Triple-horizon spherically symmetric spacetime and holographic principle. *Int. J. Mod. Phys. D* **2012**, *21*, 1242007.
18. Dymnikova, I.; Dobosz, A.; Soltysek B. Lemaitre dark energy model singled out by the holographic principle. *Gravit. Cosmol.* **2017**, *23*, 28–34.
19. Hooft, G.T. Dimensional reduction in quantum gravity. *arXiv* **1993**, arXiv:9310026.
20. Susskind, L. The World as a hologram. *J. Math. Phys.* **1995**, *36*, 6377–6396.
21. Dymnikova, I.; Korpusik, M. Thermodynamics of Regular Cosmological Black Holes with the de Sitter Interior. *Entropy* **2011**, *13*, 1967–1991.
22. Gibbons, G.W.; Hawking, S.W. Cosmological event horizons, thermodynamics, and particle creation. *Phys. Rev. D* **1977**, *15*, 2738–2751.
23. Dymnikova, I.; Korpusik, M. Regular black hole remnants in de Sitter space. *Phys. Lett. B* **2010**, *685*, 12–18.
24. Corasaniti, P.S.; Kunz, M.; Parkinson, D.; Copeland, E.J.; Bassett, B.A. Foundations of observing dark energy dynamics with the Wilkinson Microwave Anisotropy Probe. *Phys. Rev. D* **2004**, *70*, 083006.
25. Landau, L.D.; Lifshitz, E.M. *Classical Theory of Fields*, 4th ed.; Butterworth-Heinemann: Oxford, UK, 1975.
26. Olson, D.W.; Silk J. Primordial inhomogeneities in the expanding universe. II—General features of spherical models at late times. *Astrophys. J.* **1979**, *233*, 395–401.
27. Harko, T.; Mak, M.K. Bianchi Type I universes with dilaton and magnetic fields. *Int. J. Mod. Phys. D* **2002**, *11*, 1171–1189.
28. Wiaux, Y.; Vielva, P.; Martinez-Gonzalez, E.; Vandergheynst, P. Global universe anisotropy probed by the alignment of structures in the cosmic microwave background. *Phys. Rev. Lett.* **2006**, *96*, 151303.
29. Chang, Z.; Li, X.; Lin, H.-N.; Wang, S. Constraining anisotropy of the universe from different groups of type-Ia supernovae. *Eur. Phys. J. C* **2014**, *74*, 2821.
30. Chang, Z.; Li, X.; Lin, H.-N.; Wang, S. Constraining anisotropy of the universe from Supernovae and Gamma-ray Bursts. *Mod. Phys. Lett. A* **2014**, *29*, 1450067.
31. Rothman, T.; Matzner, R. Nucleosynthesis in anisotropic cosmologies revisited. *Phys. Rev. D* **1984**, *30*, 1649–1668.
32. Ahluwalia-Khalilova, D.V.; Dymnikova, I. A theoretical case for negative mass-square for sub-eV particles. *Int. J. Mod. Phys. D* **2003**, *12*, 1787–1796.