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Simplified Swarm Optimization for the Heterogeneous Fleet Vehicle Routing Problem with Time-Varying Continuous Speed Function

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Abstract: Transportation planning has been established as a key topic in the literature and practices of social production, especially in urban contexts. To consider traffic environment factors, more and more researchers are taking time-varying factors into account when scheduling their logistic activities. The time-dependent vehicle routing problem (TDVRP) is an extension of the classical Vehicle Routing Problem with Time Windows (VRPTW) by determining a set of optimal routes serving a set of customers within specific time windows. However, few of them use the continuous speed function to express the time-varying. In practice, many vehicle routing problems are addressed by a fleet of heterogeneous vehicles with different capacities and travel costs including fix costs and variable costs. In this study, a Heterogeneous Fleet Vehicle Routing Problem (HFPRP) Time-Varying Continuous Speed Function has been proposed. The objective is to minimize distribution costs, which contained fixed costs of acquiring and variable fuel costs. To address this problem, our research developed a mathematical model and proposed a Simplified Swarm Optimization (SSO) heuristic for HFVRP with Time-Varying Continuous Speed Function.

Keywords: vehicle routing problem; time window; heterogeneous fleet; time-varying continuous speed function; simplified swarm optimization

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1. Introduction

Throughout recent decades, transportation planning has been established as a key topic in the literature and practices of social production. Especially in urban contexts, due to industrial strategies, urbanization, city design and unforeseen accidents, weather conditions, traffic congestion, logisticians and citizens face many challenges related to delivery delays [1]. The traffic environment is dynamically changing, which increases the complexity of the problem, the most obvious one being time-varying travel time. Travel velocity is significantly reduced during peak hours. Therefore, to achieve socio-economic and environmental sustainability in city transportation, a major intervention on time-varying congestion is needed.

Problems related to the distribution of goods between the warehouse and the final customer are generally considered as the vehicle routing problem (VRP). The vehicle routing problem was first raised by Dantzig and Ramser [2]. Clarke and Wright [3] added more practical restrictions for the problems, in which the delivery of goods to each customer should have occurred in the bound. Such a problem model is called the Vehicle Routing Problem with Time Windows (VRPTW). The actual distribution process is much more complex, such as travel speeds on the road varying substantially during peak and off-peak hours in the urban areas. Consequently, Malandraki and Daskin [4] first took the existence of diversified conditions of traffic at different times of the day. They discussed the Time-Dependent Vehicle Routing Problem (TDVRP) where the travel time was based on "a step function distribution". Unfortunately, this model does not satisfy the First-In-First-Out

(FIFO) property which ensures that if a vehicle leaves a node *i* for a node *j* at a given time, any other vehicle leaving a node *i* for a node *j* at an earlier time will arrive earlier at node *j*. Without the "FIFO" property, that is, a vehicle can depart later than another vehicle and arrive earlier at its destination, even if the same path is followed by the two vehicles. This situation is contrary to reality, but it happens in the assumptions of the model, for example, if one vehicle waits just a little before departing to catch a faster travel time associated with the next interval.

Previous studies tend to use different travel link times of node *i* and node *j* to guarantee the FIFO property. Ichoua et al. [5] presented a model based on time-dependent travel speeds that they proved to be better at modeling time-dependency, and the FIFO assumption is not necessarily satisfied either. The idea of time-varying speed is adopted in our problem as well. At the same time, the time-varying continuous speed model is closer to the actual distribution situation, the speed of vehicles changes smoothly, rather than a step-change at a certain moment [6,7].

Apart from the Time-Dependent VRPs, another variant of the VRPs is named the Heterogeneous Fleet Vehicle Routing Problem (HFVRP), where a heterogeneous fleet of vehicles is used for the distribution activities; see Baldacci et al. [8]. The HFVRP with a limited number of vehicles proposed by Taillard [9] involves optimizing the vehicle routes with the available fixed fleet. The idea is not only to consider the routing of the vehicles, but also the fleet composition. Relating to the type of costs (fixed and variable) to be minimized, two different objective functions have been considered.

In the case of just-in-time (JIT) production and urban logistics distribution, time-varying speed, time windows, and heterogeneous fleet settings naturally occur and are therefore more in line with the actual distribution situation. Logistics need to find a balance between costs and quality of service. This route construction problem has been encountered by the authors in several contexts: e.g., cold chain logistics [10], fresh logistics [11], milk run [12], automated guided vehicle (AGV) systems [13], freight distribution [14], JIT production [15], B2C e-commerce [16], inter-vehicle communication [17], vaccine distribution [18], and so on. Nonetheless, there are few cases involving optimization of vehicle type selection and route distribution under the minimizing of total cost based on a continuous time-varying speed model which are closer to the actual distribution situation.

Solving VRP is computationally expensive as it is categorized as NP-Hard; see Lenstra and Kan [19]. Metaheuristics approaches are necessary to solve this problem within considerable computational time. Compared to the classical heuristic, metaheuristics carry out a more thorough search of the solution space. Thus, they are notably capable of consistently producing high-quality solutions, despite the greater computational time than early heuristics; see Cordeau et al. [20]. They have been successfully applied for different combinatorial optimization problems. However, in the scientific literature, almost all existing memetic algorithms for solving different VRP variants follow the framework of genetic algorithms (GA) [21–23] or the Particle Swarm Optimization algorithm (PSO) [24,25] and very rarely, the framework of Simplified Swarm Optimization algorithms.

As one metaheuristic methodology, the Simplified Swarm Optimization (SSO) is often used to address both discrete and continuous optimization problems (Yeh [26]; Yeh [27]; Yeh and Chuang [28]). SSO has some attractive advantages such as its ability to deal with non-linear models, chaotic and noisy data, and many constraints.

The rest of this paper is organized as follows. In Section 2, the literature review is presented, and the problem statement is described in Section 3. Simplified Swarm Optimization is presented in Section 4. Details of the computational result and analysis about the problem are presented in Section 5. Section 6 draws conclusions and possible future research.

2. Literature Review

Vehicle Routing Problems (VRP) are essential elements of distribution systems for delivering goods and services. The effective management of these vehicles could improve customers' experience of delivery and minimize the total routing costs. Due to the wide application of logistics distribution scenarios, there is more and more research on VRPs, so there are a lot of different objective types and limit types of VRPs.

Hence, we have briefly reviewed two different types of VRPs: (1) those that model Time-Dependent travel time VRP with a speed model, (2) and the Heterogeneous Fleet VRP is introduced. Time Dependent VRP (TDVRP) with Time-Varying Speeds and Heterogeneous Fixed Fleet VRP (HFFVRP) in Sections 2.1 and 2.2, respectively.

2.1. Time Dependent Vehicle Routing Problem with Time-Varying Speeds

As the Time Dependent VRP (TDVRP) was proposed, since its assumption was more in line with the realistic world, the TDVRP has attracted the attention of many researchers. However, literature on the speed model remains scarce. The pioneering work has been done by Malandraki and Daskin [4] and Malandraki and Dial [29]. In these papers, mixed-integer linear programs (MILP) and several heuristics to solve the problem are proposed. However, these models may potentially violate the First-In-First-Out (FIFO) property, which implies that for every arc, a later departure time results in a later (or equal) arrival time.

As opposed to Malandraki and Daskin [4], Ichoua, Gendreau, and Potvin [5] proposed an alternative approach model; a stepwise speed function was proposed, resulting in a piecewise linear travel time function, like the one illustrated in Figure 1a, where the day is divided into several time periods and a speed is associated with each period. For a given arc, a stepwise speed function can easily be translated into a corresponding piecewise linear travel time function, as shown in Figure 1b.

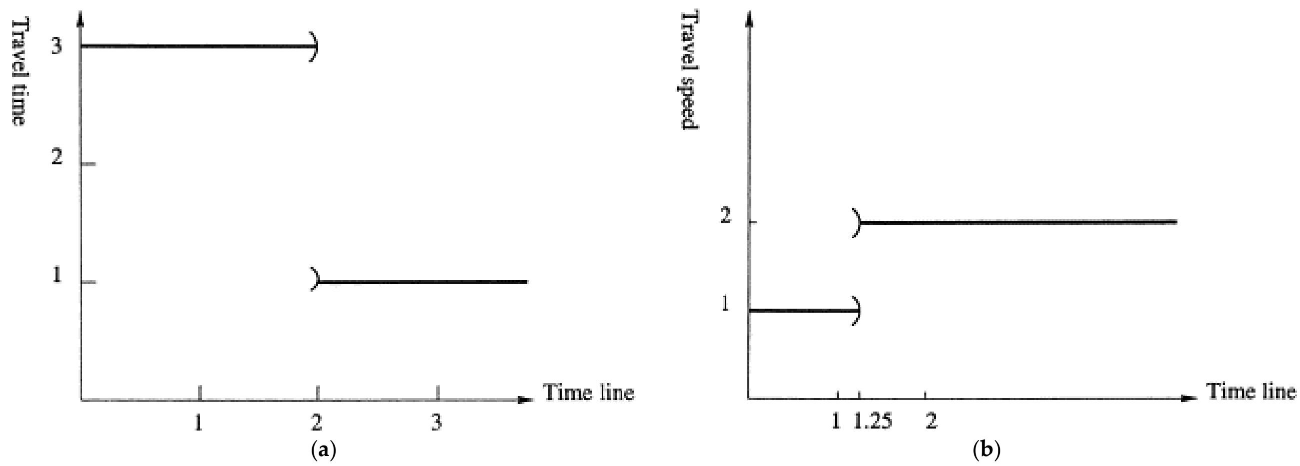

Figure 1. (**a**) Travel time function on a link; (**b**) travel speed function at a node.

This new travel time model satisfies the FIFO principle since a vehicle can only arrive later at the destination if it departs later. The time-dependent speed model is commonly adopted in later publications on VRP variants with time-dependent travel times

Çimen and Soysal [30] addressed a TDVRP with stochastic vehicle speeds and environmental concerns and formulated the problem as a Markovian Decision Process. Soysal and Çimen [31] also proposed a GTDVRP that accounts for transportation emissions, and the problem has been formulated and solved using the Dynamic Programming approach. Sun et al. [32] introduced a time-dependent capacitated profitable tour problem with the object to maximize the profit and developed a tailored labeling algorithm to find the optimal tour. Wang, Assogba, Fan, Xu, Liu, and Wang [14] presented a bi-objective model which focused on freight distribution to minimize total carbon emission and operating cost. Adriano, Montez, Novaes, and Wangham [12] also proposed a dynamic model to simulate the milk-run tours with time windows, using the auxiliary vehicle to deal with traffic jams. Liu et al. [33] proposed TDVRPTW to minimize the sum of the fixed costs of the vehicle used, as well as the costs of drivers, fuel consumption, and carbon emissions. Pan et al. [34] studied the duration-minimizing time-dependent vehicle routing problem with time windows (DM-TDVRPTW), where time-dependent travel times represent different levels of road congestion throughout the day. Pan et al. [35] also proposed a multi-trip time-dependent vehicle routing problem with time windows (MT-TDVRPTW) and formulated the time-dependent ready time function and duration function for any segment of consecutive nodes as piecewise linear functions. Gmira et al. [36] proposed a time-dependent vehicle routing problem with time windows in which travel speeds are associated with road segments in the road network.

Few papers use the time-varying speed of continuous function. Xiao and Konak [37] defined GVRSP which considers heterogeneous vehicles and time-varying traffic congestion where the speed of smooth period is variable. Huang et al. [38] proposed the time-varying speed of the linear continuous function model, but without detailed specification, and their research only studied two types of vehicles with a fixed cost. Xu, Elomri, Pokharel, and Mutlu [21] extended the period-based speed pattern to be a time-varying speed pattern. The approximate corresponding function denoting the relationship between the departure time t and the vehicle speed $\mathrm{v}(t)$ can be characterized by the trigonometric function. Fan, Zhang, Tian, Lv, and Fan [23] continued the assumption in which speed is a trigonometric function of the departure time and applied it in time-dependent multi-depot GVRP. We should still express the time-varying speed as a linear continuous function, without loss of generality, and amplify the speed transformation time to illustrate the time-varying speed of continuous function.

2.2. Heterogeneous Fixed Fleet Vehicle Routing Problem

In practice, many vehicle routing problems are addressed by a non-homogeneous vehicle fleet where vehicles of different capacities are used. Heterogeneous fleets VRPs (HFVRPs) are consider a limited or an unlimited fleet of capacitated vehicles, with a problem consisting of determining the fleet composition and vehicle routes [39].

There are two major HVRPs: the Heterogeneous Fixed Fleet Vehicle Routing Problem (HFFVRP) and the Fleet Size and Mix Vehicle Routing Problem (FSMVRP). HFFVRP was proposed by [9] in which the fleet is predetermined. FSMVRP considers an infinite availability of vehicles of different type, proposed by [40]. In HFFVRP, the number of vehicles is a constraint but in the presence of a large number of vehicles, not all of them may be used. In FSMVRP, part of the decision is to determine the composition of the fleet. FSMVRP can be treated as a particular HFFVRP [9].

Since HFFVRP models can better simulate the actual distribution situation and achieve a better balance of economic and environmental sustainability simultaneously (Micheli and Mantella [41]), HFFVRP has been extensively studied. Afshar-Nadjafi and Afshar-Nadjafi [42] introduced a time-dependent multi-depot vehicle routing problem to minimize total cost and formulated the problem as mixed-integer programming. Vincent et al. [43] developed a Heterogeneous Fleet Pollution Routing Problem (HFPRP) to minimize the total costs of fuel which contain vehicle variable cost and greenhouse gas emissions. Wang et al. [44] proposed a heterogeneous fleet to provide services for a very large transportation network, which also determined the number of vehicles of different types that facilitated on each service link, to better reflect real applications. The proposed methodology is applied to a real-world network, which shows the necessity of considering a heterogeneous fleet. De and Giri [45] studied a closed-loop supply chain with a heterogeneous fleet to minimize carbon emissions. Soman and Patil [46] introduced a heterogeneous fleet vehicle routing problem with release and due dates in the presence of consolidation of customer orders and limited warehousing capacity. Cao, Liao, and Huang [22] addressed a vehicle routing problem considering an electric heterogeneous fleet for a two-echelon recycling network, recycling stations, and recycling centers: with the goal of minimizing the total cost, a recycling heterogeneous fleet electric vehicle routing model with time windows.

The case we proposed is associated with the Heterogeneous Fixed Fleet Vehicle Routing Problem, since the size of the fleet is known for each type of the vehicle, but in our case, we determine the composition of the fleet and the route of each vehicle to serve all customers as in the FSMVRP. Recent literature review and model comparison can be seen in Table 1.

Table 1. State of the art in recent five years.

Paper	Speed		HF		TW	Test Problem	Veh Type	Tp	Objective	Solution Method
	D	C	F	V						
Çimen and Soysal [30]	√	-	-	-	-	Pollution-Routing Problem Instance Library	homo	4	carbon	Approximate Dynamic Programming (ADP) based heuristic algorithm
Soysal and Çimen [31]	√	-	-	-	-	Pollution-Routing Problem InstanceLibrary	homo	4	carbon	Simulation Based Restricted Dynamic Programming (RDP) algorithm
Sun, Veelenturf, Dabia and Van Woensel [32]	√	-	-	-	√	Instances proposed by Ropke et al. [47]	homo	5	max profit	Tailored labeling algorithm
Wang, Assogba, Fan, Xu, Liu and Wang [14]	√	-	-	-	√	Pollution-Routing Problem Instance Library	homo	5	carbon and cost	Clarke and Wright Saving, Sweep algorithm, and multi-objective PSO
Adriano, Montez, Novaes and Wangham [12]	-	-	-	-	√	Self-generated	homo	not TD	cost	Dynamic milk-run vehicle routing solution
Liu, Kou et al. [33]	√	-	-	-	√	Solomon dataset	homo	3	cost	Ant colony algorithm
Pan, Zhang et al. [34]	√	-	-	-	√	Solomon dataset	homo	5	time	Tabu search
Pan, Zhang et al. [35]	√	-	-	-	√	Solomon dataset	homo	5	dis	Tabu search
Gmira, Gendreau et al. [36]	√	-	-	-	√	NEWLET coming from [48]	homo	5	dis	Tabu search
Afshar-Nadjafi and Afshar-Nadjafi [42]	√	-	-	√	√	Self-generated	4	3	cost	Simulated annealing
Vincent, Redi et al. [43]	-	-	-	√	-	Pollution-Routing Problem Instance Library	3	not TD	cost	Simulated annealing
Wang, Qi et al. [44]	-	-	-	√	-	Instances proposed by [49]	10	not TD	cost	Linear solution provided by the column-and-cut generation with local search
Soman and Patil [46]	-	-	-	√	-	Instances proposed by [50]	2	not TD	cost	Scatter search

Table 1. *Cont.*

Paper	Speed		HF		TW	Test Problem	Veh Type	Tp	Objective	Solution Method
	D	C	F	V						
De and Giri [45]	-	-	-	√	-	Self-generated	3	not TD	carbon and cost	Mixed integer linear programming
Cao, Liao et al. [22]	-	-	√	√	-	Self-generated	3	not TD	cost	Genetic algorithm
Huang, Jiang et al. [38]	-	√	-	√	√	Self-generated	3	5	cost	Simplified swarm optimization
Xu, Elomri et al. [21]	-	√	-	√	√	Instances proposed by [51]	12	4	fuel	Genetic algorithm
Fan, Zhang et al. [23]	-	√	√	√	√	MDVRP by [52], MDVRPTW by [53]	3	4	cost	Genetic algorithm
Our study	-	√	√	√	√	Pollution-Routing Problem Instance Library	3	5	cost	Simplified swarm optimization

Note 1. Speed stands for time dependency speed mode, D for Discrete function, C for Continuous function; 2. HF stands for heterogeneous fleet, homo for homogeneous, F for fix cost, V for variable cost; 3. TW stands for time window, Tp for time period.

3. Problem Statement

3.1. Time Dependent Vehicle Routing Problem with Time-Varying Speeds of Continuous Function

The classic Time Dependent Vehicle Routing Problem with Time-Varying Speeds can be described as follow. Let G $= (V,\ A)$ be a graph, where V $= \{v_0, v_1,\ v_2,\ \ldots,\ v_N\}$, where $\{v_1,\ v_2,\ \ldots,\ v_N\}$ is the nodes set standing for customers needing to be served in a time window $tw_i = [b_i,\ e_i]$, and v_0 is the depot. Each customer is characterized by a demand D_i and service time s_i, which is the time to complete the delivery. A $= \{(v_i,\ v_j) : v_i,\ v_j \in V\}$ is the arcs set (subscript means sequence), link node i, and node j with its distance d_{ij}. The time horizon is divided into b time intervals T $= \{T_1,\ T_2,\ \ldots,\ T_b\}$. The travel speed for an arc remains constant within each time zone T_b but changes at the end of the time zones and without loss of generality, and the time zones are set to be the same for all the arcs. The travel time on a given arc $(v_i,\ v_j)$ is then derived based on its distance $d_{i,j}$ and its speed profile.

Even though many works in TDVRP have been carried out with time-varying speeds, the investigation carried out by the Texas A&M Transportation Institute Jha and Eisele [7] mentioned that the traveling speed increases or decreases smoothly, rather than treating it as a stepwise function. The drawbacks of this model are very obvious and, as shown in the Figure 2, will lead to breakpoints; although it can be said that the change in speed takes much less time than delivery at a constant speed. Hence, we present a time-varying speed model of a continuous function, as shown in the Figure 3a.

Proposition 1. *If the speed profile for an arc $(v_i,\ v_j)$ is a stepwise linear continuous function, as show in Figure 3a, then the time-dependent travel distance function can be obtained by integrating the speed function, and the derived travel distance function is a continuous function, as depicted in Figure 3b.*

Proposition 2. *Since travel distance corresponds to travel time one-to-one, then the time-dependent travel time function is the inverse function of travel distance. Using this one-to-one correspondence relationship, when the departure time and travel distance are known, the travel time can be calculated.*

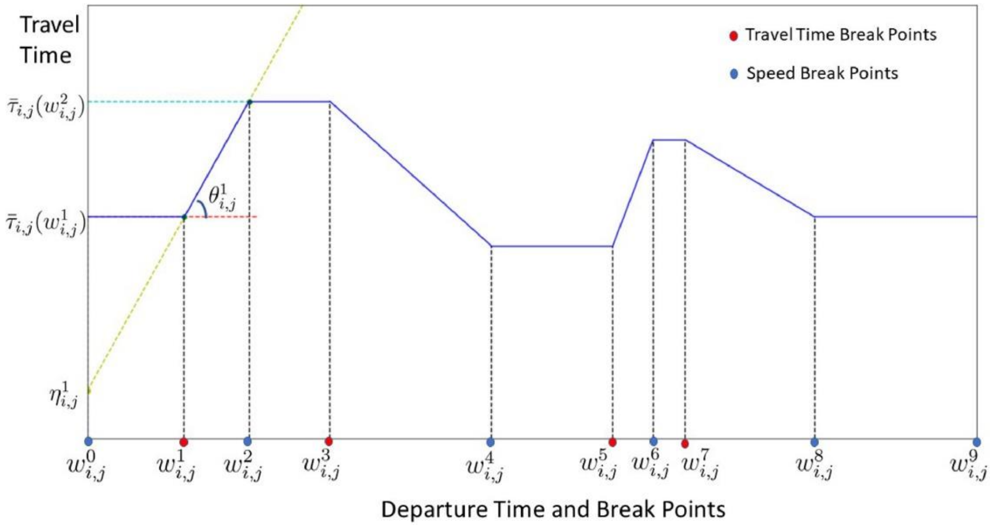

Figure 2. Travel time function.

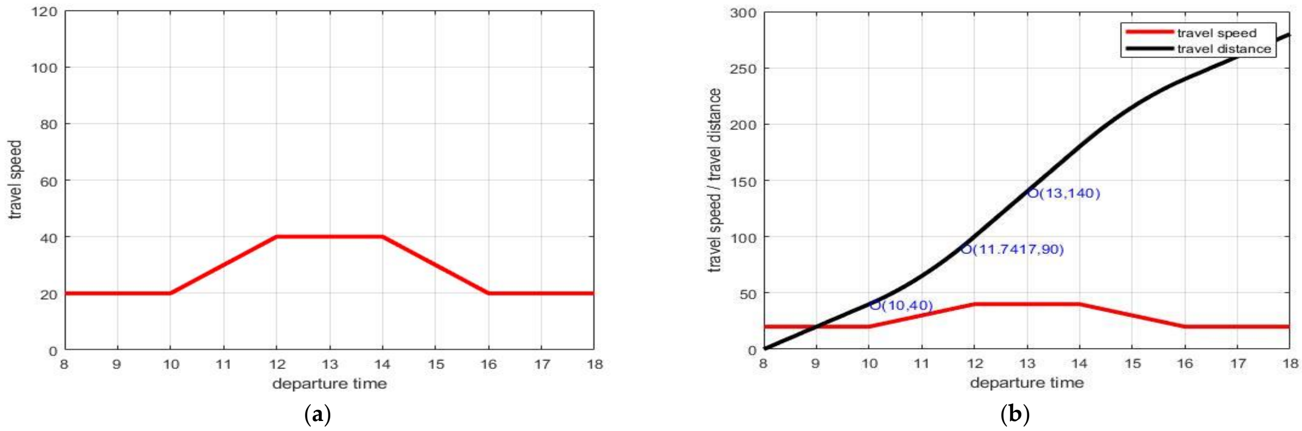

Figure 3. (**a**) Time-varying continuous travel speed function; (**b**) travel distance function.

3.2. Heterogeneous Fixed Fleet Vehicle Routing Problem

Heterogeneous fleets VRPs (HVRPs) are considered a limited or unlimited fleet of capacitated vehicles, with problems, which consist of determining the fleet composition and vehicle routes (Koç, Bektaş, Jabali, and Laporte [39]). Let M_m be the set of vehicles with m types, each type vehicle max available number be veh_m, and subscript m be the type of vehicle (which will be used in the following text). The maximum load capacity of m type vehicle is cap_m, the time-varying speed function of m type vehicle is $\mathrm{fun}-\mathrm{speed}_m\left(t_{dep,ij}\right)$, where $t_{dep,i}$ is the departure time from node i to node j, the time-varying travel time function is $\mathrm{fun}-\mathrm{time}_m\left(t_{dep,ij}, d_{ij}\right)$, and d_{ij} is the distance linking node i and node j, as mentioned before. Every type of car has a fixed cost fc_m, which is referred to as the cost of buying or renting, and a variable cost vc_m, used to express the cost per kilometer per kilogram of the weight of goods.

Assumptions

1. The depot has a demand equal to zero
2. Each customer location is serviced from only one vehicle
3. Each customer's delivery must arrive within the time window
4. The number of each type of vehicle in routing cannot exceed each type of vehicle; the maximum available number is veh_m
5. Each vehicle shall not exceed its maximum load capacity cap_m
6. The total delivery time of each vehicle shall not exceed 9 h

Notations used in problem statement are defined as follows.
Detail model sets, indices, and parameters are shown in Table 2.

Table 2. Sets, indices, and parameters.

Sets and Indices	
V	nodes set, v_0 is the depot, $\{v_1, v_2, \ldots, v_N\}$ are customers
i,j	subscript of the customer node, $i, j = 1, 2, \ldots N$
A	$A = \left\{\left(v_i, v_j\right) : v_i, v_j \in V\right\}$ is the arcs set linking node i and node j
M_m	the set of vehicles with m types
m	vehicle types, $m = 1, 2, \ldots MK$
Parameters	
D_i	demand of i customer
tw_i	$tw_i = [b_i, e_i]$ time window of i customer
s_i	service time of i customer
T	$T = \{T_1, T_2, \ldots, T_b\}$ time period
$t_{arr,i}$	the arrive time of node i
$t_{dep,ij}$	the departure time from node i to node j
$t_{tra,ij}$	the travel time from node i to node j
d_{ij}	the distance linking node i and node j
$\text{fun} - \text{speed}_m\left(t_{dep,ij}\right)$	time-varying speed function of m type vehicle
$\text{fun} - \text{time}_m\left(t_{dep,i}, d_{ij}\right)$	time-varying travel time function of m type vehicle
veh_m	each type of vehicle, maximum available number
cap_m	maximum load capacity of m type vehicle
fc_m	fixed cost of m type vehicle
vc_m	variable cost of m type vehicle
$Dm_{ij,m}$	amount carried using type m vehicle from i to j
Decision variable	
$X_{ij,m}$	one if a type m vehicle travels from node i to j; otherwise, zero

3.3. Fitness Function and Mathematical Model

The traditional logistics model is concerned with minimizing total cost in a network. This is where the concept of Vehicle Routing Problem (VRP) is best applied. We follow this concept and add the fixed cost fc_m of each type of vehicle into the total cost to minimize the total number of vehicles. We also include the variable cost vc_m of delivery of each type of vehicle to optimize the vehicle scheduling. Other constraints appear in the target calculation in the form of penalty functions to enforce the constraints of the limit. The objective of minimizing the total cost is as follows:

$$\text{Minimize} \sum_{m=1}^{M}\sum_{i=0}^{N}\sum_{j=0}^{N} X_{ij,m} fc_m + \sum_{m=1}^{M}\sum_{i=0}^{N}\sum_{j=0}^{N} d_{ij} Dm_{ij,m} vc_m \tag{1}$$

Subject to Routing

$$\sum_{i=1}^{N} X_{i0,m} = 1 \;\forall\, m \in M_m \tag{2}$$

$$\sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij,m} = 1 \,\forall\, (i,j) \in \mathrm{A} \,\forall\, m \in M_m \tag{3}$$

$$\sum_{m=1}^{M}\sum_{i=1}^{N} X_{ip,m} = \sum_{m=1}^{M}\sum_{i=1}^{N} X_{pi,m} \;\forall\, p \in V \tag{4}$$

Demand and Capacities

$$\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij,m} D_j = Dm_{ij,m} \,\forall\, (i,j) \in \mathrm{A} \,\forall\, m \in M_m \tag{5}$$

$$\sum_{i=1}^{N} Dm_{0i,m} \le cap_m \,\forall\, m \in M_m \tag{6}$$

$$\sum_{m=1}^{M} X_{ij,m} \le veh_m \,\forall\, m \in M_m \tag{7}$$

Time Windows

$$t_{arr,j} = t_{dep,ij} + t_{tra,ij} \,\forall\, i,j \in \mathrm{V} \,\forall\, (i,j) \in \mathrm{A} \tag{8}$$

$$t_{arr,j} \in \left[b_j,\ e_j\right] \tag{9}$$

$$t_{dep,ij} = t_{arr,i} + s_i \,\forall\, i,j \in \mathrm{V} \tag{10}$$

$$t_{arr,0} < 9 \forall\, m \in M_m \tag{11}$$

The objective function (1) is total cost including fixed cost and variable cost. Constraint (2) defines that each vehicle should back to the Depot that subscript stands for 0. Constraint (3) ensures that each node can only be visited once in a route. Constraint (4) states that if a vehicle arrives at a node, it must leave it, and by this way, the route continuity is ensured. Constraints (10) and (11) state the restrictions for the amount of demand and capacities. Constraint (7) defines the maximum number of available vehicles veh_m. Constraints (8) and (9) are the time window restrictions. Constraint (11) ensures that each vehicle cannot delivery over 9 h.

4. Simplified Swarm Optimization

As a generalization of VRP, the TDVRP and HFFVRP are also NP-hard and require intentional investigation for problem modeling and algorithmic design.

4.1. Simplified Swarm Optimization

SSO is one of the simplest machine-learning methods (Wang et al. [54–56] Yeh, et al. [57]) in terms of its update mechanism. It was first proposed by Yeh [27], and has been tested to be a very useful and efficient algorithm for optimization problems, including network reliability (Yeh [58] Yeh, et al. [59]), deep learning training (Yeh [60] Yeh, Lin, Liang and Lai [57]), disassembly sequencing problems (Yeh [61] Yeh [62]), energy problems (Lin et al. [63]), and so on. Owing to its simplicity and efficiency, SSO is used here to find the best values in vehicle routing of the proposed HFVRP with Time-Varying Continuous Speed Function.

The basic idea of SSO is that each variable, such as the *j*th variable in the *i*th solution $x_{i,j}$, needs to be updated based on the following stepwise function (Yeh [55,62]):

$$x_{i,j} = \begin{cases} g_j & \text{if } \rho_{[0,1]} \in [0,\ C_g) \\ p_{i,j} & \text{if } \rho_{[0,1]} \in [C_g,\ C_p) \\ x_{i,j} & \text{if } \rho_{[0,1]} \in [C_p,\ C_w) \\ x & \text{if } \rho_{[0,1]} \in [C_w,\ 1) \end{cases} \tag{12}$$

where the value $\rho_{[0,1]} \in [0,1]$ is generated randomly, the parameters C_g, $C_p - C_g$, $C_w - C_p$, $1 - C_w$ are all in [0,1] and are the probabilities of the current variable that are copied and pasted from the best of all solutions, the best *i*th solution, the current solution, and a random generated feasible value, respectively.

There are different variants of the traditional SSO that are customized to different problems from the no free lunch theorem; for example, the four items in Equation (11) are also reduced to three items to increase the efficiency; parameters C_g, C_p, and C_w are all self-adapted; special values or equations are implemented to replace g_j, $p_{i,j}$, $x_{i,j}$, and x; or only a certain number of variables is selected to be updated, etc. However, the SSO update mechanism is always based on the stepwise function.

4.2. Example for Code and Decode

To explain how the model works, a small instance from Vincent, Redi, Jewpanya, Lathifah, Maghfiroh, and Masruroh [43] was used for testing with 1 depot, 5 customers, and 3 types of vehicles. Table 1 shows the part of parameters for each vehicle from Vincent, Redi, Jewpanya, Lathifah, Maghfiroh, and Masruroh [43], since our research focus on the total cost which included fixed cost and variable cost, instead of the pollution cost in Vincent, Redi, Jewpanya, Lathifah, Maghfiroh, and Masruroh [43] study, some of the parameter settings have been omitted which can be known in detail in the Vincent, Redi, Jewpanya, Lathifah, Maghfiroh, and Masruroh [43] paper. Tables 3 and 4 present the corresponding values for the distance matrix (in meters), demand, time window, and service time.

Table 3. Vehicle specification.

Notation	Description	Typical Values of a Type of m Vehicle		
		Vehicle Type 1	**Vehicle Type 2**	**Vehicle Type 3**
cap_m	Maximum load capacity of m type vehicle	1000	2000	3650
vc_m	Variable cost of m type vehicle (£/m)	0.0001	0.00015	0.0002

Note: vc_m in the Vincent, Redi, et al. study did not consider the load of delivery

Table 4. Parameters for the small problem size.

	D	V_1	V_2	V_3	V_4	V_5	**Demand**	b_i	e_i	s_i
D	0	41,150	25,680	23,000	32,450	22,500	0	0	22,400	0
V_1	40,660	0	51,980	40,000	23,000	32,000	900	752	21,289	200
V_2	25,010	51,780	0	30,000	32,000	23,000	727	270	24,050	2000
V_3	20,000	30,000	300,000	0	23,000	25,000	800	250	22,500	1500
V_4	32,500	23,000	32,000	23,000	0	30,000	580	700	28,000	200
V_5	22,500	32,000	23,000	25,000	30,000	0	600	300	27,000	300

The route result given by the Vincent, Redi, et al. study is shown as follow

Route is 0-5-4-3-0, total distance is 95,500, vehicle type is type 2; Route **II** is 0-1-0, total distance is 81,810, vehicle type is type 1; Route **III** is 0-2-0, total distance is 50,690, vehicle type is type 1. Total distance of Route **I**, Route **II**, and Route **III** is 228,000, and total variable cost is 27.575

If code, this route result using SSO, first, should give each vehicle type the maximum available number veh_m. Assume that each type of vehicle can complete the distribution task separately; since the total demand of customers is 3607, which requires $veh_1 = 4, veh_2 = 2, veh_3 = 1$. According to the HFFVRP problem mentioned above, now, we need to decide the routing problem of 7 cars for 3 types of vehicles. And the representation using SSO is shown in Table 5.

Table 5. Representation using SSO.

V_1	V_2	V_3	V_4	V_5
1.2	2.1	5.3	5.2	5.1

The integer part of each number stands for the vehicle that will service, the same integer parts represent the same vehicle for serving the customers, and the order depends on value. For example, V_3, V_4, V_5 have the same integer parts, which means they are all delivered by vehicle 5 belong to type 2. The route is dependent on the value, since $5.1 < 5.2 < 5.3$, so the route of vehicle 5 is 0-5-4-3-0.

After an iterative update of SSO, the final solution is shown in the Table 6.

Table 6. Cumulative result of small problem size; assume the velocity is 25 m/s.

From to	Distance	Travel time	Arrive T	TW	Service Time	Departure T
0–2	25,680	10,272	10,272	$[270, 24,050]$	2000	12,272
2–5	23,000	920	13,192	$[300, 27,000]$	300	13,492
5–1	32,000	1280	14,772	$[752, 21,289]$	200	14,972
1–4	23,000	920	15,892	$[700, 28,000]$	200	16,092
4–3	23,000	920	17,012	$[752, 21,289]$	1500	18,512
3–0	20,000	800	19,312	$[0, 22,400]$	0	/

The route is 0-2-5-1-4-3-0, service is by vehicle type 3, and the total distance is 146,680, which is much smaller than Vincent and Redi's result. More importantly, the variable cost of Vincent and Redi's model does not reflect the distribution cost caused by the load of the delivered goods, which is inconsistent with the actual distribution process

In practice, logistics companies will buy or rent vehicles with large volume ratings because the average variable cost per kilogram and per meter is lower due to the scale effect. For example, the average demand is 721.4, the average distance is 24,446.7 (take the best solution as an example), the variable cost will be a thousand, so we present a new parameters setting, as shown in Table 7, which will be used in the subsequent test indices, and new optimum route results as Table 8.

Table 7. New parameters setting.

Notation	Description	Typical Values of a Type of m Vehicle		
		Vehicle Type 1	Vehicle Type 2	Vehicle Type 3
cap_m	Maximum load capacity of m type vehicle	1000	2000	3650
vc_m	Variable cos t of m type vehicle $(£/\text{kg}\cdot\text{m})$	0.00002	0.000015	0.00001
fc_m	Fixed cost of m type vehicle	50	100	180

New optimum route is shown as follow

Route **I** 0-5-0, delivery by vehicle type 1; Route **II** 0-2-0, delivery by vehicle type 2; Route **III** 0-4-0, delivery by vehicle type; Route **IV** 0-3-1-0, delivery by vehicle type 3.

4.3. SSO Algorithm Pseudo Code

In order to increase the speed of searching for the best solution, the part of inheriting the solution from the previous generation is cancelled, and the update mechanism is changed to

$$x_{i,j} = \begin{cases} g_j & \text{if } \rho_{[0,1]} \in [0,\ C_g) \\ p_{i,j} & \text{if } \rho_{[0,1]} \in [C_g,\ C_p) \\ x & \text{otherwise} \end{cases} \tag{13}$$

Table 8. After new parameters setting-new optimum route.

From to	Distance	Total Dis	Variable Cost	Demand	Total VC
0–5	22,500	-	0.00002	600	270
5–0	22,500	-		1	0.45
0–2	25,680	-	0.000015	727	280.0404
2–0	25,010	-		1	0.3752
0–4	32,450	-	0.000015	580	282.315
4–0	32,500	-		1	0.4875
0–3	23,000	-	0.00001	800	184
3–1	30,000	53,000		900	477
1–0	40,660	-		1	0.4066
Total $\cos$ t = total VC + fix $\cos$ t for vehicle					1924.6995

STEP 0. Initialize parameters; let $t = 1$, randomly initial solution S^1.

Each variable for S^t is $s^t_{nm} \in [1,\ mk]$, there are in total mk vehicles, subscript n stands for n customer nodes, subscript m stands for m particle size, d_i stands for customer i needs, $[b_i,\ e_i]$ stands for customer i time window.

STEP 1. Let $m = 1$, first row of solution matrix.

STEP 1.1 Let v = vehicle order = 1

STEP 1.2 Find $veh_v = S^t_{nm} \in [v, v+1)$

STEP 1.3.1 Rank veh_v get X^1_{ij}, then read the distance d_{ij} link node i and node j, add the veh_v vehicle fix cost;

STEP 1.3.2 calculate arrive time $t_{arr,i}$ for each node, and departure time $t_{dep,ij} = t_{arr,i} + s_i$ check whether the arrival time is in the time window.

STEP 1.3.3 calculate variable cost for each node,

STEP 1.3.4 check total weight not over cap_m

STEP 1.4 Calculate $object_v$ for vehicle veh_v

$$object_v = \text{fix } \cos \text{t} + \text{variable } \cos \text{t}$$

STEP 1.5

if $veh_v < mk$

$veh_v = veh_v + 1$, go back to STEP 1.2

Otherwise

Calculate $object^t_m = \sum object_v$

If $Pbest^t_m > object^t_m$

$Pbest^t_m = object^t_m,\ \ PbestS^t_m = S^t_m$ Otherwise $Pbest^t_m = Pbest^t_m,\ \ PbestS^t_m = PbestS^t_m$ STEP 2.

if $m <$ popsize

$m = m + 1$, go back to STEP 1.1

Otherwise

If $Gbest^t > min\left(Pbest^t_m\right)$

$Gbest^t = min\left(Pbest^t_m\right),\ \ GbestS^t = min\left(PbestS^t_m\right)$ Otherwise $Gbest^t = Gbest^t,\ \ GbestS^t = GbestS^t$ STEP 3.

if $t <$ iteration and CPU time is not met, randomly $\rho \in [node,\ popsize]$ and x

switch (ρ_{nm})

Case1 $\rho_{nm} \in [0, C_w = c_w)$

$s^{t+1}_{nm} = GbestS^t$ Case2 $\rho_{nm} \in \left[C_w, C_p = C_w + c_p\right)$

$s^{t+1}_{nm} = Pbest^t_m$ Case3 $\rho_{nm} \in \left[C_p, 1\right]$

$s^{t+1}_{nm} = xt = t + 1$

go back to STEP1

Otherwise

Halt

5. Computational Experiments

In this section, the experimental results are reported. First, the details for the results of the case study are presented. Next, the current solution obtained by the GA and PSO is used to compare with the results obtained by both the mathematical model and the algorithm. The benchmark instance is based on the Pollution Routing Problem dataset. The datasets are taken from http://www.apollo.management.soton.ac.uk/prplib.htm, accessed on 1 June 2021 (operating environment: Intel(R) Core(TM) i-74770K CPU@3.50 GHz 3.50 GHz, memory 16.0 GB).

Vehicle setting as mentioned before, vc_m; unit change to (£/kg·km) as shown in Table 7.

Vehicle type 1 belongs to "slow vehicle, travel speed type 1"; vehicle type 2, 3 belong to "quick vehicle, travel speed type 2"; the time-varying continuous travel speed function and time-varying travel time function are as in Figures 4–6.

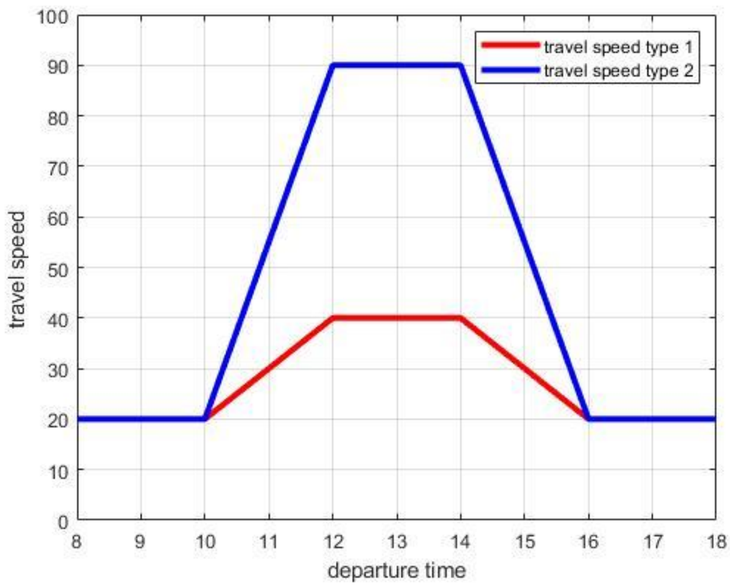

Figure 4. Time-varying continuous travel speed function for 2 types of vehicles.

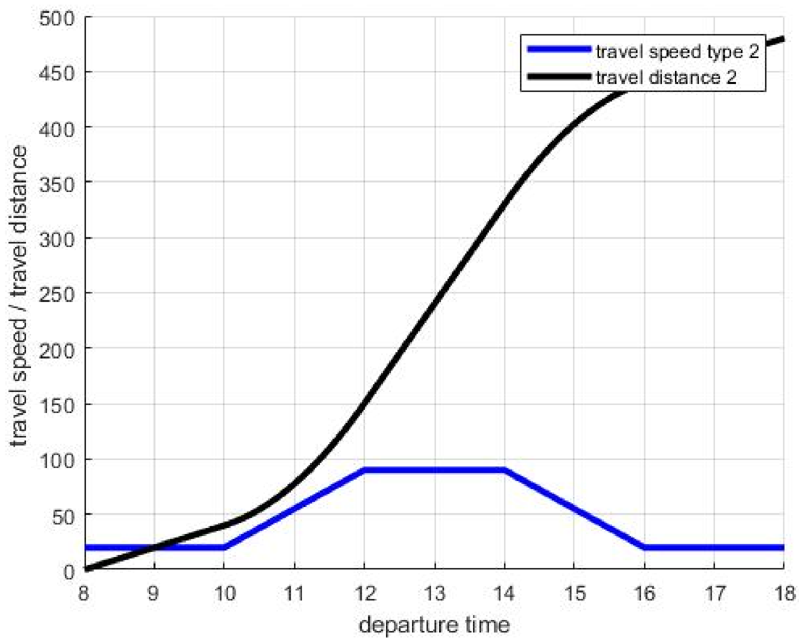

Figure 5. Time-varying travel time function for "slow vehicle".

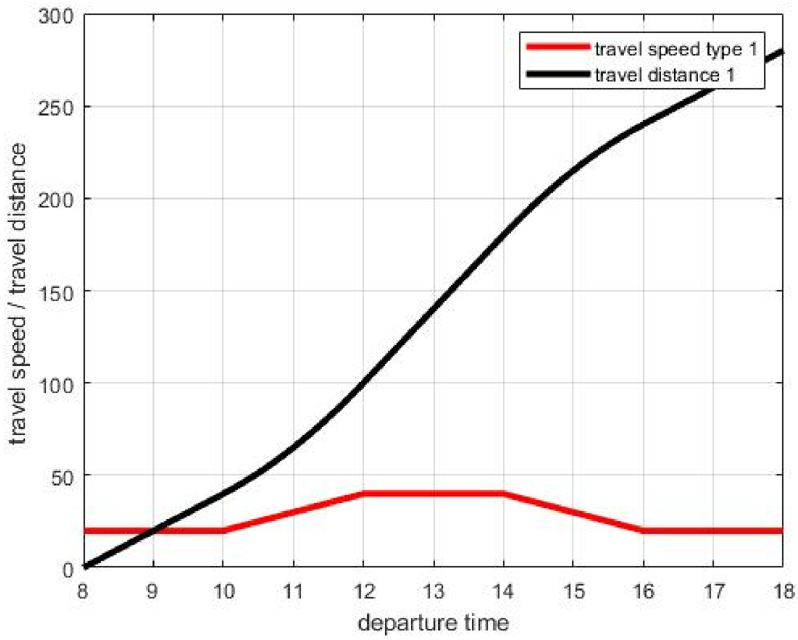

Figure 6. Time-varying travel time function for "quick vehicle'.

Take UK10_01, for example, to illustrate the calculation; the units are adjusted to kilometers and hours. After the adjustment, the service time can be ignored and is not considered in this study. The parameters after adjusted as show in the Table 9. The result comparison is shown is Table 10.

Table 9. Parameters for the UK10_01.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	41	26	54	95	16	89	74	26	88	66	0	0	9
V_1	41	0	52	33	100	42	76	64	24	72	26	721	0.60	6.15
V_2	25	52	0	62	74	13	69	53	43	73	77	814	0.18	5.85
V_3	54	33	62	0	77	52	43	32	49	40	30	620	0.29	5.67
V_4	95	100	74	77	0	81	56	46	112	62	106	311	1.42	6.73
V_5	16	43	13	52	81	0	78	61	34	82	68	167	0.65	6.03
V_6	89	76	69	43	55	78	0	17	92	7	69	513	1.02	6.70
V_7	73	63	52	32	46	61	17	0	76	21	61	568	1.22	6.96
V_8	26	24	44	50	112	34	91	76	0	89	49	763	0.97	6.76
V_9	88	72	73	39	61	82	7	21	88	0	64	558	1.04	6.68
V_{10}	65	26	77	29	106	67	69	61	49	64	0	636	1.47	7.28

SSO is compared with GA and PSO, which have been widely used. The updating methods of GA and PSO can be referred to the papers of other authors: [21,22,24].

Parameter setting of GA algorithm, the crossover rate and mutation rate are 0.6 and 0.4
Parameter setting of PSO algorithm, w $= 0.5,\ C_1 = C_2 = 2$
Parameter setting of SSO algorithm, $C_g = 0.5,\ C_p = C_w = 0.75$,
More detailed information can be found in Appendix A.

Table 10. Result comparison.

	GA		PSO		SSO	
	Average	**Runtime**	**Average**	**Runtime**	**Average**	**Runtime**
1	4744.389	28.758	5085.359	20.51	**4734.099**	18.478
2	4822.404	29.674	4959.387	20.772	**4572.63**	19.234
3	5155.445	29.27	5335.604	20.818	**4856.664**	18.736
4	4618.963	30.378	4959.715	20.876	**4758.297**	18.936
5	**3786.287**	29.118	4073.978	19.468	3866.133	19.302
6	4466.445	29.3425	4498.934	19.816	**4345.842**	19.004
7	3779.422	29.43	4002.631	19.738	**3737.188**	18.468
8	5735.288	29.826	5681.62	19.658	**5673.156**	18.868
9	4277.396	30.118	4349.053	19.214	**3955.826**	19.06
10	5411.929	28.9	5540.31	19.706	**5383.005**	19.288
average	4679.797	29.48145	4845.599	20.0576	**4589.337**	18.9374

From the result, bold is a better solution, shows that the SSO outperforms the original GA and PSO with respect to the average cost (run 5 times independently to take the average value) and runtime. In terms of runtime, SSO and PSO are close, slightly better than PSO. The objective value of the GA is also good, but it takes longer to run, due to the update mechanism of the GA.

6. Conclusions

In urban areas, traffic conditions show a discrepancy throughout the day and the travel time between two locations depends on the time of the day. Even though there are many discussions about the time-dependent model, few adopted the time-varying speed model which can better reflect different road time conditions and naturally satisfy the FIFO property. Thus, we present a novel model using the Time-Varying Continuous Speed Function to explain the time-varying traffic condition effect on delivery.

Moreover, in real conditions, a logistics company can choose a different vehicle to deliver the goods to minimize the total cost including fixed cost and variable cost. In our research, we proposed three types of vehicles with different capacities and costs. By comparing with [43], it is proved that our model can better reflect the actual distribution process and optimize vehicle selection and vehicle routing in the process of minimizing the total cost.

Owing to the complexity of the problem, we use SSO to address the proposed problem and use a Pollution Routing Problem dataset to validate it and compare the result with GA and PSO, which have been widely used in solving VRP problems. The results obtained from the analysis show that the SSO outperforms the original GA and PSO with respect to the total cost and runtime. We acknowledge some limitations in the current study, but these limitations also provide opportunities for further research: for example, applying the SSO to solve larger-scale problems. For the problem of the wide interval time window, path planning is not obviously constrained by the time window, and service time has little impact on distribution planning. Therefore, the problem of a narrow time window can be considered in the future.

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Appendix A

Table A1. Uk10_02.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	83	80	32	86	79	57	57	44	86	61	0	0	9
V_1	82	0	25	88	135	9	87	47	97	96	29	403	1.45	7.16
V_2	80	24	0	90	113	31	98	28	80	111	37	411	0.73	6.12
V_3	32	88	90	0	117	82	48	75	74	77	64	596	0.75	6.29
V_4	86	135	112	116	0	139	140	91	44	169	125	582	0.24	6.1
V_5	78	9	32	82	139	0	79	49	98	87	23	212	0.26	5.73
V_6	58	87	98	48	140	79	0	95	98	39	65	330	0.96	6.28
V_7	57	47	28	75	91	49	94	0	55	113	38	687	0.45	5.91
V_8	43	97	78	73	44	98	97	55	0	126	83	210	0.89	6.55
V_9	87	96	111	77	169	86	39	113	127	0	78	330	0.88	6.19
V_{10}	59	30	38	62	124	22	63	39	83	76	0	697	0.84	6.51

Table A2. Uk10_03.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	93	53	48	34	63	80	99	51	16	31	0	0	9
V_1	93	0	46	55	106	106	139	9	140	109	124	187	0.96	6.47
V_2	53	46	0	33	73	92	114	47	98	67	84	806	0.33	6.12
V_3	48	54	33	0	53	62	86	59	96	63	77	672	1.16	6.62
V_4	34	106	73	53	0	38	50	110	67	40	41	465	1.01	6.42
V_5	63	106	92	61	38	0	34	114	103	76	77	628	1.16	6.94
V_6	80	139	113	86	50	34	0	143	113	87	86	735	1.1	6.55
V_7	99	9	47	59	110	114	143	0	144	113	129	207	1.23	6.7
V_8	51	140	98	96	67	103	113	145	0	35	29	824	0.3	5.9
V_9	16	108	67	63	40	76	87	113	35	0	18	348	0.97	6.19
V_{10}	31	124	83	77	42	77	87	130	29	19	0	672	0.23	5.79

Table A3. Uk10_04.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	48	51	34	95	59	32	71	14	97	59	0	0	9
V_1	48	0	91	14	120	71	16	95	38	119	101	691	1.49	6.92
V_2	51	91	0	78	137	51	77	113	64	150	24	692	0.67	6.48
V_3	34	14	78	0	106	65	3	81	25	106	88	190	0.85	6.04
V_4	95	119	138	106	0	150	103	25	84	30	136	613	0.65	6.28
V_5	58	71	51	64	152	0	66	127	66	152	72	528	0.28	5.62
V_6	32	16	77	3	104	66	0	78	22	103	87	375	1.3	6.56
V_7	71	94	113	81	25	124	78	0	59	35	111	718	1.15	6.59
V_8	14	38	64	25	84	66	22	59	0	85	73	203	1.34	6.46
V_9	97	118	149	105	30	151	102	35	85	0	151	414	0.86	6.35
V_{10}	59	100	25	88	136	73	86	111	73	150	0	168	0.95	6.31

Table A4. Uk10_05.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	50	24	34	49	86	24	28	52	73	93	0	0	9
V_1	51	0	36	19	33	115	39	32	45	115	124	169	0.56	5.91
V_2	24	36	0	22	31	83	6	20	48	84	92	508	0.92	6.22
V_3	35	19	22	0	30	102	27	13	33	99	112	702	1.09	6.55
V_4	49	33	31	30	0	105	35	36	61	111	116	406	0.66	6.17
V_5	86	117	84	103	106	0	80	101	128	59	12	269	0.39	5.63
V_6	25	40	6	27	36	79	0	25	52	80	88	279	1.02	6.62
V_7	28	32	20	13	35	100	25	0	31	96	109	659	1.36	6.76
V_8	51	45	48	33	60	128	52	31	0	123	137	215	0.19	5.62
V_9	73	114	84	99	111	58	80	96	123	0	66	589	0.71	6.19
V_{10}	94	126	93	112	117	13	89	109	137	66	0	541	1.32	6.97

Table A5. Uk10_06.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	76	53	84	35	76	63	31	80	63	74	0	0	9
V_1	77	0	95	26	55	141	22	50	92	78	148	575	0.37	6.12
V_2	53	94	0	113	42	111	91	76	128	32	97	675	1.13	6.57
V_3	84	26	113	0	73	142	22	53	72	101	153	194	1.33	6.92
V_4	35	55	43	73	0	108	52	41	97	35	99	438	0.63	6.05
V_5	76	141	111	143	108	0	122	92	94	131	39	147	1.09	6.26
V_6	63	25	91	26	51	123	0	33	73	79	130	651	0.82	6.51
V_7	31	50	76	53	41	91	33	0	58	75	97	712	1	6.77
V_8	80	92	127	72	97	94	70	58	0	130	110	533	0.93	6.46
V_9	64	77	32	101	34	131	79	75	130	0	119	298	0.7	6.07
V_{10}	74	149	97	154	99	40	132	97	111	118	0	125	0.41	5.95

Table A6. Uk10_07.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	94	29	65	71	87	99	100	42	73	18	0	0	9
V_1	94	0	76	150	150	10	9	44	69	46	82	697	1.09	6.54
V_2	29	76	0	74	98	69	81	92	49	69	34	816	0.2	5.89
V_3	65	149	73	0	94	144	155	164	105	137	81	110	0.58	6.16
V_4	71	150	98	93	0	142	159	135	82	112	72	413	0.6	6.15
V_5	87	10	69	144	142	0	18	43	61	38	75	343	0.6	6.12
V_6	100	9	81	155	159	19	0	44	78	56	91	146	1.14	6.48
V_7	100	44	92	163	135	43	44	0	61	28	83	335	0.51	6.02
V_8	41	69	49	104	82	61	78	61	0	35	24	366	0.81	6.57
V_9	73	46	69	137	112	38	55	28	35	0	57	146	0.83	6.07
V_{10}	18	83	34	81	72	75	90	84	24	57	0	668	0.84	6.73

Table A7. Uk10_08.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	54	97	71	88	48	68	48	79	73	38	0	0	9
V_1	54	0	139	109	139	98	94	39	97	75	54	629	0.52	6.01
V_2	96	139	0	134	43	50	58	138	163	92	133	114	0.62	5.69
V_3	71	110	135	0	109	87	135	76	39	143	55	475	1.39	7.15
V_4	88	139	43	109	0	42	76	132	137	109	118	370	0.72	6.35
V_5	48	98	51	88	41	0	51	94	116	75	83	596	0.5	6.3
V_6	69	94	58	136	77	52	0	109	146	36	106	679	0.38	6.15
V_7	48	39	139	75	132	94	108	0	63	102	21	512	1.26	6.6
V_8	79	97	164	39	137	114	146	63	0	150	45	781	0.34	5.9
V_9	73	75	92	143	109	75	36	102	149	0	105	392	1.38	6.82
V_{10}	38	55	133	55	118	82	105	21	45	105	0	771	0.72	6.39

Table A8. Uk10_09.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	91	45	36	97	89	72	63	74	95	75	0	0	9
V_1	91	0	111	87	145	36	121	117	32	49	144	486	1.32	6.78
V_2	45	111	0	24	41	91	38	19	104	94	31	682	0.59	6.2
V_3	35	87	24	0	63	70	39	32	86	75	61	533	1.46	7.1
V_4	97	145	41	63	0	113	26	49	147	111	10	556	1.37	6.78
V_5	89	36	90	69	112	0	89	85	58	15	111	257	0.56	6.01
V_6	73	122	37	39	26	89	0	24	123	93	23	227	0.75	6.15
V_7	63	117	19	33	49	85	24	0	117	88	47	128	1.08	6.61
V_8	74	32	104	86	147	58	123	117	0	72	146	365	0.27	5.56
V_9	95	49	94	75	111	14	92	88	72	0	115	126	1.49	7.08
V_{10}	75	144	31	61	10	112	23	47	145	115	0	404	0.92	6.16

Table A9. Uk10_10.

	D	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	**Dem**	b_i	e_i
D	0	57	65	38	100	82	59	93	38	86	59	0	0	9
V_1	57	0	9	80	122	107	54	39	45	126	101	431	1.23	6.95
V_2	66	9	0	88	130	115	57	33	54	135	110	236	1.38	6.86
V_3	37	79	88	0	84	62	75	116	54	49	22	426	0.27	5.73
V_4	100	122	130	84	0	22	79	141	77	61	89	550	1.16	6.76
V_5	82	107	115	62	23	0	70	129	64	42	66	687	0.72	6.14
V_6	59	54	57	75	79	70	0	60	22	100	96	578	0.19	6.06
V_7	94	39	33	117	138	127	60	0	70	155	138	403	1.48	6.86
V_8	38	45	53	54	77	63	22	70	0	83	74	409	0.43	5.71
V_9	86	127	136	49	62	42	100	159	83	0	41	597	1.43	7.15
V_{10}	59	101	109	21	89	67	95	138	74	41	0	441	1.07	6.73

Table A10. Detail result of GA.

Dataset	Object	Runtime	Dataset	Object	Runtime	Dataset	Object	Runtime
UK10_1	4579.4	28.02	UK10_5	3800.175	28.66	UK10_9	4404.37	31.04
	4578.21	29.21		3626.59	30.17		4694.27	30.46
	4857.73	26.86		3916.33	27.38		4167.22	29.78
	4650.555	30.03		3636.265	29.52		4125.075	28.61
	5056.05	29.67		3952.075	29.86		3996.045	30.7
UK10_2	4990.565	29.78	UK10_6	4277.8	31.49	UK10_10	5374.835	27.83
	4423.505	30.36		4618.615	29.93		5444.72	30.14
	4848.19	28.71		4776.03	27.87		5243.825	29.49
	4918.935	28.97		4426.16	29.12		5399.75	29.27
	4930.825	30.55		4233.62	28.89		5596.515	28.14
UK10_3	5283.625	29.06	UK10_7	3665.455	28.96			
	5532.12	27.72		3859.02	30.25			
	5106.665	27.83		3727.06	28.77			
	5178.215	31.24		3979.495	29.8			
	4676.6	30.5		3666.08	29.37			
UK10_4	4438.18	31.17	UK10_8	5821.58	30.58			
	4925.145	29.44		5944.335	29.17			
	4617.705	30.74		5556.985	29.87			
	4536.81	30.38		5550.52	29.34			
	4576.975	30.16		5803.02	30.17			

Table A11. Detail result of PSO.

Dataset	Object	Runtime	Dataset	Object	Runtime	Dataset	Object	Runtime
UK10_1	5299.55	20.34	UK10_5	3942.56	18.93	UK10_9	4372.21	18.34
	4991.555	20.92		4660.105	19.12		4416.71	18.67
	5005.5	19.68		3983.105	20.31		4248.505	19.03
	5130.775	20.55		3864.295	19.56		4342.815	20.29
	4999.415	21.06		3919.825	19.42		4365.025	19.74
UK10_2	5045.55	21.65	UK10_6	4537.515	18.97	UK10_10	5539.805	20.31
	5104.19	20.92		4452.69	19.77		5476.845	19.72
	4849.96	19.68		4477.275	20.05		5426.95	19.86
	4748.23	20.55		4464.92	19.94		5621.52	18.81
	5049.005	21.06		4562.27	20.35		5636.43	19.83
UK10_3	5412.62	22.27	UK10_7	3978.84	20.33			
	5416.275	19.76		3863.26	18.67			
	5451.84	21.66		4011.7	20.1			
	5221.17	20.46		4107.485	19.85			
	5176.115	19.94		4051.87	19.74			
UK10_4	4942.085	21.52	UK10_8	5564.86	19.76			
	4788.575	20.12		5762.02	20.3			
	4947.73	22.02		5663.74	19.45			
	5062.86	19.97		5657.115	19.23			
	5057.325	20.75		5760.365	19.55			

Table A12. Detail result of SSO.

Dataset	Object	Runtime	Dataset	Object	Runtime	Dataset	Object	Runtime
UK10_1	4729.38	19.05	UK10_5	3862.26	19.82	UK10_9	3998.045	19.12
	4664.14	17.68		3851.745	20.58		3998.045	19.36
	4664.14	19.25		3877.2	18.21		3998.045	19.07
	4664.14	18.73		3877.2	18.23		3786.95	19.42
	4948.695	17.68		3862.26	19.67		3998.045	18.33
UK10_2	4423.505	18.53	UK10_6	4357.945	18.97	UK10_10	5393.96	20.34
	4554.775	20.1		4319.66	20.1		5316.535	18.78
	4753.095	19.58		4316.145	19.03		5320.675	19.23
	4423.505	18.79		4444.69	18.42		5554.535	19.36
	4708.27	19.17		4290.77	18.5		5329.32	18.73
UK10_3	4876.6	18.11	UK10_7	3727.06	18.67			
	4876.6	19.51		3727.06	18.39			
	4768.49	18.76		3727.06	17.89			
	4876.6	19.29		3777.7	19.02			
	4885.03	18.01		3727.06	18.37			
UK10_4	4639.595	18.11	UK10_8	5655.395	19.02			
	4858.115	19.51		5624.36	17.88			
	4854.055	18.76		5724.765	19.27			
	4676.96	20.29		5610.88	18.76			
	4762.76	18.01		5750.38	19.41			

References

1. Cattaruzza, D.; Absi, N.; Feillet, D.; González-Feliu, J. Vehicle routing problems for city logistics. *EURO J. Transp. Logist.* **2017**, *6*, 51–79. [CrossRef]
2. Dantzig, G.; Ramser, J. The Truck Dispatching Problem. *Manag. Sci.* **1959**, 80–91. [CrossRef]
3. Clarke, G.; Wright, J.W. Scheduling of vehicles from a central depot to a number of delivery points. *Oper. Res.* **1964**, *12*, 568–581. [CrossRef]
4. Malandraki, C.; Daskin, M.S. Time dependent vehicle routing problems: Formulations, properties and heuristic algorithms. *Transp. Sci.* **1992**, *26*, 185–200. [CrossRef]
5. Ichoua, S.; Gendreau, M.; Potvin, J.-Y. Vehicle dispatching with time-dependent travel times. *Eur. J. Oper. Res.* **2003**, *144*, 379–396. [CrossRef]
6. Margiotta, R.; Eisele, B.; Short, J. Freight Performance Measure Approaches for Bottlenecks, Arterials, and Linking Volumes to Congestion Report. 2015. Available online: https://rosap.ntl.bts.gov/view/dot/41268 (accessed on 1 June 2021).
7. Jha, K.; Eisele, B. Freight Multimodal Performance Measures. 2015. Available online: https://static.tti.tamu.edu/tti.tamu.edu/documents/TTI-2015-19.pdf (accessed on 1 June 2021).
8. Baldacci, R.; Battarra, M.; Vigo, D. Routing a heterogeneous fleet of vehicles. In *The Vehicle Routing Problem: Latest Advances and New Challenges*; Springer: Berlin/Heidelberg, Germany, 2008; pp. 3–27.
9. Taillard, É.D. A heuristic column generation method for the heterogeneous fleet VRP. *RAIRO Oper. Res. Rech. Opér.* **1999**, *33*, 1–14. [CrossRef]
10. Wang, Z.; Wen, P. Optimization of a low-carbon two-echelon heterogeneous-fleet vehicle routing for cold chain logistics under mixed time window. *Sustainability* **2020**, *12*, 1967. [CrossRef]
11. Anh, P.T.; Cuong, C.T.; Phuc, P.N.K. The vehicle routing problem with time windows: A case study of fresh food distribution center. In Proceedings of the 2019 11th International Conference on Knowledge and Systems Engineering (KSE), Da Nang, Vietnam, 24–26 October 2019; pp. 1–5.
12. Adriano, D.D.; Montez, C.; Novaes, A.G.; Wangham, M. DMRVR: Dynamic milk-run vehicle routing solution using fog-based vehicular ad hoc networks. *Electronics* **2020**, *9*, 2010. [CrossRef]
13. Yuan, Z.; Yang, Z.; Lv, L.; Shi, Y. A Bi-Level Path Planning Algorithm for Multi-AGV Routing Problem. *Electronics* **2020**, *9*, 1351. [CrossRef]
14. Wang, Y.; Assogba, K.; Fan, J.; Xu, M.; Liu, Y.; Wang, H. Multi-depot green vehicle routing problem with shared transportation resource: Integration of time-dependent speed and piecewise penalty cost. *J. Clean. Prod.* **2019**, *232*, 12–29. [CrossRef]
15. Emde, S.; Schneider, M. Just-in-time vehicle routing for in-house part feeding to assembly lines. *Transp. Sci.* **2018**, *52*, 657–672. [CrossRef]

16. Liu, L.; Li, K.; Liu, Z. A capacitated vehicle routing problem with order available time in e-commerce industry. *Eng. Optim.* **2017**, *49*, 449–465. [CrossRef]
17. Nakazawa, T.; Tang, S.; Obana, S. CCN-based inter-vehicle communication for efficient collection of road and traffic information. *Electronics* **2020**, *9*, 112. [CrossRef]
18. Sujaree, K.; Samattapapong, N. A Hybrid Chemical Based Metaheuristic Approach for a Vaccine Cold Chain Network. *Oper. Supply Chain Manag. Int. J.* **2021**, *14*, 351–359. [CrossRef]
19. Lenstra, J.K.; Kan, A.R. Complexity of vehicle routing and scheduling problems. *Networks* **1981**, *11*, 221–227. [CrossRef]
20. Cordeau, J.-F.; Gendreau, M.; Laporte, G.; Potvin, J.-Y.; Semet, F. A guide to vehicle routing heuristics. *J. Oper. Res. Soc.* **2002**, *53*, 512–522. [CrossRef]
21. Xu, Z.; Elomri, A.; Pokharel, S.; Mutlu, F. A model for capacitated green vehicle routing problem with the time-varying vehicle speed and soft time windows. *Comput. Ind. Eng.* **2019**, *137*, 106011. [CrossRef]
22. Cao, S.; Liao, W.; Huang, Y. Heterogeneous fleet recyclables collection routing optimization in a two-echelon collaborative reverse logistics network from circular economic and environmental perspective. *Sci. Total Environ.* **2021**, *758*, 144062. [CrossRef]
23. Fan, H.; Zhang, Y.; Tian, P.; Lv, Y.; Fan, H. Time-dependent multi-depot green vehicle routing problem with time windows considering temporal-spatial distance. *Comput. Oper. Res.* **2021**, *129*, 105211. [CrossRef]
24. Alinaghian, M.; Ghazanfari, M.; Norouzi, N.; Nouralizadeh, H. A novel model for the time dependent competitive vehicle routing problem: Modified random topology particle swarm optimization. *Netw. Spat. Econ.* **2017**, *17*, 1185–1211. [CrossRef]
25. Iswari, T.; Asih, A.M.S. Comparing genetic algorithm and particle swarm optimization for solving capacitated vehicle routing problem. *IOP Conf. Ser. Mater. Sci. Eng.* **2018**, *337*, 012004. [CrossRef]
26. Yeh, W.-C. A hybrid heuristic algorithm for the multistage supply chain network problem. *Int. J. Adv. Manuf. Technol.* **2005**, *26*, 675–685. [CrossRef]
27. Yeh, W.-C. A two-stage discrete particle swarm optimization for the problem of multiple multi-level redundancy allocation in series systems. *Expert Syst. Appl.* **2009**, *36*, 9192–9200. [CrossRef]
28. Yeh, W.-C.; Chuang, M.-C. Using multi-objective genetic algorithm for partner selection in green supply chain problems. *Expert Syst. Appl.* **2011**, *38*, 4244–4253. [CrossRef]
29. Malandraki, C.; Dial, R.B. A restricted dynamic programming heuristic algorithm for the time dependent traveling salesman problem. *Eur. J. Oper. Res.* **1996**, *90*, 45–55. [CrossRef]
30. Çimen, M.; Soysal, M. Time-dependent green vehicle routing problem with stochastic vehicle speeds: An approximate dynamic programming algorithm. *Transp. Res. Part D Transp. Environ.* **2017**, *54*, 82–98. [CrossRef]
31. Soysal, M.; Çimen, M. A simulation based restricted dynamic programming approach for the green time dependent vehicle routing problem. *Comput. Oper. Res.* **2017**, *88*, 297–305. [CrossRef]
32. Sun, P.; Veelenturf, L.P.; Dabia, S.; Van Woensel, T. The time-dependent capacitated profitable tour problem with time windows and precedence constraints. *Eur. J. Oper. Res.* **2018**, *264*, 1058–1073. [CrossRef]
33. Liu, C.; Kou, G.; Zhou, X.; Peng, Y.; Sheng, H.; Alsaadi, F.E. Time-dependent vehicle routing problem with time windows of city logistics with a congestion avoidance approach. *Knowl. Based Syst.* **2020**, *188*, 104813. [CrossRef]
34. Pan, B.; Zhang, Z.; Lim, A. A hybrid algorithm for time-dependent vehicle routing problem with time windows. *Comput. Oper. Res.* **2021**, *128*, 105193. [CrossRef]
35. Pan, B.; Zhang, Z.; Lim, A. Multi-trip time-dependent vehicle routing problem with time windows. *Eur. J. Oper. Res.* **2021**, *291*, 218–231. [CrossRef]
36. Gmira, M.; Gendreau, M.; Lodi, A.; Potvin, J.-Y. Tabu search for the time-dependent vehicle routing problem with time windows on a road network. *Eur. J. Oper. Res.* **2021**, *288*, 129–140. [CrossRef]
37. Xiao, Y.; Konak, A. The heterogeneous green vehicle routing and scheduling problem with time-varying traffic congestion. *Transp. Res. Part E Logist. Transp. Rev.* **2016**, *88*, 146–166. [CrossRef]
38. Huang, C.-L.; Jiang, Y.-Z.; Tan, S.-Y.; Yeh, W.-C.; Chung, V.Y.Y.; Lai, C.-M. Simplified swarm optimization for the time dependent competitive vehicle routing problem with heterogeneous fleet. In Proceedings of the 2018 IEEE Congress on Evolutionary Computation (CEC), Rio de Janeiro, Brazil, 8–13 July 2018; pp. 1–8.
39. Koç, Ç.; Bektaş, T.; Jabali, O.; Laporte, G. Thirty years of heterogeneous vehicle routing. *Eur. J. Oper. Res.* **2016**, *249*, 1–21. [CrossRef]
40. Golden, B.; Assad, A.; Levy, L.; Gheysens, F. The fleet size and mix vehicle routing problem. *Comput. Oper. Res.* **1984**, *11*, 49–66. [CrossRef]
41. Micheli, G.J.; Mantella, F. Modelling an environmentally-extended inventory routing problem with demand uncertainty and a heterogeneous fleet under carbon control policies. *Int. J. Prod. Econ.* **2018**, *204*, 316–327. [CrossRef]
42. Afshar-Nadjafi, B.; Afshar-Nadjafi, A. A constructive heuristic for time-dependent multi-depot vehicle routing problem with time-windows and heterogeneous fleet. *J. King Saud Univ. Eng. Sci.* **2017**, *29*, 29–34. [CrossRef]
43. Vincent, F.Y.; Redi, A.P.; Jewpanya, P.; Lathifah, A.; Maghfiroh, M.F.; Masruroh, N.A. A simulated annealing heuristic for the heterogeneous fleet pollution routing problem. In *Environmental Sustainability in Asian Logistics and Supply Chains*; Springer: Berlin/Heidelberg, Germany, 2019; pp. 171–204.
44. Wang, Z.; Qi, M.; Cheng, C.; Zhang, C. A hybrid algorithm for large-scale service network design considering a heterogeneous fleet. *Eur. J. Oper. Res.* **2019**, *276*, 483–494. [CrossRef]

45. De, M.; Giri, B. Modelling a closed-loop supply chain with a heterogeneous fleet under carbon emission reduction policy. *Transp. Res. Part E Logist. Transp. Rev.* **2020**, *133*, 101813. [CrossRef]
46. Soman, J.T.; Patil, R.J. A scatter search method for heterogeneous fleet vehicle routing problem with release dates under lateness dependent tardiness costs. *Expert Syst. Appl.* **2020**, *150*, 113302. [CrossRef]
47. Ropke, S.; Cordeau, J.F. Branch and cut and price for the pickup and delivery problem with time windows. *Transp. Sci.* **2009**, *43*, 267–286. [CrossRef]
48. Ben Ticha, H.; Absi, N.; Feillet, D.; Quilliot, A. Vehicle routing problems with road-network information: State of the art. *Networks* **2018**, *72*, 393–406. [CrossRef]
49. Crainic, T.G.; Frangioni, A.; Gendron, B. Bundle-based relaxation methods for multicommodity capacitated fixed charge network design. *Discret. Appl. Math.* **2001**, *112*, 73–99. [CrossRef]
50. Shelbourne, B.C.; Battarra, M.; Potts, C.N. The vehicle routing problem with release and due dates. *INFORMS J. Comput.* **2017**, *29*, 705–723. [CrossRef]
51. Xu, Z.; Ming, X.; Zheng, M.; Li, M.; He, L.; Song, W. Cross-trained workers scheduling for field service using improved NSGA-II. *Int. J. Prod. Res.* **2015**, *53*, 1255–1272. [CrossRef]
52. Vidal, T.; Crainic, T.G.; Gendreau, M.; Lahrichi, N.; Rei, W. A hybrid genetic algorithm for multidepot and periodic vehicle routing problems. *Oper. Res.* **2012**, *60*, 611–624. [CrossRef]
53. Zhong, S.-Q.; Du, G.; He, G.-G. Study on open vehicle routing problem with time windows limits and its genetic algorithm. *Jisuanji Gongcheng Yu Yingyong Comput. Eng. Appl.* **2006**, *42*, 201–204.
54. Wang, M.; Yeh, W.-C.; Chu, T.-C.; Zhang, X.; Huang, C.-L.; Yang, J. Solving multi-objective fuzzy optimization in wireless smart sensor networks under uncertainty using a hybrid of IFR and SSO algorithm. *Energies* **2018**, *11*, 2385. [CrossRef]
55. Yeh, W.-C.; Lai, C.-M.; Peng, Y.-F. Multi-objective optimal operation of renewable energy hybrid CCHP system using SSO. *J. Phys. Conf. Ser.* **2019**, *1411*, 012016. [CrossRef]
56. Lai, C.-M.; Chiu, C.-C.; Liu, W.-C.; Yeh, W.-C. A novel nondominated sorting simplified swarm optimization for multi-stage capacitated facility location problems with multiple quantitative and qualitative objectives. *Appl. Soft Comput.* **2019**, *84*, 105684. [CrossRef]
57. Yeh, W.-C.; Lin, Y.-P.; Liang, Y.-C.; Lai, C.-M. Convolution Neural Network Hyperparameter Optimization Using Simplified Swarm Optimization. *arXiv* **2021**, arXiv:2103.03995.
58. Yeh, W.-C. A novel boundary swarm optimization method for reliability redundancy allocation problems. *Reliab. Eng. Syst. Saf.* **2019**, *192*, 106060. [CrossRef]
59. Yeh, W.-C.; Su, Y.-Z.; Gao, X.-Z.; Hu, C.-F.; Wang, J.; Huang, C.-L. Simplified swarm optimization for bi-objection active reliability redundancy allocation problems. *Appl. Soft Comput.* **2021**, *106*, 107321. [CrossRef]
60. Yeh, W.-C. New parameter-free simplified swarm optimization for artificial neural network training and its application in the prediction of time series. *IEEE Trans. Neural Netw. Learn. Syst.* **2013**, *24*, 661–665. [PubMed]
61. Yeh, W.-C. Optimization of the disassembly sequencing problem on the basis of self-adaptive simplified swarm optimization. *IEEE Trans. Syst. Man Cybern. Part A Syst. Hum.* **2011**, *42*, 250–261. [CrossRef]
62. Yeh, W.-C. Simplified swarm optimization in disassembly sequencing problems with learning effects. *Comput. Oper. Res.* **2012**, *39*, 2168–2177. [CrossRef]
63. Lin, P.; Cheng, S.; Yeh, W.; Chen, Z.; Wu, L. Parameters extraction of solar cell models using a modified simplified swarm optimization algorithm. *Sol. Energy* **2017**, *144*, 594–603. [CrossRef]