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Communication

Nonuniform Slip Effect in Wetting Films

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Abstract: The slip effect in wetting films is theoretically studied, and a nonlinear dependence of the hydrodynamic velocity on the slip length is discovered. It is demonstrated that the hydrodynamic flow is essentially affected by the presence of a nonuniform slip length distribution, leading also to enhancement of the energy dissipation in the films. This effect could dramatically slow the usually quick hydrodynamic flows over superhydrophobic surfaces, for instance.

Keywords: nonlinear slip effect; nonuniform surfaces; wetting films; Reynolds hydrodynamics

1. Introduction

The interest in thin liquid films has gradually increased from the middle of the previous century due to the importance of disperse systems—such as foams, emulsions and suspensions—to technology. The film studies split naturally into thermodynamic forces and hydrodynamic stability or rupture [1–3]. By the onset of nanotechnology in the present century, the flow in wetting films has attracted enormous attention as an essential process in the modern micro- and nano-fluidics [4–6]. An important aspect here is the violation of the classical no-slip boundary condition of hydrodynamics near solid surfaces [7]. This, and other effects, are especially significant on hydrophobic surfaces [8,9], repelling the water molecules. Nowadays, the engineering interest turns toward superhydrophobic surfaces, where the slip effect is even more pronounced [10,11]. Because such surfaces are intrinsically nonuniform, it is a challenge to develop a theory for the slip effect oven nonuniform surfaces. The existing models [12–14] are linearized for small slip length due to mathematical complications. The present study aims to explore the nonlinear effect of a nonuniform slip on a structured surface to the flow in thin wetting films. The complexity of the latter causes an increase in the energy dissipation, which is an important parameter to control during wetting, spreading, coating, etc. This could essentially affect some modern systems such as ionic liquids [15] and industrial polymers [16,17].

2. Wetting Film Hydrodynamics

Let us consider a thin wetting film placed on a slippery solid surface at $z = 0$. The upper film surface at $z = h$ is tangentially moving in the x-direction with a constant velocity u. In the frames of the relevant Reynolds lubrication approximation, the Stokes equations of hydrodynamics of incompressible fluids reduce to [2]

$$\begin{gathered}\partial_x v_x + \partial_z v_z = 0 \\ \partial_x p = \eta \partial_z^2 v_x \\ \partial_z p = 0\end{gathered} \tag{1}$$

where p is the local hydrodynamic pressure in the film of a liquid with dynamic viscosity η. Integrating the dynamic equations above—by employing the tangential boundary condition $v_x = u$ on the upper film surface—leads to an expression for the tangential hydrodynamic velocity in the film

$$v_x = u + (z - h)u/a + (z^2 - h^2)\partial_x p/2\eta \tag{2}$$

where the nonuniform length $a(x)$ is not specified, yet. Substituting Equation (2) in the Navier boundary condition $v_x = b\partial_z v_x$, on the bottom solid surface [6], yields the pressure gradient:

$$\partial_x p = -2\eta u(h + b - a)/ah^2. \tag{3}$$

Here, $b(x)$ is the slip length, which is nonuniform in general. If the latter is constant, there is no pressure gradient. In this case, $a = h + b$ follows from $\partial_x p = 0$, and the velocity field (2) reduces to the Couette flow:

$$v_x = u(z + b)/(h + b) \tag{4}$$

In the general case, substituting Equation (3) in Equation (2) results in

$$v_x = u[z^2 - (z - h)(hz + bz + bh)/a]/h^2. \tag{5}$$

To determine the length a, one should employ the normal boundary condition $v_z = 0$ on both surfaces of the non-thinning film. Thus, integrating of the continuity Equation (1) yields

$$\partial_x \int_0^h v_x dz = 0 \tag{6}$$

Introducing here Equation (5) and performing the integration provides the functional $a[b(x)]$

$$a = (h + \bar{b})(h + 4b)/(h + 4\bar{b}) \tag{7}$$

which is expressed by the surface-averaged slip length $\bar{b}$ in such a way that the Couette expression (4) is recovered in the case of a uniform slip length $b = \bar{b}$. Note that always $\bar{a} = h + \bar{b}$. Thus, the pressure gradient $\partial_x p = 6\eta u(b - \bar{b})/h(h + \bar{b})(h + 4b)$ from Equation (3) depends nonlinearly on the slip length. Introducing Equation (7) back into Equation (5) yields the tangential hydrodynamic velocity:

$$v_x = u[z^2 - (z - h)(hz + bz + bh)(h + 4\bar{b})/(h + \bar{b})(h + 4b)]/h^2 \tag{8}$$

As expected, Equation (8) reduces to Equation (4) if $b = \bar{b}$ everywhere. To visualize the tangential velocity, its relative v_x/u is plotted in Figure 1 for a particular slip length model with $\bar{b} = h$.

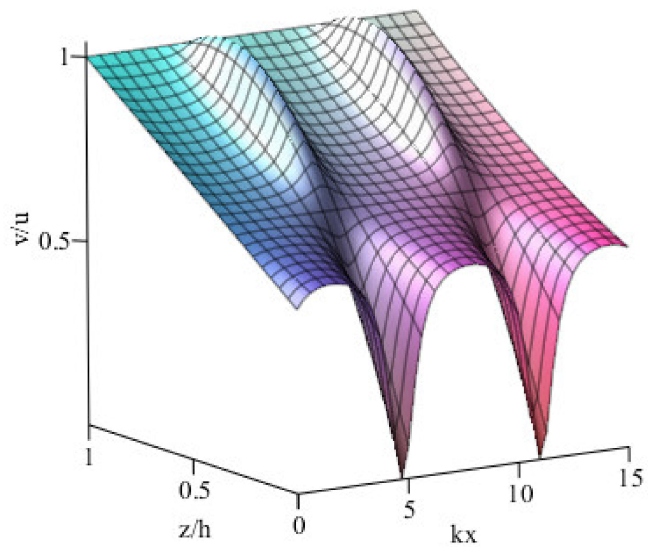

Figure 1. The relative tangential velocity v_x/u at different height z/h for $b/h = 1 + \sin(kx)$.

As is seen, the tangential velocity is no more a linear function on the vertical position z as in the Couette flow. Due to the nonlinear dependence of the velocity on the slip length, the average velocity $\overline{v}_x$ will not depend merely on the average slip length $\overline{b}$. The velocity on the bottom solid surface at z = 0, where the slip effect is most pronounced, is given by

$$v_x = ub(h + 4\overline{b})/(h + 4b)(h + \overline{b}) \tag{9}$$

It becomes uniform $v_x = u/(1 + h/\overline{b})$ at the large slip length $b > h/4$. One can estimate the effect of the ratio $h/\overline{b}$ from the plot on Figure 2.

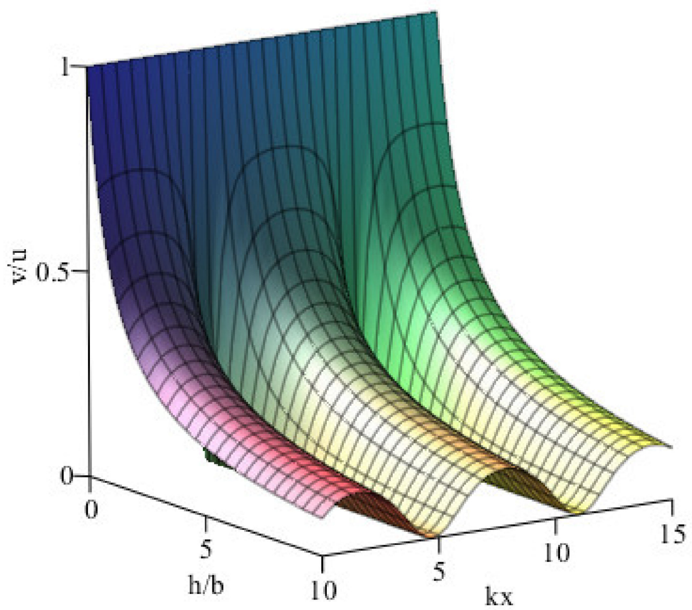

Figure 2. The relative tangential velocity v_x/u at $z = 0$ for $b/\overline{b} = 1 + \sin(kx)$ at different $h/\overline{b}$.

3. Hydrodynamics of Thicker Wetting Films

The nonlinearity of Equation (8) is important for thinner films because it reduces to the Couette flow $v_x = u(z + b)/h$ at $h > 4b$. This is misleading, however, since the Reynolds approximation is no more valid for thick films, the Stokes Equation (1) should be rewritten as

$$\begin{gathered} \partial_x v_x + \partial_z v_z = 0 \\ \partial_x p = \eta(\partial_x^2 v_x + \partial_z^2 v_x) \\ \partial_z p = \eta(\partial_x^2 v_z + \partial_z^2 v_z) \end{gathered} \tag{10}$$

Because the mathematical problem is much more complicated now, we are looking for an approximate linearized solution in the form:

$$\begin{gathered} v_x = uz/h + \overline{b}\partial_z w \\ v_z = -\overline{b}\partial_x w \end{gathered} \tag{11}$$

which is valid for the relatively small slip length b. Substituting these expressions in Equation (10) leads to the following differential equation for the unknown velocity potential w:

$$\begin{gathered} \partial_x^4 w + 2\partial_x^2 \partial_z^2 w + \partial_z^4 w = 0 \\ \partial_z^4 w_q - 2q^2 \partial_z^2 w_q + q^4 w_q = 0 \end{gathered} \tag{12}$$

where the second equation is the Fourier transform of the first one. The relevant boundary conditions are $w_q = 0$ and $\partial_z w_q = 0$ on the upper surface at $z = h$, $w_q = 0$ and $\bar{b}\partial_z w_q = ub_q/h$ on the bottom surface at $z = 0$, where b_q is the Fourier image of $b(x)$ [14]. Note that due to the small slip length b the contribution of w on the right-hand side of the Navier condition vanishes. Thus, the exact solution of Equation (12) reads

$$w_q = \frac{ub_q}{h\bar{b}} \frac{z\cosh(qz)\sinh^2(qh) + [(z-h)qh - z\sinh(qh)\cosh(qh)]\sinh(qz)}{\sinh^2(qh) - (qh)^2} \tag{13}$$

Because h is larger now, the period of the slip length variations can be considered smaller than the thickness of the liquid film. Hence, one can simplify Equation (13) at $qh > 1$ to obtain

$$\begin{gathered} w_q = ub_q z\exp(-qz)/h\bar{b} \\ w = \frac{uz^2}{\pi h\bar{b}} \int_{-\infty}^{\infty} \frac{b(y)}{(x-y)^2+z^2} dy. \end{gathered} \tag{14}$$

The corresponding hydrodynamic velocity components from Equation (11) acquire the integral forms:

$$\begin{gathered} v_x = \frac{uz}{h}\{1 + \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{(x-y)^2 b(y)}{[(x-y)^2+z^2]^2} dy\} \\ v_z = \frac{uz}{h}\frac{2}{\pi} \int_{-\infty}^{\infty} \frac{z(x-y)b(y)}{[(x-y)^2+z^2]^2} dy \end{gathered} \tag{15}$$

As expected, $v_x = u(z + \bar{b})/h$ and $v_z = 0$ in the case of a uniform slip length $b = \bar{b}$. To estimate the effect of the slip nonuniformity, let us consider again the periodic model $b/\bar{b} = 1 + \sin(kx)$, where the hydrodynamic velocities (15) acquire the periodic expressions:

$$\begin{gathered} v_x = u(z + \bar{b})/h + u\bar{b}\sin(kx)(1 - kz)\exp(-kz)/h \\ v_z = u\bar{b}\cos(kx)kz\exp(-kz)/h \end{gathered} \tag{16}$$

The corresponding mean velocities $\bar{v}_x = u(z + \bar{b})/h$ and $\bar{v}_z = 0$ describe an averaged Couette flow. This is also the case at $z = 0$, where $v_x = ub/h$ depends linearly on the slip length due to the used linearization of Equation (9). Substituting Equation (16) in Equation (10) yields the pressure distribution in the film in a periodic form as well, with a zero-mean value:

$$p = 2\eta u\bar{b}k\cos(kx)\exp(-kz)/h = 2\eta v_z/z \tag{17}$$

Using it, one can calculate the additional energy dissipation in the wetting film due to the nonuniform distribution of the slip length

$$-\overline{v_x\partial_x p} - \overline{v_z\partial_z p} = \eta(u\bar{b}k/h)^2\exp(-2kz) \tag{18}$$

As is seen, it is negligible near the upper film surface, while at the bottom slippery surface it depends strongly on the slip nonuniformity via the wave vector k. Note that for a uniform slip with $k = 0$ the energy dissipation vanishes, which is typical for the Couette flow due to the lack of pressure gradient.

4. Conclusions

Due to complexity of the thin liquid film hydrodynamics, the flow in wetting films depends nonlinearly on the slip length on the film surfaces. This effect becomes very important in the case of a nonuniform slip length distribution, which dramatically changes the flow profile in the film. As a result, the energy dissipation increase could prevail over the uniform friction in the case of very structured surfaces, such as the superhydrophobic ones. Traditionally, the latter are considered as almost frictionless due to the entrapped air, but according to our analysis the distribution of the slip length could change the energy dissipation significantly.

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