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# Study of Shiny Film Coating on Multi-Fluid Flows of a Rotating Disk Suspended with Nano-Sized Silver and Gold Particles: A Comparative Analysis

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**Abstract:** The current effort is devoted to investigate the shiny thin film with a metallic tactile covering of nanoparticles over the surface of a rotating disk. To decorate, glowing silver and gold particles were chosen. Four illustrative base liquids, namely (i) ethanol, (ii) methanol, (iii) ethylene-glycol, and (iv) water were considered with different geometries, which have great importance in industrial usage. An emphasis on comparative multi nanofluid analysis was used to make a sound judgment on which one of the fluids best suited the metallic glittering process of spin coating. The film thickness process highly depends on the process of evaporation, which takes some time to settle on the disk's surface. It was found that of the base fluids, the best choices were ethanol alloys with silver. Hence, one can conclude that from an experimental point of view, if silver alloy is used for coating, then only those liquids can be considered that exhibit ethanol-like properties. The impact of pertinent parameters with different aspects are graphically illustrated in each case.

Keywords: thin film; spin coating; rotating disk; nanoparticles; Newtonian fluids

# 1. Introduction

The mechanical process of covering the surface of an object/substrate with the help of a very thin layer is known as "Coating". This layer can be of some sort of paint, lacquer or a thin polymer sheet, which may be used for protective or decorative purposes. Nowadays, most of the engineered products go through the process of coating to prevent corrosion and to make them attractive [1,2]. From an industrial point of view, coating involves the development of a thin film layer (which can be polymeric or lacquer) on a substrate or fabric etc. If the substrate starts and ends the process wound up in a roll, the process may be termed "roll-to-roll" or "web-based" coating. Apart from a process of simple coating, developing a uniform and thin film or covering to a spinning sample or substrate, is called "spin coating". In the latter sort of coating, a small amount of liquid solution is placed at the center of the highly rotating disk, with the help of a pipette or syringe, resulting in the solution spreading uniformly and evenly in all directions as elaborated in [3]. This is all because of centrifugal forces, which cause liquid solution to spread across the surface uniformly. Application of spin coating is mainly used to fabricate tiny structures, usually of micrometer size or even much smaller, known as the microfabrication process. Manufacturing of solar cells, integrated circuits, insulators, nanomaterials,



compact disks, magnetic disk coating and microfluidic devices are a few examples of such technology, which depend upon the process of high quality spin coating. The simplicity and relative ease that helps to process any set up is regarded as the main advantage of spin coating. The spinning causes fast airflow around the rotating substrates or disks which results in quick drying of a thin layer of coating. Hence, this saves time and reduces the consumption of energy but a consistent efficiency is also achieved at the scale of nanolength or macroscopic level. Surprisingly, spin coating deals with a process which involves single substrates. This leads to a major drawback of spin coating and puts this on the back foot as compared to the "roll-to-roll" coating process. This disadvantage causes low performance. Also, the fast-drying times mean the actual material usage in a spin coating process is typically very low, around 10% or less, with the rest being flung off the side and wasted. Despite these drawbacks, spin coating is usually the starting point and benchmark for most academic and industrial processes that require a thin and uniform coating. Moreover, nanofluids can simply be termed as the liquid containing the tiny metallic particles. These tiny particles are invisible to the naked eye, for these range between 1 nm and 100 nm. In the later phase of the twentieth century, the concept of nanoparticles was introduced by Choi and Eastman [4] as a supporting agent. The initial intention was to obtain immense thermal conductivity of the base fluid. However, Buongiorno [5,6] performed his role focusing on convective heat transfer involving nanoparticles in the concerned base liquids. However, he negated some previous conclusions inferred by different researchers in their investigations. Since then, nanoparticles are being in used in different ways by scientists in their endeavors [7–37]. One cannot deny the beneficial application of nanoparticles from electronics to electrical appliances, from the energy sector to medical sciences working towards the remedy of some fatal disease, it is all mainly due to the blessing of the perfect utility of nanoparticles. Primarily, nanoparticles were meant to enhance the thermal features of a phenomenon involved in it, but recently, nanofluids are being applied in a new dimension, which is in solar collectors. In this application, nanofluids are employed for their tunable optical properties. Consequently, graphene-based nanofluid increases the performance of polymerase chain reaction. As a matter of fact, in some cases nano-technology has improved the performance of spin coating, which requires time to self-assemble or crystallize as the nanoparticles such as gold, silver, zinc oxide, copper and aluminum have significant potential in conductive metal as compared to conventional conductive materials. Especially, incorporating the said nanomaterials into thin films would always pool together electrical and optical properties for multipurpose features that play a key role in fabricating stretchable conductive thin films and coatings, since their mechanical properties include greater flexibility, stretch ability and designed structures. These materials can be easily incorporated into thin films with simple inexpensive solution-based testimony techniques like spin coating, ink-jet printing and spray coating [38,39].

What makes this paper so special is that in this study more than one base fluids suspended with a couple of different nanoparticles have been comparatively studied altogether which, so far, is a novel innovation in the field of applying a thin film of spin coating. To form this shiny silver and gold metallic layer of coating, four different types of base liquids (i.e., water, ethanol, methanol and ethylene-glycol) were brought in to use. It was found that evaporation of the liquid suggested rapidly settling down a shiny metallic layer of silver or gold on the surface of a rotating disk.

#### 2. Formulation

Let  $V = [\overline{u}(\overline{t}, \overline{r}, \overline{\theta}, \overline{z}) \ \overline{v}(\overline{t}, \overline{r}, \overline{\theta}, \overline{z}) \ \overline{w}(\overline{t}, \overline{r}, \overline{\theta}, \overline{z})]$  be the velocity of unsteady, incompressible and viscous multi nanofluids axi-symmetrically rotating disk having an angular velocity  $\overline{\Psi}$ , as shown in Figure 1.



Figure 1. Physical configuration of the rotating disk.

The layer of nanofluid across the surface is evenly spread out; thus, appropriate assumptions can be enlisted as:

- i. The nanofluid is assumed to be diluted and an impact of evaporation of a thin layer of the liquid is negligible as the solution is behaving "non-volatile".
- ii. The nanoparticles and the base fluid are in equilibrium, therefore, no slip condition is considered.

The governing equations in components form are:

$$\frac{\partial \overline{u}}{\partial \overline{t}} + \frac{\overline{u}}{\overline{r}} + \frac{\partial \overline{w}}{\partial \overline{z}} = 0 \tag{1}$$

$$\overline{\rho}_{nf}\left(\frac{\partial\overline{u}}{\partial\overline{t}} + \overline{u}\frac{\partial\overline{u}}{\partial\overline{r}} + \overline{w}\frac{\partial\overline{u}}{\partial\overline{z}} - \frac{\overline{v}^2}{\overline{r}}\right) = -\frac{\partial\overline{p}}{\partial\overline{r}} + \overline{\mu}_{nf}\left\{\frac{\partial^2\overline{u}}{\partial\overline{r}^2} + \frac{\partial}{\partial\overline{r}}\left(\frac{\overline{u}}{\overline{r}}\right) + \frac{\partial^2\overline{u}}{\partial\overline{z}^2}\right\}$$
(2)

$$\overline{\rho}_{nf}\left(\frac{\partial\overline{v}}{\partial\overline{t}} + \overline{u}\frac{\partial\overline{v}}{\partial\overline{r}} + \overline{w}\frac{\partial\overline{v}}{\partial\overline{z}} + \frac{\overline{u}\overline{v}}{\overline{r}}\right) = -\frac{\partial\overline{p}}{\partial\overline{\theta}} + \overline{\mu}_{nf}\left\{\frac{\partial^{2}\overline{v}}{\partial\overline{r}^{2}} + \frac{\partial}{\partial\overline{r}}\left(\frac{\overline{v}}{\overline{r}}\right) + \frac{\partial^{2}\overline{v}}{\partial\overline{z}^{2}}\right\}$$
(3)

$$\overline{\rho}_{nf}\left(\frac{\partial \overline{w}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{w}}{\partial \overline{r}} + \overline{w}\frac{\partial \overline{w}}{\partial \overline{z}}\right) = -\frac{\partial \overline{p}}{\partial \overline{z}} + \overline{\mu}_{nf}\left\{\frac{\partial^2 \overline{w}}{\partial \overline{r}^2} + \frac{\overline{w}}{\overline{r}} + \frac{\partial^2 \overline{w}}{\partial \overline{z}^2}\right\}$$
(4)

$$\left(\overline{\rho}C_{p}\right)_{nf}\left(\frac{\partial\overline{T}}{\partial\overline{t}} + \overline{u}\frac{\partial\overline{T}}{\partial\overline{r}} + \overline{w}\frac{\partial\overline{T}}{\partial\overline{z}}\right) = \overline{k}_{nf}\left\{\frac{\partial^{2}\overline{T}}{\partial\overline{r}^{2}} + \frac{1}{\overline{r}}\frac{\partial\overline{T}}{\partial\overline{r}} + \frac{\partial^{2}\overline{T}}{\partial\overline{z}^{2}}\right\}$$
(5)

Initial and boundary conditions associated with Equations (1)–(5) are defined in the following sub sections:

# 2.1. Initial Conditions

$$(i). \ \overline{u} = 0,$$

$$(ii). \ \overline{v} = 0,$$

$$(iii). \ \overline{w} = 0,$$

$$(iv). \ \overline{T} = \overline{T}_0,$$

$$(v). \ \overline{h}(\overline{t}) = h_0$$

$$(iv) = (iv) = (iv)$$

$$(iv) = (iv) = (iv)$$

$$(iv) = (iv)$$

here,  $\overline{h}$ ,  $h_0$  and  $\overline{T}_0$  represent thickness of the film, initial thickness of the film and room temperature respectively.

# 2.2. Boundary Conditions

• At the surface of the rotating disk

$$\begin{array}{c} (\mathbf{i}). \, \overline{u} = \mathbf{0}, \\ (\mathbf{i}\mathbf{i}). \, \overline{v} = \overline{r} \, \overline{\Psi}, \\ (\mathbf{i}\mathbf{i}\mathbf{i}). \, \overline{w} = \mathbf{0}, \\ (\mathbf{i}\mathbf{v}). \, \overline{T} = \overline{T}_a - \frac{\overline{T}_b}{2} \overline{r}^2 \end{array} \right\}; \text{ when } \overline{z} = 0$$

$$(7)$$

• At the free surface of the rotating disk

$$\begin{array}{c} (\mathrm{i}). \frac{\partial \overline{h}}{\partial t} = \overline{w}, \\ (\mathrm{ii}). \overline{p} + 2\overline{\mu}_{nf} \frac{\partial \overline{w}}{\partial \overline{z}} = 0, \\ (\mathrm{iii}). \overline{\mu}_{nf} \left( \frac{\partial \overline{u}}{\partial \overline{z}} + \frac{\partial \overline{w}}{\partial \overline{r}} \right) = \frac{\partial \overline{T}}{\partial \overline{r}} \frac{\partial \overline{\sigma}}{\partial \overline{T}}, \\ (\mathrm{iv}). \overline{\mu}_{nf} \left( \frac{\partial \overline{v}}{\partial \overline{z}} \right) = \frac{\partial \overline{T}}{\partial \overline{z}} \frac{\partial \overline{\sigma}}{\partial \overline{T}}, \\ (\mathrm{v}). \frac{\partial \overline{T}}{\partial \overline{z}} + L(\overline{T} - \overline{T}_g) = 0. \end{array} \right\}; \text{ when } \overline{z} = \overline{h}(\overline{t})$$

$$(8)$$

where *L* denotes heat transfer coefficient and  $\overline{\sigma}$  stands for surface tension.

By using suitable transformations [40], the governing equations can be obtained as:

$$2F + \frac{\partial W}{\partial z} = 0 \tag{9}$$

$$\operatorname{Re}\mathscr{D}_1\left(\frac{\partial F}{\partial t} + F^2 + W\frac{\partial F}{\partial z}\right) = \frac{\partial^2 F}{\partial z^2} + G^2$$
(10)

$$\operatorname{Re} \varnothing_1 \left( \frac{\partial G}{\partial t} - G \frac{\partial W}{\partial z} + W \frac{\partial G}{\partial z} \right) = \frac{\partial^2 G}{\partial z^2}$$
(11)

$$\operatorname{RePr} \varnothing_2 \left( \frac{\partial \Gamma}{\partial t} - \Gamma \frac{\partial W}{\partial z} + W \frac{\partial \Gamma}{\partial z} \right) = \frac{k_{nf}}{k_f} \frac{\partial^2 \Gamma}{\partial z^2}$$
(12)

$$\operatorname{RePr} \varnothing_2 \left( \frac{\partial \tau}{\partial t} + W \frac{\partial \tau}{\partial z} \right) = \frac{k_{nf}}{k_f} \left( \frac{\partial^2 \tau}{\partial z^2} + 2\Gamma \right)$$
(13)

(i). 
$$F(z,t) = 0$$
,  
(ii).  $G(z,t) = 0$ ,  
(iii).  $W(z,t) = 0$ ,  
(iv).  $\Gamma(z,t) = 0$ ,  
(v).  $\tau(z,t) = 0$ ,  
(vi).  $H(t) = 1$ ,  
}; at  $t = 0$  (14)

(i). 
$$F(z,t) = 0$$
,  
(ii).  $G(z,t) = 1$ ,  
(iii).  $W(z,t) = 0$ ,  
(iv).  $\Gamma(z,t) = 1$ ,  
(v).  $\tau(z,t) = 0$ ,  
(15)

here, Re is the Reynolds number and Pr denotes the Prandtl number, whereas  $\emptyset_1$  and  $\emptyset_2$  represent dimensionless constants.

For free surface

$$(i). \frac{\partial F}{\partial z} = \alpha (1 - \phi)^{2.5} \Gamma, (ii). \frac{\partial G}{\partial z} = 0, (iii). \frac{\partial \Gamma}{\partial z} = 0, (iv). \frac{\partial \tau}{\partial z} = 0, (v). \frac{dH}{dt} = W.$$
 }; at  $z = H(t)$  (16)

#### 2.3. Thermophysical Properties

The present investigation is concerned with the development of a thin film of liquid on a rotating disk with different metallic particles that require effective thermo-physical properties of nanofluids and nanoparticles. Two distinctive models proposed by Khanafer and Vafai [41] were chosen to analyze the density and specific heat of the nanofluids; whereas to estimate the thermal conductivity and viscosity of fluids, the thermophysical model [42] is utilized. In view of the thermophysical model in the presence of multi fluids containing two different types of nano-sized metallic particles, the realistic properties were developed as follows:

#### 2.3.1. For Water as the Base Fluid

The most significant and highly utilized fluid on this planet is water that contains 997.1 kg·m<sup>-3</sup>, density, 0.89 mPa·S. viscosity, 4179 J/Kg m heat capacity and 0.569 W·m<sup>-1</sup>·K<sup>-1</sup> thermal conductivity.

# • For gold nanoparticles

The mathematical expressions that describe the thermophysical properties of water and gold nanofluids are given as:

$$\overline{\rho}_{nf} = (1 - \phi)\overline{\rho}_f - \phi\overline{\rho}_p \tag{17}$$

$$\left(\overline{\rho}_{nf}\right)_{\text{water/gold}} = 997.1(1-\phi) - 19300\phi \tag{18}$$

$$\overline{\mu}_{nf} = \left(1.013 + 0.092\phi - 0.015\phi^2\right)\overline{\mu}_f$$
(19)

$$\left(\overline{\mu}_{nf}\right)_{\text{water/gold}} = 0.89 \left(1.013 + 0.092\phi - 0.015\phi^2\right)$$
 (20)

$$(\overline{\rho}C_P)_{nf} = (C_P)_f \overline{\rho}_f (1-\phi) - (C_P)_p \phi \overline{\rho}_p$$
(21)

$$(\bar{\rho}C_P)_{\text{water/gold}} = (4179)(997.1)(1-\phi) - (126)(19300)\phi$$
(22)

$$\bar{k}_{nf} = (1.0204 + 0.0249\phi)\bar{k}_f \tag{23}$$

$$\left(\bar{k}_{nf}\right)_{\text{water/gold}} = 0.569(1.0204 + 0.0249\phi)$$
 (24)

where the gold density is 19,300 kg·m<sup>-3</sup> and heat capacity and thermal conductivity are 126 J/kg m and 317 W·m<sup>-1</sup>·K<sup>-1</sup>, respectively.

#### • For silver nanoparticles

The thermophysical properties of water and silver nanofluids are given as:

$$\left(\overline{\rho}_{nf}\right)_{\text{water/silver}} = 997.1(1-\phi) - 10490\phi \tag{25}$$

$$\left(\overline{\mu}_{nf}\right)_{\text{water/silver}} = 0.89 \left(1.013 + 0.092\phi - 0.015\phi^2\right)$$
 (26)

$$(\bar{\rho}C_P)_{\text{water/silver}} = (4179)(997.1)(1-\phi) - (233)(10490)\phi$$
(27)

$$\left(\bar{k}_{nf}\right)_{\text{water/silver}} = 0.569(1.0204 + 0.0249\phi)$$
 (28)

where the density of silver is 10,490 kg·m<sup>-3</sup> while heat capacity and thermal conductivity are respectively 233 J/Kg m and 429 W·m<sup>-1</sup>·K<sup>-1</sup>

#### 2.3.2. For Methanol as the Base Fluid

The features displayed by methanol at room temperature according to the System International (SI) units is of density 790 kg·m<sup>-3</sup> whereas viscosity takes the numerical value 0.543 mPa·S, heat capacity is 2534 J/Kg m and thermal conductivity is 0.201 W·m<sup>-1</sup>·K<sup>-1</sup>.

• For gold nanoparticles

The mathematical expressions describing the thermophysical properties are given as:

$$\left(\bar{\rho}_{nf}\right)_{\text{methanol/gold}} = 790(1-\phi) - 19300\phi$$
 (29)

$$\left(\overline{\mu}_{nf}\right)_{\text{methanol/gold}} = 0.543 \left(1.013 + 0.092\phi - 0.015\phi^2\right)$$
 (30)

$$(\overline{\rho}C_P)_{\text{methanol/gold}} = (2543)(790)(1-\phi) - (126)(19300)\phi$$
(31)

$$\left(\bar{k}_{nf}\right)_{\text{methanol/gold}} = 0.201(1.0204 + 0.0249\phi) \tag{32}$$

• For silver nanoparticles

For the methanol and silver nanofluids suspension, the physical properties of silver, heat capacity and thermal conductivity are 10,490,790 kg·m<sup>-3</sup>, 233 J/Kg m and 429 W·m<sup>-1</sup>·K<sup>-1</sup> respectively. Thus, thermophysical properties corresponding to this model are:

$$\left(\overline{\rho}_{nf}\right)_{\text{methanol/silver}} = 790(1-\phi) - 10490\phi$$
 (33)

$$\left(\overline{\mu}_{nf}\right)_{\text{methanol/silver}} = 0.543 \left(1.013 + 0.092\phi - 0.015\phi^2\right)$$
 (34)

$$(\bar{\rho}C_P)_{\text{methanol/silver}} = (2543)(790)(1-\phi) - (233)(10490)\phi$$
(35)

$$\left(\overline{k}_{nf}\right)_{\text{methanol/silver}} = 0.201(1.0204 + 0.0249\phi)$$
(36)

In order to make a methanol and gold nanofluids suspension, gold density is 19,300 kg·m<sup>-3</sup>. The heat capacity and thermal conductivities are 126 J/Kg m and 317 W·m<sup>-1</sup>·K<sup>-1</sup> respectively.

### 2.3.3. For Ethanol as the Base Fluid

The features displayed by ethanol at room temperature have a density of 789 kg·m<sup>-3</sup>. The viscosity is 1.074 mPa·S, heat capacity is 2500 J/Kg m and thermal conductivity is 0.0235 W·m<sup>-1</sup>·K<sup>-1</sup>.

• For gold nanoparticles

For an ethanol and gold nanofluids suspension, the density of gold is 19,300 kg·m<sup>-3</sup>, and heat capacity and thermal conductivity are 126 J/Kg m and 317 W·m<sup>-1</sup>·K<sup>-1</sup>, respectively. Mathematically, it can be written as:

$$\left(\overline{\rho}_{nf}\right)_{\text{ethanol/gold}} = 789(1-\phi) - 19300\phi \tag{37}$$

$$(\overline{\mu}_{nf})_{\text{ethanol/gold}} = 1.074 \left( 1.013 + 0.092\phi - 0.015\phi^2 \right)$$
(38)

$$(\bar{\rho}C_P)_{\text{ethanol/gold}} = (2500)(789)(1-\phi) - (126)(19300)\phi$$
 (39)

$$\left(\bar{k}_{nf}\right)_{\text{ethanol/gold}} = 0.0235(1.0204 + 0.0249\phi)$$
 (40)

# • For silver nanoparticles

To make an ethanol and silver nanofluids suspension, the density of silver is 10,490 kg·m<sup>-3</sup>, heat capacity is 233 J/Kg m and thermal conductivity is 429 W·m<sup>-1</sup>·K<sup>-1</sup>. The mathematical expressions can be written as:

$$\left(\overline{\rho}_{nf}\right)_{\text{ethanol/silver}} = 789(1-\phi) - 10490\phi$$
 (41)

$$\left(\overline{\mu}_{nf}\right)_{\text{ethanol/silver}} = 1.074 \left(1.013 + 0.092\phi - 0.015\phi^2\right)$$
 (42)

$$(\overline{\rho}C_P)_{\text{ethanol/silver}} = (2500)(789)(1-\phi) - (233)(10490)\phi$$
(43)

$$\left(\bar{k}_{nf}\right)_{\text{ethanol/silver}} = 0.0235(1.0204 + 0.0249\phi)$$
 (44)

#### 2.3.4. For Ethylene-Glycol as the Base Fluid

The density of ethylene-glycol at room temperature by System International (SI) units system is 1101 kg·m<sup>-3</sup>. The viscosity takes the numerical value 0.0162 mPa·S, heat capacity and thermal conductivity are respectively 2400 J/Kg m and 0.256 W·m<sup>-1</sup>·K<sup>-1</sup>.

• For gold nanoparticles

For an Ethylene-glycol and Gold nanofluids suspension, the physical property of gold density is 19,300 kg·m<sup>-3</sup>, heat capacity is 126 J/Kg m and thermal conductivity is 317 W·m<sup>-1</sup>·K<sup>-1</sup>. Accordingly, the mathematical expression can be written as:

$$\left(\overline{\rho}_{nf}\right)_{\text{ethylene/gold}} = 1101(1-\phi) - 19300\phi$$
 (45)

$$\left(\overline{\mu}_{nf}\right)_{\text{ethylene/gold}} = 0.0162 \left(1.013 + 0.092\phi - 0.015\phi^2\right)$$
 (46)

$$(\bar{\rho}C_P)_{\text{ethylene/gold}} = (2400)(1101)(1-\phi) - (126)(19300)\phi$$
(47)

$$\left(\bar{k}_{nf}\right)_{\text{ethylene/gold}} = 0.256(1.0204 + 0.0249\phi)$$
 (48)

• For silver nanoparticles

For the suspension of ethylene-glycol and silver nanofluids the physical property of silver density is 10,490 kg·m<sup>-3</sup>. The heat capacity and thermal conductivity are 233 J/Kg m and 429 W·m<sup>-1</sup>·K<sup>-1</sup>, respectively. On the previous contrast, mathematical expression can be attained as:

$$\left(\overline{\rho}_{nf}\right)_{\text{ethylene/silver}} = 1101(1-\phi) - 10490\phi$$
 (49)

$$\left(\overline{\mu}_{nf}\right)_{\text{ethylene/silver}} = 0.0162\left(1.013 + 0.092\phi - 0.015\phi^2\right)$$
 (50)

$$\left(\bar{\rho}C_P\right)_{\text{ethylene/silver}} = (2400)(1101)(1-\phi) - (233)(10490)\phi \tag{51}$$

$$\left(\bar{k}_{nf}\right)_{\text{ethylene/silver}} = 0.256(1.0204 + 0.0249\phi)$$
 (52)

For the best understating of readers, the realistic physical properties of base fluids and nanoparticles are offered in Tables 1 and 2.

Base Fluids	Density $ ho$ (kg·m <sup>-3</sup> )	Viscosity $\mu$ (mPa·S)	Heat Capacity $C_p$ (J·kg <sup>-1</sup> ·K <sup>-1</sup> )	Thermal Conductivity $k$ (W·m <sup>-1</sup> ·K <sup>-1</sup> )
Water [43] (H <sub>2</sub> O)	997	0.89	4179	0.569
Ethanol [44] ( $C_2H_6O$ )	789	1.074	2500	0.0235
Methanol $[45]$ (CH <sub>4</sub> O)	790	0.543	2534	0.200
Ethylene-Glycol [46] ( $C_2H_6O_2$ )	1101	0.0162	2400	0.256

Table 1. Thermo-physical properties of the base fluids.

Nano ParticlesDensity<br/> $\rho$  (kg·m<sup>-3</sup>)Heat Capacity<br/> $C_p$  (J·kg<sup>-1</sup>·K<sup>-1</sup>)Thermal Conductivity<br/>k (W·m<sup>-1</sup>·K<sup>-1</sup>)Gold [47] (Au)19300126317

10490

Table 2. Physical properties of the nanoparticles.

#### 3. Analytical Results

Silver [48] (Ag)

Analytical solutions of nonlinear and coupled Equations (9)–(13) subject to (14)–(16) are obtained as:

233

429

$$F(z,t) = f_1(t)z + f_2(t)z^2 + f_3(t)z^3 + f_4(t)z^4 + f_5(t)z^5 + f_6(t)z^6 + f_7(t)z^7 + f_8(t)z^8 + f_9(t)z^9 + f_{10}(t)z^{10} + f_{11}(t)z^{11} + f_{12}(t)z^{12} + f_{13}(t)z^{13} + f_{14}(t)z^{14} + f_{15}(t)z^{15}$$
(53)

$$G(z,t) = 1 + g_1(t)z + g_2(t)z^2 + g_3(t)z^3 + g_4(t)z^4 + g_5(t)z^5 + g_6(t)z^6 + g_7(t)z^7 + g_8(t)z^8 + g_9(t)z^9$$
(54)

$$W(z,t) = w_{1}(t)z + w_{2}(t)z^{2} + w_{3}(t)z^{3} + w_{4}(t)z^{4} + w_{5}(t)z^{5} + w_{6}(t)z^{6} + w_{7}(t)z^{7} + w_{8}(t)z^{8} + w_{9}(t)z^{9} + w_{10}(t)z^{10} + w_{11}(t)z^{11} + w_{12}(t)z^{12} + w_{13}(t)z^{13} + w_{14}(t)z^{14} + w_{15}(t)z^{15} + w_{16}(t)z^{16}$$
(55)

$$\Gamma(z,t) = 1 + m_1(t)z + m_2(t)z^2 + m_3(t)z^3 + m_4(t)z^4 + m_5(t)z^5 + m_6(t)z^6 + m_7(t)z^7 + m_8(t)z^8$$
(56)

$$\tau(z,t) = n_1(t)z + n_2(t)z^2 + n_3(t)z^3 + n_4(t)z^4 + n_5(t)z^5 + n_6(t)z^6 + n_7(t)z^7 + n_8(t)z^8 + n_9(t)z^9 + n_{10}(t)z^{10}$$
(57)

where the expressions  $f_1, f_2 \dots f_{15}, g_1, g_2 \dots g_9, m_1, m_2 \dots m_8, n_1, n_2 \dots n_{10}$  and  $w_1, w_2 \dots w_{16}$  are given in the Appendix A.

#### 4. Discussion

The process of coating heavily depends upon the time taken by any fluid to settle down on the surface of the material; a fluid can only be considered more suitable for the coating if it takes less time to leave its effects on the surface. Moreover, the engaged nanoparticles are of very small size and of a concentration of at most 2%. The effects on viscosity, thermal conductivity, density and heat capacity are evaluated experimentally in many communications. It is now a well-established fact that in the presence of such a small quantity of nanosized particles, the nature of fluid does not change but changes in physical properties are evident. For that, many correlations are presented for different situations and particles. To serve the purpose of this study, four different kinds of Newtonian fluids having diverse physical and chemical properties are considered instead of non-Newtonian fluids because coatings with such types of fluids would have a tremendous impact on the cost, volume, weight, and mechanical properties of electronic, optoelectronic, and photovoltaic devices; thus, this portion is dedicated to the parametric study of the proposed model in which four kinds of Newtonian fluids, such as water, ethanol, methanol and ethylene-glycol are opted for as the base fluids. The gold and silver nanoparticles are used to furnish the thin metallic and shiny coating on the surface of the

rotating disk. The main reason to carry out this graphical work is to confirm whether or not the obtained mathematical results are in complete coherence with the physical expectation of the spin coatings. Moreover, the graphic illustrations will help to make a sound judgement about the role and contribution of field variables. Major parameters which have been comprehensively focused on are the concentration of the metallic particles and the thermocapillary parameter. Furthermore, the presented parametric study unlike the customary results and discussion have been delicately divided into three following sub sections to make this comparative analysis more clear and fathomable.

#### 4.1. Thickness of the Film

The key emphasis in this article is on furnishing a shiny metallic layer of nanoparticles, suspended with different base fluids displaying distinct physical and chemical features altogether. Here, the sole aim is to decide which one of the base fluids is the best suitable choice for this metallic covering over the disk with spin coatings that can quickly spread on the disk in a short span of time. As shown in Figure 2, it can clearly be seen that ethanol is the sole liquid which shows a rapid action with both metals as compared to the other base fluids. It is in accordance with their physical prospects, due to their densities, which help them evaporate quickly and results in a shiny metallic nanoliquids coating on the disk. On the other hand, silver particles' coating is much faster than gold, as Figure 3 shows.



Figure 2. Behaviour of film thickness for different base fluids containing gold nanoparticles.



Figure 3. Behaviour of film thickness for different base fluids containing silver particles

In Figures 4–7, thermocapillary parameters and the concentration of the metallic particles' influence on nanofluid coating have been displayed. It is a well-recognized fact that the thicker solution yields to the thicker layer of the film. The thermocapillary parameter depletes and attenuates this metallic layer as shown in Figures 4 and 5. From the above given facts, it is inferred that ethanol and silver particles share a great deal of mutual compatibility. Thickness of the film increases in size upon the additional supply of metallic particles as shown in Figures 6 and 7. This confirms the above preceding claim that an increase of the particles will enlarge the film thickness in size. Therefore, it can be concluded that any fluids and particles which exhibit different characteristics like ethanol and silver are regarded as the most suitable option for this metallic process of coating. Consequently, to see the effects of thermal, radial and azimuthal velocity, ethanol was chosen as a base fluid.



Figure 4. Behaviour of film thickness for thermocapillary parameter for gold particles.



Figure 5. Behaviour of film thickness for thermocapillary for silver particles parameter.



Figure 6. Effects of the concentration of particles on film thickness for the case of gold.



Figure 7. Effects of the concentration of particles on film thickness for the case of silver.

# 4.2. Radial Velocity and Azimuthal Velocity

In Figures 8–21, the radial and azimuthal velocities have been sketched for all base fluids, the thermocapillary parameter and the concentration of the particles. In view of suitable transformation, the mathematical expressions take the following final form:

$$U(z,t) = R F(z,t)$$
(58)

$$V(z,t) = R G(z,t)$$
<sup>(59)</sup>



Figure 8. Behavior of radial velocity for each fluid comprising gold particles.



Figure 9. Behavior of radial velocity for each fluid comprising silver particles.



Figure 10. Behavior of radial velocity for the thermocapillary parameter comprising gold particles.



Figure 11. Behavior of radial velocity for the thermocapillary parameter comprising gold particles.



Figure 12. Effects of concentration particles on radial velocity with gold particles.



Figure 13. Effects of concentration particles on radial velocity with silver particles.



Figure 14. Behavior of azimuthal velocity for each fluid with gold particles.



Figure 15. Behavior of azimuthal velocity for each fluid with silver particles.



Figure 16. Behavior of azimuthal velocity for the thermocapillary parameter with gold particles.



Figure 17. Behavior of azimuthal velocity for the thermocapillary parameter with silver particles.



Figure 18. Effects of concentration particles on azimuthal velocity for gold.



**Figure 19.** Effects of concentration particles suspended with ethanol on *N* for silver.

10

8

0.0

0.2

H



0.6

0.8

1.0

Z

0.4



Figure 21. Temperature effects of thermocapillary parameter.

In Equations (58) and (59),  $R = \frac{r}{h_0}$  is the initial thickness of the film. A similar trend in the behavior of both types of velocities is observed in the presence of silver and gold particles.

In Figures 8–19, the behavior of ethanol is quite prominent for all cases. It is observed that the radial velocity and azimuthal velocity increase for silver and gold. However, radial velocity and azimuthal velocity react quite differently for the thermocapillary parameter and the concentration of the particles. It is seen that temperature increases by increasing the values of thermocapillary parameter, as shown in Figures 20 and 21. It is in accordance with the physical expectation because radial velocity does not allow the fluid to move with full strength. However, the radial velocity is supported by the thermocapillary parameter. On the other hand, a complete reverse trend can be noted for the azimuthal velocity by varying both  $\alpha$  and  $\varphi$ .

#### 4.3. Thermal Analysis

In this section, the temperature of nanofluid was examined vertical to the disk. The mathematical relationships for temperature and temperature gradient were respectively denoted by the following relations:

$$T(z,t) = \frac{R^2}{2}\Gamma(z,t) + \tau(z,t)$$
(60)

$$T_z(z,t) = \frac{R^2}{2}\Gamma_z(z,t) + \tau_z(z,t)$$
(61)

Here smooth and organized curves are drawn in Figures 22–27. It is found that an addition of extra nanoparticles strengthens the drag force between the particles. However, thermocapillary parameter  $\alpha$  works altogether differently by reducing the heat of the nanofluid that ultimately affirms the earlier preceding claim regarding the addition of metallic particles to the base fluid ethanol.



Figure 22. Temperature effects of concentration particles.



Figure 23. Temperature effects of concentration particles.



**Figure 24.** Variation of  $T_z$  for the thermocapillary parameter.



**Figure 25.** Variation of  $T_z$  for the thermocapillary parameter.



**Figure 26.** Variation of  $T_z$  on concentration particles.



**Figure 27.** Variation of  $T_z$  on concentration particles.

# 5. Conclusions

A comparative study for silver and gold nanoparticles was comprehensively carried out to form a thin and shiny metallic layer over the surface of a rotating disk via spin coatings. Moreover, a detailed analysis of nanofluids suspended with four different types of base fluids, namely water, ethanol, methanol and ethylene-glycol has also been examined under the assumptions of nanofluids to be diluted and non-volatile. Finally, a parametric study on the basis of obtained expressions of results was made to apprehend the effects of the main parameters involved. Some significant findings are enlisted below:

- Silver metallic coating quickly settles down on the surface of the disk than to develop a gold coating.
- Thickness of the film increases with the addition of extra metallic particles.
- Radial velocity is hampered by adding more nanoparticles.
- Increase in the quantity of particles surges the thermal effects of the nanofluid.
- It is worth investigating that these results will help to choose the optimum base fluid with gold or silver particles.
- The graphical results show depletion of the fluid layer with time and one can hardly find such an evaluation in the available literature.
- Finally, it is concluded that the base fluid is the best choice for ethanol alloys with silver in the process of coating. In this way, it can be concluded that from the experimental point of view if silver alloy is used for coating then only such liquids should be considered which exhibit ethanol-like properties. Now, this effort is available for further experimental studies for those who are working in this regime for the validation of their lab results.

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# Nomenclatures

$h_0$	Initial film thickness
р	Pressure
t	Spinning time
U	Radial velocity component
V	Velocity
W	Axial velocity component
z	Axial coordinate
Pr	Prandtl number
Re	Reynolds number
$\overline{h}$	Film thickness
$\overline{r}$	Radial coordinate
$\overline{T}$	Temperature of nanofluid
$\overline{T}_0$	Initial room temperature
$\overline{T}_{g}$	Temperature in gas phase
$\overline{u}$	Radial velocity component
$\overline{v}$	Azimuthal velocity component
$\overline{w}$	Axial velocity component
k <sub>nf</sub>	Thermal conductivity of nanofluid
k <sub>f</sub>	Thermal conductivity of fluid
-	

Greek Symbols	
$\overline{ heta}$	Azimuthal coordinate of the velocity
$\overline{\Psi}$	Rotational velocity
ξ	Constant
$\phi_1$	Dimensionless constant
$\overline{ ho}_p$	Density particle
$(C_P)_f$	Heat capacity of base fluid
$\overline{\mu}_{nf}$	Dynamic viscosity of nanofluid
μ	Dynamic viscosity
α	Thermocapillary parameter
$\overline{\sigma}_0$	Initial surface tension
$\overline{\sigma}$	Surface tension
$\phi$	Concentration of particles
$\phi_2$	Dimensionless constant
$\overline{\rho}_{f}$	Density of fluid
$(C_P)_f$	Heat capacity of particle
$\overline{\rho}_{nf}$	Density of nanofluid
$(\overline{\rho}C_P)_{nf}$	Heat capacity of nanofluid
Subscripts	
f	Base fluid
nf	Nanofluid

# Appendix A

$$\begin{split} f_{1}(t) &= H + \alpha (1-\phi)^{2.5} - \frac{3H^{4}Re\phi_{1}}{4} + \frac{3H^{5}Re\phi_{1}}{20} - \frac{H^{2}\frac{dH}{2}Re\phi_{1}}{2} + \frac{13H^{5}\frac{dH}{2}Re^{2}\phi_{1}}{60} - \frac{53H^{6}\frac{dH}{2}Re^{2}\phi_{1}}{72} + \\ \frac{H^{3}(\frac{dH}{2})^{2}Re^{2}\phi_{1}}{24} + \frac{5H^{4}\frac{d^{2}H}{dt^{2}}Re^{2}\phi_{1}}{24} + \frac{H^{3}Re\alpha(1-\phi)^{2.5}\phi_{1}}{45} - \frac{5H^{4}Re\alpha(1-\phi)^{2.5}\phi_{1}}{4} - \frac{H^{4}\frac{dH}{dt}Re^{2}\alpha(1-\phi)^{2.5}\phi_{1}}{181200} + \frac{67H^{6}\frac{dH}{dt}Re^{2}\phi_{1}^{2}}{180} + \\ \frac{63H^{5}\frac{dH}{dt}Re^{2}\alpha(1-\phi)^{2.5}\phi_{1}}{20} + \frac{31H^{7}Re^{2}\phi_{1}^{2}}{1260} - \frac{293H^{8}Re^{2}\phi_{1}^{2}}{2016} + \frac{12869H^{9}Re^{2}\phi_{1}^{2}}{90720} - \frac{3751H^{10}Re^{2}\phi_{1}^{2}}{181200} + \frac{67H^{6}\frac{dH}{dt}Re^{2}\phi_{1}^{2}}{180} + \\ \frac{8H^{7}\frac{dH}{dt}Re^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{1260} - \frac{131H^{8}Re^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{1120} + \frac{18679H^{9}Re^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{90720} + \frac{7H^{5}\frac{dH}{dt}Re^{2}\alpha_{1}(1-\phi)^{2.5}\phi_{1}^{2}}{1} \\ \frac{8H^{7}\frac{dH}{dt}Re^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{1260} - \frac{4H^{8}Re^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{2}}{1260} - \frac{667H^{7}Re^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{2}}{11200} - \frac{418679H^{8}Re^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{2}}{1} + \frac{18679H^{9}Re^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{2}}{10080} - \frac{11H^{11}Re^{2}\phi_{1}^{2}}{1296} - \\ \frac{407H^{13}Re^{2}\phi_{1}^{2}}{1} + \frac{1369H^{14}Re^{2}\phi_{1}^{3}}{396900} + \frac{55H^{10}\frac{dH}{dt}Re^{2}\phi_{1}^{3}}{1} - \frac{37H^{11}\frac{dH}{dt}Re^{2}\phi_{1}^{3}}{1} - \frac{25H^{10}\frac{dH}{dt}Re^{2}\phi_{1}^{3}}{1} - \frac{11H^{11}Re^{2}\alpha^{2}(1-\phi)^{2.5}\phi_{1}^{3}}{1} + \frac{95H^{10}\frac{dH}{dt}Re^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{3}}{1} + \frac{$$

 $f_{5}(t) = \frac{\text{Re}\phi_{1}}{60} - \frac{H\text{Re}\phi_{1}}{30} + \frac{H\frac{dH}{dt}\text{Re}^{2}\phi_{1}}{15} - \frac{H^{2}\frac{dH}{dt}\text{Re}^{2}\phi_{1}}{30} + \frac{\frac{d^{2}H}{dt^{2}}\text{Re}^{2}\phi_{1}}{120} - \frac{\text{Re}\alpha(1-\phi)^{2.5}\phi_{1}}{60} + \frac{H\frac{dH}{dt}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}}{15} + \frac{H^{4}\text{Re}^{2}\phi_{1}^{2}}{16} - \frac{H^{4}\frac{dH}{dt}\text{Re}^{2}\phi_{1}}{16} - \frac{H^{4}\frac{dH}{dt}\text{R$  $\frac{H^5 \text{Re}^2 \phi_1^2}{80} + \frac{H^2 \frac{dH}{dt} \text{Re}^2 \phi_1^2}{24} - \frac{H^3 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{36} + \frac{17 H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{120} - \frac{3 H^5 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{400} + \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{40} - \frac{3 H^5 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{400} + \frac{2 H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{40} + \frac{2 H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{40} + \frac{2 H^2 \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{40} + \frac{2 H^2 \frac{H^2 \frac{H}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{40} + \frac{2 H^2 \frac{H^2 \frac{H}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{40} + \frac{2 H^2 \frac{H^2 \frac{H}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{40} + \frac{2 H^2 \frac{H}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{40} + \frac{2 H^2 \frac{H}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{40} + \frac{2 H^2 \frac{H}{dt}$  $\frac{H^3 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^2}{60} + \frac{H^4 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^2}{16} - \frac{9H^8 \text{Re}^2 \phi_1^3}{320} + \frac{9H^9 \text{Re}^2 \phi_1^3}{800} - \frac{9H^{10} \text{Re}^2 \phi_1^3}{8000} - \frac{3H^6 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{80} + \frac{3H^7 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{400} - \frac{3H^6 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{8000} + \frac{3H^7 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{8000} + \frac{3H^7 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{8000} - \frac{3H^6 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{8000} + \frac{3H^7 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{800} + \frac{3H^7 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{800} + \frac{3H^7 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{800} + \frac{3H^7 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{80} + \frac{3H^7 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{800} + \frac{3H^7 \frac{\text{dH}}{dt} \text{Re}^2 \phi_1^3}{80} + \frac{3H^7 \frac{\text{dH}$  $\frac{H^4 \left(\frac{dH}{dt}\right)^2 \text{Re}^2 \phi_1^3}{80} + \frac{H^7 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{40} - \frac{79 H^8 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{800} + \frac{3 H^9 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{160} + \frac{H^5 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{60} - \frac{3 H^9 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{160} + \frac{2 H^5 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{60} + \frac{2 H^5 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{160} + \frac{2 H^5 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{16$  $\frac{H^{6}\frac{dH}{dt}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{3}}{16} - \frac{H^{6}\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{3}}{180} + \frac{H^{7}\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{3}}{24} - \frac{5H^{8}\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{3}}{64};$  $f_6(t) = \frac{\text{Re}\phi_1}{360} - \frac{\frac{dH}{dt}\text{Re}^2\phi_1}{360} + \frac{H\frac{dH}{dt}\text{Re}^2\phi_1}{180} + \frac{\frac{dH}{dt}\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1}{360} + \frac{H^2\text{Re}^2\phi_1^2}{30} + \frac{H^3\text{Re}^2\phi_1^2}{180} - \frac{H^4\text{Re}^2\phi_1^2}{72} + \frac{H^5\text{Re}^2\phi_1^2}{600} + \frac{H^4\text{Re}^2\phi_1^2}{600} + \frac{H^4\text{Re}^2\phi_1^2}{180} + \frac{H^4\text{Re}^2\phi_1^2$  $\frac{H\frac{dH}{dt}\text{Re}^{2}\phi_{1}^{2}}{120} - \frac{H^{2}\frac{dH}{dt}\text{Re}^{2}\phi_{1}^{2}}{180} + \frac{H^{2}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{12} + \frac{H^{3}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{270} - \frac{H^{4}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{72} + \frac{\frac{dH}{dt}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{120} + \frac{H^{2}}{120} +$  $\frac{H^2 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^2}{22};$ 20  $f_7(t) = -\frac{\text{Re}^2 \phi_1 \frac{dH}{dt}}{1260} - \frac{8H^2 \text{Re}^2 \phi_1^2}{315} + \frac{2H^3 \text{Re}^2 \phi_1^2}{189} - \frac{17\text{Re}^2 \phi_1^2 \frac{dH}{dt}}{2520} - \frac{2H\text{Re}^2 \alpha (1-\phi)^{25} \phi_1^2}{315} - \frac{11H^2 \text{Re}^2 \alpha (1-\phi)^{25} \phi_1^2}{420} + \frac{11H^2 \text{Re}^2 \alpha$  $\frac{13H^{3}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{3780} - \frac{\frac{dH}{dt}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{2}}{252} - \frac{\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{2}}{315} + \frac{H\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{2}}{315} - \frac{13H^{2}\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{2}}{1260} + \frac{H^{6}\text{Re}^{2}\phi_{1}^{3}}{112} - \frac{H^{6}}{315} + \frac{H^{6}}{315}$  $\frac{H^7 \text{Re}^2 \phi_1^3}{210} + \frac{H^8 \text{Re}^2 \phi_1^3}{1680} + \frac{H^4 \frac{dH}{dt} \text{Re}^2 \phi_1^3}{96} - \frac{29 H^5 \frac{dH}{dt} \text{Re}^2 \phi_1^3}{10080} + \frac{H^2 \left(\frac{dH}{dt}\right)^2 \text{Re}^2 \phi_1^3}{336} - \frac{H^5 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{252} + \frac{19 H^6 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{756} - \frac{H^4 (1-\phi)^{2.5} \phi_1^3}{756} + \frac{H^4 (1-\phi)^{2.5$  $\frac{17H^7 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{2520} - \frac{H^3 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{504} + \frac{3H^4 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{224} - \frac{H^5 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{252} + \frac{5H^6 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{336};$  $f_8(t) = -\frac{\text{Re}^2\phi_1^2}{360} + \frac{31H\text{Re}^2\phi_1^2}{10080} - \frac{H^2\text{Re}^2\phi_1^2}{840} - \frac{H^3\text{Re}^2\phi_1^2}{1260} + \frac{\text{Re}^2\phi_1^2\frac{dH}{dt}}{1008} - \frac{5\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{2016} - \frac{13H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{5040} + \frac{13H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1$  $\frac{H^2 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{1120} - \frac{\text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^2}{560} - \frac{H \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^2}{672} - \frac{H^5 \text{Re}^2 \phi_1^3}{640} + \frac{3H^6 \text{Re}^2 \phi_1^3}{1600} - \frac{H^7 \text{Re}^2 \phi_1^3}{3200} - \frac{H^3 \frac{\text{dH}}{\text{dt}} \text{Re}^2 \phi_1^3}{960} + \frac{H^3 \frac{\text{dH}}{\text{dt}} \text{Re}^2 \phi_1^3}{960} + \frac{H^3 \frac{\text{dH}}{\text{dt}} \text{Re}^2 \phi_1^3}{1200} - \frac{H^3 \frac{\text{dH}}{\text{dt}} \text{Re}^2 \phi_1^3}{960} + \frac{H^3 \frac{\text{dH}}{\text{dt}} \text{Re}^2 \phi_1^3}{1200} - \frac{H^3 \frac{\text{dH}}{\text{dt}} \text{Re}^2 \phi_1^3}{1200} + \frac{H^3 \frac{\text{dH}}{\text{dt}} \text{Re}^2 \phi_1^3}{120} + \frac{H^3 \frac{\text{d$  $\frac{H^4 \frac{dH}{dt} \text{Re}^2 \phi_1^3}{960} - \frac{H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{1152} - \frac{41 H^5 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{28800} + \frac{11 H^6 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{4800} - \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{960} + \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{960}$  $\frac{H^3 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{960} + \frac{H^3 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{1440} - \frac{19 H^4 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{5760} + \frac{H^5 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{384};$  $f_9(t) = \frac{83\text{Re}^2\phi_1^2}{90720} + \frac{H\text{Re}^2\phi_1^2}{2016} + \frac{H^2\text{Re}^2\phi_1^2}{2592} + \frac{127\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{90720} + \frac{11H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{12960} + \frac{41\text{Re}^2\alpha^2(1-\phi)^5\phi_1^2}{90720} - \frac{7H^4\text{Re}^2\phi_1^3}{12960} + \frac{11H^2}{12960} + \frac{11H^2}{1$  $\frac{H^5 \text{Re}^2 \phi_1^3}{194400} + \frac{H^6 \text{Re}^2 \phi_1^3}{145800} - \frac{H^2 \frac{dH}{dt} \text{Re}^2 \phi_1^3}{1620} - \frac{H^3 \frac{dH}{dt} \text{Re}^2 \phi_1^3}{19440} - \frac{\left(\frac{dH}{dt}\right)^2 \text{Re}^2 \phi_1^3}{5184} - \frac{H^3 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{9720} - \frac{23H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{19440} - \frac{H^3 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{9720} - \frac{23H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{19440} - \frac{H^3 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{9720} - \frac{23H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{19440} - \frac{H^3 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{9720} - \frac{23H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{19440} - \frac{H^3 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{9720} - \frac{23H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{19440} - \frac{H^3 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{9720} - \frac{23H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{19440} - \frac{H^3 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{9720} - \frac{23H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{19440} - \frac{H^3 \text$  $\frac{41 H^5 \mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{194400} - \frac{H^2 \frac{\mathrm{d} H}{\mathrm{d} t} \mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{1080} + \frac{H^3 \mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{9720} - \frac{H^4 \mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{864};$  $f_{10}(t) = -\frac{41\text{Re}^2\phi_1^2}{226800} - \frac{H\text{Re}^2\phi_1^2}{8100} - \frac{11\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{64800} + \frac{H^3\text{Re}^2\phi_1^3}{3600} - \frac{103H^4\text{Re}^2\phi_1^3}{302400} + \frac{131H^5\text{Re}^2\phi_1^3}{1512000} + \frac{H\frac{dH}{dt}\text{Re}^2\phi_1^3}{7200} - \frac{H^3}{302400} + \frac{H^3}{30240} + \frac{H^3}{30240$  $\frac{H\frac{dH}{dt}\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{7200} + \frac{H^2\text{Re}^2\alpha^2(1-\phi)^5\phi_1^3}{3600} - \frac{H^3\text{Re}^2\alpha^2(1-\phi)^5\phi_1^3}{3600};$  $f_{11}(t) = \frac{\text{Re}^2 \phi_1^2}{64800} - \frac{H^2 \text{Re}^2 \phi_1^3}{14850} + \frac{53H^3 \text{Re}^2 \phi_1^3}{356400} - \frac{19H^4 \text{Re}^2 \phi_1^3}{356400} - \frac{\text{Re}^2 \phi_1^3 \frac{\text{d}H}{\frac{1}{dt}}}{47520} + \frac{H \frac{\text{d}H}{\frac{\text{d}}{dt}} \text{Re}^2 \phi_1^3}{23760} - \frac{H \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{19800} + \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} - \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} + \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} - \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} + \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} - \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} + \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} - \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} + \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} + \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} - \frac{H \frac{\text{d}}{\frac{1}{dt}} \text{Re}^2 \phi_1^3}{19800} + \frac{H \frac{\text{d}}{\frac{1}{3}} \text{Re}^2 \phi_1^3$  $\frac{H^2 \mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{9900} + \frac{7 H^3 \mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{356400} + \frac{\mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3 \frac{\mathrm{d}H}{\mathrm{d}t}}{47520} - \frac{\mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{39600} + \frac{H \mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{19800} + \frac{4 \mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{19800} + \frac{\mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{198$  $\tfrac{H^2 \mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{59400};$  $f_{12}(t) = \frac{H \text{Re}^2 \phi_1^3}{129600} - \frac{13H^2 \text{Re}^2 \phi_1^3}{453600} + \frac{47H^3 \text{Re}^2 \phi_1^3}{2721600} - \frac{\text{Re}^2 \phi_1^3 \frac{dH}{dt}}{362880} + \frac{\text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{129600} - \frac{H \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{32400} + \frac{H \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{3240} + \frac{H \text{Re}^2 \alpha (1-\phi)$  $\frac{H^2 \mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{56700} - \frac{\mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{129600} + \frac{H \mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{129600};$  $f_{13}(t) = -\frac{\mathrm{Re}^2 \phi_1^3}{1684800} + \frac{H\mathrm{Re}^2 \phi_1^3}{294840} - \frac{H^2 \mathrm{Re}^2 \phi_1^3}{294840} + \frac{\mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{453600} - \frac{H\mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{294840} - \frac{\mathrm{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{1684800};$  $f_{14}(t) = -\frac{\mathrm{Re}^2 \phi_1^3}{6350400} + \frac{\mathrm{HRe}^2 \phi_1^3}{3175200} + \frac{\mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{6350400};$  $f_{15}(t) = -\frac{\mathrm{Re}^2 \phi_1^3}{95256000};$ 

$$\begin{split} g_{1}(t) &= -H^{2}\text{Re}\phi_{1} + H^{3}\text{Re}^{2}\phi_{1}\frac{dH}{36} - H^{2}\text{Re}a(1-\phi)^{2.5}\phi_{1} + H^{3}\text{Re}^{2}a_{1}\phi_{1}\frac{dH}{11}(1-\phi)^{2.5} - \frac{H^{4}\text{Re}^{2}\phi_{1}^{4}}{36} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{36} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{4} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{4} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{4} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{4} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{4} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{12} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{18} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{18} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{18} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{18} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{16} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{18} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{16} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{18} + \frac{g_{1}H^{2}\text{Re}^{2}\phi_{1}^{4}}{14} + \frac{g_{1}H^{2}}\text{Re}^{2}\phi_{1}^{4}}{14} + \frac{g_{1}H^{2}$$

$$\begin{split} & \eta_{3}(t) = \frac{h^{2}E_{0}}{3\lambda_{2}} = \frac{h^{2}E_{0}}{3\lambda_{2}} + \frac{h^{2}E_{0}}{3\lambda_{2}} - \frac{2h^{2}E_{0}}{10\lambda_{2}} + \frac{11h^{2}E_{0}}{22\lambda_{2}} - \frac{2h^{2}E_{0}}{3\lambda_{2}} + \frac{2h^{2}E_{0}}{3\lambda_{2}} - \frac{2h^{2}E_{0}}{3\lambda_{2}} + \frac{h^{2}E_{0}}{3\lambda_{2}} - \frac{h^{2}E_{0}}{3\lambda_{2}} + \frac{h^{2}E_{0}}}{3\lambda_{2}} + \frac$$

 $\frac{4H^5 \operatorname{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^2}{45} - \frac{7H^6 \operatorname{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^2}{216};$ 

$$\begin{split} w_5(t) &= \frac{H\text{Re}\phi_1}{30} - \frac{H^2\text{Re}\phi_1}{30} + \frac{\text{Re}\alpha(1-\phi)^{2.5}\phi_1}{30} - \frac{H\text{Re}\alpha(1-\phi)^{2.5}\phi_1}{30} + \frac{H^4\text{Re}^2\phi_1^2}{30} + \frac{H^5\text{Re}^2\phi_1^2}{360} - \frac{7H^6\text{Re}^2\phi_1^2}{5400} + \frac{H^3\frac{dt}{dt}\text{Re}^2\phi_1^2}{60} + \frac{29H^4\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{360} + \frac{13H^5\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{900} + \frac{H^2\frac{dt}{dt}\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{60} - \frac{H^3\text{Re}^2\alpha^2(1-\phi)^{5}\phi_1^2}{90} + \frac{3H^4\text{Re}^2\alpha^2(1-\phi)^{5}\phi_1^2}{40} ; \\ w_6(t) &= -\frac{\text{Re}\phi_1}{180} + \frac{H\text{Re}\phi_1}{90} - \frac{H\frac{dt}{dt}\text{Re}^2\phi_1}{45} + \frac{H^2\frac{dt}{dt}\text{Re}^2\phi_1}{90} - \frac{\frac{d^2H}{dt^2}\text{Re}^2\phi_1}{360} + \frac{\text{Re}\alpha(1-\phi)^{2.5}\phi_1}{180} - \frac{H\frac{dt}{dt}\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1}{40} - \frac{H\frac{dt}{dt}\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1}{45} - \frac{H^4\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{360} + \frac{H^5\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1}{40} - \frac{H^3\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1}{40} - \frac{H^4\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{48} - \frac{H^5\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{48} - \frac{H^5\text{Re}^2\alpha(1-\phi)^{5.5}\phi_1^2}{48} - \frac{H^4\text{Re}^2\alpha^2(1-\phi)^{5.5}\phi_1^2}{48} - \frac{H^4\text{Re}^2\alpha^2(1-\phi)^{5.5}\phi_1^3}{4800} - \frac{H^6\text{Re}^2\alpha^2(1-\phi)^{5.5}\phi_1^3}{160} - \frac{H^6\text{Re}^2\alpha^2(1-\phi)^{5.5}\phi$$

 $w_8(t) = \frac{\text{Re}^2\phi_1\frac{dH}{dt}}{5040} + \frac{2H^2\text{Re}^2\phi_1^2}{315} - \frac{H^3\text{Re}^2\phi_1^2}{378} + \frac{17\frac{dH}{dt}\text{Re}^2\phi_1^2}{10080} + \frac{H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{630} + \frac{11H^2\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{1680} - \frac{11H^2}{1680} + \frac{$  $\frac{13H^3\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{15120} + \frac{\frac{dH}{dt}\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{1008} + \frac{\text{Re}^2\alpha^2(1-\phi)^5\phi_1^2}{1260} - \frac{H\text{Re}^2\alpha^2(1-\phi)^5\phi_1^2}{1260} + \frac{13H^2\text{Re}^2\alpha^2(1-\phi)^5\phi_1^2}{5040} - \frac{H^6\text{Re}^2\phi_1^3}{448} + \frac{H^6}{448} + \frac{13H^2}{448} + \frac{13H^2}{1260} + \frac{13H^2}$  $\frac{H^7 \text{Re}^2 \phi_1^3}{840} - \frac{H^8 \text{Re}^2 \phi_1^3}{6720} - \frac{H^4 \frac{dH}{dt} \text{Re}^2 \phi_1^3}{384} + \frac{29 H^5 \frac{dH}{dt} \text{Re}^2 \phi_1^3}{40320} - \frac{H^2 \left(\frac{dH}{dt}\right)^2 \text{Re}^2 \phi_1^3}{1344} + \frac{H^5 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{1008} - \frac{19 H^6 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{3024} + \frac{H^5 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{1008} - \frac{19 H^6 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{3024} + \frac{H^5 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{1008} - \frac{19 H^6 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{3024} + \frac{H^5 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{1008} - \frac{19 H^6 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{3024} + \frac{H^6 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{1008} - \frac{H^6 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{1008} -$  $\frac{17H^7 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{10080} + \frac{H^3 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{2016} - \frac{3H^4 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{896} + \frac{H^5 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{1008} - \frac{5H^6 \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{1344};$  $w_9(t) = \frac{\text{Re}^2\phi_1^2}{1620} - \frac{31H\text{Re}^2\phi_1^2}{45360} + \frac{H^2\text{Re}^2\phi_1^2}{3780} + \frac{H^3\text{Re}^2\phi_1^2}{5670} - \frac{\frac{dH}{dt}\text{Re}^2\phi_1^2}{4536} + \frac{5\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{9072} + \frac{13H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{22680} - \frac{31H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{4536} + \frac{31H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{22680} + \frac{31H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{22680} + \frac{31H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{4536} + \frac{31H\text{Re}^2\alpha(1-\phi)^{2.5$  $\frac{H^2 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^2}{5040} + \frac{\text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^2}{2520} + \frac{H \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^2}{3024} + \frac{H^5 \text{Re}^2 \phi_1^3}{2880} - \frac{H^6 \text{Re}^2 \phi_1^3}{2400} + \frac{H^7 \text{Re}^2 \phi_1^3}{14400} + \frac{H^3 \frac{\text{dH}}{\text{dI}} \text{Re}^2 \phi_1^3}{4320} - \frac{H^6 \frac{1}{3} (1-\phi)^2 (1-\phi)^2 (1-\phi)^2}{4320} + \frac{H^3 \frac{\text{dH}}{\text{dI}} \text{Re}^2 \phi_1^3}{4320} + \frac{H^3 \frac{\text{dH}}{\text{dI}} \text{Re}^2 \phi_1^3}{4400} + \frac{H^3 \frac{\text{dH}}{\text{dI}} \text{Re}^2 \phi_1^3}{440} + \frac{H^3 \frac{\text{dH}}{\text{dI}} \text{Re}^2 \phi_1^3}{440} + \frac{H^3 \frac{\text{dH}}{\text{dI}} \text{Re}^2 \phi_1^3}{440} + \frac{H^3 \frac{\text{dH}}{\text{dI}} \text{Re}^2 \phi_1^3}{4$ 2400  $\frac{H^4 \frac{dH}{dt} \text{Re}^2 \phi_1^3}{4320} + \frac{H^4 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{5184} + \frac{41 H^5 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{129600} - \frac{11 H^6 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{21600} + \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{4220} - \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{21600} + \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{4220} - \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{21600} + \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{4220} - \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{21600} + \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{4220} - \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{21600} - \frac{H^2 \frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.$ 129600  $\frac{H^{3}\frac{dH}{dt}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{3}}{4320} - \frac{H^{3}\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{3}}{6480} + \frac{19H^{4}\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{3}}{25920} - \frac{H^{5}\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{3}}{1728};$  $w_{10}(t) = -\frac{83\text{Re}^2\phi_1^2}{453600} - \frac{H\text{Re}^2\phi_1^2}{10080} - \frac{H^2\text{Re}^2\phi_1^2}{12960} - \frac{127\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{453600} - \frac{11H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{64800} - \frac{41\text{Re}^2\alpha^2(1-\phi)^5\phi_1^2}{453600} + \frac{11}{12}\frac{1$  $\frac{7H^4 \mathrm{Re}^2 \phi_1^3}{64800} - \frac{H^5 \mathrm{Re}^2 \phi_1^3}{972000} - \frac{H^6 \mathrm{Re}^2 \phi_1^3}{729000} + \frac{H^2 \frac{\mathrm{dH}}{\mathrm{dt}} \mathrm{Re}^2 \phi_1^3}{8100} + \frac{H^3 \frac{\mathrm{dH}}{\mathrm{dt}} \mathrm{Re}^2 \phi_1^3}{97200} + \frac{\left(\frac{\mathrm{dH}}{\mathrm{dt}}\right)^2 \mathrm{Re}^2 \phi_1^3}{25920} + \frac{H^3 \mathrm{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{48600} + \frac{H^3 \mathrm{$  $\frac{23H^4\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{97200} + \frac{41H^5\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{972000} + \frac{H^2\frac{dH}{dt}\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{5400} - \frac{H^3\text{Re}^2\alpha^2(1-\phi)^5\phi_1^3}{48600} + \frac{H^4\text{Re}^2\alpha^2(1-\phi)^5\phi_1^3}{4320};$  $w_{11}(t) = \frac{41\text{Re}^2\phi_1^2}{1247400} + \frac{H\text{Re}^2\phi_1^2}{44550} + \frac{\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^2}{32400} - \frac{H^3\text{Re}^2\phi_1^3}{19800} + \frac{103H^4\text{Re}^2\phi_1^3}{1663200} - \frac{131H^5\text{Re}^2\phi_1^3}{8316000} - \frac{H\frac{dH}{dt}\text{Re}^2\phi_1^3}{39600} + \frac{H\frac{dH}{dt}\text{R}^2\phi_1^3}{3$  $\frac{H^2\frac{dH}{dt}\text{Re}^2\phi_1^3}{46200} - \frac{H^2\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{19800} + \frac{H^3\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{51975} + \frac{41H^4\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{1663200} - \frac{\frac{dH}{dt}\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{39600} + \frac{H^2}{39600} + \frac{H^2}{3960} + \frac{H^2}{39600} + \frac{H^2}{3960} + \frac{H^2$ 46200  $\frac{H\frac{dH}{dt}\text{Re}^{2}\alpha(1-\phi)^{2.5}\phi_{1}^{3}}{39600}-\frac{H^{2}\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{3}}{19800}+\frac{H^{3}\text{Re}^{2}\alpha^{2}(1-\phi)^{5}\phi_{1}^{3}}{19800};$  $w_{12}(t) = -\frac{\mathrm{Re}^2\phi_1^2}{38800} + \frac{H^2\mathrm{Re}^2\phi_1^3}{89100} - \frac{53H^3\mathrm{Re}^2\phi_1^3}{2138400} + \frac{19H^4\mathrm{Re}^2\phi_1^3}{2138400} + \frac{\frac{dH}{dt}\mathrm{Re}^2\phi_1^3}{285120} - \frac{H\frac{dH}{dt}\mathrm{Re}^2\phi_1^3}{142560} + \frac{H\mathrm{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{118800} - \frac{H^2}{118800} + \frac{$  $\frac{H^2 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{59400} - \frac{7H^3 \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{2138400} - \frac{\frac{dH}{dt} \text{Re}^2 \alpha (1-\phi)^{2.5} \phi_1^3}{285120} + \frac{\text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{237600} - \frac{H \text{Re}^2 \alpha^2 (1-\phi)^5 \phi_1^3}{118800} - \frac{H (1-\phi)^{2.5} \phi_1^3}{118800} - \frac{H (1 \frac{H^2 {\rm Re}^2 \alpha^2 {(1-\phi)}^5 \phi_1^3}{356400};$ 

$$\begin{split} w_{13}(t) &= -\frac{H\text{Re}^2\phi_1^3}{842400} + \frac{H^2\text{Re}^2\phi_1^3}{226800} - \frac{47H^3\text{Re}^2\phi_1^3}{17690400} + \frac{\frac{dH}{dL}\text{Re}^2\phi_1^3}{2358720} - \frac{\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{842400} + \frac{H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{210600} - \frac{H\text{Re}^2\alpha(1-\phi)^{5.5}\phi_1^3}{842400} + \frac{W\text{Re}^2\alpha(1-\phi)^{5.5}\phi_1^3}{842400}; \\ w_{14}(t) &= \frac{\text{Re}^2\phi_1^3}{11793600} - \frac{H\text{Re}^2\phi_1^3}{2063880} + \frac{H^2\text{Re}^2\phi_1^3}{2063880} - \frac{\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{3175200} + \frac{H\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{2063880} + \frac{\text{Re}^2\alpha^2(1-\phi)^{5.5}\phi_1^3}{11793600}; \\ w_{15}(t) &= \frac{\text{Re}^2\phi_1^3}{47628000} - \frac{H\text{Re}^2\phi_1^3}{23814000} - \frac{\text{Re}^2\alpha(1-\phi)^{2.5}\phi_1^3}{47628000}; \end{split}$$

 $w_{16}(t) = rac{\mathrm{Re}^2 \phi_1^3}{762048000};$ 

Similarly, some constants have also been enlisted as

$$\begin{split} A_1 &= \frac{k_{nf}}{k_f}; \\ A_2 &= \frac{A_1}{\Pr \phi_2}; \\ \phi_1 &= \left(1.013 + 0.092\phi - 0.015\phi^2\right) \left[1 - \phi \left(1 + \frac{\rho_p}{\rho_f}\right)\right]; \\ \phi_2 &= \left(1 - \phi\right) - \phi \frac{(\rho C_p)_p}{(\rho C_p)_f}. \end{split}$$

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