Spherical Aberration-Corrected Metalens for Polarization Multiplexed Imaging

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Supplementary Materials for "the derivation of Equation (5)"

The generalized Snell's law of refraction: $\frac{d\varphi(\rho)}{d\rho} = \frac{2\pi}{\lambda}(\sin\theta_t - \sin\theta_i)$ show in Equation (1). The phase delay $\phi(\rho)$ between rays respectively at position ϱ and at the center in the radial dimension is: $\phi(\rho) = \varphi(\rho) - \varphi(0) = \frac{2\pi}{\lambda}\left(-\sqrt{s^2 + \rho^2} - \sqrt{s'^2 + \rho^2} + s + s'\right)$, as shown in Equation (4). By substituting s' into the upper equation and use the Gaussian imaging formation ($\frac{sf}{s-f} = s'$), the detailed specifications show as:

$$\phi(\rho) = \frac{2\pi}{\lambda}\left(-\sqrt{s^2 + \rho^2} - \sqrt{\left(\frac{sf}{s-f}\right)^2 + \rho^2} + s + \frac{sf}{s-f}\right) = \frac{2\pi}{\lambda}\left(-\sqrt{s^2 + \rho^2} - \sqrt{\left(\frac{sf}{s-f}\right)^2 + \rho^2} + \frac{s(s-f) + sf}{s-f}\right).$$

Deduction steps are given:

$$\Rightarrow \frac{2\pi s}{\lambda(f-s)}\left(\frac{(s-f)\sqrt{s^2+\rho^2}}{s} + \frac{s-f}{s}\sqrt{\left(\frac{sf}{s-f}\right)^2 + \rho^2} - s\right) = \frac{2\pi s}{\lambda(f-s)}\left(\frac{(s-f)\sqrt{s^2+\rho^2}}{s} + \sqrt{\left(\frac{s-f}{s}\right)^2\left[\left(\frac{sf}{s-f}\right)^2 + \rho^2\right]} - s\right).$$

A clear simplified form is: $\phi(\rho) = \frac{2\pi s}{\lambda(f-s)}\left(\frac{(s-f)\sqrt{s^2+\rho^2}}{s} + \sqrt{f^2 + \frac{(s-f)^2}{s^2}\rho^2} - s\right)$.

We rewrite the expression in the second sqrt of the above formula to include only the numerator and denominator, as shown $\phi(\rho) = \frac{2\pi s}{\lambda(f-s)}\left(\frac{(s-f)\sqrt{s^2+\rho^2}}{s} + \sqrt{\frac{s^2f^2 + s^2\rho^2 - 2sf\rho^2 + f^2\rho^2}{s^2}} - s\right)$. To express the above formula more clearly, we extract the two parts of $s^2 + \rho^2$ and $s^2 + \rho^2$ from the second sqrt, as shown: $\phi(\rho) = \frac{2\pi s}{\lambda(f-s)}\left(\frac{(s-f)\sqrt{s^2+\rho^2}}{s} + \sqrt{\frac{s^2f^2 + f^2\rho^2}{s^2} + \frac{(s^2\rho^2 + s^4) + (s^2\rho^2 - s^4) + (2s^3f - 2sf\rho^2) + (-2s^3f - 2sf\rho^2)}{2s^2}} - s\right)$. We next give the steps of the intermediate process from the above formula to Equation (5) in the main body.

$$\Rightarrow \frac{2\pi s}{\lambda(f-s)}\left(\frac{(s-f)\sqrt{s^2+\rho^2}}{s} + \sqrt{\frac{(s^2+\rho^2)(2f^2+s^2-2sf)}{2s^2} + \frac{(s^2-\rho^2)(2sf-s^2)}{2s^2}} - s\right)$$

$$\Rightarrow \frac{2\pi s}{\lambda(f-s)}\left(\frac{(s-f)\sqrt{s^2+\rho^2}}{s} + \frac{1}{\sqrt{2}}\sqrt{\left[2f^2 + s^2 - 2sf + s(2f-s)\frac{s^2-\rho^2}{s^2+\rho^2}\rho^2\right]\frac{s^2+\rho^2}{s^2}} - s\right)$$

Finally, we intuitively derive the above formula to Equation (5).