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Development and Simulation of Motion Control System for Small Satellites Formation

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Abstract: In the paper, the problem of forming and maintaining the small satellites formation in the near-earth projected circular orbits is considered. The satellite formation reconfiguration and formation-keeping control laws are proposed by employing the passivity-based output feedback control. For the complete nonlinear and time-dependent dynamics of the relative motion of a pair of satellites in elliptical orbits, new combined control algorithms, including a consensus protocol, are proposed and analyzed. A comparison of the control modes using the passivity-based output feedback control and the proportional-differential controller with and without the consensus algorithm is given. On the basis of the passification method, the algorithm is obtained ensuring the stable motion of the slave satellite relative to the orbit of the master satellite. To improve the accuracy of the satellites' positioning, a consensus protocol based on measurements of the relative positions of the satellites is proposed and studied. Computer simulations of the proposed algorithms for options to construct formations are provided for two projected circular orbits of 8 satellites, demonstrating the efficiency of the proposed control schemes. It is shown that the resulting passivity-based output feedback control provides better accuracy than the PD controller. It is also shown that the use of the consensus protocol further increases the positioning accuracy of the satellite constellation.

Keywords: satellite formation; keeping; control; passification; consensus; projected circular orbit

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1. Introduction

Space research has led to the emergence of the idea of spacecrafts' motion in a group. Group motion makes it possible to perform a wider range of tasks. This approach has reduced the costs of production and operation, and increased the reliability of the entire system. There are several satellite constellation formation methods for creating time-invariant and time-varying configurations. In the literature, various options to create satellite constellations are proposed, cf. [1–3]. The constant tuning method [4] implies periodic adjustment of active satellites relative to the spatial position. The variable formation method [5] forms a swarm, which periodically changes its configuration as satellites move in non-coplanar orbits. It is possible to form the configuration of satellites by means of their tether connection; such formations are possible in low orbits [6]. A grouping can be formed using several methods; this method of formation is called *combined* [2]. Among the groupings with a constant structure, a class stands out that uses projected circular orbits (PCO) [7] to form the structure. When using this construction method, a group is considered, which consists of the head and surrounding slave satellites [8]. The orbit of the master satellite is considered to be the reference orbit, and the remaining orbits are called projection circular orbits, which are circular when projected onto the local horizontal plane [9,10]. The first successful missions were based on the results for a group of two satellites [11]. Based on these results, a mission was implemented to study the formation of satellites in near-earth orbit. Two satellites, CanX-4 and CanX-5 [12], were launched. The satellites maintained a maximum controllable range of about 50 m. The satellites under study are classified as small. For these satellites, various options were proposed to solve the problem of building a permanent group structure. However, there are still problems associated with their control. One of them is the need to keep the satellites in a prescribed order. A number of studies were carried out to address this problem. As an example, in [13–15], various methods of adaptive control of the spacecraft formation were considered. In [16], the variable structure model reference adaptive control technique for spacecraft formation flying was proposed. Zou and Kumar [17] presented the fast terminal sliding manifold, using a control term based on the hyperbolic tangent function to suppress bounded external disturbances. The control system for such plants is designed to solve many problems. One of these tasks is to keep a constant satellite configuration. For this, the method of stabilizing only the unstable subsystem can be applied. In [18–20], a partial stabilization method with an algorithm to compensate for fluctuations was applied. These studies consider the stabilization of a linearly varying component caused by a zero root of multiplicity two. Additionally, a method to maintain satellite configuration employs the PCO.

In [21,22], the control of a formation of two satellites in a near-earth circular orbit is discussed. It is proposed to use adaptive neurocontrol. One of the vehicles is considered to be controllable, while the second is uncontrolled and moves in PCO orbit relative to the first satellite. To solve the adaptation problem, a non-gradient optimization method is used. Adaptation takes 2.5 to 5 days. This approach is time consuming to control a large group of satellites. In [5], a feedback control law was developed to regulate the relative motion of satellites in a swarm in a low circular near-earth orbit, taking into account the speed of data transmission over communication channels. The control law is based on classical modal control. In [23], algorithms were developed for decentralized stabilization of the position of two satellites moving in a near circular orbit using a modal approach, passification method, sliding mode control, linear and speed-optimal partial stabilization. In the study of a new control algorithm based on the passification method, convergence was obtained at a lower initial speed than the modal control algorithm. Ref. [24] discussed maintaining a working orbit and a given configuration. For the orbit of the lead satellite, rigid (absolute) support is used. For the slave satellite, rigid and flexible (relative) support is applied. The dependence of the costs of the total characteristic velocity to maintain the master and slave vehicles on the accuracy of maintaining the working orbit was investigated, which showed the particular importance of the correct choice of the maximum accuracy of maintaining. In [25], the problem of calculating the parameters of the maneuvers of the satellite system configuration was considered. For convenience, the orbits of a group of six satellites (STARSIS) are reduced to projection circular orbits. Ref. [26] complemented the task of calculating the parameters of satellite maneuvers by examining high-elliptic orbits for four satellites. The satellite array model is brought into the main orbital plane to simplify calculations. In [27], to solve the control problem, the influence of the given initial values of the projection circular orbit for different approaches to control of a group of satellites was investigated. The design takes into account the minimization of error and fuel consumption.

When performing tasks with a large number of satellites, it is more efficient to use multi-agent and decentralized control [28]. To this aim, in [4], using local relative dimensions was suggested. Another approach was used in the article [29]. It proposed a combined consensus method with a robust H_∞ control. Using this approach, it is possible to provide an arbitrary scalability of the system. Comparisons were made between various controllers [30]. In the course of the study, data were obtained, on the basis of which it was shown that the nonlinear controller based on the Riccati equations (*state-dependent Riccati equation*—SDRE) has better accuracy.

The paper is devoted to the problem of forming and maintaining the small satellites formation in the near-earth PCO orbits. On the basis of the passification method, an algorithm is obtained that ensures the stable motion of the slave satellite relative to the orbit of the master satellite. To improve the accuracy of satellites positioning, a consensus protocol based on measurements of the relative positions of satellites is proposed and studied.

In this paper, for the complete nonlinear and time-dependent dynamics of the relative motion of a pair of satellites in elliptical orbits, new combined control algorithms, including a consensus protocol, are proposed and analyzed. A comparison of the control modes using the PD controller with and without the consensus algorithm is given. The dynamic structure of the proposed control algorithm is quite simple and avoids the use of a large number of blocks. Its implementation is simple, and the computational cost is low. The new result of the paper is also a rigorous mathematical proof of the use of control laws based on the passification method for the satellite system under consideration.

The rest of the paper is organized as follows. Section 2 is devoted to modeling the satellite formation. The problem statement is given in Section 3. Section 4 deals with application of the passivity concept to the control law design. The notion of projection circular orbit is described in Section 5. Section 6 presents designing the control law for PCO constellation. The simulation results are given in Section 8. Concluding remarks in Section 9 finalize the paper.

2. Satellite Formation Modeling

In satellite formation flying, the relationship of the positions between satellites is very important. Hence, the rotating reference coordinate is used: a chief satellite is centrally located and circles the Earth. Note that it is assumed that the main satellite is located at the origin of the coordinate system because it provides a reference point for the formation. The local-vertical local-horizontal (LVLH) coordinate system [1] is used to describe the relative motion in the formation.

The coordinate systems are illustrated in Figure 1. In this figure, $\vec{r}_c$ is the position vector of the master satellite, $\vec{r}$ is the position vector of the slave satellite, and $\vec{\rho}$ is the relative position vector between the master and slave satellites. A general non-linear equation of relative motion is expressed as in [1]:

$$\begin{bmatrix} \ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x + \frac{\mu}{\gamma}x + \frac{\mu}{\gamma}r_c - \frac{\mu}{r_c^2} \\ \ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y + \frac{\mu}{\gamma}y \\ \ddot{z} + \frac{\mu}{\gamma}z \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \tag{1}$$

$$\ddot{r}_c = r_c\dot{\theta}^2 - \frac{\mu}{r_c^2}, \quad \ddot{\theta} = -\frac{2\dot{r}_c\dot{\theta}}{r_c} \tag{2}$$

where x, y and z are state variables to describe the relative position vector, $\vec{\rho}$, in the x, y and z axes, respectively, and a_x, a_y and a_z are the orbital perturbation terms, such as the aspherical geopotential perturbation, thrust, air drag, and solar radiation pressure. $a_j = (f_j + d_j)/m_f$ for all $j \in \{x, y, z\}$ with m_f denoting the mass of the deputy spacecraft and f_j denoting the control input applied by the deputy spacecraft, d_j is for the perturbation term, θ is latitude angle of the chief, r_c is the radius of the chief orbit and μ is a gravitational parameter. Finally, γ is defined as

$$\gamma \equiv |\vec{r}_c + \vec{\rho}|^3 = \left((r_c + x)^2 + y^2 + z^2\right)^{3/2}. \tag{3}$$

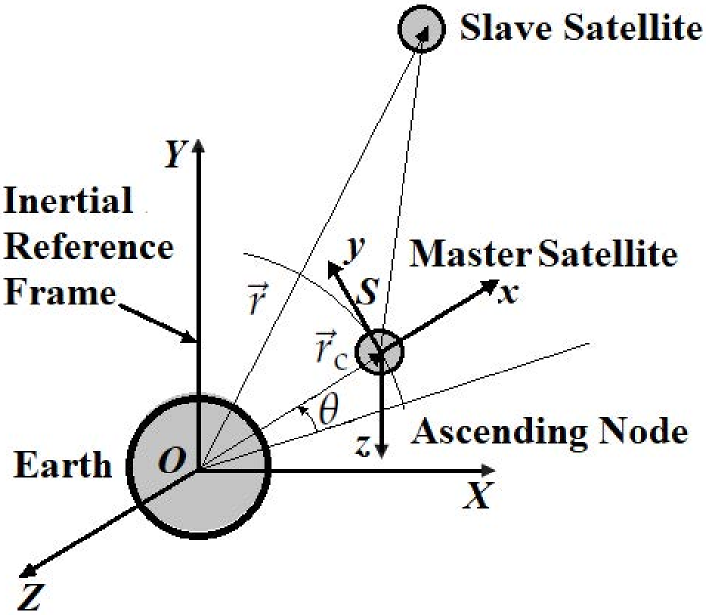


Figure 1. Reference systems.

For controller design, it is desirable to rewrite system (1) in the following state-space form

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x}, t)\mathbf{x} + \mathbf{B}\mathbf{u}, \tag{4}$$

where the state variables are the relative positions and relative velocities of the deputy satellite with regard to the chief satellite

$$\mathbf{x} = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{u} = \begin{bmatrix} f_x & f_y & f_z \end{bmatrix}^{\mathrm{T}}$$

It is assumed in this paper that all the system states are measured. Therefore, the full state feedback control can be implemented. Let us rewrite the term of $\left(\frac{\mu}{\gamma} r_{\mathrm{c}} - \frac{\mu}{r_{\mathrm{c}}^2}\right)$ in (1) to preserve the non-linearity as much as possible and to avoid a singularity [30]. Using (3), this term can be expressed in the following form:

$$\frac{\mu}{\gamma} r_{\mathrm{c}} - \frac{\mu}{r_{\mathrm{c}}{}^2} = \mu \left(\frac{r_{\mathrm{c}}}{\left(1 + \frac{2}{r_{\mathrm{c}}} x + \frac{x^2+y^2+z^2}{r_{\mathrm{c}}{}^2}\right)^{3/2}} \frac{1}{r_{\mathrm{c}}{}^3} - \frac{1}{r_{\mathrm{c}}{}^2} \right) \tag{5}$$

$$= \frac{\mu}{r_{\mathrm{c}}{}^2} \left(\left(1 - \left(-\frac{2}{r_{\mathrm{c}}} x - \frac{x^2 + y^2 + z^2}{r_{\mathrm{c}}^2} \right) \right)^{-\frac{3}{2}} - 1 \right) \tag{6}$$

Let us define

$$\xi \equiv -\frac{2}{r_{\mathrm{c}}} x - \frac{x^2 + y^2 + z^2}{r_{\mathrm{c}}{}^2} = \left(-\frac{2}{r_{\mathrm{c}}} - \frac{x}{r_{\mathrm{c}}{}^2} \right) x + \left(-\frac{y}{r_{\mathrm{c}}{}^2} \right) y + \left(-\frac{z}{r_{\mathrm{c}}{}^2} \right) z \tag{7}$$

Then, by negative binomial series and (5), (7) becomes:

$$\frac{\mu}{\gamma} r_{\mathrm{c}} - \frac{\mu}{r_{\mathrm{c}}{}^2} = \frac{\mu}{r_{\mathrm{c}}{}^2} \left((1 - \xi)^{-\frac{3}{2}} - 1 \right) = \frac{\mu}{r_{\mathrm{c}}{}^2} \left(1 + \frac{3}{2} \Psi \xi - 1 \right) = \frac{3}{2} \frac{\mu}{r_{\mathrm{c}}{}^2} \Psi \xi \tag{8}$$

where Ψ is defined as a series $\Psi \equiv 1 + \psi_1 + \psi_2 + \cdots$, where:

$$\psi_1 \equiv \frac{(\frac{3}{2}+1)}{2}\xi, \quad \psi_2 \equiv \frac{(\frac{3}{2}+2)}{3}\psi_1\xi, \quad \psi_3 \equiv \frac{(\frac{3}{2}+3)}{4}\psi_2\xi, \quad \ldots$$

Consequently, (1) becomes:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \dot{\theta}^2 - \frac{\mu}{\gamma} + \frac{3}{2}\frac{\mu}{r_c^3}\left(2 + \frac{x}{r_c}\right)\Psi & 0 & \ddot{\theta} + \frac{3}{2}\frac{\mu}{r_c^4}y\Psi & 2\dot{\theta} & \frac{3}{2}\frac{\mu}{r_c^4}z\Psi & 0 \\ -\ddot{\theta} & -2\dot{\theta} & \dot{\theta}^2 - \frac{\mu}{\gamma} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\mu}{\gamma} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_f} & 0 & 0 \\ 0 & \frac{1}{m_f} & 0 \\ 0 & 0 & \frac{1}{m_f} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \tag{9}$$

where state variables x_1, x_2, x_3, x_4, x_5 and x_6 are x, $\dot{x}$, y, $\dot{y}$, z and $\dot{z}$ in the LVLH coordinates, respectively. Equation (9) has the form of (4).

3. Problem Statement

The following problem is posed. Let the master satellite move freely along its trajectory around the Earth, and let the slave satellite move along a given trajectory in relative coordinates $x - y - z$. Let the trajectory be given by the functions:

$$\mathbf{x}_d(t) = \begin{bmatrix} x_d(t) \\ y_d(t) \\ z_d(t) \end{bmatrix} \quad \dot{\mathbf{x}}(\mathbf{t})_d = \begin{bmatrix} \dot{x}_d(t) \\ \dot{y}_d(t) \\ \dot{z}_d(t) \end{bmatrix} \quad \ddot{\mathbf{x}}_d(t) = \begin{bmatrix} \ddot{x}_d(t) \\ \ddot{y}_d(t) \\ \ddot{z}_d(t) \end{bmatrix}. \tag{10}$$

Let us introduce the following error vector:

$$\mathbf{e} \triangleq \mathbf{x} - \mathbf{x}_d. \tag{11}$$

$$\mathbf{e} = \begin{bmatrix} e_x & e_y & e_z & \dot{e}_x & \dot{e}_y & \dot{e}_z \end{bmatrix}^{\mathrm{T}}.$$

The following control aim is taken:

$$\|\mathbf{e}\| \to 0 \text{ as } t \to \infty \tag{12}$$

where $\|\cdot\|$ corresponds to the standard L^2 norm. Let $\mathbf{x}_{\mathrm{d}}(t)$ be a stable trajectory for (9). Using (11), let us make the variable substitution in (9) [31,32], which has the structure described by (4). This leads to the following expression

$$\dot{\mathbf{e}} = \mathbf{A}(\mathbf{e} + \mathbf{x}_{\mathrm{d}}, t)\mathbf{e} + \mathbf{B}(\mathbf{u} - \ddot{\mathbf{x}}_{\mathrm{d}}) \tag{13}$$

Let us make the changes: $\bar{\mathbf{A}}(\mathbf{e}, t) = \mathbf{A}(\mathbf{e} + \mathbf{x}_{\mathrm{d}}, t)$ and $\bar{\mathbf{u}} = \mathbf{u} - \ddot{\mathbf{x}}_d$. Note that the structure of the matrix $\mathbf{A}$ in (9) remains the same. Then system (13) takes the form:

$$\dot{\mathbf{e}} = \bar{\mathbf{A}}(\mathbf{e}, t)\mathbf{e} + \mathbf{B}\bar{\mathbf{u}} \tag{14}$$

4. Control Law Design Based on Passivity Concept

Let us introduce additional system outputs (14) as

$$\mathbf{y} = \begin{bmatrix} \alpha_1 e_x + \dot{e}_x & \alpha_2 e_y + \dot{e}_y & \alpha_3 e_z + \dot{e}_z \end{bmatrix}^{\mathrm{T}}, \tag{15}$$

where $\alpha_1, \alpha_2, \alpha_3$ is a known positive-definite scaling gains. Now the system (14) takes the following general form:

$$\dot{\mathbf{e}} = \bar{\mathbf{A}}(\mathbf{e}, t)\mathbf{e} + \mathbf{B}\bar{\mathbf{u}}, \quad \mathbf{y} = \mathbf{C}\mathbf{e} \tag{16}$$

where the matrices are as follows:

$$\bar{\mathbf{A}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \dot{\theta}^2 - \frac{\mu}{\gamma} + \frac{3}{2}\frac{\mu}{r_c^3}\left(2 + \frac{x}{r_c}\right)\Psi & 0 & \ddot{\theta} + \frac{3}{2}\frac{\mu}{r_c^4} y\Psi & 2\dot{\theta} & \frac{3}{2}\frac{\mu}{r_c^4} z\Psi & 0 \\ -\ddot{\theta} & -2\dot{\theta} & \dot{\theta}^2 - \frac{\mu}{\gamma} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\mu}{\gamma} & 0 \end{bmatrix} \tag{17}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{m_f} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{m_f} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{m_f} \end{bmatrix}^T, \quad \mathbf{C} = \begin{bmatrix} \alpha_1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 1 & 0 \\ 0 & 0 & \alpha_3 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

Let us show that the outputs (15) passify [33] system (16). To do this, it is necessary to show that the zero dynamics [31] of system (16) are uniformly asymptotically stable. To isolate the zero dynamics, it is necessary to find the constant matrices $\mathbf{M} \in \mathbb{R}^{6\times 3}$ and $\mathbf{N} \in \mathbb{R}^{3\times 6}$ which satisfy the relations [34]

$$\mathbf{CM} = \mathbf{0}_3 \quad \mathbf{NB} = \mathbf{0}_3 \quad \mathbf{NM} = \mathbf{I}_3 \tag{19}$$

Then, the equation of zero dynamics is obtained in the form:

$$\dot{\eta} = \mathbf{A}_\eta(\mathbf{e})\eta \quad \in \mathbb{R}^3, \tag{20}$$

where $\mathbf{A}_\eta(\mathbf{e}) \in \mathbb{R}^{3\times 3}$ and is calculated by the formula:

$$\mathbf{A}_\eta(\mathbf{e}) = N\bar{\mathbf{A}}(\mathbf{e}, t)M \tag{21}$$

Matrices $\mathbf{M}$ and $\mathbf{N}$ satisfying conditions (19) have the form [13]:

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_1 & 0 & 0 \\ 0 & -\alpha_2 & 0 \\ 0 & 0 & -\alpha_3 \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

Then because of (21), one obtains

$$\mathbf{A}_\eta = \begin{bmatrix} -\alpha_1 & 0 & 0 \\ 0 & -\alpha_2 & 0 \\ 0 & 0 & -\alpha_3 \end{bmatrix} \tag{23}$$

Considering that the coefficients $\alpha_1, \alpha_2, \alpha_3$ are positive, it can be argued that the system (20) is uniformly asymptotically stable. This immediately implies [33] that system (16) is strictly the minimal phase. Additionally, let us check the result of the product of matrices **CB**:

$$\mathbf{CB} = \begin{bmatrix} \alpha_1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 1 & 0 \\ 0 & 0 & \alpha_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_f} & 0 & 0 \\ 0 & \frac{1}{m_f} & 0 \\ 0 & 0 & \frac{1}{m_f} \end{bmatrix} = \frac{1}{m_c}\mathbf{I}_3 \tag{24}$$

We obtain the result that **CB** is a symmetric positive definite matrix. Then, from the results of [34], it follows that there exists a constant matrix of feedback gains **K** in outputs **y**, which makes the system (16) asymptotically stable, and control aim (12) is achieved. The passification-based control law has a form

$$\bar{\mathbf{u}}(t) = -\mathbf{K}\mathbf{y}(t) \tag{25}$$

where matrix $\mathbf{K} \in \mathbb{R}^{3\times 3}$.

5. Projection Circular Orbits

When the master satellite moves in a circular Earth orbit, its angular acceleration is equal to zero [1]. In this case, it is valid for (1) that $\ddot{\theta}y = \ddot{\theta}x = 0$, $r_c = \text{const}$, $\dot{\theta} = \text{const} = n$, where n is the average angular velocity of the master satellite, and Equations (1) and (2) are of the following form [35]:

$$\begin{aligned} &\ddot{x} - 2n\dot{y} - n^2 x = -\frac{\mu(r_c + x)}{\gamma} + \frac{\mu}{r_c^2} + a_x, \\ &\ddot{y} + 2n\dot{x} - n^2 y = -\frac{\mu y}{\gamma} + a_y, \\ &\ddot{z} = -\frac{\mu z}{\gamma} + a_z. \end{aligned} \tag{26}$$

Linearized equations of (26) are called the Hill–Clohessy–Wiltshire (HCW) equations [1,36]:

$$\begin{aligned} &\ddot{x} - 2n\dot{y} - 3n^2 x = a_x, \\ &\ddot{y} + 2n\dot{x} = a_y, \\ &\ddot{z} + n^2 z = a_z. \end{aligned} \tag{27}$$

The HCW equations have the following limited periodic solutions [9]:

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} (c_1/2)\sin(nt + \phi) \\ c_1\cos(nt + \phi) + c_3 \\ c_2\sin(nt + \phi) \end{bmatrix}, \tag{28}$$

where x, y and z are the coordinates of the relative motion, ϕ is the phase angle in the plane between the master satellite and the slave satellite (the initial phase angle is determined when the equator of the l master satellite's in the locally horizontal plane yz) (the initial phase angle is defined, at the time of the equator crossing of the master satellite, in the local horizon yz plane). Constants c_1, c_2, c_3, α_0 are determined by the initial conditions as follows:

- The choice $c_1 = c_2 = \rho$, $c_3 = 0$ sets the configuration in which the slave satellite moves around the master satellite so that the projection of its movement on the local horizontal plane is the circle $y^2 + z^2 = \rho^2$, where ρ is a constant. Such an orbit is called a projection circular orbit (see Figure 2a).

- Selection $c_1 = \rho$, $c_2 = \dfrac{\sqrt{3}}{2}$, $c_3 = 0$ sets the configuration in which the slave satellite moves around the master satellite in circles in the axes x, y and z of the local coordinate system: $x^2 + y^2 + z^2 = \rho^2$. This orbit is called the general circular orbit (GCO).
- Choice $c_1 = c_2 = 0$, $c_3 = \rho$ sets the master–slave or along-track orbit (ATO) configuration, where the slave satellite follows the master satellite along its orbit with a constant offset ρ (see Figure 2b).

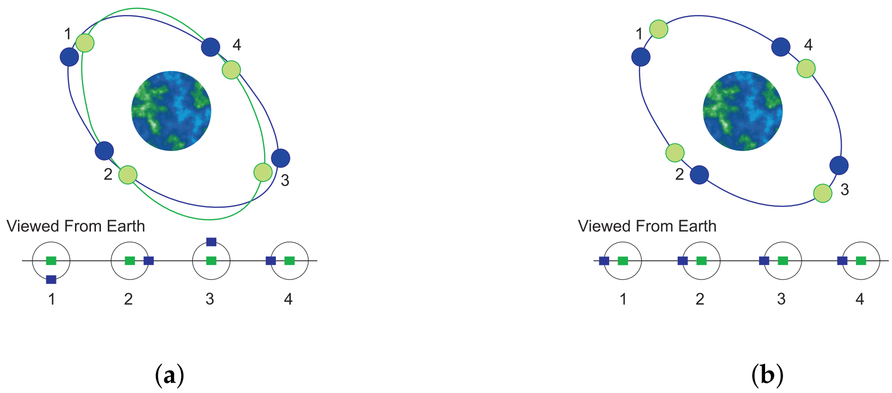


Figure 2. Configurations of satellite orbits. (**a**) Projected circular orbit. (**b**) Along-track orbit.

Substituting the values $c_1 = c_2 = r_{dpc}$, $c_3 = 0$ into equations (28), one obtains the following PCO equation:

$$\begin{bmatrix} x_d(t) \\ y_d(t) \\ z_d(t) \end{bmatrix} = \begin{bmatrix} \left(r_{\text{dpc}}/2\right)\sin(nt+\phi) \\ r_{\text{dpc}}\cos(nt+\phi) \\ r_{\text{dpc}}\sin(nt+\phi) \end{bmatrix}, \tag{29}$$

where r_{dpc} is the projected circular formation size, ϕ is the in-plane phase angle between the master and the slave satellites, and n is the mean angular velocity, $n = \sqrt{\mu/a_c^3}$, where a_c is the semi-major axis of the master satellite [1].

6. Control of PCO Constellation

Let the following aim of controlling a constellation of satellites be posed: there is a master satellite that moves in an elliptical orbit, but with a small eccentricity, i.e., its orbit is close to a circular orbit. It is necessary to create and maintain a certain constellation of satellites, which are located in a certain formation relative to the master satellite and keep this formation over time. In the case of a circular master satellite orbit, the most fuel-efficient slave satellite orbits would be general circular orbit (GCO), PCO, and along-track orbit (ATO) since such orbits are stable solutions of the HCW (27) equations. Then, control will be spent only on counteracting disturbances and countering unaccounted nonlinearities in the model (27).

The most interesting is the problem of positioning and maintaining the orbits of a constellation of satellites in the relative orbits of the PCO, even in the case of the master satellite's elliptical orbits. It is also interesting to investigate the possibility of simultaneously storing a formation of a large number of satellites in different PCO orbits. In this variant set, several orbits have different r_{dpc} in (29), and on the circles, the satellites are located with different offsets ϕ so that a symmetric distribution of satellites is obtained.

To maintain the slave satellite position in the formation, it should use control law in the form of (25) where expressions (29) are used as the target path $\mathbf{x}_d(t)$. Configurable parameters are matrix $\mathbf{K} \in \mathbb{R}^{3\times3}$ and vector $(\alpha_1, \alpha_2, \alpha_3)$. A special case of the control law (25) is proportional-differential (PD) controllers of the following form:

$$\begin{aligned} \bar{u}_x &= -K_{px}e_x - K_{dx}\dot{e}_x \\ \bar{u}_y &= -K_{py}e_y - K_{dy}\dot{e}_y \\ \bar{u}_z &= -K_{pz}e_z - K_{dz}\dot{e}_z \end{aligned} \tag{30}$$

where K_{px}, K_{dx}, K_{py}, K_{dy}, K_{pz}, K_{dz} are the positive adjustable gains. In the vector notation (30) can be written as

$$\bar{\mathbf{u}} = -\mathbf{K_{pd}}\mathbf{e} \tag{31}$$

where matrix $\mathbf{K_{pd}} \in \mathbb{R}^{3\times6}$ has the following form

$$\mathbf{K_{pd}} = \begin{bmatrix} K_{px} & 0 & 0 & K_{dx} & 0 & 0 \\ 0 & K_{py} & 0 & 0 & K_{dy} & 0 \\ 0 & 0 & K_{pz} & 0 & 0 & K_{dz} \end{bmatrix}. \tag{32}$$

7. Multi-Agent Control and Consensus Algorithm

The obtained control laws (25) and (31) are applied separately to each satellite of the constellation and use information about the relative position in satellite form. To improve the accuracy of the formation positioning, you can apply multi-agent control based on the consensus algorithm (protocol) [37,38].

7.1. Basic Information on Consensus Algorithm

Graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = 1, \ldots, n$ is a set of nodes (agents), $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is a set of edges in which each edge is represented by an ordered pair of different nodes. Edge (i, j) shows that node i is a neighboring to node j, and node j can receive information from node i. A graph is called *undirected* if for each $(i, j) \in \mathcal{E}$, $(j, i) \in \mathcal{E}$. The path from node i_1 to node i_l is a sequence of ordered edges of the form (i_k, i_{k+1}), $k = 1, \ldots, l-1$. An undirected graph is connected if for any $i \in \mathcal{V}$, there are paths to all other nodes.

Let graph $\mathcal{G}$ contain n nodes. The compatibility matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is defined as $a_{ii} = 0, a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, and 0 otherwise. The Laplace matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ is defined as $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$.

Let us assume that communication graph $\mathcal{G}$ is undirected and connected (some examples of undirected connected graphs are depicted in Figure 3). Assume that $\xi_i \in \mathbb{R}$ and $\zeta_i \in \mathbb{R}$ contain information about the state of the i-th agent. For information states with second-order dynamics, the following fundamental second-order consensus algorithm was proposed by Ren and Atkins in [39]:

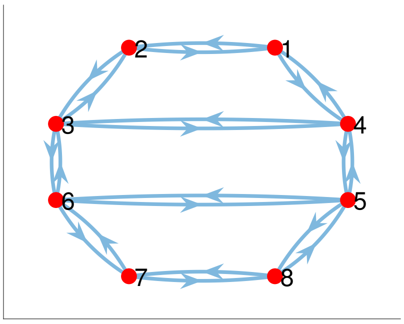
$$\begin{aligned} \dot{\xi}_i &= \zeta_i \\ \dot{\zeta}_i &= u_i \end{aligned} \tag{33}$$

where $u_i \in \mathbb{R}$ has the following form:

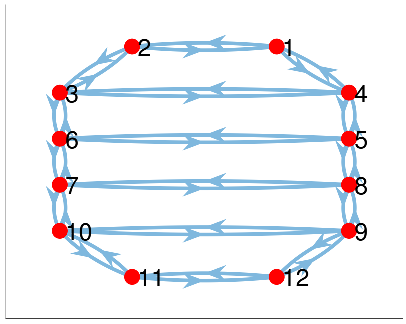
$$u_i = -\sum_{j=1}^{n} a_{ij}\big(\gamma_0(\xi_i - \xi_j) + \gamma_1(\zeta_i - \zeta_j)\big), \tag{34}$$

where $\gamma_0 > 0, \gamma_1 > 0$.

For consensus algorithms (33) and (34), it is assumed that consensus is achieved asymptotically among several agents if for any $\xi_i(0)$ and $\zeta_i(0)$ it is valid that $\|\xi_i(t) - \xi_j(t)\| \to 0$ and $\|\zeta_i(t) - \zeta_j(t)\| \to 0$, for all $i \neq j$ as $t \to \infty$. In the case when $\dot{\xi}_i$ and $\dot{\zeta}_i$ represent the moving agent position and velocity, expression (34) defines its acceleration.



(**a**) Communication graph of 8 agents



(**b**) Communication graph of 12 agents

Figure 3. Examples of communication graphs of 8 and 12 agents.

Denoting $\xi = [\xi_1, \ldots, \xi_n]^{\mathrm{T}}$ and $\zeta = [\zeta_1, \ldots, \zeta_n]^{\mathrm{T}}$, one can write the system model in the following vector form

$$\left[\begin{array}{c} \dot{\xi} \\ \dot{\zeta} \end{array} \right] = \Gamma \left[\begin{array}{c} \xi \\ \zeta \end{array} \right] \tag{35}$$

where

$$\Gamma = \left[\begin{array}{cc} 0_{n \times n} & I_n \\ -\gamma_0 \mathcal{L} & -\gamma_1 \mathcal{L} \end{array} \right] \tag{36}$$

In the vector form, the control law is as

$$U_c = -[\gamma_0 \mathcal{L} \xi + \gamma_1 \mathcal{L} \zeta] \tag{37}$$

where $U_c \in \mathbb{R}^n$.

7.2. Consensus-Based Satellite Formation Control Law

The multi-agent satellite formation control law can be obtained by combining expressions (30) and (34). It should be noted that (34) is a control law for a single coordinate. Let us introduce the following notation: n is a number of agents in the formation (number of satellites); $i = 1, \ldots, n$ is the agent number; $\mathbf{e}_i = \left[\begin{array}{cccccc} e_{xi} & e_{yi} & e_{zi} & \dot{e}_{xi} & \dot{e}_{yi} & \dot{e}_{zi} \end{array} \right]^{\mathrm{T}}$ is the error vector of i-th agent. Then the control law of agent i can be written as:

$$\begin{aligned} \bar{u}_{xi} &= -K_{px} e_{xi} - K_{dx} \dot{e}_{xi} - \sum_{j=1}^{n} a_{ij} \big(\gamma_0 \big(e_{xi} - e_{xj} \big) + \gamma_1 \big(\dot{e}_{xi} - \dot{e}_{xj} \big) \big), \\ \bar{u}_{yi} &= -K_{py} e_{yi} - K_{dy} \dot{e}_{yi} - \sum_{j=1}^{n} a_{ij} \big(\gamma_0 \big(e_{yi} - e_{yj} \big) + \gamma_1 \big(\dot{e}_{yi} - \dot{e}_{yj} \big) \big), \\ \bar{u}_{zi} &= -K_{pz} e_{zi} - K_{dz} \dot{e}_{zi} - \sum_{j=1}^{n} a_{ij} \big(\gamma_0 \big(e_{zi} - e_{zj} \big) + \gamma_1 \big(\dot{e}_{zi} - \dot{e}_{zj} \big) \big). \end{aligned} \tag{38}$$

Another variant of multi-agent control of the formation of satellites can be obtained based on (25). To do this, let us introduce the matrix of coefficients $\mathbf{K} = \left[\mathbf{k}_x \quad \mathbf{k}_y \quad \mathbf{k}_z \right]^{\mathrm{T}}$, where $\mathbf{k}_x, \mathbf{k}_y, \mathbf{k}_z \in \mathbb{R}^{1 \times 3}$, and vector $\mathbf{y} = \left[\begin{array}{ccc} y_x & y_y & y_z \end{array} \right]^{\mathrm{T}}$. In new notations, (25) can be written as

$$\begin{aligned} \bar{u}_x &= -\mathbf{k}_x \mathbf{y}, \\ \bar{u}_y &= -\mathbf{k}_x \mathbf{y}, \\ \bar{u}_z &= -\mathbf{k}_x \mathbf{y}. \end{aligned} \tag{39}$$

Expressions (39) represent control vector components for a single agent. For the multi-agent control case, it is possible to employ the consensus algorithm for first-order systems since the combined output includes derivatives, and the zero-dynamics [31] of the system is stable. Then the control law based on the passivity of i-th agent can be written as:

$$\begin{aligned} \bar{u}_{xi} &= -\mathbf{k}_x \mathbf{y}_i - \gamma_0 \sum_{j=1}^{n} a_{ij}(y_{xi} - y_{xj}), \\ \bar{u}_{yi} &= -\mathbf{k}_y \mathbf{y}_i - \gamma_0 \sum_{j=1}^{n} a_{ij}(y_{yi} - y_{yj}), \\ \bar{u}_{zi} &= -\mathbf{k}_z \mathbf{y}_i - \gamma_0 \sum_{j=1}^{n} a_{ij}(y_{zi} - y_{zj}). \end{aligned} \tag{40}$$

8. Simulation Results

For examining the developed control systems dynamics, a series of simulations was carried out in the MATLAB/Simulink software framework. The system considered was (1), (2) and four control laws, (25), (30), (38) and (40). In what follows, passification-based control (25) law control is denoted as PB law, law (30) is denoted as PD law, (38) as proportional-differential-consensus (PDC) law and (40) as passification-based consensus (PBC) law. One situation was simulated for all control laws: 8 satellites in two PCO orbits.

The leader's satellite orbit was taken to be the same for all simulation runs. The leader's orbit parameters are as follows: $m_F = 10$ kg; $\mu_e = 398.600$ km$^3 \cdot$ s^{-2}; perigee radius $R_p = 6971$ km; eccentricity $e = 0.2$; $\Omega, \omega = i = M = 0$ rad/s. Disturbances $d_j(t)$ for $j = \{x, y, z\}$, acting on (1), (2), and including gravitational perturbations J_2, atmospheric resistance and solar radiation pressure perturbation forces were taken into account. The following disturbances model of [7,36] was adopted:

$$\begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} = 1.2 \times 10^{-3} \begin{bmatrix} 1 - 1.5\sin(nt) \\ 0.5\sin(2nt) \\ \sin(nt) \end{bmatrix}. \tag{41}$$

It is worth mentioning that model (41) gives the disturbances magnitudes which are slightly higher than the actual ones under the similar conditions.

8.1. Simulation for Motion of 8 Satellites in Two PCO Orbits

The radii of PCO orbits of 1000 m and 2000 m were set for the simulations. Each orbit contains four satellites. The phase shift between the agents located in the first orbit is $\pi/2$. The phase shift between small spacecraft located on the first orbit is as $\pi/2$, the second four, relative to the first $\pi/4$. The initial positions for 8 satellites are shown in Table 1.

8.2. Simulation the PD Laws and PDC Control Laws

The coefficients for the PD laws and PDC control laws (in SI units) are $K_{p_x} = K_{p_y} = K_{p_z} = 0.025$, $K_{d_x} = K_{d_y} = K_{d_z} = 15$. They are obtained by the trial-and-error method. The criterion is the positioning accuracy after two orbits around the Earth. For PDC, the consensus algorithm coefficients $\gamma_0 = 0.01$ and $\gamma_1 = 0.04$ were additionally set for the proportional and differential components, respectively. These coefficients are the same for the x, y, z channels of all 8 satellites. The communication graph is plotted in Figure 3a.

The simulation results for the flight of a group of 8 satellites with PD control are depicted in Figures 4 and 5. The time histories of errors and control signals along x, y, z axes are shown in Figure 6. Figures 7 and 8 demonstrate the simulation results of the motion of a group of 8 satellites with PDC control. The time histories of errors and control signals along x, y, z axes are shown in Figure 9.

Comparison of simulation results for PD and PDC laws is given in Table 2.

8.3. Simulation the PB Laws and PBC Control Laws

The coefficients for the PB laws and PBC control laws (in SI units) are $\alpha_1 = \alpha_2 = \alpha_3 = 0.0025$, $\mathbf{k}_x = \begin{bmatrix} 15 & 1 & 1 \end{bmatrix}$, $\mathbf{k}_x = \begin{bmatrix} 1 & 15 & 1 \end{bmatrix}$, $\mathbf{k}_x = \begin{bmatrix} 1 & 1 & 15 \end{bmatrix}$.

They are obtained by the trial-and-error method. The criterion is the positioning accuracy after two orbits around the Earth. For PBC, the consensus algorithm coefficients $\gamma_0 = 15.5$. These coefficients are the same for the x, y, z channels of all 8 satellites. The communication graph is plotted in Figure 3a. Figures 10 and 11 show the simulation results of the flight of a group of 8 satellites with PB control. The time histories of errors and control signals along the axes x, y, z are depicted in Figure 12. Figures 13 and 14 show the simulation results of the flight of a group of 8 satellites with PBC control. Time histories of errors and control signals along the axes x, y, z are shown in Figure 15.

Comparison of simulation results for PB and PBC algorithms for 8 satellites is demonstrated in Table 2.

Table 1. Initial conditions for 8 satellites.

Satellite #	1	2	3	4	5	6	7	8
X_x m	1000	666.7	−1000	−500	1000	−250	−666.7	500
X_y m	2000	−2000	−2000	1000	2000	−3000	−1000	2000
X_z m	2000	2000	−2000	−2000	4000	4000	−4000	−4000
$\dot{X}_x$ m/s	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
$\dot{X}_y$ m/s	0.0002	0.0002	−0.0002	0.0002	−0.0002	0.0004	−0.0002	−0.0005
$\dot{X}_z$ m/s	0.0008	−0.0008	−0.0008	0.0008	0.0008	−0.0008	0.0008	−0.0008

Table 2. Position tracking errors for formation of 8 satellites.

Mode	e_x, cm	e_y, cm	e_z, cm
PD	33	12	31
PDC	21	7.5	20
PB	23	9	23
PBC	11	4	9

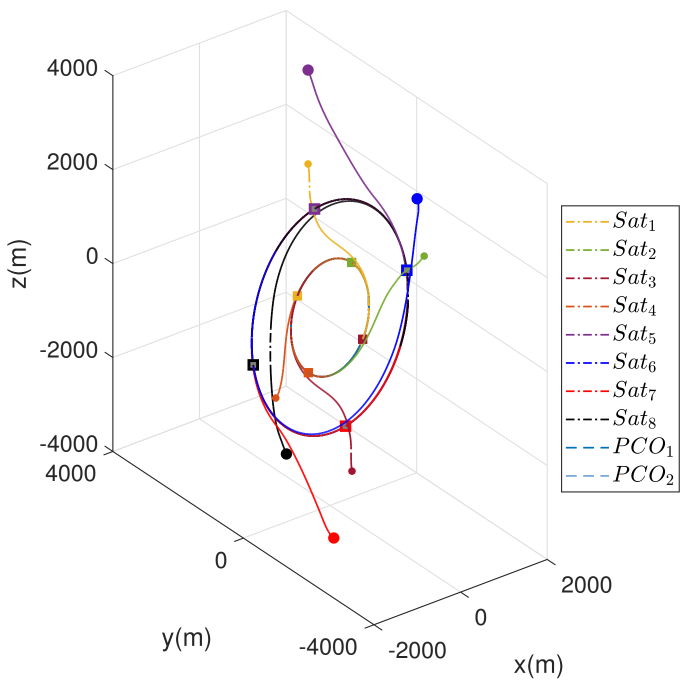


Figure 4. PD law: Plot of 3D trajectory.

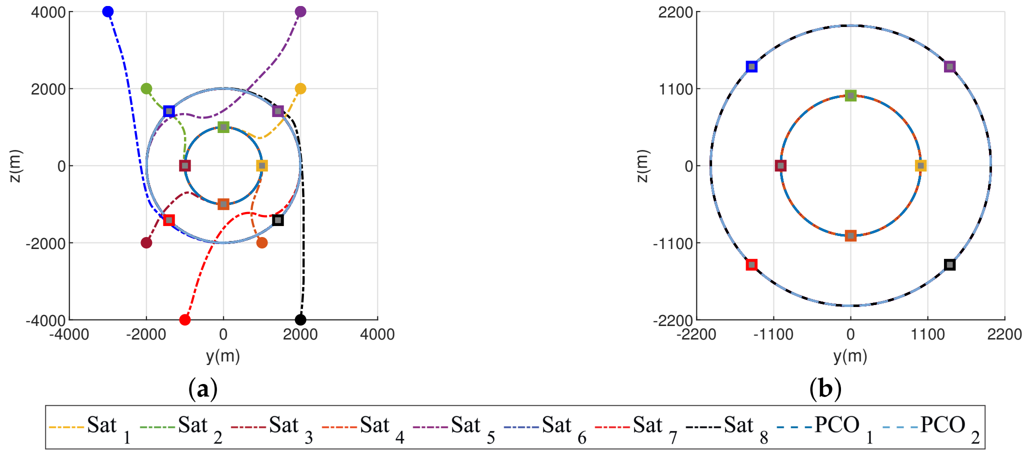


Figure 5. PD law. Trajectories plots: (**a**) on plane YZ; (**b**) on plane YZ after orbiting.

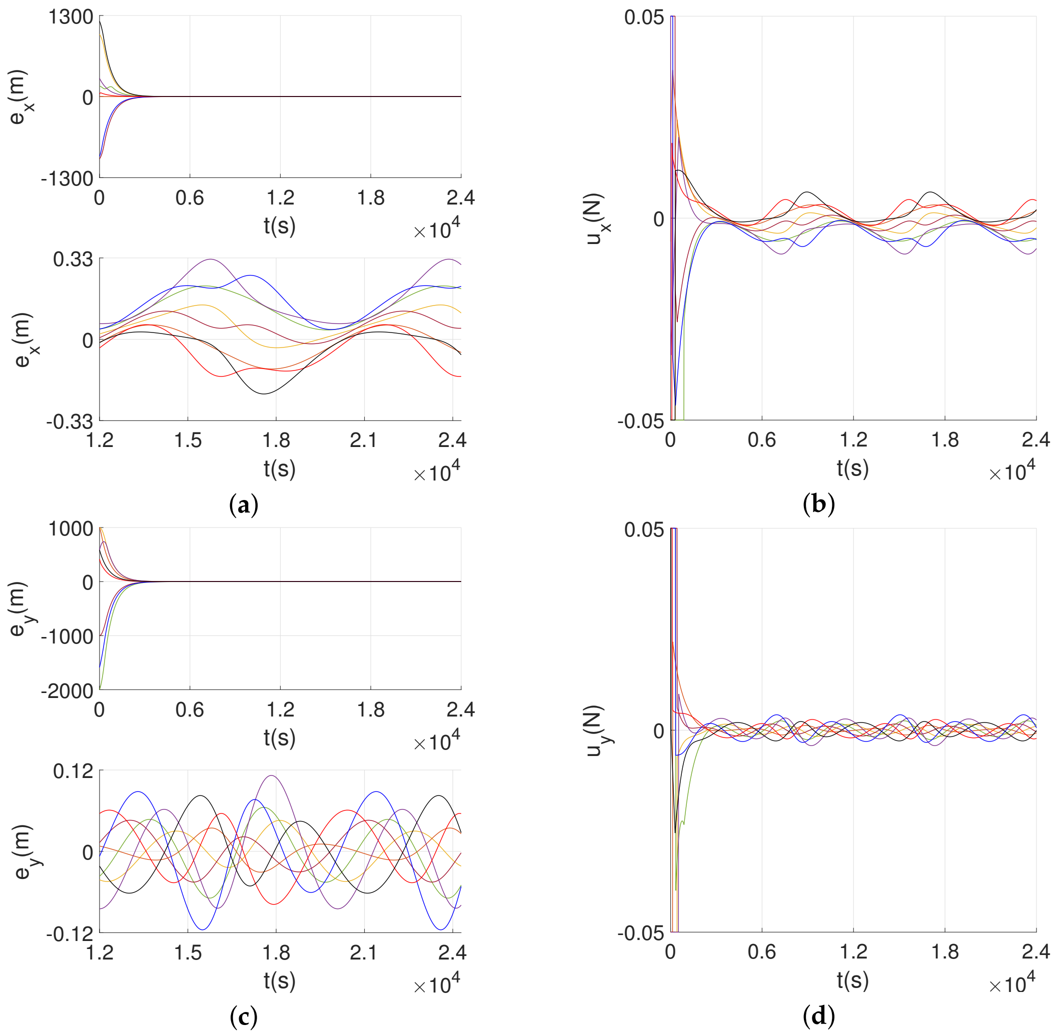


Figure 6. *Cont.*

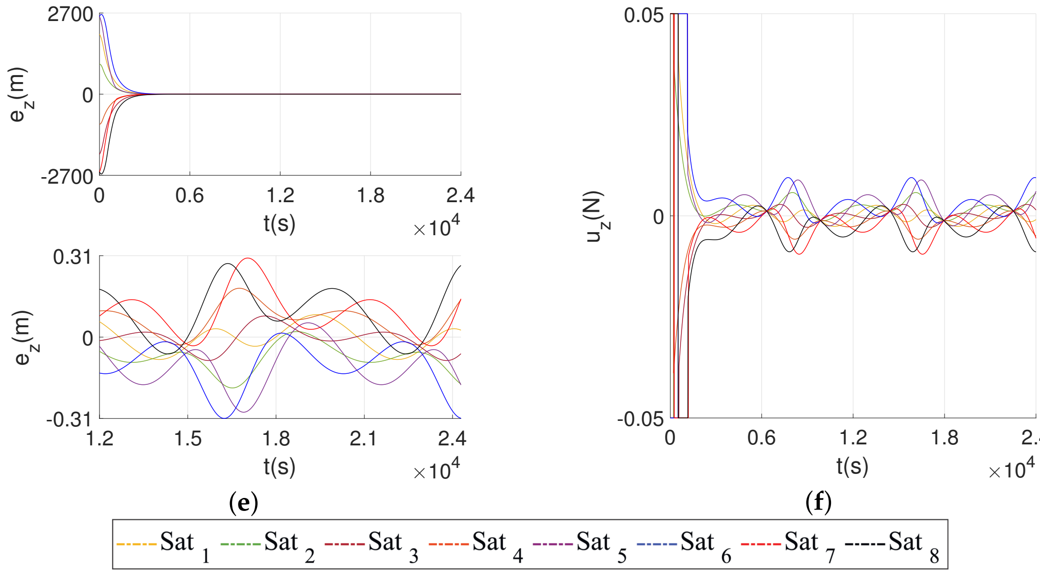


Figure 6. PD law. Error and control time histories: (**a**) errors along axis x; (**b**) controls along axis x; (**c**) errors along y; (**d**) controls along y; (**e**) errors along axis z; (**f**) control along axis z.

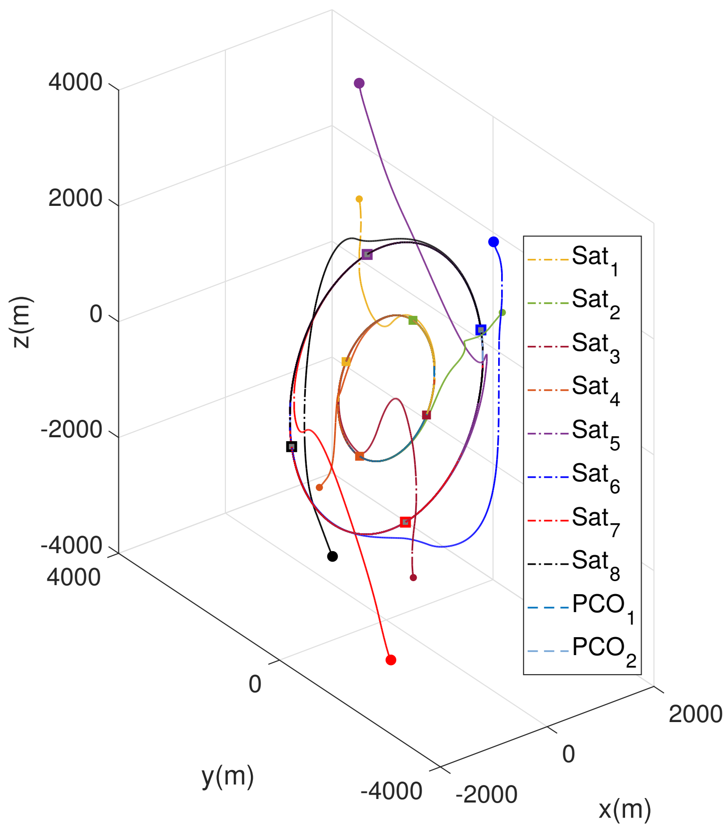


Figure 7. PDC law: Plot of 3D trajectory.

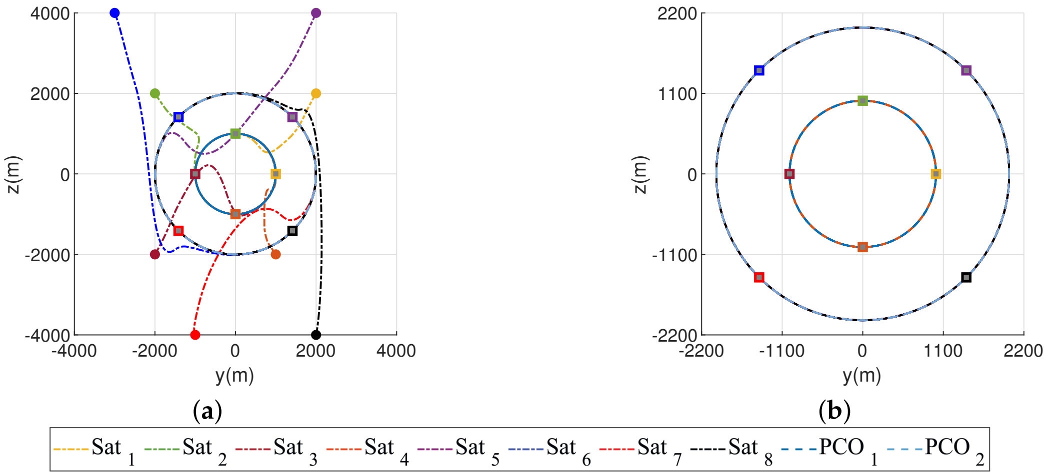


Figure 8. PDC law. Trajectories' plots: (**a**) on plane YZ; (**b**) on plane YZ after orbiting.

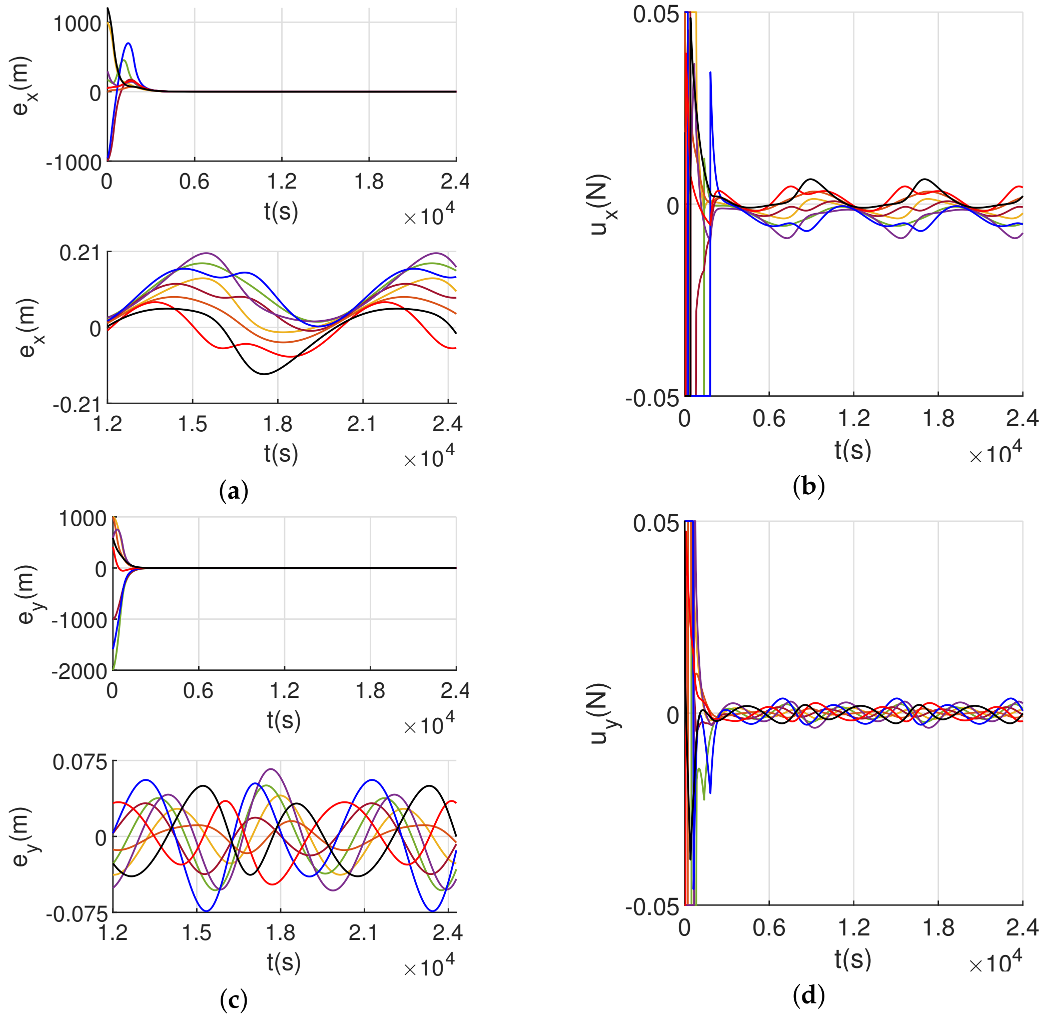


Figure 9. *Cont*.

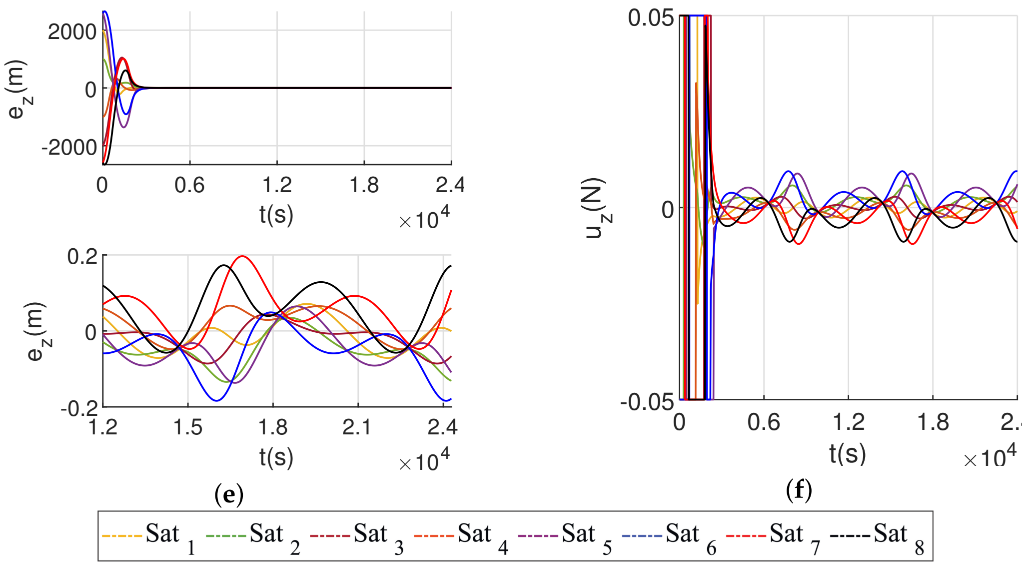


Figure 9. PDC law. Error and control time histories: (**a**) errors along axis x; (**b**) controls along axis x; (**c**) errors along axis y; (**d**) controls along axis y; (**e**) errors along axis z; (**f**) control along axis z.

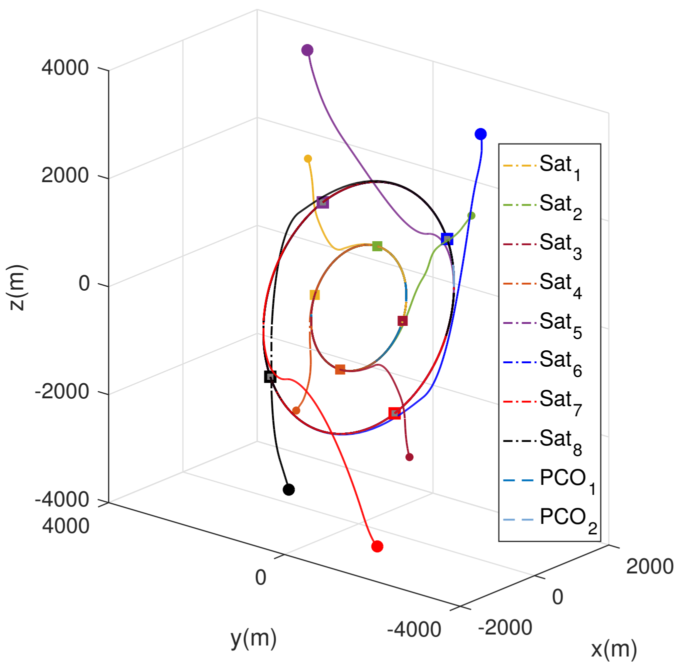


Figure 10. PB law. Plot of 3D trajectory.

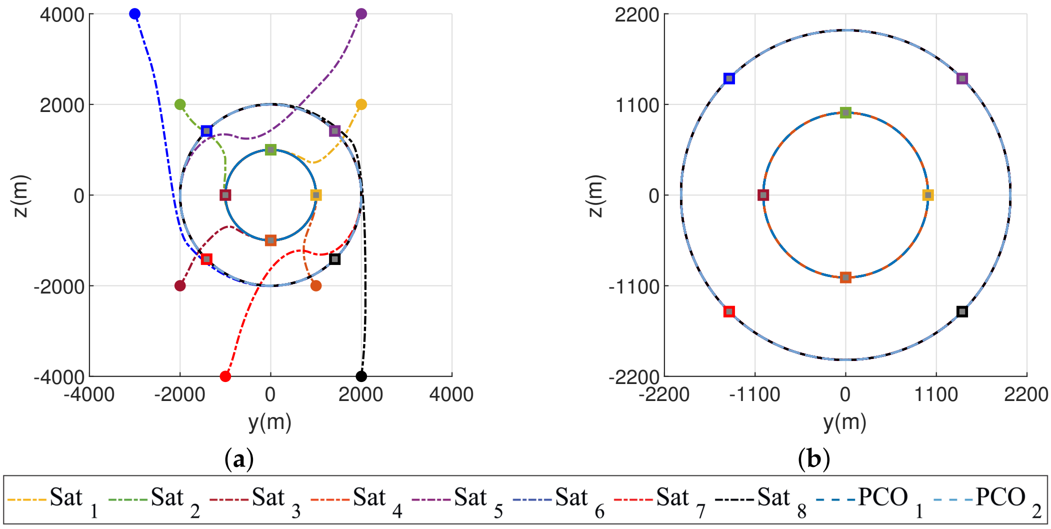


Figure 11. PB law. Plots of trajectories: (**a**) on plane YZ; (**b**) on plane YZ after orbiting.

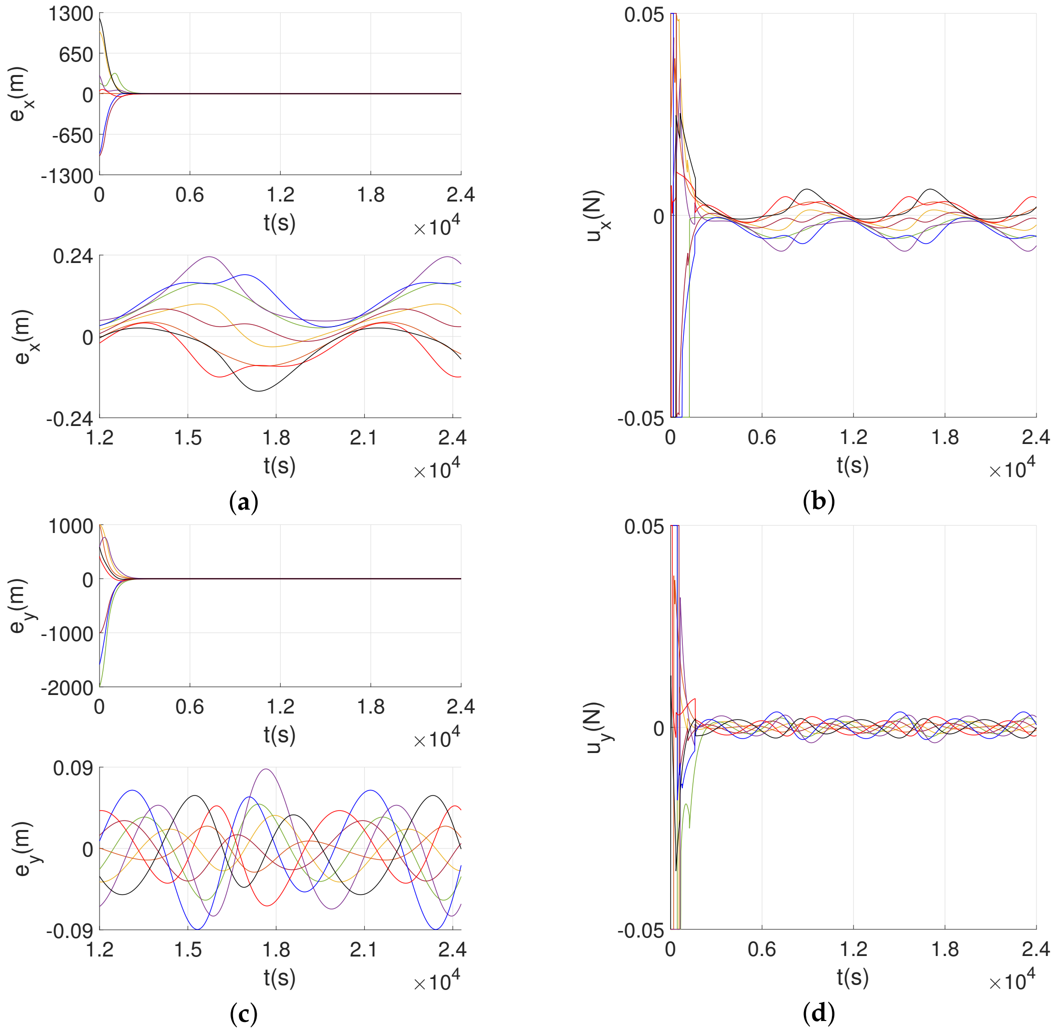


Figure 12. *Cont.*

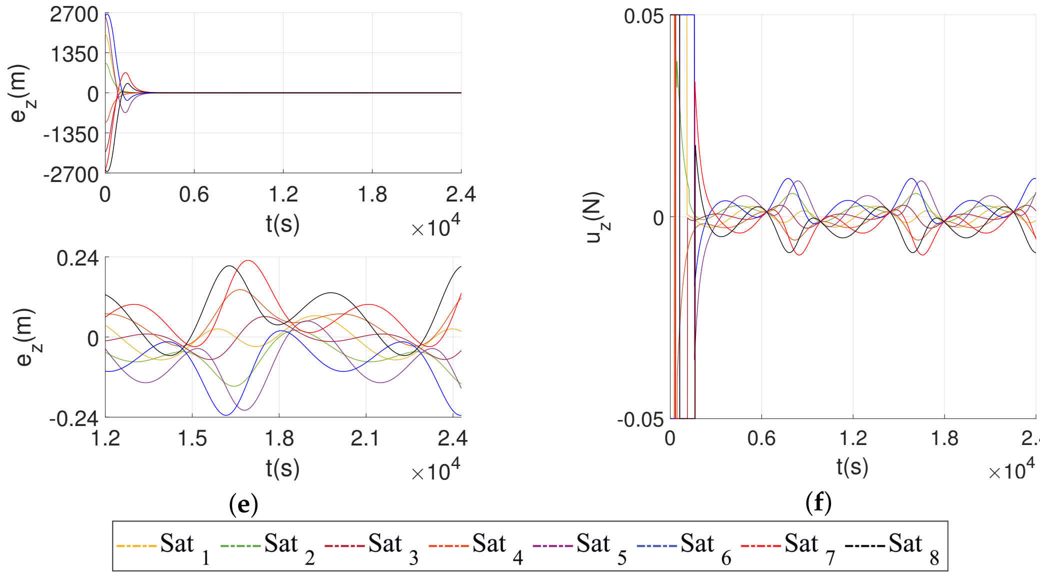


Figure 12. PB law. Error and control time histories: (**a**) errors along axis x; (**b**) controls along axis x; (**c**) errors along axis y; (**d**) controls along axis y; (**e**) errors along axis z; (**f**) control along axis z.

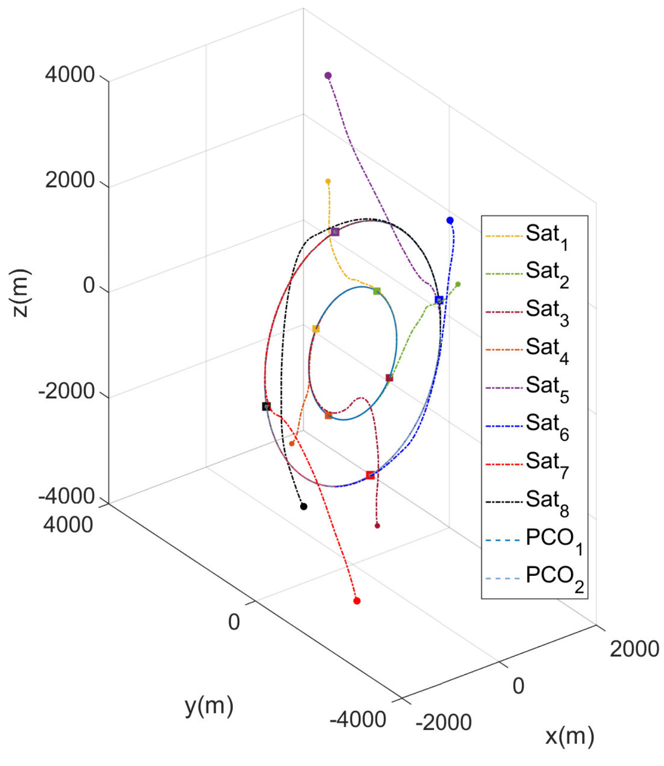


Figure 13. PBC law. Plot of 3D trajectory.

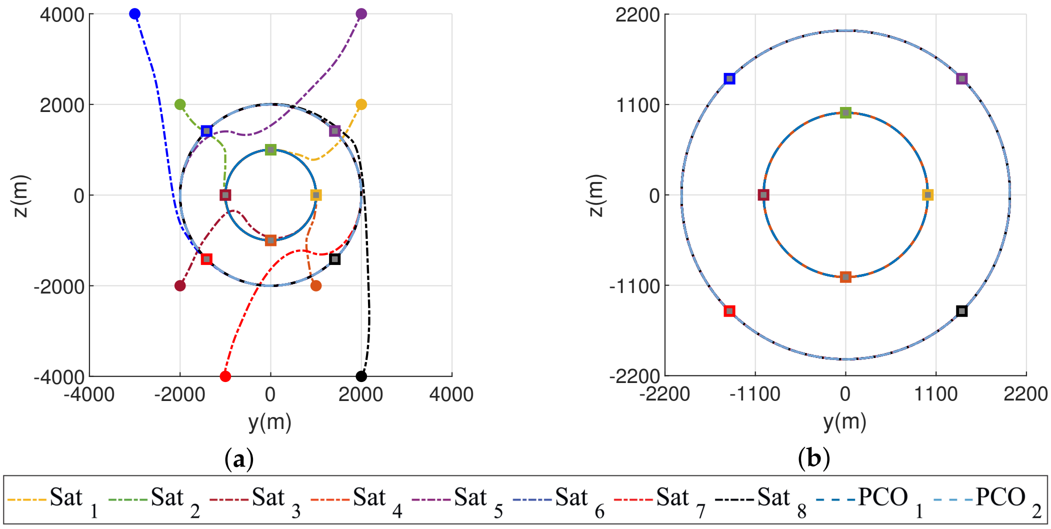


Figure 14. PBC law. Plots of trajectories: (**a**) on plane YZ; (**b**) on plane YZ after orbiting.

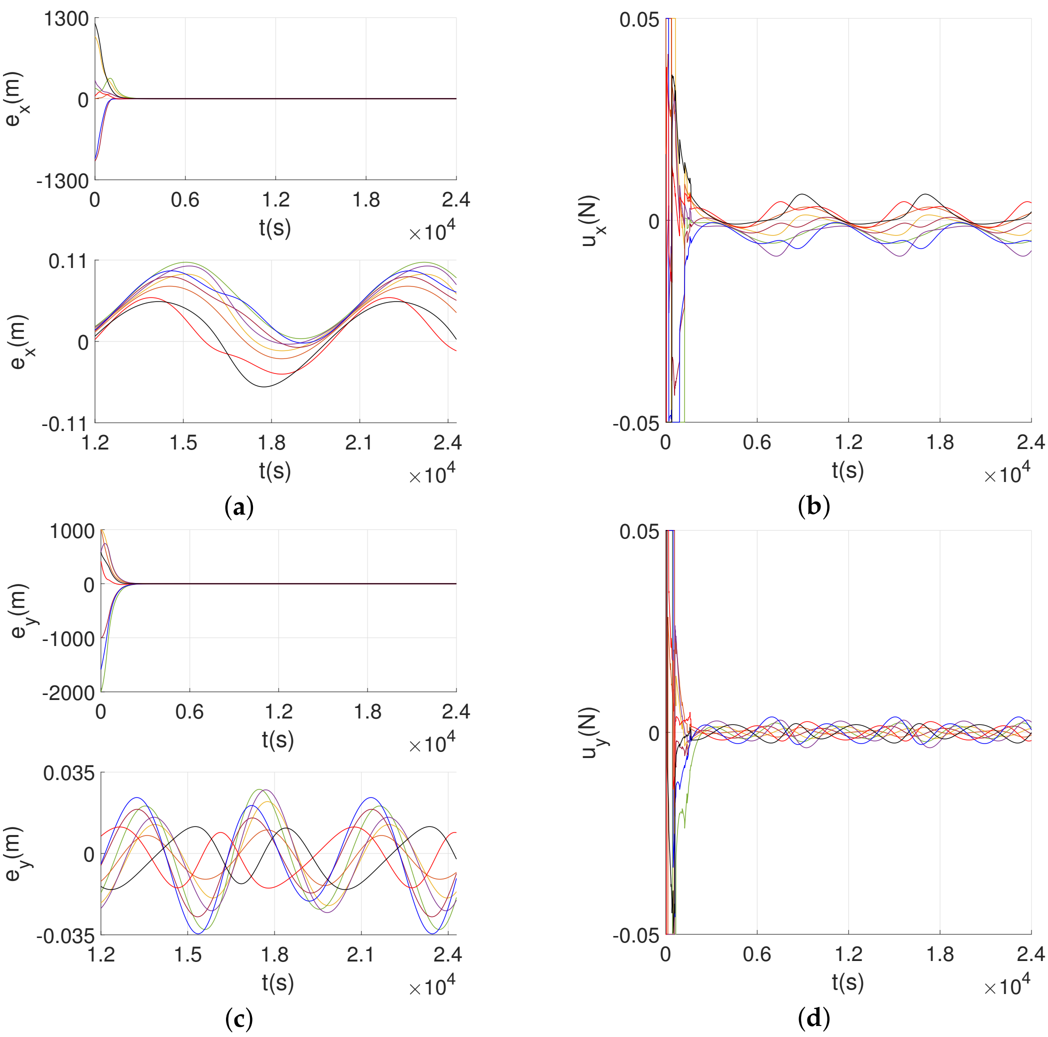


Figure 15. *Cont*.

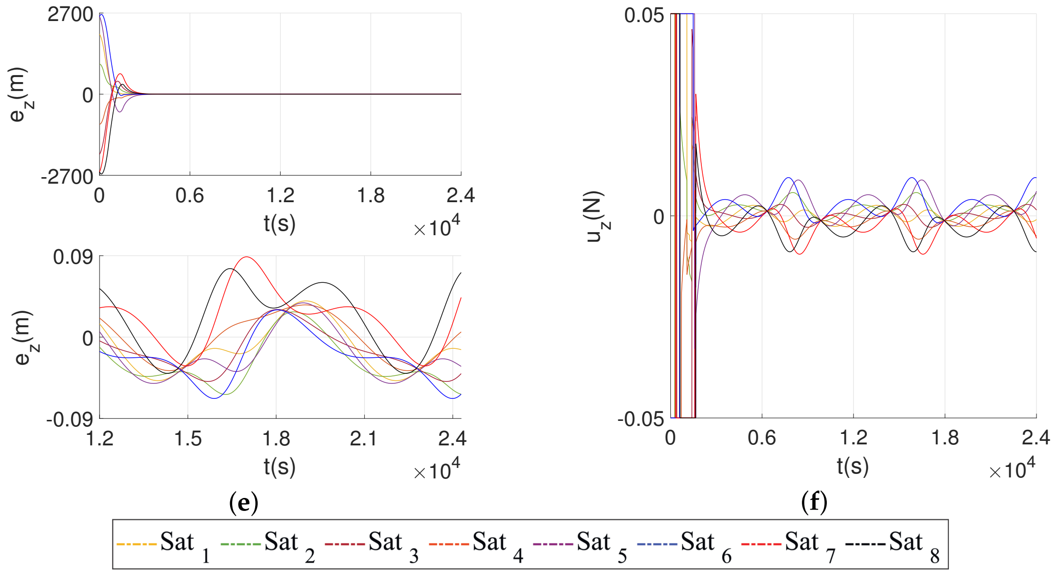


Figure 15. PBC law. Error and control time histories: (**a**) errors along axis x; (**b**) controls along axis x; (**c**) errors along axis y; (**d**) controls along axis y; (**e**) errors along axis z; (**f**) control along axis z.

9. Conclusions

In this paper, satellite formation reconfiguration and formation-keeping control laws are proposed by employing the passivity-based output feedback concept. For the complete nonlinear and time-dependent dynamics of the relative motion of a pair of satellites in elliptical orbits, new combined control algorithms, including a consensus protocol, are proposed and analyzed. A comparison of the control modes using passivity-based output feedback control and the proportional-differential controller with and without the consensus algorithm is given. The new result of the paper is also a rigorous mathematical proof of the use of control laws based on the passification method for the satellite system under consideration. The simplified version of the control law, which requires fewer calculations, is also considered. To increasing the accuracy of positioning satellites in the formation, in addition to the obtained algorithms, it is proposed to use multi-agent control algorithms based on the consensus protocol. Various methods of constructing satellite formations require the lowest energy costs to maintain the system. Mathematical modeling and computer simulations of the proposed algorithms for options for constructing formations are carried out in two PCO orbits of 8 satellites. The results obtained show good performance of the proposed satellites formation control laws for the problem of interest. It is demonstrated that the proposed formation-keeping control laws have robustness with respect to a variety of perturbations, including aspherical geopotential perturbation, air drag and solar radiation pressure.

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Abbreviations

The following abbreviations are used in this manuscript:

ATO	Along-Track Orbit
GCO	General Circular Orbit
HCW	Hill–Clohessy–Wiltshire
LVLH	Local-Vertical Local-Horizontal
PCO	Projected Circular Orbit
PD	Proportional-Differential
PDC	Proportional-Differential-Consensus
PB	Passification-Based
PBC	Passification-Based Consensus
SDRE	State-Dependent Riccati Equation

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