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Seismoelectric Coupling Equations of Oil-Wetted Porous Medium Containing Oil and Water

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Abstract: For porous medium containing multiphase fluid, such as oil-wetted porous medium with oil–water dual phase fluid, its fluid interface will also produce electric double layer (EDL), which will play a role in the seismoelectric effects. At this time, the principle of seismoelectric effects is more complex. The existing theory for the seismoelectric effects is the Pride theory used in the water-saturated porous formation, which cannot meet the actual needs of the theoretical research of seismoelectric exploration in the porous formation with multiphase fluid. Carbonate porous formations are often oil-wetted; therefore, it is necessary to study the electrokinetic effects of oil-wetted porous medium containing multiphase fluid. In this paper, we treated the oil–water mixture as an effective fluid, and solved the effective elastic parameters and extended the Biot equations to the case of oil-wetted porous medium with oil–water dual phase fluid. We calculated the effective electromagnetic parameters and derived the macroscopic coupling equations of seismoelectric effects and electroseismic effects, and proposed the new electrokinetic coupling coefficients of the oil-wetted porous medium with dual phase fluid. We also deduced the coupling functions of electric and magnetic fields relative to the solid displacement in the homogeneous porous medium, and studied the polarization characteristics of the electric field. We use the derived coupling equations to simulate the seismoelectric logging while drilling in the model of oil-wetted porous formation with dual phase fluid under the excitation of multipole sources. The influence of drill collar wave on the acoustic field and electric field under the excitation of different sources was investigated, which has a certain guiding role in the selection of electrokinetic logging tools.

Keywords: two-phase flow; oil-wetted porous medium; effective parameters; seismoelectric coupling equations; electrokinetic coupling coefficients; seismoelectric logging while drilling

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1. Introduction

The seismoelectric effects refer to the mutual coupling process of the energy of the acoustic field and the energy of the electromagnetic field in the porous medium. Carbonate reservoirs are often oil-wetted porous media [1]. In the oil well logging, the invasion zone of such reservoirs is oil-wetted porous medium with oil–water dual phase fluid. Therefore, the study on the electrokinetic effects of oil-wetted porous medium with dual phase fluid is of great significance for oil exploration.

At present, the research object of seismoelectric effects is mainly water-saturated porous medium. In 1994, Pride proposed the Pride equations to describe the electrokinetic effects of water-saturated porous medium, and gave the expression of macroscopic electrokinetic coupling coefficients [2]. In 1996, Pride and Haartsen studied the reciprocity of electrokinetic fields in saturated porous medium, and deduced the coupling functions of the electromagnetic field relative to the solid displacement of homogeneous porous medium [3]. A large number of studies have also been carried out on water-wetted porous media with multiphase flow. In 2013, Warden and Garambois. et al. treated the gas–liquid mixture as a fluid, extended the Pride theory to the unsaturated porous medium case, and gave the expressions of the electrokinetic coupling coefficients of the unsaturated porous medium by analogy with the saturated porous medium [4]. In 2017, Zyserman and Monachesi. et al. deduced the coupling functions of the electromagnetic field relative to the solid phase displacement in homogeneous unsaturated porous media using the seismoelectric coupling equations proposed by Warden, and carried out numerical simulation of SH-TE mode wave in horizontal layered unsaturated porous medium [5].

Some research has been carried out on the electrokinetic effects of oil-wetted porous formation; however, the existing theory has not studied the seismoelectric coupling equations and the corresponding electrokinetic coupling coefficients in the time-varying case. Alkafeef studied the microcosmic mechanism of acousto-electric conversion in oil saturated pores, and deduced the microscopic coupling coefficient [6]. Jackson used Dullen's multiphase flow pore model [7] to derive the streaming potential coupling coefficient of oil-wetted porous medium with dual phase fluid [8,9]. However, the used multiphase flow model is too simple, and is far from the actual model in pores.

Zhan and Zhu carried out the numerical simulation of electrokinetic logging while drilling in a water-saturated porous medium [10,11]. The electromagnetic field was calculated by solving the Poisson equation satisfied by the electric potential. In actual seismoelectric logging, the seismoelectric effects produce a time-varying electric field. According to the comparative analysis of Gao, in some cases, the quasi-static field approximation method will produce errors [12]. In 2012, Ren carried out the numerical simulation of the coseismic electromagnetic fields excited by the seismic waves in saturated porous medium [13]. He found that in saturated porous formations, the acousto-electric conversion efficiency of fast longitudinal waves is higher than that of other mode waves. In 2013, Guan conducted numerical simulations of electrokinetic logging while drilling in saturated porous formations under multipole sources excitation, and investigated the velocity-time correlation characteristics of sound and electric fields, as well as the influence of drill collar waves [14]. Guan's research indicates that the electric field is less affected by the drill collar wave in the electrokinetic logging while drilling in saturated porous formation. In 2015, Zheng conducted a similar study using the quasi-static field method and obtained the same conclusion [15].

Based on the above background, we adopted the Warden's method, regarded the oil–water mixture as an effective fluid, and extended the Biot theory to the case of oil-wetted porous medium with dual phase fluid. In this paper, we deduced the macroscopic coupling equations of seismoelectric effects in time-varying case, and derived the corresponding electrokinetic coupling coefficients. We derived the coupling functions of the electromagnetic field relative to the solid displacement in homogeneous oil-wetted porous medium with dual phase fluid, and analyzed the polarization characteristics of electric field. We carried out the simulation of the electrokinetic logging while drilling in oil-wetted porous formation with dual phase fluid. We introduced the Hertz vector potential, and treated the electromagnetic field as a time-varying field. We also investigated the influence of the drill collar wave on the acoustic and electric fields under the excitation of multipole sources.

2. Methods

2.1. Seismoelectric Coupling Equations

When oil-wetted porous medium contains dual phase fluid, the acoustic field model is more complicated than that of a water-saturated porous medium. In order to simplify the model, the method of warden is adopted in this paper to equate to the oil–water fluid mixture to a fluid, and the Biot equations are extended to the case of oil-wetted porous medium containing dual phase fluid:

$$\begin{cases} \nabla \cdot \boldsymbol{\tau} = -\omega^2(\rho \mathbf{u}_s + \rho_f \mathbf{u}_{fs}) \\ \boldsymbol{\tau} = (H-2G)(\nabla \cdot \mathbf{u}_s)\mathbf{I} + C(\nabla \cdot \mathbf{u}_{fs})\mathbf{I} + G(\nabla \mathbf{u}_s + \nabla {\mathbf{u}_s}^T) \\ -p = C\nabla \cdot \mathbf{u}_s + M\nabla \cdot \mathbf{u}_{fs} \\ \mathbf{u}_{fs} = \phi(\mathbf{u}_f - \mathbf{u}_s) \end{cases} \tag{1}$$

The Biot equations can be found in Ref. [2]. Where $\boldsymbol{\tau}$ denotes the stress tensor, ρ denotes the density of the oil-wetted porous medium with dual phase fluid, $\mathbf{u}_s$ denotes the solid displacement, ρ_f denotes the effective fluid density, $\mathbf{u}_{fs}$ denotes the seepage displacement of the equivalent fluid, G denotes the formation shear modulus, H, C and M denote the elastic modulus of the porous medium, K_s, K_f and K_b are the bulk modulus of solid matrix, liquid and solid skeleton, respectively, p denotes the acoustic pressure, ϕ denotes the porosity, and $\mathbf{u}_f$ denotes the displacement of the equivalent fluid.

The electromagnetic field satisfies the following equations:

$$\begin{cases} \nabla \times \mathbf{E} = -j\omega \mathbf{B} \\ \nabla \times \mathbf{H} = j\omega \mathbf{D} + \mathbf{J} \\ \mathbf{D} = \varepsilon \mathbf{E} \\ \mathbf{B} = \mu \mathbf{H} \end{cases} \tag{2}$$

Equation (2) represents the laws that macroscopic electromagnetic fields satisfy at frequency domain, including induction law, Ampere's law, the constitutive relationship of electric field, and the constitutive relationship of magnetic field. $\mathbf{E}$ represents the electric field intensity, $\mathbf{H}$ represents the magnetic field intensity, $\mathbf{B}$ represents the magnetic induction intensity, $\mathbf{D}$ represents the electric displacement vector, and μ is the magnetic permeability. $\mathbf{J}$ represents the macroscopic current in the medium, including streaming current and conduction current, and is used to describe the macroscopic acousto-electric conversion.

The effective acoustic and electric parameters are calculated in Table 1.

Table 1. Effective parameters.

Effective Parameters	
Effective density [16]	$\rho_f(s_w) = (1-s_w)\rho_o + s_w\rho_w$
Effective viscosity [17]	$\eta_f(s_w) = \eta_o(\eta_w/\eta_o)^{s_w}$
Effective fluid bulk modulus [18]	$K_f(s_w) = (K_w - K_o)s_w^5 + K_o$
Effective critical frequency [19]	$\omega_c(s_w) = \frac{\phi\eta_f(s_w)}{\alpha_\infty k_0 \rho_f(s_w)}$
Conductivity [20]	$\sigma(s_w) = \frac{s_w^n}{F}\sigma_w$
Dielectric constant [21]	$\varepsilon(s_w) = \varepsilon_0[(1-\phi)\sqrt{\varepsilon_s} + \phi s_w\sqrt{\varepsilon_w} + \phi(1-s_w)\sqrt{\varepsilon_o}]^2$

Where ρ_o and ρ_w are the densities of oil and water, respectively, s_w represents the water saturation, η_o and η_w denote the viscosity of oil and water, respectively, K_w denotes the bulk modulus of the water, K_o denotes the bulk modulus of the oil, α_∞ denotes the tortuosity of the porous medium, k_0 denotes the effective static permeability of the mixed fluid, σ_w is the conductivity of the water, $F = \alpha_\infty/\phi$, ϕ is the porosity, ε_0 denotes the vacuum dielectric constant, ε_s, ε_w and ε_o are the dielectric constants corresponding to the three phases of solid, water, and oil, respectively.

The seismoelectric coupling equations of oil-wetted porous medium with oil–water dual phase fluid are derived below.

The microscopic current in pores satisfies the local equation [22]:

$$\mathbf{J}_w = \mathbf{J}_{ws} + \mathbf{J}_{wc} \tag{3}$$

where $\mathbf{J}_{ws}$ is the microscopic streaming current in pores, and $\mathbf{J}_{wc}$ denotes the microscopic conduction current in pores. According to the research of Ref [23], the streaming current in the oil-wetted pores with dual phase fluid is mainly caused by the electric double layer (EDL) at the fluid interface; therefore, $\mathbf{J}_{ws}$ can be expressed as follows:

$$\mathbf{J}_{ws} = Q_{wo}\mathbf{v}_{wo} \tag{4}$$

Additionally, $\mathbf{J}_{wc}$ can be expressed as follows:

$$\mathbf{J}_{wc} = \sigma_w \mathbf{e}_w \tag{5}$$

where Q_{wo} is the net charge density of the EDL near the fluid interface, $\mathbf{v}_{wo}$ is the microscopic relative velocity of water to the liquid interface in pores, and $\mathbf{e}_w$ denotes the microscopic electric field in pores.

According to Slattery's volume averaging theory, the macroscopic conduction current can be expressed as follows:

$$\bar{\mathbf{J}}_c = \sigma\overline{\mathbf{E}} \tag{6}$$

$\overline{\mathbf{E}}$ is the macroscopic electric field. According to Ref [23], by defining the flux averaging of Q_{wo} relative to $\mathbf{v}_{wo}$, the macroscopic streaming current of oil-wetted porous medium with dual phase fluid can be expressed as follows:

$$\bar{\mathbf{J}}_{ws} = Q_{wo}^{eff}\overline{\mathbf{v}}_{wo} \tag{7}$$

where $\overline{\mathbf{v}}_{wo}$ denotes the macroscopic relative velocity of water to the liquid interface in the porous medium, and Q_{wo}^{eff} denotes the effective net residual charge density (ERCD) of two-phase oil-wetted porous medium.

According to Ref [23], it can be deduced that:

$$\overline{\mathbf{v}}_{wo} = l\overline{\mathbf{v}}_{ws} \tag{8}$$

where $l = \frac{s_w\eta_o}{s_w\eta_o + 2\eta_w(1-s_w)}$, and $\overline{\mathbf{v}}_{ws}$ denotes the macroscopic seepage velocity of water in porous medium.

According to Darcy's law, the effective seepage velocity of fluid can be expressed as follows:

$$\overline{\mathbf{v}}_f = \frac{k_0}{\eta_f}(-\nabla\overline{P} + i\omega\rho_f\overline{\mathbf{v}}_s) \tag{9}$$

where $\nabla\overline{P}$ denotes the sound pressure gradient, $\overline{\mathbf{v}}_s$ denotes the solid phase displacement.

Then, $\overline{\mathbf{v}}_{ws}$ can be deduced as follows:

$$\overline{\mathbf{v}}_{ws} = \frac{k_0 k_{wr}}{\eta_f}(-\nabla\overline{P} + i\omega\rho_f\overline{\mathbf{v}}_s) \tag{10}$$

where k_{wr} is the aqueous relative permeability.

It can be further deduced that the macroscopic streaming current in porous medium can be expressed as follows:

$$\bar{\mathbf{J}}_{ws} = \frac{k_0 k_{wr} Q_{wo}^{eff}}{\eta_f} l(-\nabla\overline{P} + i\omega\rho_f\overline{\mathbf{v}}_s) \tag{11}$$

The seismoelectric coupling coefficient is defined as follows:

$$L_m = \frac{k_0 k_{wr} Q_{wo}^{eff}}{\eta_f} l \tag{12}$$

Finally, the coupling equation of acousto-electric conversition can be deduced as follows:

$$\mathbf{J} = \sigma \mathbf{E} + L_m(-\nabla P + \omega^2 \rho_f \mathbf{u}_s) \tag{13}$$

According to the generalized Darcy permeability [24], we can get:

$$\overline{\mathbf{v}}_f = \frac{k_0}{\eta_f}(-\nabla \overline{P} + i\omega\rho_f \overline{\mathbf{v}}_s + \overline{F}_e) \tag{14}$$

where $\overline{F}_e = Q_{wo}^{eff} \overline{\mathbf{E}}$ denotes the electric field force.

Similarly, the macroscopic seepage velocity of water in porous medium is obtained:

$$\overline{\mathbf{v}}_{ws} = \frac{k_0 k_{wr}}{\eta_f}(-\nabla \overline{P} + i\omega\rho_f \overline{\mathbf{v}}_s + \overline{F}_e) \tag{15}$$

The expression of $\overline{\mathbf{v}}_{wo}$ can be further deduced:

$$\overline{\mathbf{v}}_{wo} = \frac{k_0 k_{wr}}{\eta_f} l(-\nabla \overline{P} + i\omega\rho_f \overline{\mathbf{v}}_s + \overline{F}_e) \tag{16}$$

The electroseismic coupling coefficient is defined as follows:

$$L_e = \frac{k_0 k_{wr} Q_{wo}^{eff}}{\eta_f} l \tag{17}$$

Finally, the electroseismic coupling equation of two-phase oil-wetted porous medium can be expressed as follows:

$$-i\omega \mathbf{u}_{wo} = \frac{k_0 k_{wr}}{\eta_f} l(-\nabla P + \omega^2 \rho_f \mathbf{u}_s) + L_e \mathbf{E} \tag{18}$$

2.2. Coupling Functions of the Oil-Wetted Porous Medium with Oil–water Dual Phase Fluid

Referring to the method of Garambois [25], it is assumed that a uniform plane acoustic wave propagates in the oil-wetted porous medium with dual phase fluid, and the electric and magnetic field amplitudes generated by the seismoelectric effect were calculated. According to the seismoelectric coupling equations obtained earlier, we can deduce the wave equations satisfied by the acoustic field, the electric field and the magnetic field, solve the eigenmodes of the wave equations, and we can obtain the following results. When the acoustic field is a longitudinal wave, we assume that $\mathbf{e}_k = \mathbf{e}_s = \mathbf{e}_{fs} = \mathbf{e}_e$, where $\mathbf{e}_s$ denotes the direction of solid displacement, $\mathbf{e}_{fs}$ denotes the direction of seepage velocity of equivalent liquid, and $\mathbf{e}_e$ denotes the direction of electric field. The direction of all fields is $\mathbf{e}_k$.

The corresponding coupling functions are as follows:

$$\begin{cases} u_{fs0} = \beta_{L\xi} u_{s0} \\ E_0 = i\omega \frac{\overline{\rho} L_m}{\overline{\varepsilon}} \beta_{L\xi} u_{s0} \end{cases} \tag{19}$$

where $\beta_{L\xi} = -(\frac{Hs_\xi^2 - \rho}{Cs_\xi^2 - \rho_f})$, s_{pf} and s_{ps} represent the slowness of fast longitudinal wave and slow longitudinal wave, respectively, $\rho_t = \rho - \rho_f^2/\overline{\rho}$, and $\overline{\rho} = \frac{i}{\omega}\frac{\eta_f}{k_0}$.

When the acoustic field is SH wave, the direction of each field is $\mathbf{e}_k \cdot \mathbf{e}_s = \mathbf{e}_k \cdot \mathbf{e}_{fs} = \mathbf{e}_k \cdot \mathbf{e}_e = 0$. The electromagnetic field is TE wave, and the direction of other field is: $\mathbf{e}_{fs} = \mathbf{e}_e = \mathbf{e}_s = \mathbf{y}$, $\mathbf{e}_h = \mathbf{e}_k \times \mathbf{e}_s$. The corresponding coupling functions are as follows:

$$\begin{cases} u_{fs0} = \frac{G}{\rho_f}(s_\xi^2 - \frac{\rho}{G})u_{s0} \\ E_0 = i\omega\mu\frac{\overline{\rho}}{\rho_f}L_m G\beta_{T\xi}u_{s0} \\ H_0 = i\omega s_\xi \frac{\overline{\rho}}{\rho_f}L_m G\beta_{T\xi}u_{s0} \end{cases} \tag{20}$$

When the sound field is SV wave, the electromagnetic field is TM wave, and the direction of other field is: $\mathbf{e}_{fs} = \mathbf{e}_e = \mathbf{e}_s = \mathbf{y} \times \mathbf{e}_k$, $\mathbf{e}_h = \mathbf{y}$. The coupling functions are as follows:

$$\begin{cases} u_{fs0} = \frac{G}{\rho_f}(s_\xi^2 - \frac{\rho}{G})u_{s0} \\ E_0 = i\omega\mu\frac{\overline{\rho}}{\rho_f}L_m G\beta_{T\xi}u_{s0} \\ H_0 = i\omega s_\xi \frac{\overline{\rho}}{\rho_f}L_m G\beta_{T\xi}u_{s0} \end{cases} \tag{21}$$

where $\beta_{T\xi} = -(\frac{s_\xi^2 - \rho/G}{s_\xi^2 - \mu\bar{\varepsilon}})$, s_s and s_{em} represent the slowness of acoustic field shear wave and electromagnetic field, respectively.

It can be seen from the above derivation that for oil-wetted porous medium with dual phase fluid, each mode wave of the sound field can excite the electric field of the corresponding mode, and the polarization direction of the electric field is determined by the solid displacement of the acoustic field.

2.3. Seismoelectric Logging While Drilling

The solution method of the acoustic field in the electrokinetic logging while drilling in the oil-wetted porous medium with dual phase fluid is the same as that of Zhan and Zhu [10,11], except that the formation parameters of the acoustic field adopt the effective parameters obtained above. The solution process of electromagnetic field is different from the two.

Ignoring the coupling effect of electric field on acoustic field, and according to Equations (11) and (16), the streaming current in the porous medium can be expressed as follows:

$$\mathbf{J}_s = -i\omega L_m \eta_f/(kk_{wr}l)\mathbf{u}_{wo} \tag{22}$$

Therefore, the streaming current in porous formation can be expressed with the potential function of acoustic field:

$$\mathbf{J}_s = \nabla\psi_1 + \nabla\psi_2 + \nabla\times\mathbf{F}_1 + \nabla\times\nabla\times\mathbf{F}_2 \tag{23}$$

where ψ_1, ψ_2, $\mathbf{F}_1$ and $\mathbf{F}_2$ are the potential functions of sound field.

According to the theory of electrodynamics, any current source $\mathbf{J}$ can be expressed as follows:

$$\mathbf{J} = \frac{\partial\mathbf{P}}{\partial t} + \nabla\times\mathbf{M} \tag{24}$$

where $\mathbf{P}$ is the current source without divergence, $\mathbf{M}$ is the current source without curl.

If the Hertz electric vector $\mathbf{\Pi}^e$ and the Hertz magnetic vector $\mathbf{\Pi}^m$ satisfy, respectively:

$$\begin{cases} (\nabla^2 - \mu\bar{\varepsilon}\frac{\partial^2}{\partial t^2})\mathbf{\Pi}^e = -\frac{\mathbf{P}}{\bar{\varepsilon}} \\ (\nabla^2 - \mu\bar{\varepsilon}\frac{\partial^2}{\partial t^2})\mathbf{\Pi}^m = -\mathbf{M} \end{cases} \tag{25}$$

Then, the electric field and magnetic field expressed by Hertz vector are as follows:

$$\begin{cases} \mathbf{E} = \nabla\nabla \cdot \mathbf{\Pi}^e + k_{em}^2 \mathbf{\Pi}^e + i\omega\mu\nabla \times \mathbf{\Pi}^m \\ \mathbf{H} = -i\omega\bar{\varepsilon}\nabla \times \mathbf{\Pi}^m + \nabla \times \nabla \times \mathbf{\Pi}^m \end{cases} \tag{26}$$

When the excitation source is streaming current, there is:

$$\begin{cases} \frac{\partial \mathbf{P}}{\partial t} = \nabla\psi_1 + \nabla\psi_2 \\ \nabla \times \mathbf{M} = \nabla \times \mathbf{F}_1 + \nabla \times \nabla \times \mathbf{F}_2 \end{cases} \tag{27}$$

By substituting Equation (27) into Equation (25), the problem of solving Maxwell equations excited by streaming current is transformed into vector Helmholtz equations about Hertz vector.

$$\begin{cases} (\nabla^2 - \mu\bar{\varepsilon}\frac{\partial^2}{\partial t^2})\mathbf{\Pi}^e = -\frac{\nabla\psi_1 + \nabla\psi_2}{i\omega\bar{\varepsilon}} \\ (\nabla^2 - \mu\bar{\varepsilon}\frac{\partial^2}{\partial t^2})\mathbf{\Pi}^m = -\mathbf{F}_1 - \nabla \times \mathbf{F}_2 \end{cases} \tag{28}$$

By solving the special solution and substituting the result into Equation (26), we can obtain the special solution of electromagnetic field in porous formation as follows: $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ and $\mathbf{E}_4$.

The electromagnetic field in the passive case only needs two independent scalar functions to represent. Let $\mathbf{\Pi}^e = \Pi^e \mathbf{e}_z$, $\mathbf{\Pi}^m = \Pi^m \mathbf{e}_z$; then, we have:

$$\begin{cases} \nabla^2 \Pi^e + k_{em}^2 \Pi^e = 0 \\ \nabla^2 \Pi^m + k_{em}^2 \Pi^m = 0 \end{cases} \tag{29}$$

Then, the Hertz vector potential of the passive electromagnetic field in the borehole fluid is as follows:

$$\begin{cases} \Pi_0^e = R_e A_e I_n(\eta_{em1} r) + R_e B_e K_n(\eta_{em1} r) \\ \Pi_0^m = R_e A_m I_n(\eta_{em1} r) + R_e B_m K_n(\eta_{em1} r) \end{cases} \tag{30}$$

Substitute the expressions of Π_0^e and Π_0^m into Equation (26) to obtain the expression of the electromagnetic field ($\mathbf{E}_b$ and $\mathbf{H}_b$) in the borehole.

The Hertz vector potential of the passive electromagnetic field in the porous medium is as follows:

$$\begin{cases} \Pi_{20}^e = R_e B_{e2} K_n(\eta_{em2} r) \\ \Pi_{20}^m = R_e B_{m2} K_n(\eta_{em2} r) \end{cases} \tag{31}$$

Substitute the expressions of Π_{20}^e and Π_{20}^m into Equation (26) to obtain the passive electromagnetic field in the porous medium as $\mathbf{E}_0$ and $\mathbf{H}_0$.

The expression of the electromagnetic field induced by the acoustic wave in the porous medium is as follows:

$$\begin{cases} \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 + \mathbf{E}_0 \\ \mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 + \mathbf{H}_3 + \mathbf{H}_4 + \mathbf{H}_0 \end{cases} \tag{32}$$

In the LWD model, the boundary conditions satisfied by the electromagnetic fields are as follows:

The tangential electric field on the surface of the tool layer is 0:

$$E_{bz}(R_t) = 0 \tag{33}$$

The tangential components of the electric and magnetic fields on the wall of the well are continuous:

$$\begin{cases} E_{bz}(R_b) = E_{fz}(R_b) \\ H_{b\varphi}(R_b) = H_{f\varphi}(R_b) \end{cases} \tag{34}$$

R_t is the outer surface radius of the tool layer and R_b is the wellbore radius. According to the boundary conditions, list the equations, solve the equations to obtain the unknown coefficients, and substitute them into the expression of the electromagnetic field in the wellbore to obtain the electromagnetic field ($\mathbf{E}_b$ and $\mathbf{H}_b$) on the wall of the well.

The sound pressure in the borewhole is as follows:

$$P(r) = \rho_w \omega^2 [A_n I_n(k_\rho r) + B_n(k_\rho r)] \cos n\theta \tag{35}$$

A_n and B_n are the coefficients of acoustic potential functions, which are solved in Ref [11]. k_ρ is the radial wavenumber.

3. Simulation and Analysis

In this section, we carried out numerical simulation on the electrokinetic logging while drilling in oil-wetted porous medium with dual phase fluid under the excitation of multipole sources. We have compared and analyzed the received acoustic field and electric field, investigated their frequency–velocity correlation characteristics, and analyzed the influence of drill collar wave on the electrokinetic fields.

The center frequency of the excitation source is 7 KHz, which is excited by a monopole source, a dipole source, and a quadrupole source, respectively. There is a total of seven receiving sites on the wall of the well: the first site is at z = 1.3713 m, and the distance between adjacent sites is 0.1524 m. The parameters of the formation are shown in Table 2.

Table 2. Parameters of oil wetted porous formation with dual phase fluid.

Parameter	Value	Unit
Static permeability k_0	10^{-12}	m^2
Porosity ϕ	0.2	Dimensionless
Water phase viscosity η_w	0.001	$Pa \cdot m$
Oil phase viscosity η_o	0.1	$Pa \cdot m$
Water phase density ρ_w	1000	kg/m^3
Oil phase density ρ_o	776	kg/m^3
Solid matrix density ρ_s	2650	kg/m^3
Skeleton shear modulus G	13.99	GPa
Skeleton bulk modulus K_b	14.4	GPa
Elastic modulus of skeleton K_s	35.73	GPa
Bulk modulus of water K_w	2.25	GPa
Bulk modulus of oil K_o	1.29	GPa
Water saturation s_w	0.9	Dimensionless
Tortuosity α_∞	2	Dimensionless

In the following figures, t represents the time, P represents the acoustic pressure, Ez represents the axial electric field, V represents the group velocity, and f represents the frequency.

The frequency–velocity correlation characteristics of sound and electric fields are obtained through the principle of multiple filtering [26]. First, preprocess the original seismic signal by removing zero drift and tendency. Transform the signal into the frequency domain through Fourier transform and then use a narrowband bandpass digital filter to filter the signal, the frequency components outside the passband are filtered out. Then, the signal is transformed into the time domain by inverse Fourier transform. The amplitude is calculated using Hilbert transform, and find the time corresponding to the maximum amplitude, that is, the arrival time of the wave packet corresponding to the central frequency. Through the above process, the amplitude and group velocity of the signal can be found.

Repeat the above steps until the group velocity values of all the desired frequencies (periods) are obtained.

Figure 1 shows the comparison of electric field and acoustic field in the electrokinetic logging while drilling in the oil-wetted porous formation with dual phase fluid under the excitation of monopole source. It can be seen from the figure that the waveforms of electric field and acoustic field are very similar, which shows that under the excitation of monopole source, each mode wave of acoustic field will excite the corresponding mode electric field. The electro-acoustic ratio of the fast longitudinal wave is relatively larger, which indicates that the acousto-electric conversion efficiency of the fast longitudinal wave is higher.

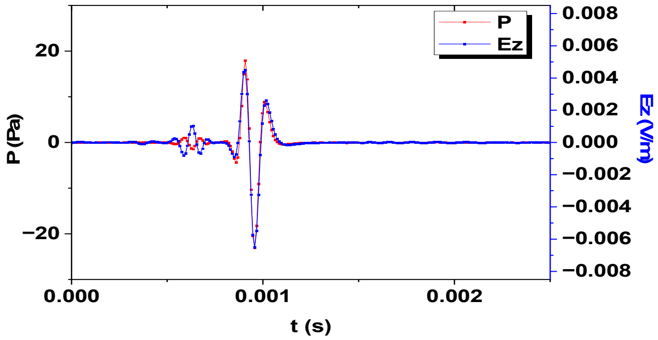

Figure 1. Comparison of electric field and acoustic field (monopole source).

Figure 2a,b shows the frequency–velocity correlation characteristics of sound field and electric field in the electrokinetic logging while drilling in the oil-wetted porous formation with dual phase fluid under the excitation of monopole source. The drill collar wave velocity is about 5940 m/s. It can be seen from Figure 2a that the bright spots near the velocity of the drill collar wave are very obvious, and the sound field is greatly affected by the drill collar wave. It can be seen from Figure 2b that the bright spots near the drill collar wave velocity are much smaller, and the electric field is less affected by the drill collar wave than the acoustic field. The velocity of the fast longitudinal wave in the formation is 4000 m/s. Compared with the acoustic field, the bright spot of the electric field near the fast longitudinal wave is larger, the electro-acoustic ratio of the fast longitudinal wave is relatively larger, and the acousto-electric conversion ability of the fast longitudinal wave is stronger than other mode waves. By comparing Figure 2a,b, it can be found that under the excitation of monopole source, measuring electric field can avoid the influence of drill collar wave to a certain extent, and makes it easier to measure the longitudinal wave velocity.

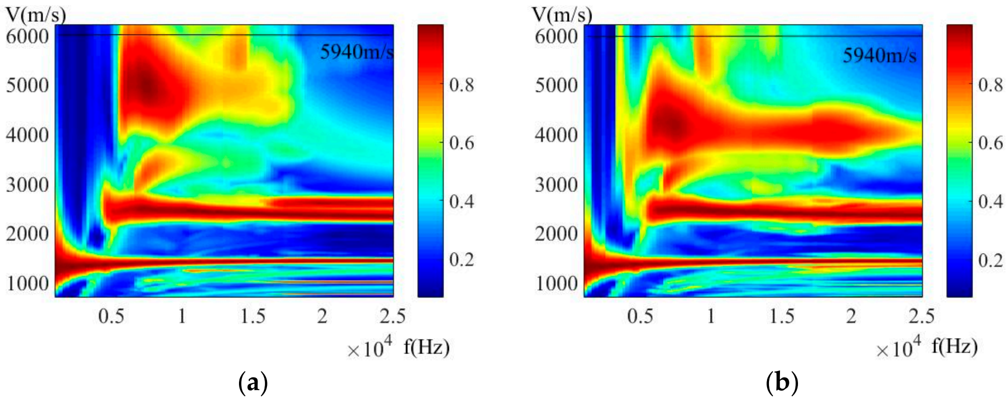

Figure 2. Correlation characteristics of the seismoelectric fields (monopole source): (**a**) acoustic field; (**b**) electric field.

Figure 3 shows the comparison of the electric field and the acoustic field in the electrokinetic logging while drilling in the oil-wetted porous formation with dual phase fluid under the excitation of dipole source. It can be seen from the figure that the waveforms of the electric field and the acoustic field are very similar, which shows that under the excitation of the dipole source, each mode wave of the acoustic field will stimulate the electric field of the corresponding mode. The electro-acoustic ratio of the fast longitudinal wave is relatively larger, which indicates that the acousto-electric conversion efficiency of the fast longitudinal wave is higher.

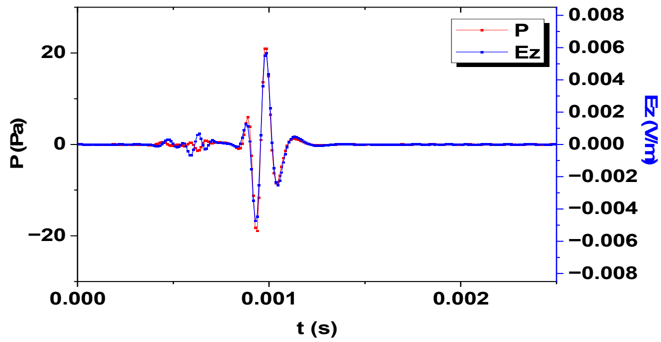

Figure 3. Comparison of electric field and acoustic field (dipole source).

Figure 4a,b shows the frequency–velocity correlation characteristics of sound field and electric field in the electrokinetic logging while drilling in the oil-wetted porous formation with dual phase fluid under the excitation of dipole source. The drill collar wave velocity is about 5940 m/s. It can be seen from Figure 4a that the bright spot near the velocity of the drill collar wave is very dark, and the drill collar wave has very little effect on the acoustic field. It can be seen from Figure 4b that the bright spot near the drill collar wave is smaller, and the electric field is less affected by the drill collar wave, which can be almost ignored. Through analysis, it can be deduced that the influence of drill collar wave can be ignored in the electrokinetic logging while drilling under the excitation of dipole source.

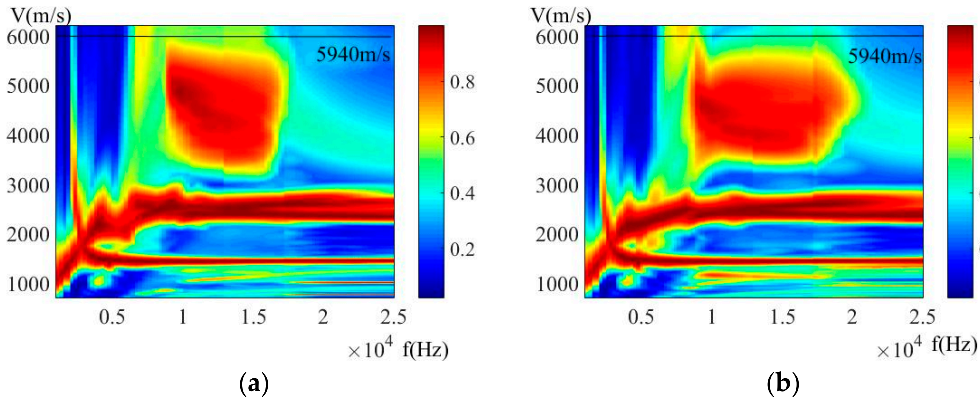

Figure 4. Correlation characteristics of the seismoelectric fields (dipole source): (**a**) acoustic field; (**b**) electric field.

Figure 5 shows the comparison of electric field and acoustic field in the electrokinetic logging while drilling in the oil-wetted porous formation with dual phase fluid under the excitation of quadrupole source. It can be seen from the figure that the waveforms of the electric field and the acoustic field are very similar, which shows that under the excitation of the quadrupole source, each mode wave of the acoustic field will stimulate the electric field of the corresponding mode. The electro-acoustic ratio of the fast longitudinal wave is relatively larger, which indicates that the acousto-electric conversion efficiency of the fast longitudinal wave is higher.

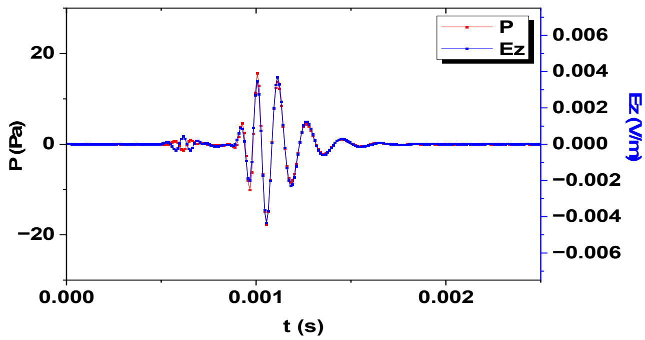

Figure 5. Comparison of electric field and acoustic field (quadrupole source).

Figure 6a,b shows the frequency–velocity correlation characteristics of sound field and electric field in the electrokinetic logging while drilling in the oil-wetted porous formation with dual phase fluid under the excitation of quadrupole source. The drill collar wave velocity is about 5940 m/s. It can be seen from Figure 6a that the bright spots near the velocity of the drill collar wave are very obvious, and the sound field is greatly affected by the drill collar wave. It can be seen from Figure 6b that the bright spots near the drill collar wave velocity are also obvious, but are much smaller compared with that of acoustic field. The drill collar wave also has influence on the electric field; however, compared with the acoustic field, the drill collar wave has much less influence on the electric field.

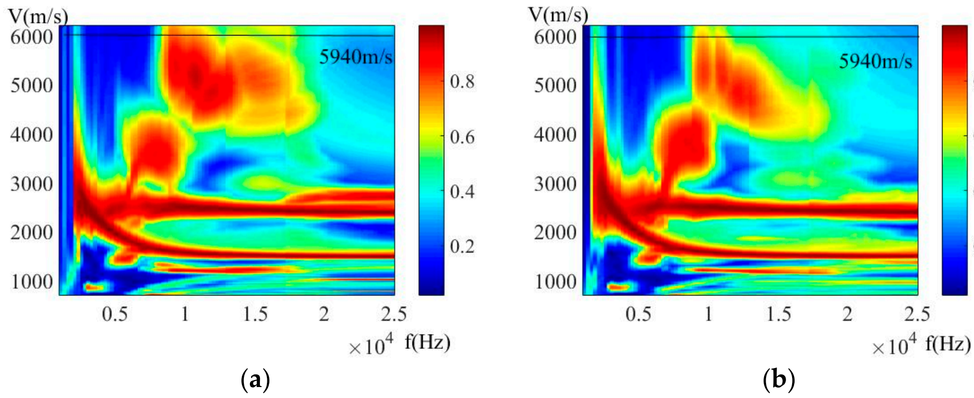

Figure 6. Correlation characteristics of the seismoelectric fields (quadrupole source): (**a**) acoustic field; (**b**) electric field.

4. Discussion

From Equations (19)–(21), it can be seen that each mode wave of the acoustic field will excite the electric field of the corresponding mode. The comparison of sound field and electric field in Figures 1, 3 and 5 can also prove that under the excitation of multipole sources, each mode wave of acoustic field in the stratum can excite the accompanying electric field.

By observing Figures 1, 3 and 5, we can find that the electro-acoustic ratio of fast longitudinal wave is relatively larger. This is consistent with the research results of saturated porous medium [13]. By comparing Figure 2a,b, Figure 4a,b and Figure 6a,b, it can also be found that the bright spot near the fast longitudinal wave velocity is brighter in the correlation characteristic diagram of the electric field, which can also verify the above conclusion.

From Figures 1, 3 and 5, it can be seen that for the fast longitudinal wave part, the electric field and the acoustic field are reversed, while for other mode wave parts, the electric field and the acoustic field are in the same direction. The conduction current and streaming current should be in the opposite direction; therefore, the electric field and streaming current are in the opposite direction. According to Equation (9), the longitudinal wave part of seepage velocity is mainly composed of liquid phase velocity, and the liquid is positively charged; therefore, the electric field corresponding to the longitudinal wave part is opposite to the direction of the acoustic field. According to Equation (9), the other mode wave part of seepage velocity is mainly determined by the solid phase velocity, and the solid phase surface is negatively charged; therefore, the electric field corresponding to the other mode wave parts is in the same direction with the acoustic field.

By comparing Figure 2a,b, Figure 4a,b and Figure 6a,b, it can be found that the electric field in the electrokinetic logging while drilling in the oil-wetted porous formation with dual phase fluid is much less affected by the drill collar wave than the acoustic field, which is an advantage of electrokinetic logging. This is consistent with the research results of saturated porous medium [14,15]. At the same time, it can also be found that under the excitation of dipole source, the influence of drill collar wave can be almost ignored. This proves that the use of dipole source excitation can effectively avoid the impact of drill collar wave in the electrokinetic logging while drilling.

5. Conclusions

In this paper, we studied the electrokinetic effects in a oil-wetted porous medium with dual phase fluid, derived the seismoelectric coupling equations, deduced the coupling functions of electric field and magnetic field relative to solid displacement of acoustic field in a homogeneous porous medium, and investigated the polarization characteristics of electric field. We also conducted numerical simulation of the electrokinetic logging while drilling in an oil-wetted porous formation with dual phase fluid. We can draw the following conclusions through the research:

(1) The polarization direction of the electric field generated by the electrokinetic conversion in the oil-wetted porous medium with dual phase fluid is determined by the acoustic field. For the longitudinal wave part, the direction of the electric field is opposite to the sound field, while for other mode waves, the direction of the electric field is the same as the sound field.
(2) The electrokinetic conversion in the oil-wetted porous medium with dual phase fluid will stimulate the electric field that accompanies each mode wave of the acoustic field; therefore, the electrokinetic logging can measure the acoustic velocity in the formation.
(3) In the electrokinetic logging, the electrokinetic conversion efficiency of fast longitudinal wave is higher than that of other mode waves.
(4) In electrokinetic logging while drilling, the electric field is less affected by the drill collar wave than the acoustic field, which is the advantage of electrokinetic logging.
(5) In electrokinetic logging, while drilling in the oil-wetted porous formation with dual phase fluid, using dipole source excitation can effectively avoid the influence of drill collar wave.

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