



Article An Integrated EDAS Model for Fermatean Fuzzy Multi-Attribute Group Decision Making and Its Application in Green-Supplier Selection

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Abstract: The environment and economy benefit from the sustained growth of a high-quality green supplier. During a supplier evaluation and selection process, DMs tend to use fuzzy tools to express evaluation information due to complex practical problems. Therefore, this study explores the green-supplier evaluation method in a complex Fermatean fuzzy (FF) environment. First, a group of indicators was created to evaluate the green capabilities and the social impact of suppliers. Second, by combining the merits of the Heronian mean and power average approaches, a FF power Heronian mean and its weighted framework were developed, and their related properties and special families were then presented. Third, to acquire the relative importance of indicators, a marvelous unification of the best–worst method (BWM) and FF entropy is then introduced. The challenge of choosing a green supplier was finally solved using an integrated evaluation based on distance from the average solution (EDAS) evaluation framework in the FF environment. Finally, the presented tool's viability and robustness were confirmed by actual case analysis.

Keywords: Fermatean fuzzy set; BWM; green supplier selection; power Heronian mean; EDAS



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Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). 1. Introduction

Owing to the rapid advancement in science and technology since the industrial age, domestic output has been high, resulting in an unprecedented level of material riches for a nation and its citizens. Quality lifestyle requirements are becoming extremely important with the rise in residents' standards of living. Simultaneously, people's purchasing power has increased, propelling the domestic economy to a new level. However, various significant social issues have emerged as well [1]. Rapid resource consumption has led to an increase in environmental issues such as resource wastage and air pollution, which are extremely challenging for the environment in which people live. Social unrest, economic upheaval, and other unforeseen issues can eventually result from an ecological imbalance. If the economy continues to develop without taking environmental protection measures, the environment will inevitably become polluted, posing a threat to ecological security. Environmentalism and economic growth are never mutually exclusive; rather, they are interdependent. Economic growth will eventually be affected by ecological issues. Therefore, prioritizing environmental conservation should go hand in hand with economic development [2]; this is currently the global perspective. For example, the idea of sustainable development was explained in a global report as early as 1987. China developed and implemented a sustainable development strategy in March 1994 based on its unique national circumstances [3]. Since then, a global green wave has begun, which is necessary to advance sustainable development, raise awareness among citizens on the need to safeguard the environment, and encourage synchronized economic and environmental growth [4]. In this context, the concept of green supply chain management (GSCM) has emerged [5].

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All supply chain workstations and activities are intended to have a lower adverse ecological impact through the use of GSCM [6]. A crucial aspect of GSCM is selecting the right suppliers [7]. Green suppliers are in the upstream of the green supply chain, influencing the downstream of procurement and production, etc. [8]. Offering green products from green suppliers of the highest caliber can cut expenses while simultaneously preserving the environment and has the potential to attract customers for enterprises. This is highly beneficial to the enterprises and thereby enhances the abilities of both suppliers and businesses to compete [9,10]. Responding to social needs and selecting appropriate green suppliers for themselves is a strategic step toward scientific GSCM for business.

Green supplier selection (GSS) is a complex decision issue in a multi-attribute domain since it involves multiple options, attributes, and decision-makers (DMs) [11]. However, there is a significant amount of uncertainty in the green-supplier selection (GSS). DMs may not be able to provide exact evaluation values based on each assessment criterion in the context of GSS and evaluation. Fuzzy sets (FSs) [12], intuitionistic fuzzy sets (IFSs) [13], and Pythagorean fuzzy sets (PFSs) [14] were all used as tools to evaluate the green suppliers. However, the membership u and non-membership degree v of IFSs and PFSs need to satisfy the constraints $u + v \le 1$ [15] and $u^2 + v^2 \le 1$ [16], respectively, that prevent them from fully expressing uncertain information. Senapati and Yager [17] further reduced the limitations to $u^3 + v^3 \le 1$, and proposed Fermatean fuzzy sets (FFSs) to describe additional information.

A set of succinct and precise evaluation index systems is crucial to the GSS. Green suppliers have numerous intricate evaluation standards. Economical details in traditional supplier selection and environmental standards had been considered [18]. Scholars are particularly concerned about the impact of green issues on GSS. For example, Tavana et al. built an environmental capability evaluation index system for suppliers that included pollution controls, pollution products, green protection, and environmental protection [19].

Research on decision-making methods and models of GSS has also received increasing attention. Ref. [20] identified the importance of each green supplier evaluation index using the best-worst method (BWM). Ref. [21] constructed an analytic hierarchy process (AHP)-based model for determining the weight of green supplier indicators for steel companies. Evaluation information from a single DM is not convincing. Several multi-attribute group decision-making (MAGDM) models are proposed to obtain reliable GSS results. After integrating the fuzzy green evaluation information using the weighted average (WA) operator, VIKOR (ViseKriterijumska Optimizacija I Kompromisno Resenje) and TOPSIS obtained the ideal green suppliers for edible oil [14] and agricultural tool [22] enterprises, respectively.

With the above review, several research gaps continue to exist regarding GSS, as follows:

- Uncertainty exists in the evaluation standards of green suppliers. The information conveyed by IFSs and PFSs was limited. Few GSS studies considered the usefulness of Fermatean fuzzy (FF) evaluation information.
- (2) More factors need to be taken into account rather than just stating the characteristics of traditional green suppliers. Refs. [18,19] focused only on the economic and environmental benefits of green suppliers but neglected their social responsibility.
- (3) In the group decision model related to GSS, the WA operator utilized to assemble assessment fuzzy data in refs. [14,22] did not take into account the link between attributes and the incorrect judgment brought on by extreme data.
- (4) BWM [20] and AHP [21] were subjective weighting methods dominated by experts' subjective judgments. It is impossible to make a fully reasonable judgment on the importance of indicators without the joint participation of objective weighting methods. Furthermore, it is crucial to apply precise and consistent evaluation methods when ranking alternative solutions. Decision methods such as VIKOR [14] and TOPSIS [22] may increase the negative impact of extreme value decision results.

Driven by the above research gaps, this work's central thrust is to support the GSS using a comprehensive FF MAGDM approach, which combines the power Heronian mean (PHM) operator, BWM, the entropy weight method (EWM), and evaluation based on distance from the average solution (EDAS). In this context, information regarding expert evaluations is aggregated using the FFPHM operator. A novel BWM based on entropy measures is combined with the EWM to provide an integrated assignment approach. The best green supplier is found by using EDAS to rank the considered alternatives. The following are the study's significant contributions:

- (1) The GSS problem in the FF environment will be examined, where FFSs have a broader range of information representation.
- (2) Create a comprehensive set of index systems for evaluating green suppliers. This study developed a set of index systems combining traditional qualities, green attributes, and social attributes based on references and analysis of the existing index system.
- (3) We propose the FFPHM and FFWPHM operators by applying the PHM operator to the FF environment. The proposed operators consider the consistency and correlation of data when aggregating evaluation information.
- (4) For the GSS problem that it is unknown how important the various indicators are, an integrated weight calculation method is offered in the foundations of EWM and BWM. This integrated technique successfully lowers the disparity between subjective and objective information.
- (5) A FF MAGDM framework based on the integrated weight determination model and EDAS is developed. EDAS simplifies the calculation process while reducing the impact of extreme values on decision results. The method improves and deepens fuzzy decision theory and gives specialists technical direction for resolving GSS issues.

The remaining portions of this work are divided into several parts. The literature review component is given in Section 2. Section 3 summarizes and examines the current green supplier evaluation criteria before proposing a new set of criteria that consider social issues. Section 4 provides the definitions and properties of FFPHM and FFWPHM operators, while the fundamentals of FFSs, power average (PA), and Heronian mean (HM) operators are discussed in the Appendix A. In Section 5, an extended BWM based on the entropy measure is introduced along with the EWM. In an FF environment, the unique procedure of MAGDM based on an integrated EDAS is described. Through a case study of the GSS, Section 6 offers a sensitivity and comparison analysis to show the viability of the suggested strategy. The complete text is finally summarized in Section 7.

2. Literature Review

2.1. FFSs

Since Senapati first proposed the FFSs, scholars have recommended several aggregation operators to aggregate the FF information. Zeng et al. [23] introduced a FF Dombiweighted partitioned Muirhead mean operator and used it to aggregate Fermatean fuzzy numbers (FFNs) for evaluating the quality of online instructions while considering the complex correlation of attributes and calculation flexibility. Wei et al. [24] described Schweizer-Sklar algorithms for FFNs and created an FF Schweizer-Sklar weighted average operator; the operator has some flexibility because of its parameters. Similarly, Tan et al. [25] presented FF frank aggregation operators and operational principles of FFNs to increase the flexibility of fusing information. Mishra and Rani [26] created a FF-weighted aggregated sum product assessment (WASPAS) with the use of a unique score and entropy function to strategically support the Indian government's choice of appropriate medical waste disposal sites. The integration of FFSs with the additive ratio assessment (ARAS) and VIKOR methods by Gül [27] in response to the global health crisis allowed for the proper selection of laboratories for performing health testing because of the stark differences between the two approaches. The above analysis clearly shows that the FFSs have been cleverly used in combination with decision methods to solve decision problems in several domains. However, the use of EDAS to solve GSS problems in the FF environment has not been studied so far.

2.2. Power Heronian Mean Aggregation Operators

Owing to cognitive bias or personal preferences, DMs in a MAGDM process may give certain irrational assessment values (maximum or minimum). Yager [28] suggested the PA operator consider the degree of support between the data to lessen the effects of erroneous data by considering the integrity between the data. Many academics have conducted studies and consider academic promotion on this topic. Researchers have extended the PA operator to various fuzzy environments to aggregate information in various fuzzy contexts.

In addition, there are often cases in which attributes are correlated but cannot be solved by the PA operator in MAGDM problems. To effectively address interrelated issues, Beliakov et al. [29] introduced the HM operator to handle similar challenges efficiently. The HM operator was subsequently expanded upon by academics. Many academics merged the PA and HM operators to propose the PHM operator, which successfully reduced the detrimental effects of linked relationships while considering the interconnected relationships and integrity of the data. Shi et al. [30] developed a power geometry Heronian average operator in an intuitionistic fuzzy environment and provided related theorems and properties. Liu et al. [31] proposed a linguistic neutrosophic PHM operator.

It is observed that many scholars have recognized that PHM operators can compensate for the shortcomings of PA and HM operators, and they are widely extended to linguistic and fuzzy sets. However, no studies using PHM operators to integrate FF information have been found.

2.3. BWM and EWM for Attribute Weights

Attribute-confirmation methods are also significantly important in MAGDM problems with unknown attribute weights. Common weight determination methods include subjective and objective weight determination methods. However, owing to the limitations of both methods, integrated weights are often used to determine attribute weights. Since the BWM introduced by Rezaei [32] offers particular benefits when deciding on subjective weights, it is often combined with the EWM to calculate the integrated weights more comprehensively. In different fuzzy environments, research using the integrated weighting method to determine evaluation attribute weights has emerged. To solve MAGDM problems with IFSs, an integrated VIKOR model to select the best biowaste recycling channel was proposed by Liu et al. [33]. To examine the various relevance of probable factors that affect GSS, Wei et al. [24] presented a fuzzy entropy suitable for the FF environment and fused it with the traditional BWM to obtain an extended BWM. Ma et al. [34] combined the subjective and objective weights using a multiplicative integration method. An integrated model in a probabilistic linguistic fuzzy environment was proposed to evaluate online recycling platforms. The integrated weights that incorporate customer value and economic goals were determined using a combination assignment approach by Feng et al. [35] in a rough set that evaluates fuzzy information without membership functions.

Although the integrated assignment method based on BWM and EWM has received little attention, the entropy measures involved in [24] make the computation more complicated.

2.4. Evaluation Methods for GSS

There is abundant research on the evaluation methods of GSS. Wang et al. [21] suggested a complex AHP to enable DMs to measure and choose the best cooperative green suppliers in light of the vagueness of appropriate analysis information. A double-hierarchy hesitant fuzzy linguistic set was given the TODIM (an acronym in Portuguese for interactive and multi-criteria decision making) treatment by Krishankumar et al. [36] to tackle the GSS problem. After using AHP to determine the index weight, Nguyen et al. [18] used VIKOR to rank the green suppliers to be evaluated and finally gave the optimal solution. Xiong et al. [37] set elasticity as an indicator and upgraded the WASPAS to determine a solution to the multi-attribute selection problem of green suppliers based on IFSs. Liu et al. [38] utilized the BWM to assign weights to criteria based on experts' knowledge levels and familiarity with the problem. The study of a supplier selection method on the basis of a q-rung orthopair fuzzy set (QROFS) encompasses the ambiguity of the selection process. Considering the uncertainty of the selection process, a supplier selection method based on QROFS has been studied. Considering environmental criteria in traditional supply chains, Çalık [22] integrated fuzzy AHP and TOPSIS to select the most appropriate green cooperative supplier for the firm. Baki [39] employed ARAS to help with green supplier choice after investigating and determining the elements impacting GSS. Zhang et al. [40] designed an EDAS-based decision framework for optimal GSS in a picture-fuzzy environment. Zhang et al. [41] combined EDAS and cumulative prospect theory to evaluate community group purchase platforms in a probabilistic linguistic environment. Mishra et al. [42] used EDAS to select sustainable third-party reverse logistics providers. He et al. [43] extended the EDAS method to probabilistic uncertain linguistic sets to examine its applicability to GSS.

Scholars have been searching for decision models that can identify the best green suppliers. Compared with TOPSIS and VIKOR, EDAS has higher efficiency and less workload. The FFWA operator involved in the group decision of GSS [42] is prone to the loss of integrated information, which may prevent EDAS from obtaining a reasonable ranking. In summary, there is a lack of a hybrid GSS evaluation framework integrating the FFPHM operator, BWM, EWM, and EDAS.

3. Evaluation Index System for GSS

An upper-tier green supplier may support an organization's long-term expansion. The best GSS is crucial to operating a green supply chain. Contrary to typical supplier selection, the GSS should be based on both the economic and environmental benefits they may provide. It is essential to understand how to build a collection of powerful index systems. Therefore, this section presents a collection of index systems that combine environmental and economic criteria to lay the foundation for a reasonable GSS.

Although numerous studies have been conducted on choosing green suppliers, each study used a different index system. Nguyen et al. [18] set 12 sub-criteria to evaluate and select green suppliers based on five aspects: quality, cost, transportation, technology, and environment. Tavana et al. [19] proposed evaluation criteria for the best green tire recycling supplier based on the environmental dimensions, including green products, pollution output and control, and environmental management. Fazlollahtabar et al. [20] constructed a set of index systems containing eight primary elements and thirty-one sub-criteria for the GSS. To save resources and reduce pollution, Wang et al. [21] selected an optimal green supplier for Vietnamese steel manufacturing companies by considering five aspects: price, quality, transportation, service, and environment. In a study by Çalık [22], transportation, pollution control, product quality, and environmental responsibility were evaluated by different departments of the company to select optimal green suppliers. For the green limestone supplier selection, Krishankumar et al. [36] set up six benefit-type criteria containing green products and impressions and three cost-type criteria containing pollution and costs. Xiong et al. [37] determined the optimal elastic green supplier by evaluating green, elasticity, and coincidence attributes. Liu et al. [38] employed the BWM to determine the importance of the five attributes of product quality, green design, price, organizational and transportation capacity, and the supplier's environmentally friendly cooperative culture, in which product quality, transportation, and organizational capabilities were selected as the best and worst attributes, respectively. Baki [39] explored the factors that influence GSS and established eight influencing factors to be tested in three dimensions—classical, social, and environmental. The results showed that quality, social responsibility, service, cost, and green products were key variables impacting the GSS. For the textile enterprises, five primary characteristics were included in the index systems proposed by Xu et al. [44]: cost, quality, delivery, partnership, and environmental management. Wu et al. [45] selected a MAGDM method for GSS of electric vehicle charging equipment and constructed 12 sub-criteria from five levels of cost, quality, delivery, technology, and environment. Kang et al. [46] proposed seven main criteria, which include quality, cost, service cooperation, stability ability, green environment, green development, and green competition, for analyzing and choosing environmentally friendly suppliers in papermaking enterprises covering the three dimensions. For optimal GSS, Gegovska et al. [47] identified four classical criteria—quality, cost, transport, and service—and three environmental criteria, namely pollution control, green products, and environmental management. To reduce air pollution and select appropriate green suppliers for the construction of raw materials, Krishankumar et al. [48] established three benefit-type criteria, including product delivery, quality, and green design, and two cost-type criteria, including total cost, energy, and resource utilization. Table 1 summarizes the criteria in the GSS literature mentioned above and counts their frequency of occurrence.

Table 1. Evaluation criteria for the GSS.

Evaluation Criteria	[1 8]	[19]	[20]	[21]	[22]	[36]	[37]	[38]	[39]	[44]	[45]	[46]	[47]	[48]	Occurrence Percentage
Green design															64.29%
Service	v				v	•	•	•	v		•		v	v	50.00%
Green image	v			•					•					·	42.86%
Quality				\checkmark			-	\checkmark			\checkmark				78.57%
Environmental management															64.29%
Green product												\checkmark			28.57%
Delivery				\checkmark				\checkmark			\checkmark	\checkmark			71.43%
Cost			\checkmark	\checkmark				\checkmark	78.57%						
Technology				\checkmark					\checkmark		\checkmark	\checkmark	\checkmark		42.86%
Pollution control		\checkmark					\checkmark				\checkmark				50.00%
Energy resource utilization				\checkmark							\checkmark				35.71%
Social responsibility															14.29%
Cooperation							\checkmark	\checkmark		\checkmark			\checkmark		28.57%

From the literature review and the summary in Table 1, we find that cost (78.57%), quality (78.57%), delivery (71.43%), green design (64.29%), and environment management (64.29%) are the most frequently occurring indicators. Many studies place a high priority on them. Although the attribute "social responsibility" occurs only twice, the research by Baki [39] exactly showed that among the eight factors to be tested, social responsibility had a significant impact on GSS and could not be overlooked.

Hence, following the principles of wholeness, scientificity, and representativeness in selecting indicators, as well as focusing on the frequency of citations, this study combines classic attributes with green standards and social factors to establish a relatively complete green supplier evaluation index system. Environmental indicators are divided into environmental management and green design. Implementing environmental management and green product design at all levels of the supply chain helps meet consumers' environmental protection demands and achieve environmental benefit targets. Economic indicators include quality, cost, and delivery. Economic efficiency is the primary goal of enterprises and is a necessary prerequisite for achieving environmental and social benefits. Enterprises can improve economic efficiency by improving product quality, reducing costs, and shortening delivery times. Social responsibility is a social indicator. Companies should assume social responsibility and contribute to society. The achievement of social benefit objectives drives the sustainability of a company.

Environmental management (Λ_1): To minimize potential sources of pollution at all stages of production, green suppliers should formulate corresponding policies and plans to form a complete environmental management system [36]. The implementation of the policy and its continuous monitoring can be examined.

Green design (Λ_2): This is mainly measured by product design, including the total number of environment-friendly products, the use of energy-saving and consumption-reducing technologies, and the recovery and recycling of waste equipment [22,38].

Quality (Λ_3): Green suppliers should have a complete quality assurance system. Only by providing qualified products can the economic benefits of an enterprise be realized. Enterprises can assess the quality of products based on quality inspection pass rates and durability [45]. A higher quality inspection pass rate indicates better product quality, and excellent products have a lower failure rate during the warranty period [46].

Cost (Λ_4): Whether an enterprise can minimize cost and maximize profit margin depends on the price of the product provided by the supplier [21]. The price that depends on the cost can be measured by the transportation, environmental governance, and product research and development costs of green suppliers [18].

Delivery (Λ_5): Green suppliers should respond to order demands on time. Knowing the historical supply performance of green suppliers helps determine whether they have sufficient resources and production capacity to ensure the on-time delivery of products. It can also examine whether green suppliers have the flexibility to adjust their quantity and delivery time [18,47]. If the ordered products required by enterprises are provided within the shortest production cycle, it would be helpful to establish a long-term good partnership with them and help both parties to pre-empt fierce market competition.

Social responsibility (Λ_6): From a social perspective, green suppliers' social responsibility can be measured in aspects such as employee welfare [18], vocational training, and workplace mental health concerns. Understanding the compliance with laws and regulations, credit behavior, and administrative penalty records of green suppliers is also helpful.

4. Fermatean Fuzzy Power Heronian Mean Aggregation Operators

We will review the relevant knowledge of FFSs, PAs, and HM operators in preparation for suggesting new operators in this section. The definitions of PA and HM operators and the basics of FFSs are listed in Appendix A.

The PA aggregation method focuses on the integrity of the data [28], whereas the HM operator can help solve the problem of data correlation [29]. Using the benefits of PA and HM operators as a starting point, the concept of FFPHM is given below. The FFPHM operator is introduced for aggregating FF information in this study, which focuses on both integrity and correlation.

Definition 1. Let $\mathfrak{A}_i = (u_i, v_i)(i = 1, 2, ..., \lambda)$ be a set of FFNs, where $\xi, \zeta \ge 0$ and $\Psi_i = (1 + \sigma(\mathfrak{A}_i)) / \sum_{t=1}^{\lambda} (1 + \sigma(\mathfrak{A}_t))$. Then:

$$FFPHM(\mathfrak{A}_{1},\mathfrak{A}_{2},\ldots,\mathfrak{A}_{\lambda}) = \left(\frac{2}{\lambda(\lambda+1)} \mathop{\oplus}\limits_{i=1,i=j}^{\lambda} \left((\lambda \Psi_{i}\mathfrak{A}_{i})^{\zeta} \otimes (\lambda \Psi_{j}\mathfrak{A}_{j})^{\zeta} \right) \right)^{\frac{1}{\zeta+\zeta}}$$
(1)

is called FFPHM operator, where

$$\Omega(\mathfrak{A}_i,\mathfrak{A}_j) = 1 - d(\mathfrak{A}_i,\mathfrak{A}_j), \qquad (2)$$

$$\sigma(\mathfrak{A}_i) = \sum_{j=1, j \neq i}^{\lambda} \Omega(\mathfrak{A}_i, \mathfrak{A}_j).$$
(3)

Theorem 1. Let $\mathfrak{A}_i = (u_i, v_i)(i = 1, 2, ..., \lambda)$ be a set of FFNs where $\xi, \zeta \ge 0$. Hence, the aggregation result obtained by the FFPHM operator is also a FFN and can be described as:

$$FFPHM(\mathfrak{A}_1,\mathfrak{A}_2,\ldots,\mathfrak{A}_{\lambda}) = (\mu, \nu).$$
(4)

where
$$\mu = \left(1 - \left(\prod_{i=1,j=i}^{\chi} \left(1 - \left(1 - \left(1 - u_i^3\right)^{^{\chi\Psi_i}}\right)^{\xi} \left(1 - \left(1 - u_j^3\right)^{^{\chi\Psi_j}}\right)^{\zeta}\right)\right)^{\frac{2}{\chi(\chi+1)}}\right)^{\frac{1}{3(\xi+\zeta)}}$$
 and $v = \sqrt[3]{1 - \left(1 - \left(\prod_{i=1,j=i}^{\chi} \left(1 - \left(1 - v_i^{^{3\chi\Psi_i}}\right)^{\xi} \left(1 - v_j^{^{3\chi\Psi_j}}\right)^{\zeta}\right)\right)^{\frac{2}{\chi(\chi+1)}}\right)^{\frac{1}{\xi+\zeta}}}.$

Following Theorem 1, we will propose and demonstrate the properties of the FFPHM operator, laying the groundwork for its use.

Property 1 (Idempotency). Let $\mathfrak{A}_i = (u_i, v_i)(i = 1, 2, ..., \lambda)$ be a set of FFNs and $\mathfrak{A}_1 = \mathfrak{A}_2 = ... = \mathfrak{A}_{\lambda} = \mathfrak{A}$, then:

$$FFPHM(\mathfrak{A}_1,\mathfrak{A}_2,\ldots,\mathfrak{A}_{\mathfrak{X}})=\mathfrak{A}.$$

Proof.

$$FFPHM(\mathfrak{A}_{1},\mathfrak{A}_{2},\ldots,\mathfrak{A}_{\lambda}) = \left(\frac{2}{\lambda(\lambda+1)} \bigoplus_{i=1,i=j}^{\lambda} \left((\lambda \Psi_{i}\mathfrak{A}_{i})^{\zeta} \otimes (\lambda \Psi_{j}\mathfrak{A}_{j})^{\zeta} \right) \right)^{\frac{1}{\zeta+\zeta}}$$
$$= \left(\frac{2}{\lambda(\lambda+1)} \bigoplus_{i=1,i=j}^{\lambda} \left(\left(\chi \frac{(1+\sigma(\mathfrak{A}))}{\sum\limits_{t=1}^{\lambda}(1+\sigma(\mathfrak{A}))} \mathfrak{A} \right)^{\zeta} \otimes \left(\chi \frac{(1+\sigma(\mathfrak{A}))}{\sum\limits_{t=1}^{\lambda}(1+\sigma(\mathfrak{A}))} \mathfrak{A} \right)^{\zeta} \right) \right)^{\frac{1}{\zeta+\zeta}}$$
$$= \left(\frac{2}{\lambda(\lambda+1)} \bigoplus_{i=1,i=j}^{\lambda} \left(\mathfrak{A}^{\zeta+\zeta} \right) \right)^{\frac{1}{\zeta+\zeta}} = \mathfrak{A}$$

Property 2 (Commutativity). Let $\mathfrak{A}_i = (u_i, v_i)(i = 1, 2, ..., \lambda)$ be a set of FFNs and $\mathfrak{M}_1, \mathfrak{M}_2, \ldots, \mathfrak{M}_{\lambda}$ be a random permutation of $\mathfrak{A}_1, \mathfrak{A}_2, \ldots, \mathfrak{A}_{\lambda}$, then,

$$FFPHM(\mathfrak{A}_1,\mathfrak{A}_2,\ldots,\mathfrak{A}_{\chi})=FFPHM(\mathfrak{M}_1,\mathfrak{M}_2,\ldots,\mathfrak{M}_{\chi}).$$

Proof. Let $\psi_i = (1 + \sigma(\mathfrak{M}_i)) / \sum_{t=1}^{\lambda} (1 + \sigma(\mathfrak{M}_t))$, then

$$FFPHM(\mathfrak{A}_{1},\mathfrak{A}_{2},\ldots,\mathfrak{A}_{\lambda}) = \left(\frac{2}{\lambda(\lambda+1)} \underset{i=1,i=j}{\overset{\mathfrak{X}}{\bigoplus}} \left((\lambda \Psi_{i}\mathfrak{A}_{i})^{\xi} \otimes (\lambda \Psi_{j}\mathfrak{A}_{j})^{\zeta} \right) \right)^{\frac{1}{\xi+\zeta}} \\ = \left(\frac{2}{\lambda(\lambda+1)} \underset{i=1,i=j}{\overset{\mathfrak{X}}{\bigoplus}} \left((\lambda \Psi_{i}\mathfrak{M}_{i})^{\xi} \otimes (\lambda \Psi_{j}\mathfrak{M}_{j})^{\zeta} \right) \right)^{\frac{1}{\xi+\zeta}} \\ = FFPHM(\mathfrak{M}_{1},\mathfrak{M}_{2},\ldots,\mathfrak{M}_{\lambda})$$

Property 3 (Boundedness). Let $\mathfrak{A}_i = (u_i, v_i)(i = 1, 2, ..., \lambda)$ be a set of FFNs and $f_i = \lambda \frac{(1+\sigma(\mathfrak{A}_i))}{\sum\limits_{t=1}^{\lambda} (1+\sigma(\mathfrak{A}_t))} \mathfrak{A}_i$, where $f^+ = \max_i (f_i)$ and $f^- = \min_i (f_i)$. Then $f^- \leq FFPHM(\mathfrak{A}_1, \mathfrak{A}_2, ..., \mathfrak{A}_{\lambda}) \leq f^+.$ (5)

When aggregating FF decision information, it is often necessary to consider the importance of attributes. Therefore, we provide the concept of an FFWPHM operator as below.

Definition 2. Let $\mathfrak{A}_i = (u_i, v_i)(i = 1, 2, ..., \lambda)$ be a set of FFNs, where $\xi, \zeta \ge 0$. The weighted vector is $\varphi = (\varphi_1, \varphi_2, ..., \varphi_{\lambda})^T$, where $\varphi_i \in [0, 1]$ and $\sum_{i=1}^{\lambda} \varphi_i = 1$. Then,

$$FFWPHM(\mathfrak{A}_{1},\mathfrak{A}_{2},\ldots,\mathfrak{A}_{\lambda}) = \left(\frac{2}{\lambda(\lambda+1)} \underset{i=1,i=j}{\overset{\lambda}{\bigoplus}} \left(\left(\frac{\chi\varphi_{i}(1+\sigma(\mathfrak{A}_{i}))}{\overset{\lambda}{\sum}} g_{t}(1+\sigma(\mathfrak{A}_{t}))} \mathfrak{A}_{i}\right)^{\zeta} \otimes \left(\frac{\chi\varphi_{i}(1+\sigma(\mathfrak{A}_{j}))}{\overset{\lambda}{\sum}} g_{t}(1+\sigma(\mathfrak{A}_{t}))} \mathfrak{A}_{j}\right)^{\zeta} \right) \right)^{\frac{1}{\zeta+\zeta}}$$

is called the FFWPHM operator, where $\sigma(\mathfrak{A}_i) = \sum_{j=1, j \neq i}^{\chi} \varphi_j \Omega(\mathfrak{A}_i, \mathfrak{A}_j)$ and $\Omega(\mathfrak{A}_i, \mathfrak{A}_j)$ have the same definition.

Theorem 2. Let $\mathfrak{A}_i = (u_i, v_i)(i = 1, 2, ..., \lambda)$ be a set of FFNs, where $\xi, \zeta \ge 0$. The weighted vector is $\varphi = (\varphi_1, \varphi_2, ..., \varphi_{\lambda})^T$, where $\varphi_i \in [0, 1]$ and $\sum_{i=1}^{\lambda} \varphi_i = 1$. Then, the aggregation value obtained by the FFWPHM operator is also an FFN.

The FFWPHM shares the equivalent characteristics of the FFPHM operator. Since their properties are identical to those for Theorem 1 and Properties 1–3, correspondingly, the proofs are omitted.

5. Fermatean Fuzzy MAGDM Model Based on the Integrated EDAS Method

In this section, we outlined the precise phases of the suggested comprehensive MAGDM model.

5.1. Integrated Weight Based on BWM and Fermatean Fuzzy Entropy

(1) Objective weight determination based on the EWM.

Let $R = (h_{ij})_{\hbar \times \lambda}$ be the comprehensive evaluation matrix of the scheme sets $\mathbb{F} = \{\mathbb{F}_1, \mathbb{F}_2, \dots, \mathbb{F}_{\hbar}\}$ normalized under the criterion $\Lambda = \{\Lambda_1, \Lambda_2, \dots, \Lambda_{\lambda}\}$. Then, the objective weight ϕ_j for attribute $\Lambda_j (j = 1, 2, \dots, \lambda)$ can be calculated as:

$$\phi_j = \frac{1 - \widetilde{E}_j}{\lambda - \sum_{j=1}^{\chi} \widetilde{E}_j},\tag{6}$$

where $\widetilde{E}_j = \frac{1}{\hbar} \sum_{i=1}^{\hbar} E(h_{ij})$. $E(h_{ij})$ is the entropy value of h_{ij} and is calculated as follows [49]:

$$E(h_{ij}) = 1 - \left[\left(u_{ij}^3 - v_{ij}^3 \right) \left(u_{ij}^3 + v_{ij}^3 \right) \right]^2.$$
(7)

(2) Subjective weight determination method based on the BWM.

BWM significantly reduces errors that may be caused by the objective weights. Compared with AHP, BWM has fewer comparisons, less bias, and higher agreement. The BWM is widely used owing to its excellent characteristics. The specific steps of the BWM for determining the subjective weight are as follows.

Step 1. Choose the best Λ_B and worst attributes Λ_W in the attribute collection $\{\Lambda_1, \Lambda_2, \Lambda_3, \dots, \Lambda_{\lambda}\}$.

Step 2. Build comparison vectors $BO = (B_{B1}, B_{B2}, ..., B_{B\lambda})$ and $OW = (W_{1W}, W_{2W}, ..., W_{\lambda W})$, where B_{Bj} and $W_{j\lambda}(j = 1, ..., \lambda)$ represent the preference of the best attribute Λ_B over other attributes, and them over the worst attribute Λ_W . B_{Bj} and W_{jW} are expressed in FFNs.

Step 3. Compute the entropy values $E(B_{Bj})$ and $E(B_{jW})$ of B_{Bj} and W_{jW} based on Equation (7) and obtain the preference matrices *EBO* and *EOW*:

$$EBO = (E(B_{B1}), E(B_{B2}), \dots, E(B_{B\lambda})),$$
 (8)

$$EOW = (E(W_{1W}), E(W_{2W}), \dots, E(W_{\lambda W})).$$
 (9)

Step 4. The FF entropy measure is used to build the following BWM solution weight model:

$$s.t \begin{cases} \left| \frac{\varphi_B}{\varphi_B + \varphi_j} - E(B_{Bj}) \right| \le x \\ \left| \frac{\varphi_j}{\varphi_j + \varphi_W} - E(W_{jW}) \right| \le x \\ \sum_{j=1}^{\chi} \varphi_j = 1 \\ \varphi_j \ge 0 \end{cases}$$

$$(10)$$

Equation (10) can be changed subsequently to the formula below:

minv

$$\min x_{1}$$

$$s.t \begin{cases} \left| \varphi_{B} - (\varphi_{B} + \varphi_{j}) \times E(B_{Bj}) \right| \leq x_{1} \\ \left| \varphi_{j} - (\varphi_{j} + \varphi_{W}) \times E(W_{jW}) \right| \leq x_{1} \\ \sum_{j=1}^{\chi} \varphi_{j} = 1 \\ \varphi_{j} \geq 0 \end{cases}$$
(11)

By using the LINGO 18.0 software, we can easily obtain the subjective weights $\varphi_j = (\varphi_1, \varphi_2, \dots, \varphi_{\lambda})^T$.

(3) Integrated weight determination method based on the BWM and EWM.

According to the BWM and EWM, the subjective $\varphi_j = (\varphi_1, \varphi_2, ..., \varphi_\lambda)^T$ and objective weights $\phi_j = (\phi_1, \phi_2, ..., \phi_\lambda)^T$ are calculated. Therefore, the integrated weights can be calculated using the Equation (12):

$$\omega_j = \phi_j \, \varphi_j \, / \sum_{j=1}^{\lambda} \varphi_j \phi_j \,, \tag{12}$$

evidently, $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$.

5.2. Procedure of Fermatean Fuzzy Integrated EDAS Model

Consider a MAGDM issue that requires the cooperation of FFNs. Let $\mathbb{F} = {\mathbb{F}_1, \mathbb{F}_2, ..., \mathbb{F}_h}$ denote the discrete set of alternatives, while $\Lambda = {\Lambda_1, \Lambda_2, ..., \Lambda_{\lambda}}$ denotes the finite set of evaluation attributes. The expert evaluation set is $E = {e_1, e_2, ..., e_h}$ and corresponding weight is $\lambda = {\lambda_1, \lambda_2, ..., \lambda_h}$. Assume that the evaluation information of expert $e_k(k = 1, ..., h)$ about alternatives $\mathbb{F}_i \in \mathbb{F}$ under the considered attribute $\Lambda_j \in \Lambda$ is denoted by FFN $h_{ij}^k = (u_{ij}^k, v_{ij}^k)$, where $u_{ij}^k, v_{ij}^k \in [0, 1]$ and $0 \le (u_{ij}^k)^3 + (v_{ij}^k)^3 \le 1$. Therefore, the FFN evaluation matrix provided by expert $e_k(k = 1, ..., h)$ can be expressed as follows:

$$R^{k} = \left(h_{ij}^{k}\right)_{\hbar \times \mathfrak{X}} = \begin{pmatrix}h_{11}^{k} & \cdots & h_{1\mathfrak{X}}^{k} \\ \vdots & \ddots & \vdots \\ h_{\hbar 1}^{k} & \cdots & h_{\hbar \mathfrak{X}}^{k} \end{pmatrix}.$$
(13)

Considering the material above, Figure 1 depicts the workflow of the FF integrated EDAS model and the precise procedures for the FF integrated EDAS model are presented below.

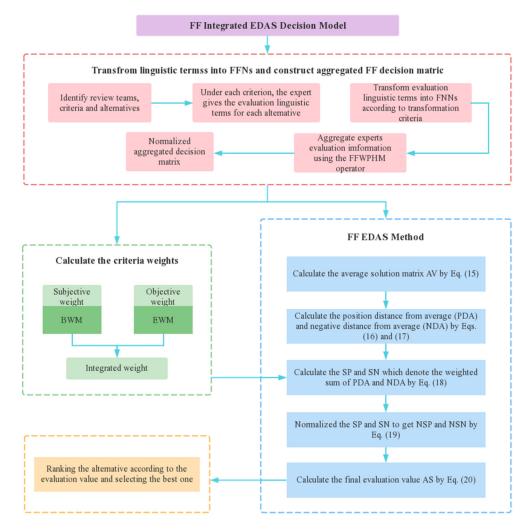


Figure 1. Process of FF's integrated EDAS model.

Step 1. To help experts better express evaluation information, each expert needs to provide personal evaluation linguistic terms for each alternative under each criterion. Using the conversion criteria in Table 2, the evaluated linguistic phrases are reduced to

FFNs. Therefore, the evaluation matrix $R^k = (h_{ij}^k)_{h \times X}$ of the expert $e_k (k = 1, 2, ..., h)$ can be obtained, where $h_{ij}^k = (u_{ij}^k, v_{ij}^k)$.

Table 2. Evaluating linguistic terms and conversion criteria for FFN.

Linguistic Term	FFN
Very Eligible (VE)	(0.9, 0.2)
Eligible (E)	(0.8, 0.3)
Medium Eligible (ME)	(0.7, 0.5)
Medium	(0.6, 0.6)
Medium Unqualified (MU)	(0.5, 0.7)
Unqualified (U)	(0.3, 0.8)
Very Unqualified (VU)	(0.2, 0.9)

Step 2. Using the FFWPHM operator proposed in Section 4, the individual evaluations of experts are integrated to obtain a comprehensive decision matrix $\tilde{R} = (\tilde{h}_{ij})_{\tilde{h} \times \tilde{\lambda}}$, where $\tilde{h}_{ij} = (\tilde{u}_{ij}, \tilde{v}_{ij})$.

Step 3. Make the decision matrix normalized. Unify attribute types with the conversion criterion (14) to further obtain the standardized decision $R = (h_{ij})_{h \times \lambda}$, where J_b and J_c are the benefit and cost-type attribute collections.

$$h_{ij} = (u_{ij}, v_{ij}) = \begin{cases} (\widetilde{u}_{ij}, \widetilde{v}_{ij}), C_j \in J_b \\ (\widetilde{v}_{ij}, \widetilde{u}_{ij}), C_j \in J_c \end{cases}$$
(14)

Step 4. Determine the average solution matrix $AV = [AV_j]_{1 \times \lambda}$:

$$AV_{j} = \frac{1}{\hbar} \stackrel{\hbar}{\underset{i=1}{\oplus}} h_{ij} = \left(\sqrt[3]{1 - \prod_{i=1}^{\hbar} \left(1 - (u_{ij})^{3}\right)^{\frac{1}{\hbar}}}, \prod_{i=1}^{\hbar} (v_{ij})^{\frac{1}{\hbar}}\right)$$
(15)

Step 5. Compute the positive distance from the average (PDA) and negative distance from the average (NDA):

$$\left(PDA_{ij}\right)_{\hbar \times \chi} = \frac{\max\left(0, S\left(h_{ij}\right) - S\left(AV_{j}\right)\right)}{S\left(AV_{j}\right)},\tag{16}$$

$$\left(NDA_{ij}\right)_{\hbar\times\chi} = \frac{\max\left(0, S\left(AV_{j}\right) - S\left(h_{ij}\right)\right)}{S\left(AV_{j}\right)},\tag{17}$$

where $S(AV_j)$ and $S(h_{ij})$ are the score values of AV_j and h_{ij} calculated by Equation (A2) in Appendix A.

Step 6. Calculate the subjective weight $\varphi_j = (\varphi_j, \dots, \varphi_{\lambda})^T$ and objective weight $\phi_j = (\phi_1, \dots, \phi_{\lambda})^T$ of the attributes based on the BWM and EWM proposed in Section 5, respectively. Thus, the integrated weight $\omega = (\omega_1, \dots, \omega_{\lambda})^T$ of the attributes according to Equation (12) is obtained.

Step 7. Aggregate the PDA and NDA to get SP_i and SN_i .

$$\begin{cases} SP_i = \sum_{j=1}^{\lambda} \omega_j PDA_{ij} \\ SN_i = \sum_{j=1}^{\lambda} \omega_j NDA_{ij} \end{cases}$$
(18)

Step 8. Normalize SP_i and SN_i .

$$NSP_{i} = \frac{SP_{i}}{\max(SP_{i})}$$

$$NSN_{i} = 1 - \frac{SN_{i}}{\max(SN_{i})}$$
(19)

Step 9. Calculate the final evaluation value AS_i of the alternative \mathbb{F}_i .

$$AS_i = \frac{NSP_i + NSN_i}{2} \tag{20}$$

Step 10. Sort the alternatives on the basis of AS_i . The larger the value of AS_i , the better the alternative \mathbb{F}_i is.

6. Case Study

A business is currently suggesting that the best partner be selected from four environmental suppliers (\mathbb{F}_1 , \mathbb{F}_2 , \mathbb{F}_3 , and \mathbb{F}_4). An expert panel consisting of the top management in this company as well as university academics and representatives from research institutes in the fields of supply chain and environmental management is formed. Four experts $e_k(k = 1, 2, 3, 4)$ in total with the weight vector (0.27, 0.2, 0.3, 0.23)^{*T*} are invited to assess the four green suppliers according to attributes $\Lambda_j(j = 1, 2, 3, 4, 5, 6)$. Among them, Λ_1 is environmental management, Λ_2 is green design, is quality, Λ_4 is cost Λ_3 , Λ_5 is delivery, and Λ_6 is social responsibility. It is easily identifiable that Λ_4 is a cost-type attribute and the rest are benefit-type attributes. The specific process steps are as follows:

Step 1. Create an individual FF assessment matrix. Based on the six constructed attributes above, experts e_k (k = 1, 2, 3, 4) provided evaluations in linguistic terms of the four green suppliers that are observed in Table 3. The linguistic words are then converted into FFNs using the conversion criteria in Table 2. Finally, Table 4 summarizes the evaluation data of the four experts.

Experts	Alternatives	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6
	\mathbb{F}_1	Е	Е	U	U	VE	Е
0.	\mathbb{F}_2	Μ	VU	Е	Μ	U	VE
e_1	\mathbb{F}_3	E	Е	U	VU	Μ	U
	\mathbb{F}_4	ME	E	E	Μ	Μ	U
	\mathbb{F}_1	VE	Е	U	М	Е	ME
<i>R</i> ₂	\mathbb{F}_2	U	MU	E	ME	Μ	E
<i>e</i> ₂	\mathbb{F}_3	E	VE	MU	U	ME	MU
	\mathbb{F}_4	VE	VE	E	U	Μ	Ε
	\mathbb{F}_1	Е	Е	М	U	Е	Е
80	\mathbb{F}_2	U	MU	E	E	Μ	VE
<i>e</i> ₃	\mathbb{F}_3	VE	VE	U	U	Μ	U
	\mathbb{F}_4	Е	ME	VE	М	E	U
	\mathbb{F}_1	VE	ME	U	ME	VE	Е
0.	\mathbb{F}_2	U	U	ME	VE	Е	Е
e_4	\mathbb{F}_3	VE	Е	U	MU	М	U
	\mathbb{F}_4	Е	М	Е	М	VE	М

Table 3. Expert evaluation of linguistic terms.

Experts	Alternatives	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6
	\mathbb{F}_1	(0.8, 0.3)	(0.8, 0.3)	(0.3, 0.8)	(0.3, 0.8)	(0.9, 0.2)	(0.8, 0.3)
0.	\mathbb{F}_2	(0.6, 0.6)	(0.2, 0.9)	(0.8, 0.3)	(0.6, 0.6)	(0.3, 0.8)	(0.9, 0.2)
e_1	\mathbb{F}_3	(0.8, 0.3)	(0.8, 0.3)	(0.3, 0.8)	(0.2, 0.9)	(0.6, 0.6)	(0.3, 0.8)
	\mathbb{F}_4	(0.7, 0.5)	(0.8, 0.3)	(0.8, 0.3)	(0.6, 0.6)	(0.6, 0.6)	(0.3, 0.8)
	\mathbb{F}_1	(0.9, 0.2)	(0.8, 0.3)	(0.3, 0.8)	(0.6, 0.6)	(0.8, 0.3)	(0.7, 0.5)
0.0	\mathbb{F}_2	(0.3, 0.8)	(0.5, 0.7)	(0.8, 0.3)	(0.7, 0.5)	(0.6, 0.6)	(0.8, 0.3)
<i>e</i> ₂	\mathbb{F}_3	(0.8, 0.3)	(0.9, 0.2)	(0.5, 0.7)	(0.3, 0.8)	(0.7, 0.5)	(0.5, 0.7)
	\mathbb{F}_4	(0.9, 0.2)	(0.9, 0.2)	(0.8, 0.3)	(0.3, 0.8)	(0.6, 0.6)	(0.3, 0.8)
	\mathbb{F}_1	(0.8, 0.3)	(0.8, 0.3)	(0.6, 0.6)	(0.3, 0.8)	(0.8, 0.3)	(0.8, 0.3)
e ₃	\mathbb{F}_2	(0.3, 0.8)	(0.5, 0.7)	(0.8, 0.3)	(0.8, 0.3)	(0.6, 0.6)	(0.9, 0.2)
63	\mathbb{F}_3	(0.9, 0.2)	(0.9, 0.2)	(0.3, 0.8)	(0.3, 0.8)	(0.6, 0.6)	(0.3, 0.8)
	\mathbb{F}_4	(0.8, 0.3)	(0.7, 0.5)	(0.9, 0.2)	(0.6, 0.6)	(0.8, 0.3)	(0.3, 0.8)
	\mathbb{F}_1	(0.9, 0.2)	(0.7, 0.5)	(0.3, 0.8)	(0.7, 0.5)	(0.9, 0.2)	(0.8, 0.3)
ρ.	\mathbb{F}_2	(0.3, 0.8)	(0.3, 0.8)	(0.7, 0.5)	(0.9, 0.2)	(0.8, 0.3)	(0.8, 0.3)
e_4	\mathbb{F}_3	(0.9, 0.2)	(0.8, 0.3)	(0.3, 0.8)	(0.5, 0.7)	(0.6, 0.6)	(0.3, 0.8)
	\mathbb{F}_4	(0.8, 0.3)	(0.6, 0.6)	(0.8, 0.3)	(0.6, 0.6)	(0.9, 0.2)	(0.6, 0.6)

Table 4. Expert evaluation information.

Step 2. Four individual assessments are integrated with the use of FFWPHM operator $(\xi = \zeta = 3)$ to obtain the collective decision matrix $\tilde{R} = (\tilde{h}_{ij})_{4 \times 6}$, as summarized in Table 5.

Table 5. Collective decision matrix \tilde{R} .

	Λ_1	Λ_2	Λ_3
\mathbb{F}_1	(0.8548, 0.2530)	(0.7926, 0.3503)	(0.5371, 0.7259)
\mathbb{F}_2	(0.5200, 0.7350)	(0.4058, 0.7929)	(0.7926, 0.3503)
\mathbb{F}_3	(0.8696, 0.2564)	(0.8655, 0.2570)	(0.4027, 0.7794)
\mathbb{F}_4	(0.8151, 0.3291)	(0.7946, 0.3963)	(0.8446, 0.2798)
	Λ_4	Λ_5	Λ_6
\mathbb{F}_1	(0.6009, 0.6741)	(0.8655, 0.2545)	(0.7960, 0.3454)
\mathbb{F}_2	(0.8092, 0.3933)	(0.6887, 0.5574)	(0.8748, 0.2570)
\mathbb{F}_3^-	(0.4238, 0.7869)	(0.6256, 0.5797)	(0.4027, 0.7794)
\mathbb{F}_4	(0.6021, 0.6201)	(0.8034, 0.4115)	(0.4948, 0.7478)

Step 3. Make the decision matrix normalized. Since Λ_4 is a cost-type attribute, it is transformed into a benefit-type attribute using Equation (14). Thus, the normalized decision matrix $R = (h_{ij})_{4\times 6}$ has been obtained and represented in Table 6.

Table 6. Normalized decision matrix *R*.

	Λ_1	Λ_2	Λ_3
\mathbb{F}_1	(0.8548, 0.2530)	(0.7926, 0.3503)	(0.5371, 0.7259)
\mathbb{F}_2	(0.5200, 0.7350)	(0.4058, 0.7929)	(0.7926, 0.3503)
\mathbb{F}_3^-	(0.8696, 0.2564)	(0.8655, 0.2570)	(0.4027, 0.7794)
\mathbb{F}_4	(0.8151, 0.3291)	(0.7946, 0.3963)	(0.8446, 0.2798)
	Λ_4	Λ_5	Λ_6
\mathbb{F}_1	(0.6741, 0.6009)	(0.8655, 0.2545)	(0.7960, 0.3454)
\mathbb{F}_2	(0.3933, 0.8092)	(0.6887, 0.5574)	(0.8748, 0.2570)
\mathbb{F}_3	(0.7869, 0.4238)	(0.6256, 0.5797)	(0.4027, 0.7794
\mathbb{F}_4	(0.6201, 0.6021)	(0.8034, 0.4115)	(0.4948, 0.7478)

Step 4. Compute the average solution matrix $AV = [AV_i]_{1 \times 6}$.

 $\left[AV_{j}\right]_{1\times6} = \left\langle \begin{array}{c} (0.8070, 0.3539), (0.7746, 0.4101), (0.7178, 0.4852), \\ (0.6618, 0.5935), (0.7710, 0.4289), (0.7333, 0.4769) \end{array} \right\rangle$

Step 5. Compute the PDA and NDA, as listed in Tables 7 and 8.

Table 7. PDA.

	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6
\mathbb{F}_1	0.2642	0.1497	0.0000	0.1067	0.6653	0.6201
\mathbb{F}_2	0.0000	0.0000	0.7800	0.0000	0.0000	1.2824
\mathbb{F}_3	0.3313	0.5955	0.0000	4.0925	0.0000	0.0000
\mathbb{F}_4	0.0511	0.1102	1.2710	0.0000	0.1831	0.0000

Table 8. NDA.

	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6
\mathbb{F}_1	0.0000	0.0000	1.8905	0.0000	0.0000	0.0000
\mathbb{F}_2	1.5330	2.0905	0.0000	6.8093	0.5956	0.0000
\mathbb{F}_3	0.0000	0.0000	2.5965	0.0000	0.8682	2.4275
\mathbb{F}_4	0.0000	0.0000	0.0000	0.7508	0.0000	2.0389

Step 6. First, using the BWM suggested in this study, the subjective weights φ_i (*i* = 1,...,6) of the attributes were determined. Following the advice of the expert panel, the greatest and worst characteristics were Λ_1 and Λ_4 , respectively. The preferences for the finest and worst attributes in relation to other attributes were ascertained using FF information to obtain FFBO and FFOW:

FFBO = ((0.5, 0.5), (0.85, 0.25), (0.92, 0.15), (0.9, 0), (0.91, 0.2), (0.93, 0.5))

FFOW = ((0.9, 0), (0.95, 0.2), (0.85, 0.25), (0.5, 0.5), (0.92, 0.2), (0.95, 0.15)).

Calculate the entropy value of each FFN using Equation (7):

EBO = (1.000, 0.8579, 0.6323, 0.7176, 0.6776, 0.6014)

EWO = (0.7176, 0.4597, 0.8579, 1.0000, 0.6324, 0.4597)

A linear model of the problem is constructed as follows:

```
\min x_1
```

```
 \begin{split} \min x_1 \\ \left\{ \begin{array}{l} |\varphi_1 - (\varphi_1 + \varphi_2) \times 0.8579| \leq x_1 \\ |\varphi_1 - (\varphi_1 + \varphi_3) \times 0.6323| \leq x_1 \\ |\varphi_1 - (\varphi_1 + \varphi_4) \times 0.7176| \leq x_1 \\ |\varphi_1 - (\varphi_1 + \varphi_5) \times 0.6776| \leq x_1 \\ |\varphi_1 - (\varphi_1 + \varphi_6) \times 0.6014| \leq x_1 \\ |\varphi_2 - (\varphi_2 + \varphi_4) \times 0.4597| \leq x_1 \\ |\varphi_3 - (\varphi_3 + \varphi_4) \times 0.8579| \leq x_1 \\ |\varphi_5 - (\varphi_5 + \varphi_4) \times 0.4597| \leq x_1 \\ |\varphi_6 - (\varphi_6 + \varphi_4) \times 0.4597| \leq x_1 \\ |\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 + \varphi_5 + \varphi_6 = 1 \\ |\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6 \geq 0 \end{split}
```

The subjective weights were then calculated using the LINGO 18.0 software.

Second, the EWM mentioned in Section 5.1 was implemented to calculate the objective weights. Finally, the integrated weights were calculated using Equation (12). Attribute weights of different types were presented in Table 9.

Table 9. Attribute weights of different types.

Method	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6
Subjective weights φ	0.288	0.090	0.223	0.078	0.189	0.132
Objective weights ϕ	0.249	0.201	0.143	0.075	0.142	0.190
Integrated weights ω	0.400	0.100	0.178	0.033	0.150	0.140

Step 7. Using the integrated weights from Table 9 as a guide, aggregate the PDA and NDA to obtain SP_i and SN_i , and then normalize them to get NSP_i and NSN_i . Finally, the final evaluation value AS_i was calculated and the alternatives were ranked. Table 10 provides a summary of the findings.

Table 10. Calculation results and ranking under integrated weights.

	SPi	SNi	NSP _i	NSN _i	AS _i	Ranking
\mathbb{F}_1	0.3104	0.3384	0.9620	0.7000	0.8310	1
\mathbb{F}_2	0.3192	1.1280	0.9890	0.0000	0.4945	4
\mathbb{F}_3	0.3227	0.0948	1.0000	0.1712	0.5856	3
\mathbb{F}_4	0.2864	0.3095	0.8875	0.7256	0.8066	2

As shown in Table 10, the alternatives are ranked $\mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_3 \succ \mathbb{F}_2$ and \mathbb{F}_1 is the optimal alternative.

6.1. Sensitivity Analysis

Attribute weights hold an important position in evaluation and decision results, and effective GSS can be aided by reasonable attribute weights. Subjective weights based on the BWM, objective weights based on the EWM, and integrated weights were then calculated in this study. Here, the impact of EDAS based on different weight types on GSS was analyzed. The results calculated using the subjective and objective weights are listed in Tables 11 and 12, respectively.

Table 11. Calculation results and ranking under subjective weights.

	SPi	SNi	NSPi	NSN _i	AS _i	Ranking
\mathbb{F}_1	0.3055	0.4216	0.6524	0.6689	0.6607	2
\mathbb{F}_2	0.3432	1.2733	0.7330	0.0000	0.3665	4
\mathbb{F}_3^-	0.4682	1.0635	1.0000	0.1648	0.5824	3
\mathbb{F}_4	0.3427	0.3277	0.7319	0.7427	0.7373	1

Table 12. Calculation results and ranking under objective weights.

	SPi	SNi	NSP _i	NSN _i	AS _i	Ranking
\mathbb{F}_1	0.3115	0.2703	0.6118	0.8103	0.7111	1
\mathbb{F}_2	0.2808	1.4252	0.5516	0.0000	0.2758	4
\mathbb{F}_3^-	0.5091	1.8558	1.0000	0.3995	0.6998	2
\mathbb{F}_4	0.2512	0.3252	0.4935	0.7717	0.6326	3

The evaluation values and change trends of each alternative under different weight types are shown in Figure 2.

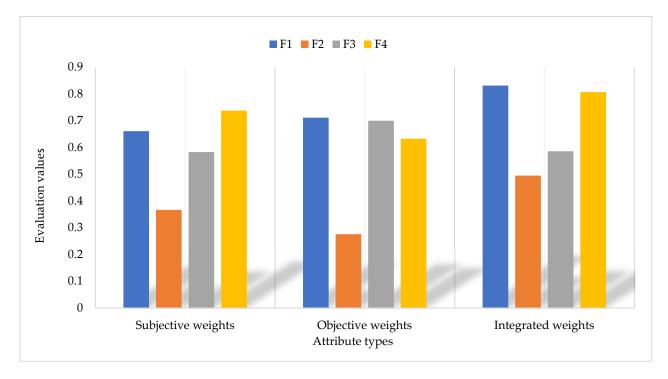


Figure 2. Evaluation values of each alternative under different types of weights.

From Tables 11 and 12 and Figure 2, we can see that the alternative under the subjective weight is ranked $\mathbb{F}_4 \succ \mathbb{F}_1 \succ \mathbb{F}_3 \succ \mathbb{F}_2$, the ranking list under the objective weight is $\mathbb{F}_1 \succ \mathbb{F}_3 \succ \mathbb{F}_1 \succ \mathbb{F}_3 \succ \mathbb{F}_2$, and the $\mathbb{F}_1 \succ \mathbb{F}_3 \succ \mathbb{F}_2$ is under the integrated method. On the basis of these ordered lists, we can see that when the attribute weights are different, the ranking of the schemes differs. There is a slight difference between the objective and integrated weights of the alternative ranking, whereas the subjective and integrated weights while evaluating and choosing green suppliers. Since the objective weight generally rests with objective data, we should focus on both the subjective awareness of evaluation experts and the objective weights leads to more rational decisions.

The decision's effectiveness is directly correlated with the change in parameters. We will conduct sensitivity analyses on several parameters involved in the integrated EDAS group decision model below. In the above numerical study, when aggregating expert evaluation information using the proposed FFWPHM operator, only case $\xi = \zeta = 3$ is considered, which cannot comprehensively demonstrate the stability of the FFWPHM operator and EDAS in evaluating green suppliers. Subsequently, we discuss the selection of the ideal solution for the green suppliers in relation to the adjustment of the parameters ξ and ζ in the FFWPHM operator.

The proposed FFWPHM operator includes two variables, ξ and ζ , thus when those values change, the way in which expert assessment data is integrated will likewise change. Due to the different decision matrices obtained by the aggregation of operators, the integrated weights based on aggregated data also change accordingly. The changes to the optimal GSS achieved by different parameters in ξ and ζ remain to be discussed. We set ξ and ζ as different real numbers and fused the rigorous assessment data. Table 13 lists the ranking outcomes using the EDAS and FFWPHM operator under different parameter combinations.

	AS ₁	AS ₂	AS ₃	AS_4	Ranking
FFWPHM ^(2,2) – EDAS	0.7856	0.5000	0.4786	0.8019	$\mathbb{F}_4 \succ \mathbb{F}_1 \succ \mathbb{F}_2 \succ \mathbb{F}_3$
FFWPHM ^(2.5,3) – EDAS	0.8348	0.5033	0.5840	0.8152	$\mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_3 \succ \mathbb{F}_2$
FFWPHM ^(3,3.5) – EDAS	0.8007	0.4551	0.5986	0.7804	$\mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_3 \succ \mathbb{F}_2$
$FFWPHM^{(3.5,4)} - EDAS$	0.7722	0.4186	0.6079	0.7558	$\mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_3 \succ \mathbb{F}_2$
$FFWPHM^{(4,4.5)} - EDAS$	0.7393	0.3778	0.6217	0.7266	$\mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_3 \succ \mathbb{F}_2$
$FFWPHM^{(4.5,5)} - EDAS$	0.7081	0.3380	0.6383	0.6974	$\mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_3 \succ \mathbb{F}_2$
FFWPHM ^(5,5) – EDAS	0.6968	0.3228	0.6445	0.6862	$\mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_3 \succ \mathbb{F}_2$
$FFWPHM^{(5.5,5.5)} - EDAS$	0.6668	0.2822	0.6672	0.6582	$\mathbb{F}_3 \succ \mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_2$
FFWPHM ^(6,6) – EDAS	0.6374	0.2446	0.6913	0.6309	$\mathbb{F}_3 \succ \mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_2$

Table 13. Calculation results under different ξ and ζ .

From Table 13, it is noted that while ξ and ζ change between [2.5, 5], the scheme's sort remains consistent $\mathbb{F}_1 \succ \mathbb{F}_4 \succ \mathbb{F}_3 \succ \mathbb{F}_2$, which is the same as the result when $\xi = \zeta = 3$. This means that the proposed integrated EDAS decision model in the FF environment has a certain stability under different values of ξ and ζ . It can also be observed that the ranking of the solutions alters as ξ and ζ are beyond the above mentioned range. However, the poorest option is always the alternative \mathbb{F}_2 , while the best choice changes from \mathbb{F}_1 to \mathbb{F}_3 . In other words, the operator can consistently aggregate decision information while remaining adaptable. To adapt to various decision situations, DMs can adjust the parameters in response to their risk preferences.

6.2. Comparative Analysis

Some current representative decision mechanisms, specifically TOPSIS [17], WAS-PAS [26], VIKOR [27], and ARAS [27], are adopted for comparative analysis to further verify the viability and applicability of the proposed framework in the FF environment. To guarantee the uniformity of the results, under the situation of $\xi = \zeta = 3$, the weight vector (0.288, 0.090, 0.223, 0.078, 0.189, 0.132)^{*T*} computed in this work is incorporated into the calculation procedure of each approach. Tables 14 and 15 display the main computing results and rankings produced by various decision techniques.

	Proposed Integrated EDAS		VIKOR [27]				ARAS [27]		
	ASi	Ranking	Si	R _i	Qi	Ranking	$sc(\mathbb{F}_i)$	K _i	Ranking
\mathbb{F}_1	0.6607	2	0.2938	0.1834	0.1588	3	0.4487	0.7358	1
\mathbb{F}_2	0.3665	4	0.6155	0.2880	1.0000	1	0.2067	0.3389	4
\mathbb{F}_3^-	0.5824	3	0.5440	0.2230	0.7102	2	0.3141	0.5150	3
\mathbb{F}_4	0.7373	1	0.3005	0.1189	0.0121	4	0.4268	0.6998	2

Table 14. Main computing results and ranking under EDAS, VIKOR, and ARAS.

Table 15. Main computing results and rankings under TOPSIS and WASPAS.

	TOPSIS [17]				WASPAS [26]			
	$D(\mathbb{F}_{i},\mathbb{F}^+)$	$D(\mathbb{F}_{i},\mathbb{F}^{-})$	$\boldsymbol{\varsigma}(\mathbb{F}_i)$	Ranking	$\mathbb{C}_i^{(1)}$	$\mathbb{C}_i^{(2)}$	\mathbb{C}_i	Ranking
\mathbb{F}_1	0.1803	0.2249	-0.0510	2	0.5380	0.4526	0.4953	1
\mathbb{F}_2	0.2013	0.2159	-0.2103	3	0.3977	0.2817	0.3397	4
\mathbb{F}_3^-	0.2140	0.2046	-0.3322	4	0.4580	0.2852	0.3716	3
\mathbb{F}_4	0.1750	0.2297	0.0000	1	0.5273	0.4613	0.4943	2

As observed by Tables 14 and 15, not only do the computing values computed by the established model differ from the current methods, but also the final green supplier ranking produced is significantly varied. Only the optimal solution obtained by [17] is compatible with the suggested model and \mathbb{F}_4 is the best green supplier. Different core ideas of each decision method cause variations in ranking.

M

Criteria weights

Parameters involved

No

Assumed

No

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By modifying the relationship between the weighted summation and the product model, WASPAS can increase decision accuracy. The core idea of the ARAS approach for choosing the appropriate solution is to identify the weighted decision matrix's optimal function for calculating the utility level of the decision object. The proximity to optimal solutions, both positive and negative, is emphasized by TOPSIS and VIKOR. However, TOPSIS only adds up the distances between negative and positive ideal alternatives, while VIKOR additionally takes into account the relative significance of these distances. The distinguishing characteristic of EDAS is that it assesses options by figuring out their distance from the typical option. With EDAS, the extremely positive and negative ideal solutions can be converted into the average solution, which has a more realistic meaning.

According to the study presented above and the thorough comparison shown in Table 16, compared to the created model, the following drawbacks of the other decision approaches exist:

	Established Model	[17]	[26]	[27]	[27]
Ranking method	EDAS	TOPSIS	WASPAS	VIKOR	ARAS
Decision process	Group	Single	Group	Single	Single
Multiple aggregation strategies	Yes	No	No	No	No
DMs' weights	Assumed	No	Computed	No	No

Integrated

Yes

Table 16. A comprehensive comparison of different approaches.

(1)In regard to the ranking approach, it is not suitable to utilize the closeness degree formula that was finally employed for ranking in [17] when an alternative to being considered is a positive ideal solution. The concept of superior and inferior solutions is transformed by EDAS into a compromise idea, which significantly improves the influence of extreme values on the decision outcome. The FF weighted average (FFWA) operator engaged in research [26,27] may result in information loss and even rank inability when membership or non-membership is equal to zero in the FF environment.

Objective

Yes

Assumed

Yes

Assumed

No

- (2) Only simple decision-making environments are covered by [17,27]. Due to insufficient information and poor consideration, a single DM might not be capable of making appropriate decisions. Meanwhile, the introduction of the FFWA operator into the MAGDM by [26] may cause incorrect initial assessment information aggregation. The decision-making model proposed assumes the participation of numerous DMs, and the choice results generated by the group of DMs with their collective wisdom are more practical to implement.
- (3)All other approaches engaged in the comparison only focus on the objective data and consider the objective weights of attributes in their investigations but neglect the subjective judgment of DMs, which is a main drawback. Subjective weights are rather realistic and aid in lowering the bias of the results. The integrated weighting technique constructed can measure the importance of attributes more comprehensively and also addresses the unscientific effects brought on by too strong subjective psychology in the calculation.

7. Conclusions

This study constructed a comprehensive series of index systems for GSS, and an innovative MAGDM strategy for selecting the ideal green supplier was created. The proposed model used the FFWPHM operator to aggregate the FF information of the reviewing experts, which overcomes the obstacles of data correlation and incompleteness. A novel BWM based on FF entropy was combined with the EWM to compute the integrated weight of each attribute, which respects the subjective judgment of DMs and relies on objective data. Finally, EDAS was used to evaluate the options, and the numerical analysis

shows that option 1 is the most reasonable green supplier. The findings of the sensitivity analysis show that the sequence of alternatives remained the same when the parameters were altered within a certain range, thus demonstrating the robustness of the proposed model. The results of the comparative analysis highlight the limitations of other methods and illustrate the strong applicability of the suggested approach.

This study provides management suggestions for companies and suppliers. In the context of green development nowadays, suppliers should properly reconcile environmental protection with economic efficiency. From the case study, we find that DMs place a high priority on environmental management. So suppliers can improve their competitiveness by upgrading their environmental management capabilities. Enterprises should also create social value while improving their profits. It costs labor and material resources to implement green technology and product innovation in a short period of time, but these investments are beneficial to boost the core competitiveness of enterprises from a development standpoint.

Although our proposed hybrid model can provide applied value for GSS, there are still limitations in the research. Green supplier evaluation involves various indicators in multiple dimensions; however, the constructed index system does not cover all the sub-criteria. Future research may expand or add some other related indicators to build a more scientific and comprehensive index system. This paper discusses the ideal situation where the DMs' weight is known; however, the social interactions between experts can be complex, and the situations where the DMs' weight is unknown exist in the actual decision process. In future research, we can introduce social networks into the calculation of expert weights.

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Appendix A

Here we will briefly review the fundamentals of FFSs, specifically the basic concepts, operations, and comparison rules between FFNs. The basic concepts of HM and PA operators are reviewed to enable laying of the groundwork for suggesting new operators.

Definition A1 ([17]). Let X be a non-empty set. An FFS is presented as follows:

$$F = \{ \langle x_j, u_F(x_j), v_F(x_j) \rangle | x_j \in X \},$$
(A1)

where $u: X \to [0,1]$ is the membership function $u_F(x_j) (0 \le u_F(x_j) \le 1)$ and $v: X \to [0,1]$ is the non-membership function $v_F(x_j) (0 \le v_F(x_j) \le 1)$. For $x_j \in X$, it satisfies the condition $0 \le (u_F(x_j))^3 + (v_F(x_j))^3 \le 1$. If $\pi_F(x_j) = \sqrt[3]{1 - (u_F(x_j))^3 - (v_F(x_j))^3}$, then $\pi_F(x_j)$ is defined to be the indeterminacy of the set F. For clarity and brevity, $F = (u_F, v_F)$ denotes an FFN.

Definition A2 ([17]). If $\lambda > 0$ exists, let $\mathfrak{A}_1 = (u_1, v_1)$ and $\mathfrak{A}_2 = (u_2, v_2)$ be two FFNs. The algorithms between FFNs are as follows:

(1)
$$\mathfrak{A}_{1}^{c} = (v_{1}, u_{1});$$

(2) $\mathfrak{A}_{1}^{c} \oplus \mathfrak{A}_{2} = \left(\sqrt[3]{u^{3} + u^{3} - u^{3}u^{3}}, u_{2}^{3}u^{3}\right);$

(2)
$$\mathfrak{A}_1 \oplus \mathfrak{A}_2 = (\sqrt{u_1 + u_2 - u_2 u_2, v_1 v_2}),$$

(3) $\mathfrak{A}_1 \otimes \mathfrak{A}_2 = (u_1 u_1, \sqrt[3]{v_1^3 + v_2^3 - v_1^3 v_2^3});$

(3)
$$\lambda \mathfrak{A}_{1} \otimes \lambda \mathfrak{A}_{2} = \left(u_{1}u_{1}, \sqrt{v_{1} + v_{2}} - v_{1}v_{2} \right)$$

(4) $\lambda \mathfrak{A}_{1} = \left(\sqrt[3]{1 - (1 - u_{1}^{3})^{\lambda}}, (v_{1})^{\lambda} \right);$

(5)
$$\mathfrak{A}_{1}^{\lambda} = \left((u_{1})^{\lambda}, \sqrt[3]{1 - (1 - v_{1}^{3})^{\lambda}} \right).$$

Definition A3 ([17]). Let $\mathfrak{A} = (u_{\mathfrak{A}}, v_{\mathfrak{A}})$ be an FFN; its score and accuracy functions are determined as follows:

$$O(\mathfrak{A}) = (u_{\mathfrak{A}})^3 - (v_{\mathfrak{A}})^3, \qquad (A2)$$

$$\Theta(\mathfrak{A}) = (u_{\mathfrak{A}})^3 + (v_{\mathfrak{A}})^3, \tag{A3}$$

where $O(F) \in [-1, 1]$ *and* $\Theta(F) \in [0, 1]$ *.*

Definition A4 ([17]). Let $\mathfrak{A}_1 = (u_1, v_1)$ and $\mathfrak{A}_2 = (u_2, v_2)$ be two FFNs; then,

- (1) if $O(\mathfrak{A}_1) > O(\mathfrak{A}_2)$, then $\mathfrak{A}_1 > \mathfrak{A}_2$;
- (2) if $O(\mathfrak{A}_1) = O(\mathfrak{A}_2)$, then,
 - (a) if $\Theta(\mathfrak{A}_1) > \Theta(\mathfrak{A}_2)$, then $\mathfrak{A}_1 > \mathfrak{A}_2$;
 - (b) if $\Theta(\mathfrak{A}_1) = \Theta(\mathfrak{A}_2)$, then $\mathfrak{A}_1 = \mathfrak{A}_2$.

Definition A5 ([50]). Let $\mathfrak{A}_1 = (u_1, v_1)$ and $\mathfrak{A}_2 = (u_2, v_2)$ be two FFNs; then, the standard Hamming distance between \mathfrak{A}_1 and \mathfrak{A}_2 is described below:

$$d(\mathfrak{A}_1,\mathfrak{A}_2) = \frac{1}{2} \left(\left| u_1^3 - u_2^3 \right| + \left| v_1^3 - v_2^3 \right| + \left| \pi_1^3 - \pi_2^3 \right| \right).$$
(A4)

Definition A6 ([29]). Let $\mathfrak{A}_i \ge 0$ ($i = 1, 2, ..., \lambda$) be the set of real values, where $\xi, \zeta \ge 0$. Then,

$$HM(\mathfrak{A}_{1},\mathfrak{A}_{2},\ldots,\mathfrak{A}_{\lambda}) = \left(\frac{2}{\lambda(\lambda+1)}\sum_{i=1,j=i}^{\lambda}\mathfrak{A}_{i}^{\xi}\mathfrak{A}_{j}^{\zeta}\right)^{\frac{1}{\zeta+\zeta}}$$
(A5)

is called the HM operator.

Definition A7 ([28]). Let $\mathfrak{A}_i \ge 0$ ($i = 1, 2, ..., \lambda$) be the set of real numbers. Then the PA operator *is defined as:*

$$PA(\mathfrak{A}_1,\mathfrak{A}_2,\ldots,\mathfrak{A}_{\lambda}) = \sum_{i=1}^{\lambda} \frac{1+\sigma(\mathfrak{A}_i)}{\sum_{t=1}^{\lambda} (1+\sigma(\mathfrak{A}_t))},$$
(A6)

where $\Omega(\mathfrak{A}_i, \mathfrak{A}_i)$ denotes the support of \mathfrak{A}_i and \mathfrak{A}_i , satisfying the properties mentioned below:

- (1) $\Omega(\mathfrak{A}_i,\mathfrak{A}_i) \in [0,1];$
- (2) $\Omega(\mathfrak{A}_i,\mathfrak{A}_i) = \Omega(\mathfrak{A}_i,\mathfrak{A}_i);$
- (3) If $|\mathfrak{A}_i \mathfrak{A}_i| \leq |\mathfrak{A}_k \mathfrak{A}_l|$, then $\Omega(\mathfrak{A}_i, \mathfrak{A}_i) \geq \Omega(\mathfrak{A}_k, \mathfrak{A}_l)$.

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