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Quiescent Optical Solitons with Cubic–Quartic and Generalized Cubic–Quartic Nonlinearity

Ahmed H. Arnous [1], Anjan Biswas [2,3,4,5,6], Yakup Yıldırım [7], Luminita Moraru [8,*], Simona Moldovanu [9] and Seithuti P. Moshokoa [10]

1 Department of Physics and Engineering Mathematics, Higher Institute of Engineering, El–Shorouk Academy, Cairo 11837, Egypt
2 Department of Mathematics and Physics, Grambling State University, Grambling, LA 71245, USA
3 Mathematical Modeling and Applied Computation (MMAC) Research Group, Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia
4 Department of Applied Mathematics, National Research Nuclear University, 31 Kashirskoe Hwy, 115409 Moscow, Russia
5 Department of Applied Sciences, Cross-Border Faculty, Dunarea de Jos University of Galati, 111 Domneasca Street, 800201 Galati, Romania
6 Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa 0204, South Africa
7 Department of Computer Engineering, Biruni University, Istanbul 34010, Turkey
8 Department of Chemistry, Physics and Environment, Faculty of Sciences and Environment, Dunarea de Jos University of Galati, 47 Domneasca Street, 800008 Galati, Romania
9 Department of Computer Science and Information Technology, Faculty of Automation, Computers, Electrical Engineering and Electronics, Dunarea de Jos University of Galati, 47 Domneasca Street, 800008 Galati, Romania
10 Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
* Correspondence: luminita.moraru@ugal.ro

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Abstract: The enhanced Kudryashov's approach retrieves quiescent bright, dark, and singular solitons to the governing model that is considered with cubic–quartic form of self-phase modulation. The algorithm however fails to retrieve stationary solitons when the nonlinearity is the generalized version of the cubic–quartic form. The current analysis is conducted with a direct approach without an intermediary phase-portrait analysis as in the past.

Keywords: solitons; Kudryashov; cubic–quartic

1. Introduction

A detrimental feature of optical soliton propagation happens when they become stalled in the middle of their propagation through an optical fiber for transcontinental distances [1–5]. One possible reason this can happen is when the chromatic dispersion (CD) turns out to be nonlinear as opposed to being linear [6–9]. Fiber non-uniformities, randomness in the pulse injection from a laser, mishandling or even rough handling of fibers and/or various other sources could lead to such an unwanted feature. Thus, it is important to take a look at the mathematical issue about this and draw conclusions about such a feature of optical solitons [10–15].

The current paper studies the formation of a quiescent optical soliton for an optical fiber having cubic–quartic (CQ) form of self-phase modulation (SPM) with nonlinear CD. Such a nonlinear form of SPM was first reported during 2009, where the phase-portrait analysis for stationary solitons was carried out [16]. The current study is conducted in two phases. In the first round, the linear temporal evolution is taken into account and, subsequently, the generalized temporal evolution is considered. Thereafter, the CQ form of SPM is generalized.

The governing model is the nonlinear Schrödinger's equation (NLSE) with CQ form of SPM [16–18] and the generalized CQ form of SPM, both with nonlinear CD. It must be noted that the governing NLSE with nonlinear CD and CQ form of SPM has been studied in the past by the aid of extended Jacobi's elliptic function approach [7]. The current paper implements the enhanced Kudryashov's scheme for both forms of SPM and bright, dark, and singular solitons are yielded only for CQ form of SPM while for the generalized CQ form of SPM, stationary optical solitons cease to exist with linear, as well as with generalized temporal evolution. The results are derived and exhibited in the rest of the paper after a quick and succinct intro to the model, as well as the integration algorithm.

To step back, the model was first studied and reported during 2009 by Inui et al. The dynamical system of the model was constructed and the phase portrait analysis was exhaustively carried out. The corresponding fixed-point analysis was also carried out for the classified fixed points that were based on the parameter regimes. The stationary soliton solution assumption was taken into consideration. A version of the model was first proposed in [19] and was later studied by Inui et al. [16]. The current work implements enhanced Kudryashov's scheme to recover the stationary solitons directly without conducting the phase portrait analysis.

2. The Enhanced Kudryashov's Procedure

Let consider a model equation

$$G(u, u_x, u_t, u_{xt}, u_{xx}, \ldots) = 0. \tag{1}$$

Here, u is the dependent variable, while t and x are the time and space variables.

Step 1 : The model (1) decreases to

$$P(U, -\mu v U', \mu U', \mu^2 U'', \ldots) = 0, \tag{2}$$

by using the restrictions

$$\xi = \mu(x - vt), \quad u(x,t) = U(\xi), \tag{3}$$

where μ and v are constants.

Step 2: Equation (2) permits the solution form [1]

$$U(\xi) = \lambda_0 + \sum_{l=1}^{N} \sum_{i+j=l} \lambda_{ij} Q^i(\xi) R^j(\xi), \tag{4}$$

along with the simplest equations

$$Q'(\xi) = Q(\xi)(\eta Q(\xi) - 1), \tag{5}$$

and

$$R'(\xi)^2 = R(\xi)^2(1 - \chi R(\xi)^2), \tag{6}$$

where N comes from the balancing technique in (2) and λ_0, $\lambda_{ij}(i,j = 0,1,\ldots,N)$ are constants.

Step 3: Inserting (4) along with (5) and (6) into (2) paves way to the much-needed constants in (2)–(6).

Step 4: By the usage of the explicit solutions of (5) and (6)

$$R(\xi) = \frac{4a}{4a^2 e^{\xi} + \chi e^{-\xi}}, \tag{7}$$

and

$$Q(\xi) = \frac{1}{\eta + be^{\xi}}, \tag{8}$$

we arrive the analytical solutions of the model (1).

This integration algorithm as proposed by Kudryashov is very welcoming in the optics community. For $\chi = \pm 4a^2$, Equation (7) would yield bright or singular solitons. This direct approach, therefore, gives way to the much-needed bright solitons that is an appreciable welcome in the electronics and telecommunications industry. Similarly, Equation (8) would yield straddled solitons whose special cases would be bright and singular solitons as well. Thus, in addition for Equation (8), bright-singular straddled solitons emerge. This is an important feature that is not quite observable in other integration techniques.

3. Cubic–Quartic Nonlinearity

3.1. Linear Temporal Evolution

In this case, the model stands as [7,16]:

$$iq_t + a\left(|q|^n q\right)_{xx} + \left(b_1|q|^2 + b_2|q|^3\right)q = 0. \tag{9}$$

In (9), b_1 and b_2 come from the CQ form of SPM, while n depicts the nonlinearity parameter. t stands for the temporal coordinate and a stems from the nonlinear CD. $q(x,t)$ depicts the wave profile, while x stands for spatial coordinate. Lastly, the first term gives the temporal evolution, where $i = \sqrt{-1}$.

The model satisfies the wave form

$$q(x,t) = U(kx)e^{i(\omega t + \theta_0)}, \tag{10}$$

where θ_0 stems from the phase constant and ω stems from the wave number.

Plugging (10) into (9) causes to

$$ak^2(n+1)U^{n+1}U'' + ak^2 n(n+1)U^n U'^2 + b_2U^5 + b_1U^4 - U^2\omega = 0. \tag{11}$$

For integrability, one obtains $n = 1$ or $n = 2$. We consider $n = 2$, then Equation (11) becomes

$$3ak^2UU'' + 6ak^2U'^2 + b_2U^3 + b_1U^2 - \omega = 0. \tag{12}$$

By using the balancing technique in (12), one secures

$$U(\xi) = \lambda_0 + \lambda_{01}R(\xi) + \lambda_{10}Q(\xi) + \lambda_{11}R(\xi)Q(\xi) + \lambda_{02}R(\xi)^2 + \lambda_{20}Q(\xi)^2. \tag{13}$$

Inserting (13) along with (5) and (6) into (12), we retrieve the results:

Result 1:

$$\lambda_0 = \mp\sqrt{\frac{9\omega}{2b_1}},\ \lambda_{01} = \lambda_{10} = \lambda_{11} = \lambda_{20} = 0,\ \lambda_{02} = -\frac{3\lambda_0\chi}{2},\ k = \frac{1}{6}\sqrt{\frac{b_1}{a}}, b_2 = \pm\frac{7\sqrt{2}b_1^{\frac{3}{2}}}{27\sqrt{\omega}}. \tag{14}$$

Substituting (14) along with (7) into (13) yields

$$q(x,t) = \pm\sqrt{\frac{9\omega}{8b_1}}\left\{3\chi\left(\frac{4a\exp\left[\frac{1}{6}\sqrt{\frac{b_1}{a}}\,x\right]}{4a^2\exp\left[\frac{1}{3}\sqrt{\frac{b_1}{a}}\,x\right] + \chi}\right)^2 - 2\right\}\ e^{i(\omega t + \theta_0)}. \tag{15}$$

Setting $ab_1 > 0$ and $\chi = \pm 4a^2$ simplifies (15) to bright and singular solitons

$$q(x,t) = \pm\sqrt{\frac{9\omega}{8b_1}}\left(3\,\text{sech}^2\left[\frac{1}{6}\sqrt{\frac{b_1}{a}}\,x\right] - 2\right)e^{i(\omega t+\theta_0)}, \tag{16}$$

and

$$q(x,t) = \mp\sqrt{\frac{9\omega}{8b_1}}\left(3\,\text{csch}^2\left[\frac{1}{6}\sqrt{\frac{b_1}{a}}\,x\right] + 2\right)e^{i(\omega t+\theta_0)}. \tag{17}$$

Figure 1 depicts the plot of a quiescent bright soliton (16). The parameter values chosen are: $\omega = 1$, $b_1 = 1$ and $a = 1$.

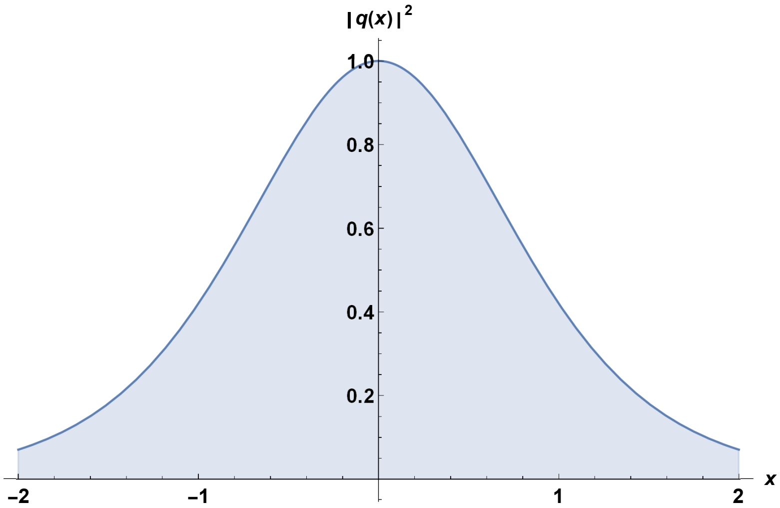

Figure 1. Profile of a quiescent bright soliton.

Result 2:

$$\lambda_0 = \pm\sqrt{\frac{9\omega}{2b_1}},\ \lambda_{01} = \lambda_{11} = \lambda_{02} = 0,\ \lambda_{20} = 6\eta^2\lambda_0,\ \lambda_{10} = -6\eta\lambda_0,\ k = \frac{1}{3}\sqrt{\frac{b_1}{a}},\ b_2 = \mp\frac{7\sqrt{2}b_1^{\frac{3}{2}}}{27\sqrt{\omega}}. \tag{18}$$

Putting (18) along with (8) into (13) gives

$$q(x,t) = \pm\sqrt{\frac{9\omega}{2b_1}}\left\{1 - \frac{6b\eta\exp\left[\frac{1}{3}\sqrt{\frac{b_1}{a}}\,x\right]}{\left(b\exp\left[\frac{1}{3}\sqrt{\frac{b_1}{a}}\,x\right] + \eta\right)^2}\right\}e^{i(\omega t+\theta_0)}. \tag{19}$$

Taking $ab_1 > 0$ and $\eta = \pm b$ changes (19) to dark and singular solitons

$$q(x,t) = \pm\sqrt{\frac{9\omega}{8b_1}}\left(3\,\tanh^2\left[\frac{1}{6}\sqrt{\frac{b_1}{a}}\,x\right] - 1\right)e^{i(\omega t+\theta_0)}, \tag{20}$$

and

$$q(x,t) = \pm\sqrt{\frac{9\omega}{8b_1}}\left(2 + 3\,\text{csch}^2\left[\frac{1}{6}\sqrt{\frac{b_1}{a}}\,x\right]\right)e^{i(\omega t+\theta_0)}. \tag{21}$$

Figure 2 depicts the plot of a quiescent dark soliton (20). The parameter values chosen are: $\omega = 1$, $b_1 = 1$, and $a = 1$.

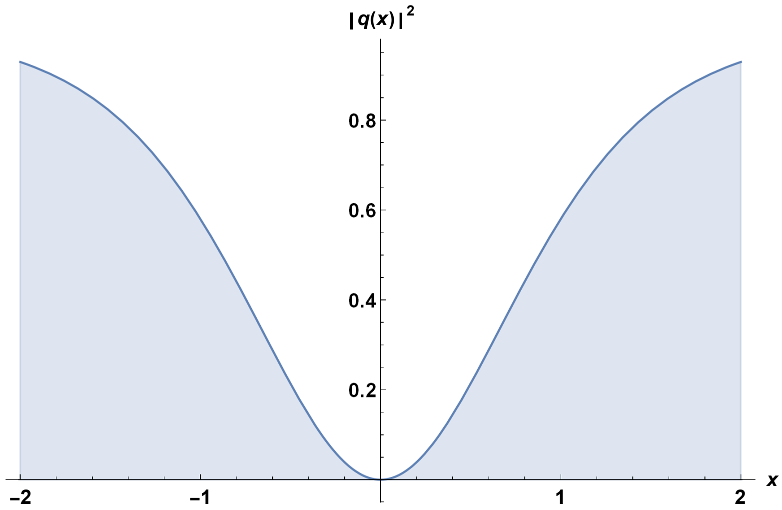

Figure 2. Profile of a quiescent dark soliton.

3.2. Generalized Temporal Evolution

Therefore, the model evolves as

$$i\left(q^l\right)_t + a\left(|q|^n q^l\right)_{xx} + \left(b_1|q|^2 + b_2|q|^3\right)q^l = 0, \tag{22}$$

where l enables us the generalized temporal evolution.

Inserting (10) into (22) leaves us with

$$ak^2\left(l^2 + l(2n-1) + (n-1)n\right)U^n U'^2 + ak^2(l+n)U^{n+1}U'' + b_1U^4 + b_2U^5 - lU^2\omega = 0. \tag{23}$$

For integrability, one acquires $n = 1$ or $n = 2$. We address $n = 2$, then Equation (23) turns into

$$ak^2(l+2)UU'' + ak^2(l+1)(l+2)U'^2 + b_2U^3 + b_1U^2 - l\omega = 0. \tag{24}$$

With the usage of the balancing approach in (24), one derives

$$U(\xi) = \lambda_0 + \lambda_{01}R(\xi) + \lambda_{10}Q(\xi) + \lambda_{11}R(\xi)Q(\xi) + \lambda_{02}R(\xi)^2 + \lambda_{20}Q(\xi)^2. \tag{25}$$

Putting (25) along with (5) and (6) into (24) enables us the results:

Result 3:

$$\begin{aligned} &\lambda_0 = \mp\sqrt{\frac{3l(l+2)\omega}{b_1(l+1)}}, \lambda_{01} = \lambda_{10} = \lambda_{11} = \lambda_{20} = 0, \lambda_{02} = -\frac{3\lambda_0\chi}{2}, \\ &k = \sqrt{\frac{b_1}{4a(l+2)^2}}, b_2 = \pm\frac{b_1 l(2l+5)\omega\sqrt{b_1(l+1)}}{3\sqrt{3}(l(l+2)\omega)^{\frac{3}{2}}}. \end{aligned} \tag{26}$$

Substituting (26) along with (7) into (25) provides us

$$q(x,t) = \pm\sqrt{\frac{3l(l+2)\omega}{4b_1(l+1)}}\left\{3\chi\left(\frac{4a\exp\left[\sqrt{\frac{b_1}{4a(l+2)^2}}\,x\right]}{4a^2\exp\left[\sqrt{\frac{b_1}{a(l+2)^2}}\,x\right] + \chi}\right)^2 - 2\right\}e^{i(\omega t+\theta_0)}. \tag{27}$$

Taking $ab_1 > 0$ and $\chi = \pm 4a^2$ decreases (27) to bright and singular solitons

$$q(x,t) = \pm\sqrt{\frac{3l(l+2)\omega}{4b_1(l+1)}}\left(3\,\mathrm{sech}^2\left[\sqrt{\frac{b_1}{4a(l+2)^2}}\,x\right] - 2\right)e^{i(\omega t+\theta_0)}, \tag{28}$$

and

$$q(x,t) = \mp\sqrt{\frac{3l(l+2)\omega}{4b_1(l+1)}}\left(3\,\mathrm{csch}^2\left[\sqrt{\frac{b_1}{4a(l+2)^2}}\,x\right] + 2\right)e^{i(\omega t+\theta_0)}. \tag{29}$$

Result 4:

$$\lambda_0 = \pm\sqrt{\frac{3l(l+2)\omega}{b_1(l+1)}},\ \lambda_1 = \lambda_{11} = \lambda_2 = 0,\ \lambda_{20} = 6\eta^2\lambda_0,\ \lambda_{10} = -6\eta\lambda_0,$$
$$k = \sqrt{\frac{b_1}{a(l+2)^2}},\ b_2 = \mp\frac{b_1 l(2l+5)\omega\sqrt{b_1(l+1)}}{3\sqrt{3}(l(l+2)\omega)^{\frac{3}{2}}}. \tag{30}$$

Plugging (30) along with (8) into (25) leaves us with

$$q(x,t) = \pm\sqrt{\frac{3l(l+2)\omega}{b_1(l+1)}}\left\{1 - \frac{6b\eta\exp\left[\sqrt{\frac{b_1}{a(l+2)^2}}\,x\right]}{\left(b\exp\left[\sqrt{\frac{b_1}{a(l+2)^2}}\,x\right]+\eta\right)^2}\right\}e^{i(\omega t+\theta_0)}. \tag{31}$$

Setting $ab_1 > 0$ and $\eta = \pm b$ translates (31) to dark and singular solitons

$$q(x,t) = \pm\sqrt{\frac{3l(l+2)\omega}{4b_1(l+1)}}\left(3\tanh^2\left[\sqrt{\frac{b_1}{4a(l+2)^2}}\,x\right] - 1\right)e^{i(\omega t+\theta_0)}, \tag{32}$$

and

$$q(x,t) = \pm\sqrt{\frac{3l(l+2)\omega}{4b_1(l+1)}}\left(2 + 3\,\mathrm{csch}^2\left[\sqrt{\frac{b_1}{4a(l+2)^2}}\,x\right]\right)e^{i(\omega t+\theta_0)}. \tag{33}$$

4. Generalized Cubic-Quartic Nonlinearity

4.1. Linear Temporal Evolution

Thus, the model sticks out as

$$iq_t + a\left(|q|^n q\right)_{xx} + \left(b_1|q|^{2m} + b_2|q|^{2m+1}\right)q = 0, \tag{34}$$

where b_1 and b_2 provide us the generalized CQ form of SPM.

Putting (10) into (34) paves way to

$$ak^2(n+1)U^{n+1}U''^2 n(n+1)U^n\left(U'\right)^2 + b_1U^{2m+2} + b_2U^{2m+3} - U^2\omega = 0. \tag{35}$$

For a non-trivial solution, we choose $n = 2m$, so (35) reads as

$$ak^2(2m+1)U^{2m+1}U''^2 m(2m+1)U^{2m}\left(U'\right)^2 + b_1U^{2m+2} + b_2U^{2m+3} - U^2\omega = 0. \tag{36}$$

If we try to use the constraint

$$U = V^{\frac{1}{m}}, \tag{37}$$

we arrive at

$$ak^2\left(2m^2+3m+1\right)\left(V'\right)^2 + ak^2m(2m+1)VV'' + b_2m^2V^{\frac{1}{m}+2} - m^2\left(\omega - b_1V^2\right) = 0. \tag{38}$$

For integrability, one needs to set $m = 1$. This shows that the integrable case is when we have the generalized CQ fom of SPM to collapse to CQ form of nonlinearity. Thus, with SPM being of the generalized CQ nonlinearity, the governing NLSE is not integrable!

4.2. Generalized Temporal Evolution

In this case, the model shapes up as

$$i\left(q^l\right)_t + a\left(|q|^n q^l\right)_{xx} + \left(b_1|q|^{2m} + b_2|q|^{2m+1}\right)q^l = 0. \tag{39}$$

Inserting (10) into (39) exposes

$$ak^2((l+n-1)(l+n))U^n\left(U'\right)^2 + ak^2(l+n)U^{n+1}U'' + b_1U^{2m+2} + b_2U^{2m+3} - lU^2\omega = 0. \tag{40}$$

For a non-trivial solution, we choose $n = 2m$, so (40) turns out to be:

$$ak^2(l+2m)U^{2m+1}U''^2((l+2m-1)(l+2m))U^{2m}\left(U'\right)^2 + b_1U^{2m+2} + b_2U^{2m+3} - lU^2\omega = 0. \tag{41}$$

The transformation

$$U = V^{\frac{1}{m}}, \tag{42}$$

leads to

$$ak^2\left(l^2 + 3lm + 2m^2\right)\left(V'\right)^2 + ak^2mV(l+2m)V''^2\left(l\omega - b_1V^2\right) + b_2m^2V^{\frac{1}{m}+2} = 0. \tag{43}$$

Again as in the previous case, setting $m = 1$ would lead to the integrable case. Thus, for generalized CQ nonlinearity for generalized temporal evolution, integrability is only possible for CQ form of SPM and not for its generalized kind.

5. Conclusions

The current paper recovers quiescent solitons that come with CQ form of SPM. The integration tool led to the extraction of stationary bright, dark and singular solitons. Subsequently, an attempt was made to recover such solitons for the NLSE with generalized CQ form of SPM. The integration algorithm failed to recover stationary solitons in this case for both linear temporal evolution, as well as generalized temporal evolution. Therefore, one can conclude that for generalized structure of CQ form of SPM, stationary solitons of any kind are not supported by the model; or perhaps, the addressed integration technique is a failure in such a situation. In future, one needs to take a look at additional integration algorithms that would make this retrieval possible. The hunt is under way and the results would be reported once success is achieved.

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